HEAVY-QUARK SYMMETRY AND LARGE-N_C OPERATOR ANALYSIS OF CHIRAL LAGRANGIAN WITH D-MESONS, CHARMED AND LIGHT BARYONS

Thanat Sangkhakrit (SUT)
In collaboration with
Thananuwat Suyuporn (SUT), Daris Samart (RMUTI),
Nopmanee Supanam (SWU), and Yupeng Yan (SUT)

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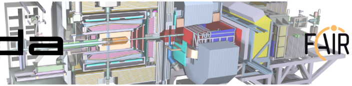
Overview

Introduction I



- In the near future, experimental studies of charmed baryon productions will be investigated at FAIR (Facility for Antiproton and Ion Research) and J-PARC (Japan Proton Accelerator Research Complex).
- In the $\bar{P}$ANDA experiment at FAIR, the beam momenta of antiprotons will be well above the threshold energy of $p\bar{p} \to \Lambda_c \bar{\Lambda}_c$ reaction (10.162 GeV)*.
- At J-PARC, the beam momenta of pions will reach over 20 GeV in the laboratory frame**.

*(Shyam, R. and Lenske, H. (2014) **Phys. Rev. D** 90: 014017)

(Kim, S.-H. et al. (2014) **PTEP 2014 10: 103D01)

Introduction II

- The production of charmed hadrons is considered to be an ideal place to study the dynamics of the light quarks in the environment of a heavy quark as well as to examine the role of both **chiral symmetry** and **heavy-quark symmetry (HQS)** in heavy-light quark systems.
- Therefore, it is a great opportunity to perform a theoretical study of charmed baryon production reactions while the experimental facilities are now under investigation.

Aims of this work I

1. Construct 3-point vertices chiral Lagrangian for light baryons (N, Δ), D-mesons (D, D^*), and charmed baryons (Σ_c, Σ_c^* , Λ_c) in the framework of SU(2) flavor symmetry
2. Correlate free parameters in our chiral Lagrangian with constraints from "Heavy-quark symmetry"
3. Correlate free parameters in our chiral Lagrangian with constraints from "Large-N_c analysis"

We will use chiral Lagrangian to calculate relevant processes of charmed hadrons with constraints from heavy-quark symmetry and large-N_c analysis.

Fundamental representations of particles I

Under SU(2) flavor symmetry, the light baryons (N, Δ), D-mesons (D, D^*), and charmed baryons (Σ_c, Σ_c^* , Λ_c) are represented by

Light Baryons

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \\ \Delta^{-} \end{pmatrix}$$

D-mesons

$$D = \left(D^0, D^+ \right)$$

Fundamental representations of particles II

Charmed baryons

$$\Sigma_c = \begin{pmatrix} \Sigma_c^{++} \\ \Sigma_c^{+} \\ \Sigma_c^{0} \end{pmatrix}, \Lambda_c = \Lambda_c$$

These representations are used to construct chiral Lagrangian, where 3 forms of coupling are included: axial-vector, vector, and tensor.

The free parameters g_i, f_i, and h_i are defined as axial-vector, vector, and tensor coupling constants, respectively.

$$
\begin{aligned}
\mathcal{L} = & \frac{1}{M_c} g_1 \partial_\mu D \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}} \cdot \vec{\boldsymbol{\tau}} \gamma^\mu \gamma_5 N + \text{H.c.} - \frac{1}{M_c} g_2 \partial_\mu D \overline{\Lambda}_c \gamma^\mu \gamma_5 N + \text{H.c.} \\
& + \frac{1}{M_c} g_3 \partial_\mu D \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}}^{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{\tau}} N + \text{H.c.} + \frac{1}{M_c} g_4 \partial_\mu D \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}} \cdot \vec{\boldsymbol{T}}^\dagger \Delta^\mu + \text{H.c.} \\
& - \frac{1}{M_c} g_5 \partial_\mu D \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}}^{\boldsymbol{\nu}} \cdot \vec{\boldsymbol{T}}^\dagger \gamma^\mu \gamma_5 \Delta_\nu + \text{H.c.} + f_1 D_\mu \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}} \cdot \vec{\boldsymbol{\tau}} \gamma^\mu N + \text{H.c.} \\
& + f_2 D_\mu \overline{\Lambda}_c \gamma^\mu N + \text{H.c.} + f_3 D_\mu \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}}^{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{\tau}} \gamma_5 N + \text{H.c.} \\
& - f_4 D_\mu \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}} \cdot \vec{\boldsymbol{T}}^\dagger i \gamma_5 \Delta^\mu + \text{H.c.} - f_5 D_\mu \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}}^{\boldsymbol{\nu}} \cdot \vec{\boldsymbol{T}}^\dagger \gamma^\mu \Delta_\nu + \text{H.c.} \\
& + \frac{i}{2M_c} h_1 D_{\mu\nu} \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}} \cdot \vec{\boldsymbol{\tau}} \sigma^{\mu\nu} N + \text{H.c.} - \frac{1}{2M_c} h_2 D_{\mu\nu} \overline{\Lambda}_c \sigma^{\mu\nu} N + \text{H.c.} \\
& - \frac{i}{M_c} h_3 D_{\mu\nu} \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}}^{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{\tau}} \gamma^\nu \gamma_5 N + \text{H.c.} + \frac{1}{M_c} h_4 D_{\mu\nu} \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}} \cdot \vec{\boldsymbol{T}}^\dagger \gamma^\mu \gamma_5 \Delta^\nu \\
& + \text{H.c.} + \frac{1}{2M_c} h_5 D_{\mu\nu} \bar{\vec{\boldsymbol{\Sigma}}}_{\boldsymbol{c}}^{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{T}}^\dagger \Delta^\nu + \text{H.c.} \qquad (1)
\end{aligned}
$$

where $D_{\mu\nu} \equiv \partial_\mu D_\nu - \partial_\nu D_\mu$

Heavy-quark symmetry

In the limit that mass of a heavy quark becomes large ($m_Q \to \infty$), the chromomagnetic interaction relates to the mass of the heavy quark by

$$\text{Chromomagnetic interaction} \propto \frac{\vec{\sigma} \cdot \vec{B}}{m_Q} \tag{2}$$

Therefore the spins of light and heavy quarks are decoupled in the heavy quark limit.

Heavy-quark symmetry is employed to construct "Heavy-quark symmetry Lagrangian", the key to reduce number of free parameters in our chiral Lagrangian.

*(Manohar, A.V. and Wise, M.B. (2000) **Heavy Quark Physics**)

Heavy-quark symmetry Lagrangian I

We follow the formalism in (M.B. Wise (1992) **Phys. Rev. D** 45: 2188) and (R. Casalbuoni et al. (1997) **Phys. Rep.** 281: 145) to introduce

$$H = \left(\frac{1+\not{v}}{2}\right)\left(i\gamma_5 D_+ + \gamma_\mu D_+^\mu\right) \tag{3}$$

$$H_T^\mu = \frac{1}{\sqrt{3}}\left(v^\mu + \gamma^\mu\right)\gamma_5\left(\frac{1+\not{v}}{2}\right)\Sigma_{c,+} + \left(\frac{1+\not{v}}{2}\right)\Sigma_{c,+}^\mu \tag{4}$$

$$H_s = \frac{1+\not{v}}{2}\Lambda_{c,+} \tag{5}$$

and their conjugate fields

$$\bar{H} = \gamma_0 H^\dagger \gamma_0 \tag{6}$$

$$\bar{H}_T^\mu = \left(H_T^\mu\right)^\dagger \gamma_0 \tag{7}$$

$$\bar{H}_s = \left(H_s\right)^\dagger \gamma_0 \tag{8}$$

Heavy-quark symmetry Lagrangian II

These fields satisfy the $SU(2)_v$ transformations,

$$H \to e^{-iJ_\alpha \theta^\alpha} H \tag{9}$$

$$\bar{H} \to \bar{H} e^{iJ_\alpha \theta^\alpha} \tag{10}$$

$$H_T^\mu \to e^{-iJ_\alpha \theta^\alpha} H_T^\mu \tag{11}$$

$$\bar{H}_T^\mu \to \bar{H} e^{iJ_\alpha \theta^\alpha} \tag{12}$$

The heavy quark spin operator (J) and its properties are

$$J^\alpha = \frac{1}{2}\gamma_5 [\not{v}, \gamma^\alpha] \tag{13}$$

$$J_\alpha^\dagger \gamma_0 = \gamma_0 J_\alpha \tag{14}$$

$$[\not{v}, J_\alpha] = 0 \tag{15}$$

Heavy-quark symmetry Lagrangian III

The heavy quark effective Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{\mathrm{HQS}} &= c_1\, Tr\left[H\gamma_\mu\gamma_\nu \bar{H}_T^\mu v^\nu \gamma_5 N\right] + H.c. + c_2\, Tr\left[H\gamma_\mu\gamma_\nu\gamma_5 \bar{H}_T^\mu v^\nu N\right] + H.c. \\ &+ c_3\, Tr\left[H\gamma_\mu \bar{H}_s v^\mu N\right] + H.c. + c_4\, Tr\left[H\gamma_5 \bar{H}_s \gamma_5 N\right] + H.c. \\ &+ c_5\, Tr\left[H\sigma_{\mu\nu}\gamma_5 \bar{H}_T^\mu \gamma_5 \Delta^\nu\right] + H.c. + c_6\, Tr\left[H\sigma_{\mu\nu} \bar{H}_T^\mu \Delta^\nu\right] + H.c. \\ &+ c_7\, Tr\left[H\gamma_5 \bar{H}_T^\mu \gamma_5 \Delta_\mu\right] + H.c. \end{aligned} \quad (16)$$

Heavy-quark sum rules

By matching structures between a non-relativistic expansion of 3-point vertices chiral Lagrangian and heavy-quark symmetry Lagrangian, we derive the following **sum rules**

$$g_1 = 3h_1$$
$$g_3 = 0$$
$$g_4 = \frac{1}{\sqrt{3}} g_5$$
$$f_2 = h_2$$

$$f_3 = h_3 = \sqrt{3} f_1 - 4\sqrt{3} h_1$$
$$f_5 = 2 f_4$$
$$h_4 = -\frac{1}{2} f_4 + \frac{1}{4\sqrt{3}} h_5$$

7 free parameters: $g_2, g_5, f_1, f_4, h_1, h_2, h_5$

Large-N_c operator analysis I

- The $1/N_c$ expansion of baryon matrix elements takes the generic form;

$$\langle \bar{p}, \bar{\chi} | \, C_{ij,a}\left(q, \bar{q}\right) | p, \chi \rangle = \sum_r c_r\left(p, \bar{p}\right) \left(\bar{\chi} | \, O^{(r)}_{ij,a} \, | \chi\right) \tag{17}$$

- The effective operators $O^{(r)}_{ij,a}$ can be written in terms of $J, \ I, \ G, \ Y, \ t$.
- The operators J^i, I^a, G^{ia}, Y^{iA}, t^A are spin, isospin, spin-isospin, charm-spin changing, and charm changing operators respectively (in $SU(2)$ flavor symmetry).
- The N_c scaling of effective operators is given by*

$$J \sim I \sim 1/N_c \,, \quad Y \sim t \sim \sqrt{N_c} \,, \quad G \sim N_c^0 \tag{18}$$

We will employ eq.(17) to correlate the coupling constants and their N_c-scaling.

*(Dashen et al. (1995) **Phys. Rev. D** 51:3697-3727)

Effective spin-flavor baryon states

Effective spin-flavor effective baryon states in our work are listed as follows;

$$\left|N; A, \chi_{1/2}\right) = \frac{1}{\sqrt{12}} i\sigma^{(2)}_{\alpha\beta}\delta_{\chi\gamma}\epsilon_{BC} q^\dagger_{B\alpha} q^\dagger_{C\beta} q^\dagger_{A\gamma} \left|0\right) \quad (19)$$

$$\left|\Lambda_c; AB_-, \chi_{1/2}\right) = \frac{1}{2} i\sigma^{(2)}_{\alpha\beta}\delta_{\chi\gamma}\left(\delta_{AC}\delta_{BD}\right)_- q^\dagger_{C\alpha} q^\dagger_{D\beta} C^\dagger_\gamma \left|0\right) \quad (20)$$

$$\left|\Sigma_c; AB_+, \chi_{1/2}\right) = \frac{1}{\sqrt{12}} \left(\vec{\sigma} i\sigma^{(2)}\right)_{\alpha\beta} \vec{\sigma}^\dagger_{\gamma\chi}\left(\delta_{AC}\delta_{BD}\right)_+ q^\dagger_{C\alpha} q^\dagger_{D\beta} C^\dagger_\gamma \left|0\right) \quad (21)$$

$$\left|\Sigma^*_c; AB_+, \chi_{3/2}\right) = \frac{1}{2} \left(\vec{\sigma} i\sigma^{(2)}\right)_{\alpha\beta} \vec{S}^\dagger_{\gamma\chi}\left(\delta_{AC}\delta_{BD}\right)_+ q^\dagger_{C\alpha} q^\dagger_{D\beta} C^\dagger_\gamma \left|0\right) \quad (22)$$

$$\left|\Delta; ABC_+, \chi_{3/2}\right) = \frac{1}{\sqrt{108}} D^{(\chi)}_{\alpha\beta\gamma}\left(\delta_{AD}\delta_{BE}\delta_{CF}\right)_+ q^\dagger_{D\alpha} q^\dagger_{E\beta} q^\dagger_{F\gamma} \left|0\right) \quad (23)$$

One-body effective operators

In our work, the following operators are used;

$$J^i = \frac{1}{2} q^\dagger_{A\mu} \sigma^i_{\mu\nu} q_{A\nu} \tag{24}$$

$$I^a = \frac{1}{2} q^\dagger_{A\mu} \tau^a_{AB} q_{B\mu} \tag{25}$$

$$G^{ia} = \frac{1}{4} q^\dagger_{A\mu} \sigma^i_{\mu\nu} \tau^a_{AB} q_{B\mu} \tag{26}$$

$$Y^{iA} = \frac{1}{2} C^\dagger_\mu \sigma^i_{\mu\nu} q_{A\nu} \tag{27}$$

$$t^A = C^\dagger_\mu q_{A\mu} \tag{28}$$

$$N_h = C^\dagger_\mu C_\mu \tag{29}$$

$$J^i_h = \frac{1}{2} C^\dagger_\mu \sigma^i_{\mu\nu} C_\nu \tag{30}$$

Therefore, we have 35 expressions for effective baryon matrix elements.

We introduce the operators

$$A^{i,A} = c_1^A q_i k_j Y^{jA} + c_2^A q^j Y^{jA} + c_3^A q_j J^j t^A \tag{31}$$

and

$$V^{i,A} = c_1^A q_i t^A + c_2^V q_j \epsilon^{ijk} Y^{kA} + c_3^V k_i t^A + c_4^V q_i q_j Y^{iA} J^j \tag{32}$$

where $p' + p = k$ and $p' - p = k$

These operators will be employed to derive large-N_c sum rules.

Large-N_c sum rules

By matching structures between a non-relativistic expansion of baryon matrix elements for axial-vector and vector currents and effective baryon matrix elements for the operators from eq.(31) and eq.(32), we derive the following **sum rules**

$$g_2 = -\frac{3}{2\sqrt{3}} g_1$$

$$g_3 = 0$$

$$g_5 = -\frac{3\sqrt{3}}{2} g_1$$

$$f_2 = \sqrt{3} f_1$$

$$f_5 = -\frac{3\sqrt{3}}{2} f_1$$

$$h_1 = \frac{\sqrt{3}}{2} h_2$$

$$h_3 = h_4 = 0$$

$$h_5 = -\sqrt{3} f_1$$

6 free parameters: g_1, g_4, f_1, f_3, f_4, h_2

Combining heavy-quark and large-N_c sum rules I

Heavy-quark sum rules

$$g_1 = 3h_1$$
$$g_3 = 0$$
$$g_4 = \frac{1}{\sqrt{3}} g_5$$
$$f_2 = h_2$$
$$f_3 = h_3 = \sqrt{3} f_1 - 4\sqrt{3} h_1$$
$$f_5 = 2f_4$$
$$h_4 = -\frac{1}{2} f_4 + \frac{1}{4\sqrt{3}} h_5$$

Large-N_c sum rules

$$g_2 = -\frac{3}{2\sqrt{3}} g_1$$
$$g_3 = 0$$
$$g_5 = -\frac{3\sqrt{3}}{2} g_1$$
$$f_2 = \sqrt{3} f_1$$
$$f_5 = -\frac{3\sqrt{3}}{2} f_1$$
$$h_1 = \frac{\sqrt{3}}{2} h_2$$
$$h_3 = h_4 = 0$$
$$h_5 = -\sqrt{3} f_1$$

Combining heavy-quark and large-N_c sum rules II

According to **heavy-quark symmetry** and **large-N_c operator analysis**, we derive following sum rules

$$g_2 = -\frac{9}{4}h_2$$
$$g_3 = 0$$
$$g_5 = -\frac{27}{4}h_2$$

$$f_1 = \frac{1}{\sqrt{3}}h_2 = -12\sqrt{3}f_4 = 4h_1$$
$$h_1 = -18\sqrt{3}f_4$$
$$f_4 = \frac{1}{2\sqrt{3}}h_5$$
$$h_3 = h_4 = 0$$

5 free parameters: g_1, g_4, f_3, f_4, h_2

Summary

- There are 15 terms of 3-point vertices chiral Lagrangian for light baryons (Nucleon and Delta), D-mesons (D, D^*), and charmed baryons (Σ_c, Λ_c) in the framework of SU(2) flavor symmetry.
- Heavy-quark symmetry reduces number of free parameters down to 7.
- Large-N_c operator analysis reduces number of free parameters down to 6.
- Heavy-quark symmetry and large-N_c operator analysis reduces number of free parameters down to 5.

Future Plan

- Performing analytical and numerical calculations of cross sections for charmed hadron production reactions near threshold.

Thank you for your attentions