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Octet baryon magnetic moments in covariant ChPT at NNLO

北京航空航天大学
Beihang University

Speaker：Yang Xiao

Supervisor：L.S. Geng

Cooperators：J.X. Lu , X.L. Ren

OUTLINE

1. INTRODUCTION
2. FORMALISM
3. RESULTS
4. CONCLUSION

1 INTRODUCTION

Magnetic moment of particles

- **Spin magnetic moment**

$$\mu = g\frac{e}{2m}S$$

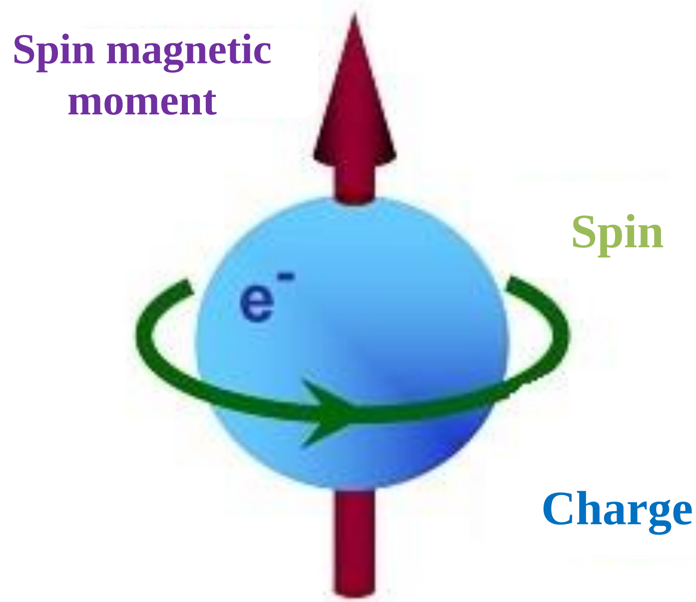
- **Nucleon magnetic moments**

$$\mu_p = \frac{e}{2m_N} \quad \mu_n = 0$$

- **Anomalous magnetic moment**

$$\mu_p = 2.793\frac{e}{2m_N}$$

$$\mu_n = -1.193\frac{e}{2m_N}$$



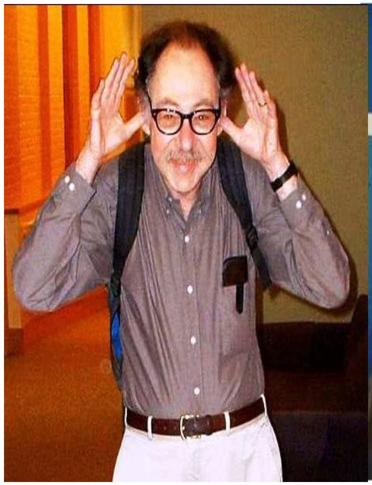
g/2=1.001159652193（10）

- **Radiative correction (electron and muon)**
- **Internal structure (proton and neutron)**

Coleman-Glashow Formula

SU(3) Symmetry

Sheldon Lee Glashow



Sidney Coleman

$$\mu(\Sigma^+) = \mu(p)$$

$$\mu(\Lambda) = \frac{1}{2}\mu(n)$$

$$\mu(\Xi^0) = \mu(n)$$

$$\mu(\Xi^-) = \mu(\Sigma^-) = -[\mu(p) + \mu(n)]$$

$$\mu(\Sigma^0) = -\frac{1}{2}\mu(n)$$

Coleman, Sidney R. et al. Phys.Rev.Lett. 6 423 (1961)

A Leading order calculation in
Chiral Perturbation Theory(ChPT)

Coleman-Glashow Formula

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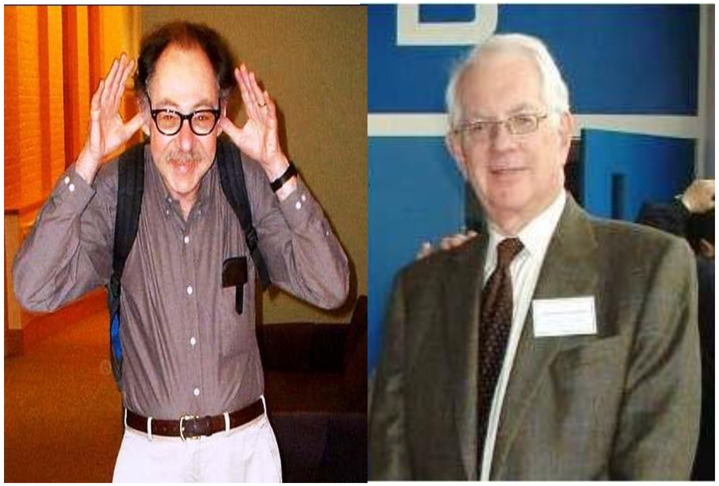

Sidney Coleman

$$2.46 = 2.793$$

$$-0.61 = -0.596$$

$$-1.25 = -1.193$$

$$-0.65 = -1.16 = -0.8$$

$$? = 0.596$$

Coleman, Sidney R. et al. Phys.Rev.Lett. 6 423 (1961)

A Leading order **calculation in**
Chiral Perturbation Theory(ChPT)

Coleman-Glashow Formula

SU(3) Symmetry

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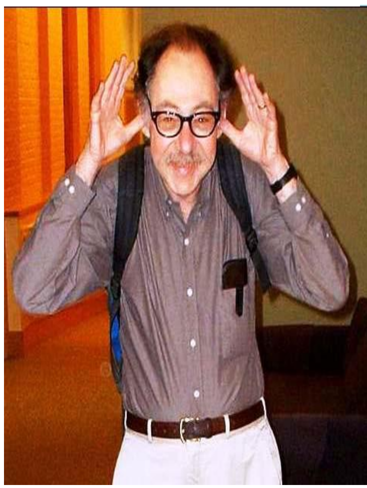


$$2.46 = 2.793$$

These relations are consistent with experiment data

$$? = 0.596$$

Sidney Coleman

Coleman, Sidney R. et al. Phys.Rev.Lett. 6 423 (1961)

A Leading order calculation in
Chiral Perturbation Theory(ChPT)

Loop correction

SU(3) symmetry breaking

1. D. G. Caldi and H. Pagels, Phys. Rev. D 10, 3739 (1974).
2. J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985).
3. J. Gasser, M. E. Sainio, and A. Svarc, Nucl. Phys. B307, 779 (1988).
4. E. E. Jenkins, M. E. Luke, A.V. Manohar, and M. J. Savage, Phys. Lett. B 302, 482 (1993); 388, 866 (1996).
5. S. Scherer, Adv. Nucl. Phys. 27, 277 (2003).
6. …………

□ The contribution of NLO turns to worsen the results

Theoretical calculation

Nonrelativistic

$M_K/\Lambda_{QCD} = 0.5$

- Heavy Baryon chiral perturbation theory
- Up to NNLO
- Bad convergence at NLO
- Excellent convergence at NNLO

$$\begin{aligned}
\mu_p &= +4.48(1-0.49+0.11) = +2.79,\\
\mu_n &= -2.47(1-0.34+0.12) = -1.91,\\
\mu_{\Sigma^+} &= +4.48(1-0.62+0.17) = +2.46,\\
\mu_{\Sigma^-} &= -2.01(1-0.31-0.11) = -1.16,\\
\mu_{\Sigma^0} &= +1.24(1-0.87+0.40) = +0.65,\\
\mu_{\Lambda} &= -1.24(1-0.87+0.37) = -0.61,\\
\mu_{\Xi^0} &= -2.47(1-0.89+0.40) = -1.25,\\
\mu_{\Xi^-} &= -2.01(1-0.64-0.03) = -0.65,\\
\mu_{\Lambda\Sigma^0} &= +2.14(1-0.53+0.19) = +1.40.
\end{aligned}$$

U. G. Meissner and S. Steininger, Nucl. Phys. B499, 349 (1997).

Relativistic

- Extended On Mass Shell ChPT
- Up to NLO
- Nice convergence properties

$$\begin{aligned}
\mu_p &= +3.47(1-0.257) = +2.58,\\
\mu_n &= -2.55(1-0.175) = -2.10,\\
\mu_{\Sigma^+} &= +3.47(1-0.300) = +2.43,\\
\mu_{\Sigma^-} &= -0.93(1+0.187) = -1.16,\\
\mu_{\Sigma^0} &= +1.27(1-0.482) = +0.66,\\
\mu_{\Lambda} &= -1.27(1-0.482) = -0.66,\\
\mu_{\Xi^0} &= -2.55(1-0.501) = -1.27,\\
\mu_{\Xi^-} &= -0.93(1+0.025) = -0.95,\\
\mu_{\Lambda\Sigma^0} &= +2.21(1-0.284) = +1.58.
\end{aligned}$$

L.S. ,Geng et al. Phys.Rev.Lett. 101 222002 (2008).

Theoretical calculation

Nonrelativistic

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$$\mu_p = +4.48(1 - 0.49 + 0.11) = +2.79,$$
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U. G. Meissner and S. Steininger, Nucl. Phys. B499, 349 (1997).

Relativistic

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- Up to NLO
- Nice convergence properties

Coincidence?

$$\mu_p = +3.47(1 - 0.257) = +2.58,$$
$$\mu_n = -2.55(1 - 0.175) = -2.10,$$
$$\mu_{\Sigma^+} = +3.47(1 - 0.300) = +2.43,$$
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L.S. ,Geng et al. Phys.Rev.Lett. 101 222002 (2008).

2

FORMALISM

The problems of QCD at Low energy region

- **QCD is the theory of the strong interaction between quarks and gluons**
- **Problems at Low energy region**
  - **The perturbative method is invalid because of large coupling constant**
  - **The basic degree of freedom does not match observables due to color confinement**
- **Two model independent theory**
  - **Chiral perturbation theory**
  - **Lattice QCD**

Chiral perturbation theory (ChPT)

Steven Weinberg

- **The low energy EFT of QCD**
  - **Perturbative treatment in terms of momentum expansion instead of coupling constant**
  - **Effective Lagrangian contains all possible terms compatible with assumed symmetry principles that consistent with the fundamental principles of quantum field theory**
    - **Effective Lagrangian**

$$\mathcal{L}_{QCD} \to \mathcal{L}_{eff} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \ldots\ldots$$

Baryon magnetic moments

Definition

Magnetic moments are defined through electromagnetic current

$$\langle \bar{B}|J_\mu|B\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^B(t) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_B} F_2^B(t)\right] u(p_i).$$

t=0

Magnetic moments= Anomalous magnetic moments + Charge

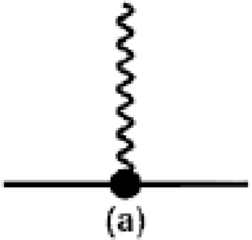
Steps

1. **Calculate all Feynman diagrams** $\mu = \mu^{(2)} + \mu^{(3)} + \mu^{(4)} + \cdots\cdots$
2. **Extract the** $F_2^B(0)$
3. **Fit LECS**

Feynman diagrams and Lagrangians

Leading order

$$\mathcal{L}_{MB}^{(2)} = \frac{b_6^D}{8m}\langle \bar{B}\sigma^{\mu\nu}\left\{F_{\mu\nu}^{+}, B\right\}\rangle + \frac{b_6^F}{8m}\langle \bar{B}\sigma^{\mu\nu}\left[F_{\mu\nu}^{+}, B\right]\rangle,$$

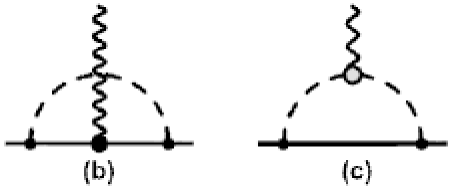


Next to leading order

$$\mathcal{L}_{B}^{(1)} = \langle \bar{B} i\gamma^{\mu} D_{\mu} B\rangle,$$

$$\mathcal{L}_{MB}^{(1)} = \frac{D}{2}\langle \bar{B}\gamma^{\mu}\gamma^{5}\left\{u_{\mu}, B\right\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^{\mu}\gamma^{5}\left[u_{\mu}, B\right]\rangle,$$

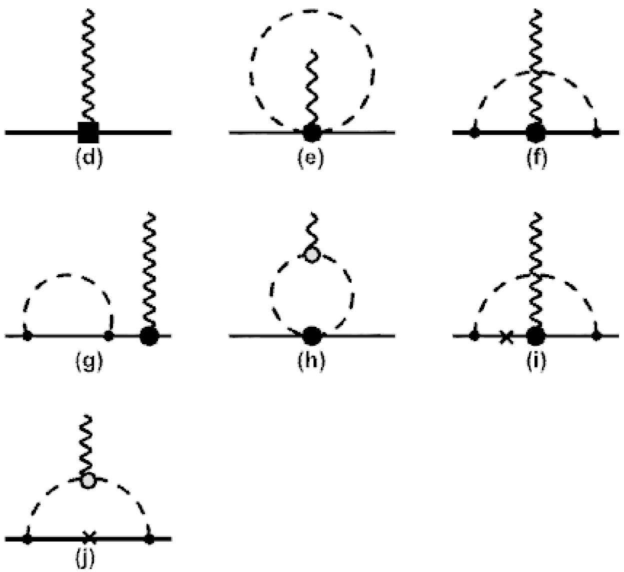
$$\mathcal{L}_{M}^{(2)} = \frac{F_0^2}{4}\langle u_{\mu}u^{\mu} + \chi^{+}\rangle,$$



Feynman diagrams and Lagrangians

Next to next to leading order

$$\mathcal{L}_{MB}^{(4)} = +\frac{b_6^{D'}}{8m}\langle\chi^+\rangle\langle\bar{B}\sigma^{\mu\nu}\{F_{\mu\nu}^+, B\}\rangle + \frac{b_6^{F'}}{8m}\langle\chi^+\rangle\langle\bar{B}\sigma^{\mu\nu}[F_{\mu\nu}^+, B]\rangle$$
$$+\frac{\alpha_1}{8m}\langle\bar{B}\sigma^{\mu\nu}\left[\left[F_{\mu\nu}^+, B\right], \chi^+\right]\rangle + \frac{\alpha_2}{8m}\langle\bar{B}\sigma^{\mu\nu}\left\{\left[F_{\mu\nu}^+, B\right], \chi^+\right\}\rangle$$
$$+\frac{\alpha_3}{8m}\langle\bar{B}\sigma^{\mu\nu}\left[\left\{F_{\mu\nu}^+, B\right\}, \chi^+\right]\rangle + \frac{\alpha_2}{8m}\langle\bar{B}\sigma^{\mu\nu}\left\{\left\{F_{\mu\nu}^+, B\right\}, \chi^+\right\}\rangle$$
$$+\frac{\beta_1}{8m}\langle\bar{B}\sigma^{\mu\nu}B\rangle\langle\chi^+F_{\mu\nu}^+\rangle.$$



$$\mathcal{L}_{MB}^{(2')} = \frac{i}{2}\left\{b_9\langle\bar{B}\sigma^{\mu\nu}u_\mu\rangle\langle u_\nu B\rangle + b_{10,11}\langle\bar{B}\sigma^{\mu\nu}\left(\left[u_\mu, u_\nu\right], B\right)_\pm\rangle\right\}.$$

$$\mathcal{L}_{MB}^{(2'')} = b_D\langle\bar{B}\{\chi^+, B\}\rangle + b_F\langle\bar{B}[\chi^+, B]\rangle.$$

Fit LECS

Problem

- **9 LECS**
  - $b_6^D, b_6^F, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, b_6^{D'}, b_6^{F'}$
- **7 experiment data**
  - $p, n, \Lambda, \Sigma^+, \Sigma^-, \Xi^-, \Xi^0$

9>7 → Cannot fit direct！ ！ ！

Solution 1

Absorb $b_6^{D'}$ and $b_6^{F'}$

$$\mathcal{L}_{MB}^{(2)} = \frac{b_6^D}{8m}\langle \bar{B}\sigma^{\mu\nu}\{F_{\mu\nu}^+, B\}\rangle + \frac{b_6^F}{8m}\langle \bar{B}\sigma^{\mu\nu}[F_{\mu\nu}^+, B]\rangle,$$

$$\mathcal{L}_{MB}^{(4)} = +\frac{b_6^{D'}}{8m}\langle \chi^+\rangle\langle \bar{B}\sigma^{\mu\nu}\{F_{\mu\nu}^+, B\}\rangle + \frac{b_6^{F'}}{8m}\langle \chi^+\rangle\langle \bar{B}\sigma^{\mu\nu}[F_{\mu\nu}^+, B]\rangle$$

$$b_6^{D,F} \to b_6^{D,F} + \langle \chi^+\rangle b_6^{D',F'}$$

U. G. Meissner and S. Steininger, Nucl. Phys. B499, 349 (1997).

Fit LECS

Solution 2

Constrain b_6^D and b_6^F according to the convergence properties

□ Start point

- **The success of BChPT**
  - **Baryon mass and sigma terms**
    PRD82:074504,2010 ;PLB766-325, (2017)
  - **$N-N$ interaction**
    arXiv:1611.08475
  - **$\pi-N$ scattering**
    PRC83:055205, 2011
  - ……

□ Assumption

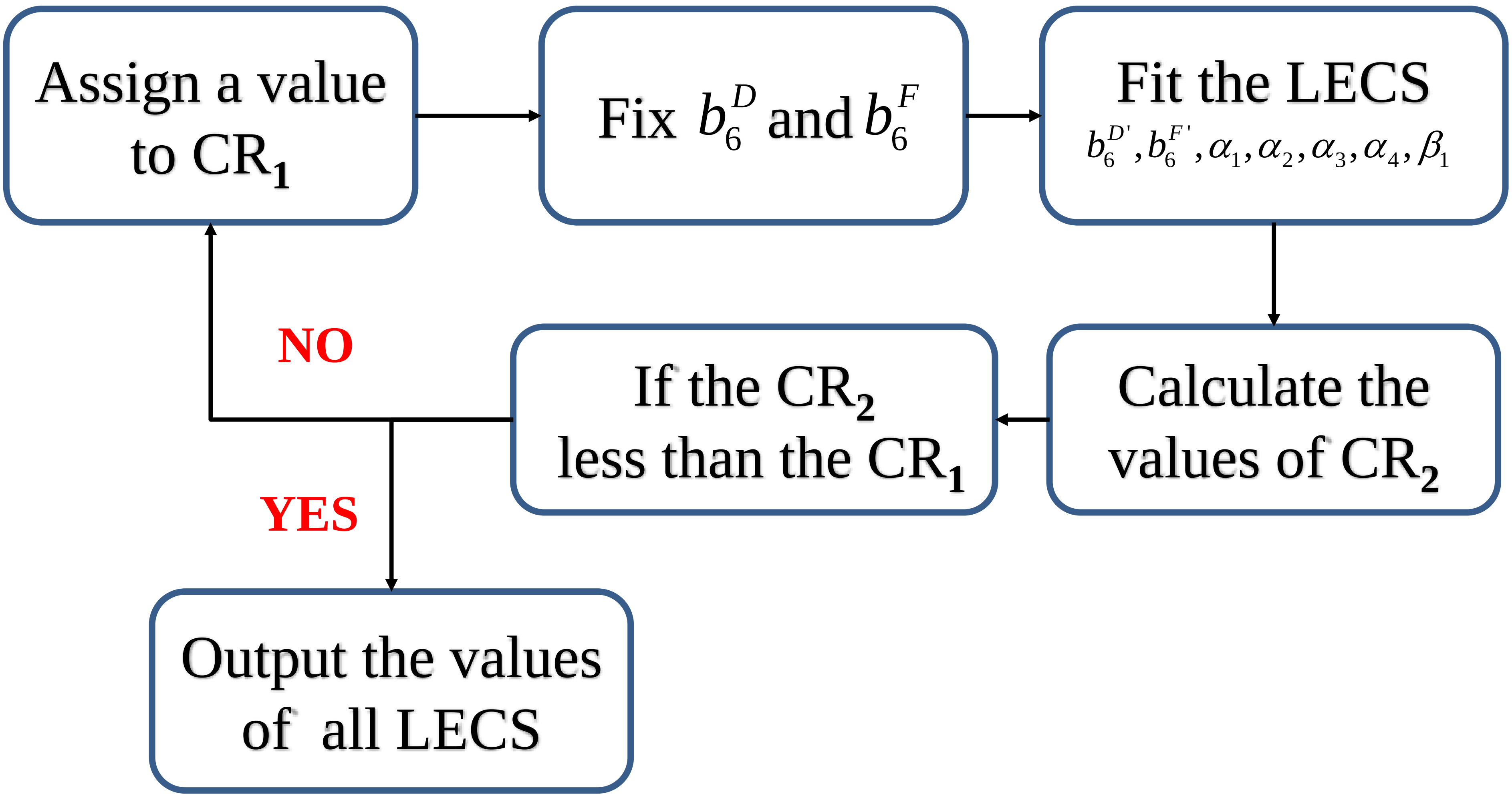
- **BChPT should present good convergence properties**

Fit LECS

Solution 2

Convergence rate

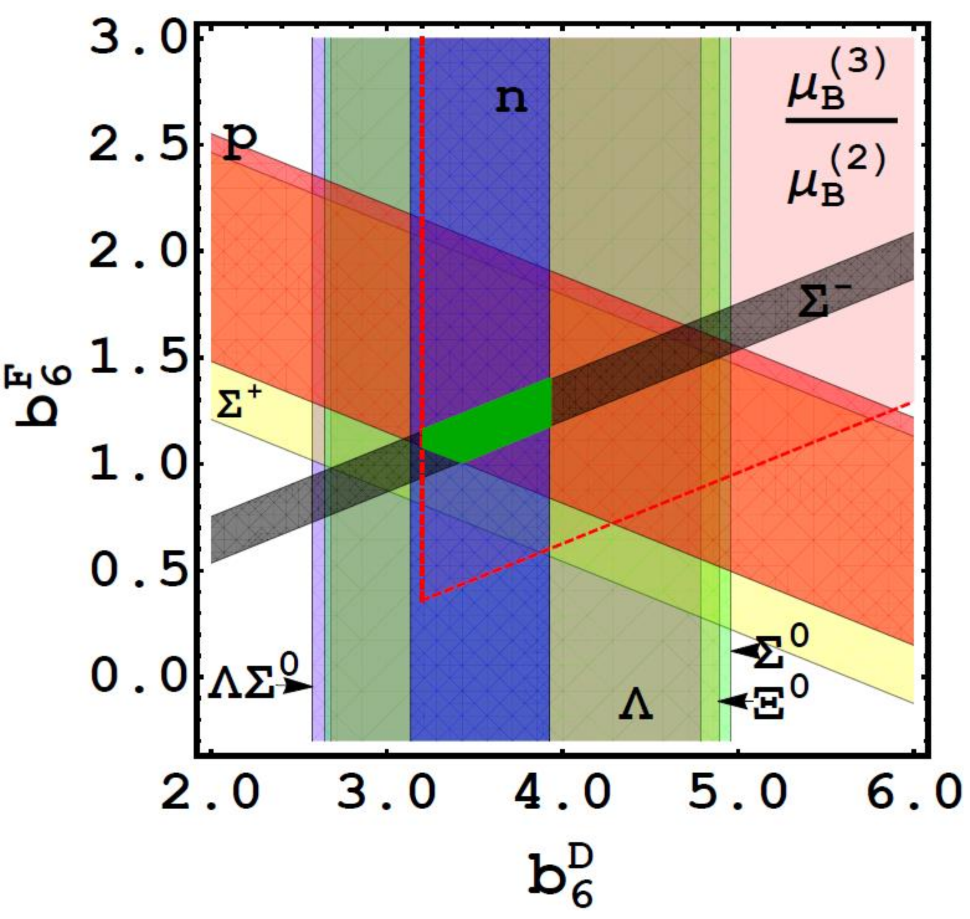
$$\mathrm{CR} = \max(\mu_B^{(3)}/\mu_B^{(2)}, \mu_B^{(4)}/\mu_B^{(3)})$$



3

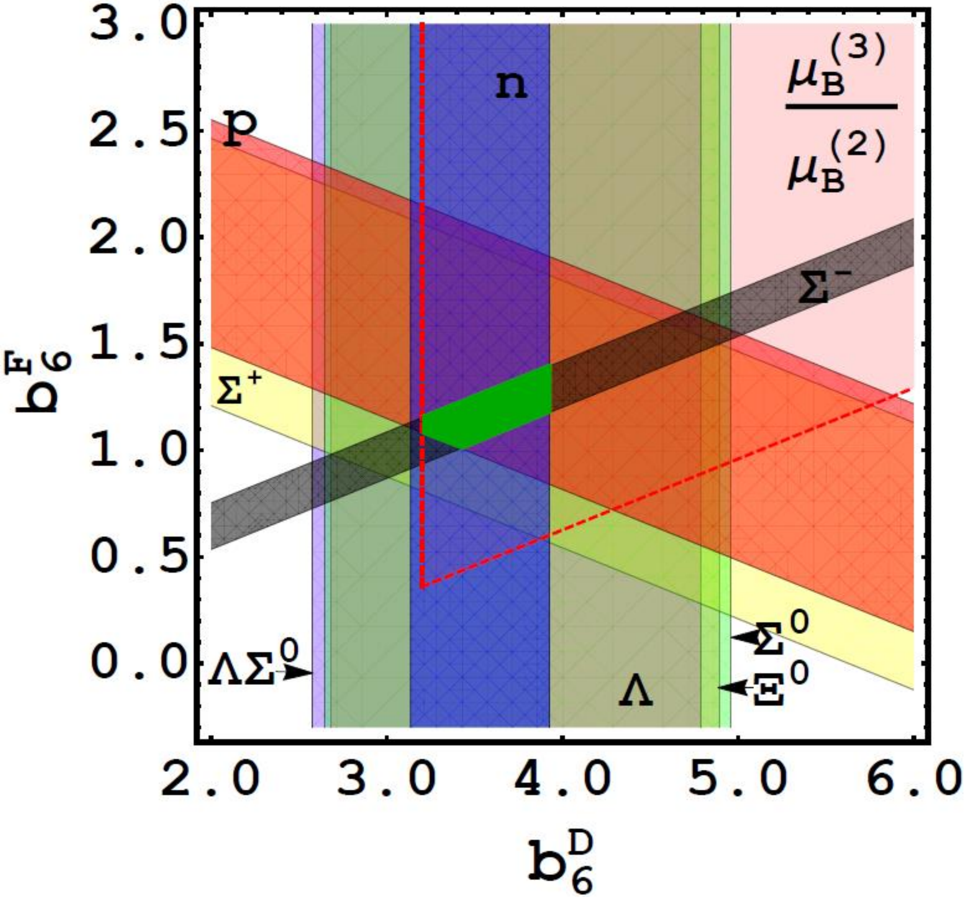
RESULTS

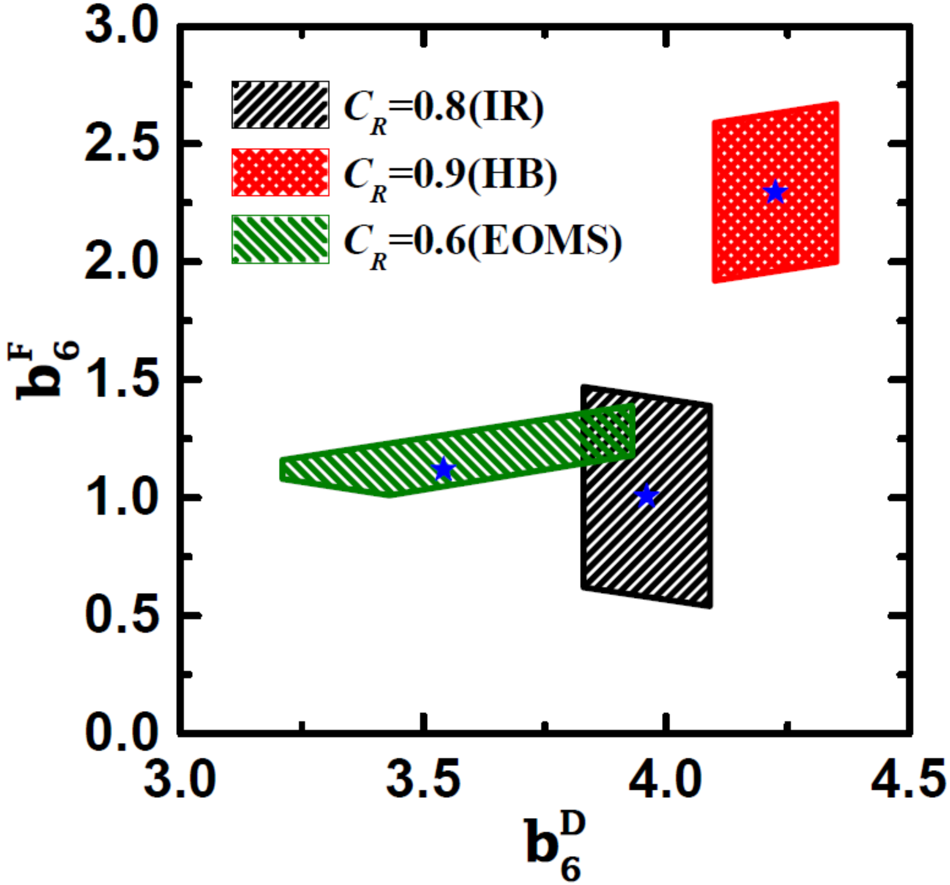
Constrain the values of b_6^D and b_6^F



$\mathrm{CR} = \max(\mu_B^{(3)}/\mu_B^{(2)}, \mu_B^{(4)}/\mu_B^{(3)})$ **CR=0.6**

Constrain the values of b_6^D and b_6^F





$\mathrm{CR} = \max(\mu_B^{(3)}/\mu_B^{(2)}, \mu_B^{(4)}/\mu_B^{(3)})$ **CR=0.6**

Convergence properties of different BChPT

TABLE I. Contributions of different chiral orders of the HB, IR, and EOMS schemes up to $\mathcal{O}(p^4)$.

Baryons	EOMS		IR		HB	
	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$
P	−0.27	−0.38	−0.16	0.01	−0.44	−0.07
N	−0.19	0.02	−0.17	0.61	−0.18	0.74
Λ	−0.52	−0.08	−0.73	−0.27	−0.83	−0.32
Σ^-	0.18	−0.04	2.58	−0.73	−0.30	0.30
Σ^+	−0.31	−0.15	−0.05	4.20	−0.61	−0.22
Σ^0	−0.52	−0.13	−0.73	−0.31	−0.83	−0.35
Ξ^-	0.03	−12.88	3.10	−1.02	−0.74	−0.12
Ξ^0	−0.54	−0.13	−0.77	−0.32	−0.87	−0.36
$\Lambda\Sigma^0$	−0.31	0.27	−0.38	−0.11	−0.43	0.46

Convergence properties of different BChPT

TABLE I. Contributions of different chiral orders of the HB, IR, and EOMS schemes up to $\mathcal{O}(p^4)$.

Baryons	EOMS		IR		HB	
	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$	$\mu_B^{(3)}/\mu_B^{(2)}$	$\mu_B^{(4)}/\mu_B^{(3)}$
P	−0.27	−0.38	−0.16	0.01	−0.44	−0.07
N	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Ξ^-	0.03	−12.88	3.10	−1.02	−0.74	−0.12
Ξ^0	−0.54	−0.13	−0.77	−0.32	−0.87	−0.36
$\Lambda\Sigma^0$	−0.31	0.27	−0.38	−0.11	−0.43	0.46

EOMS ChPT presents the best convergence properties !

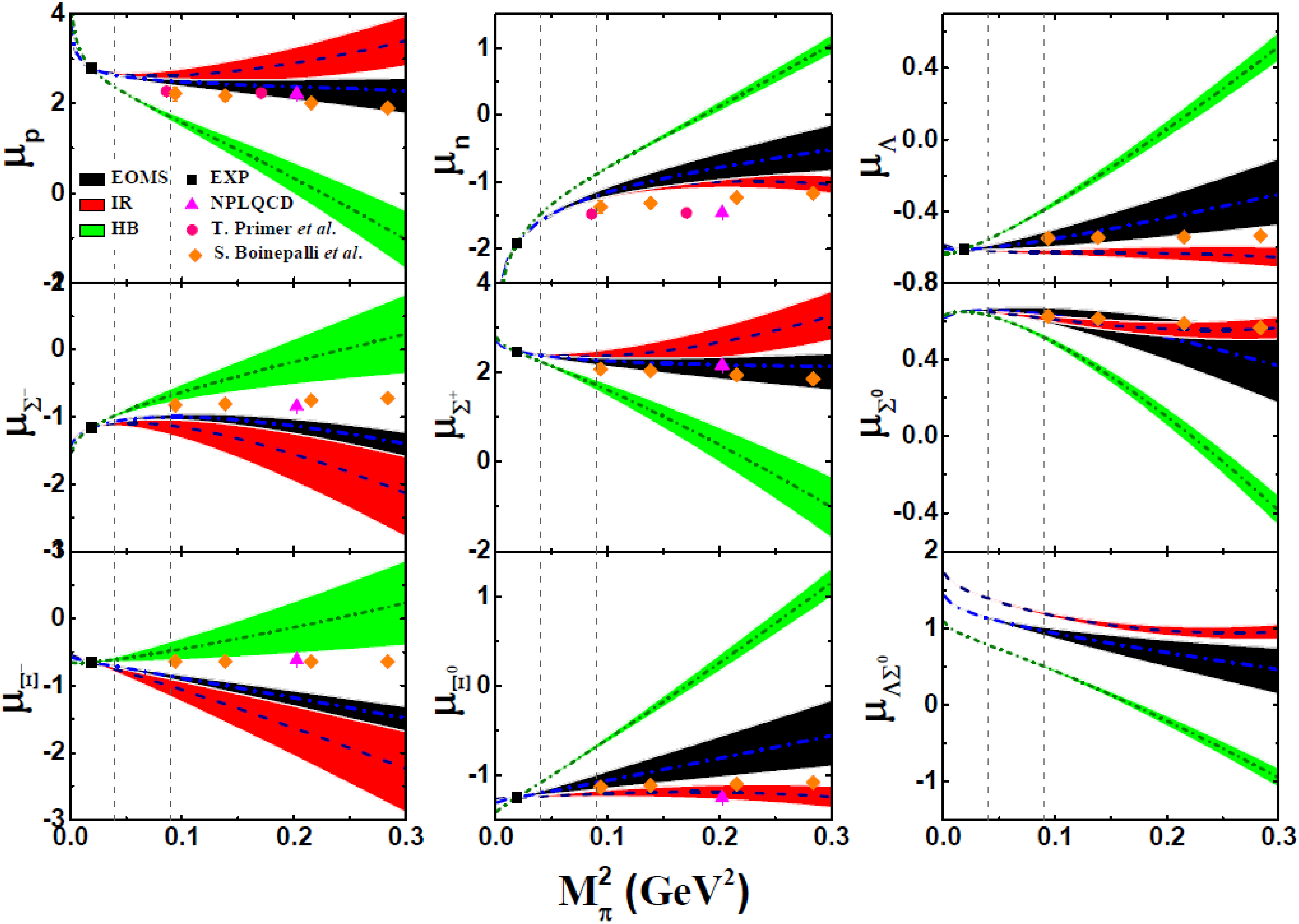
Check the reliability of the LECS

- **9 LECS**

$b_6^D, b_6^F, b_6^{D\prime}, b_6^{F\prime}, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1$

- **Are these LECS reliable?**
  - ✓ **Two model independent theory**
    - **Chiral perturbation theory**
    - **Lattice QCD**
  - ✓ **ChPT and Lattice QCD can verify each other**

Pion mass dependence



Chi-square between the ChPT results and LQCD

Chiral schemes	Chi-square		
	S.Boinepalli et al.	NPLQCD	T. Primer et al.
IR	12.01	2.48	0.69
HB	43.45	12.49	4.64
EOMS	3.66	1.11	0.5

- **Phys. Rev. D 74,093005 (2006)**
- **Phys. Rev. D 89, 034508 (2014)**
- **Phys.Rev. D 95, 114513 (2017)**

4 CONCLUSION

Summary

- Calculate the **baryon magnetic moments** in **EOMS ChPT** up to next-to-next-to-leading order
- Compare our results with the results of **HB ChPT** and **IR ChPT**
- Compare **ChPT** results with **LQCD**

- **EOMS ChPT** presents **excellent convergence properties**
- **The predictions of EOMS ChPT** are **in good agreement** with the results of **lattice QCD**

Thanks !