

Precise Few-body calculations for kaonic nuclear and atomic systems

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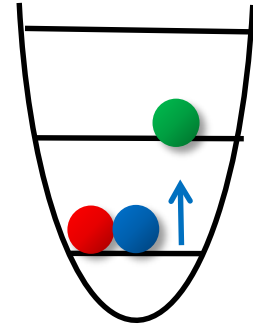
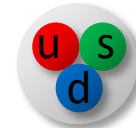
Outline

- Precise few-body calculations for kaonic systems
 - Modern $\bar{K}N$ interaction [K. Miyahara, T. Hyodo, PRC93 \(2016\)](#)
 - Kaonic nuclei: $\bar{K}NN$ to $\bar{K}NNNNNN$ (7-body)
[S. Ohnishi, WH, T. Hoshino, K. Miyahara, T. Hyodo, Phys. Rev. C95, 065202 \(2017\)](#)
 - Kaonic deuterium: $\bar{K}NN$ three-body system
[T. Hoshino, S. Ohnishi, WH, T. Hyodo, W. Weise, arXiv:1705.06857](#)
 - Unified approach to atomic and nuclear kaonic systems
 - Nucleus $\sim$ few fm
 - Atom $\sim$ several hundreds fm

Precise variational calculations with correlated Gaussian method

Kaonic nuclei (Nucleus with antikaon)

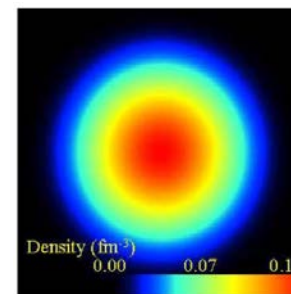
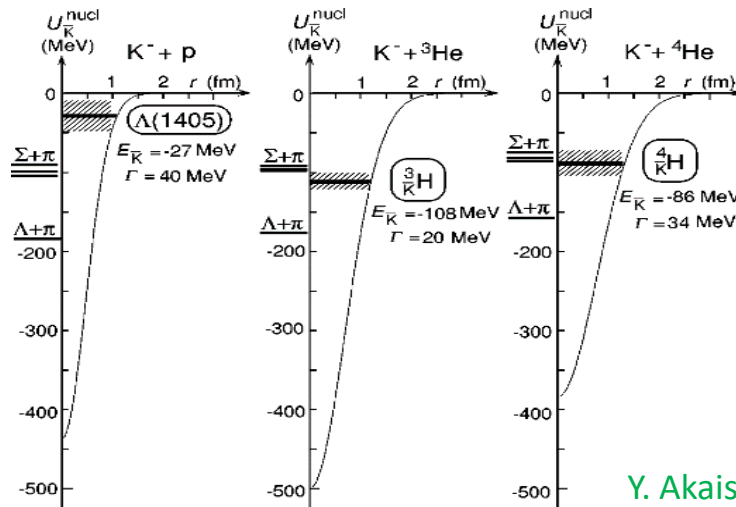
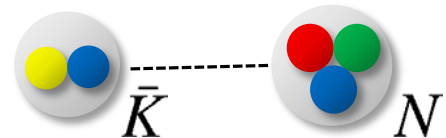
- $\Lambda(1405)$; $J^\pi=1/2^-$, $S= -1$
 - uds constituent quark model
 - Energy is too high



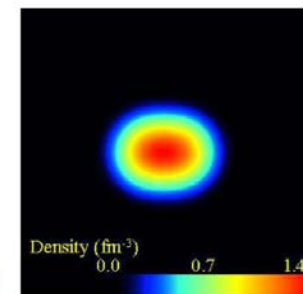
- $\bar{K}N$ quasi-bound state
 - **strongly attractive** $\bar{K}N$ interaction

Isgur, Karl, PRD 18, 4187(1978)

Dalitz, Wong, Tajasekaran,
PR 153, 1617 (1967)



(a) ${}^3\text{He}$



(b) ${}^3\text{He}\bar{K}^-$

Dote, et. al., PLB590, 51(2004).

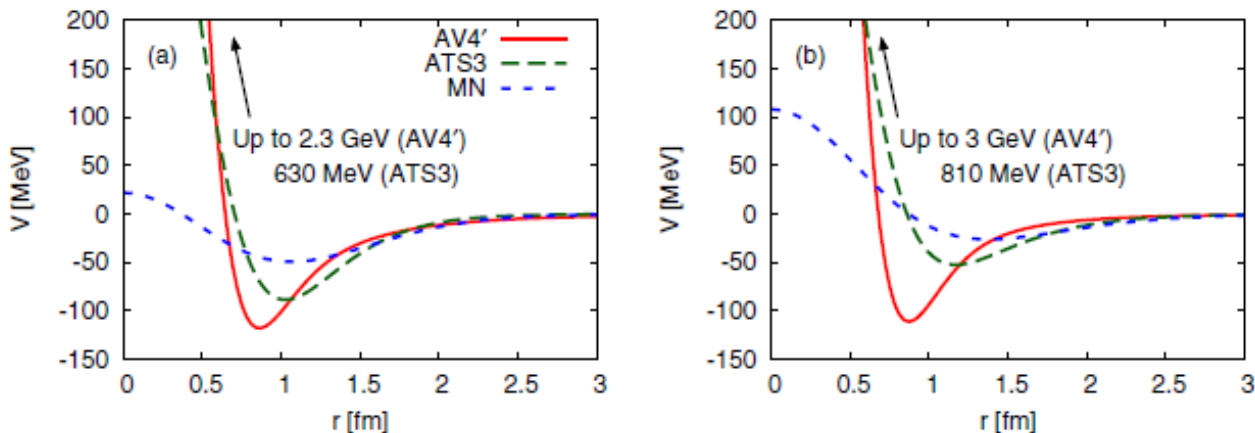
Y. Akaishi, T. Yamazaki, PRC 65, 044005 (2002).

Does Kaonic nucleus really exist? E15 exp. Y. Sada et al., Prog. Theor. Exp. Phys. **2016**, 051D01 (2016).

Can such a high density system be produced in laboratory? → $\bar{K}N$ interaction is essential!

Kaonic nuclear systems (3 to 7-body)

- Hamiltonian
$$H = \sum_{i=1}^{\mathcal{N}} T_i - T_{\text{cm}} + \sum_{i<j}^{\mathcal{N}-1} V_{ij}^{(NN)} + \sum_{i=1}^{\mathcal{N}-1} V_{i\mathcal{N}}^{(\bar{K}N)} + \sum_{i<j}^{\mathcal{N}} V_{ij}^{\text{Coul.}}$$
- Correlated Gaussian basis $\Phi_{SM_S M_T}(x, A) = \mathcal{A} \{ \exp(-\tilde{x} A x) \chi_{SM_S} \eta_{M_T} \},$
 - Many parameters $\sim (N-1)(N-2)/2 \times (\# \text{ of basis})$
 - Stochastic variational method K. Varga and Y. Suzuki, PRC52, 2885 (1995).
- Choice of NN potential (AV4', ATS3, MN)



All NN interaction models reproduce the binding energy of s -shell nuclei

Choice of $\bar{K}N$ interaction

Kyoto $\bar{K}N$ potential

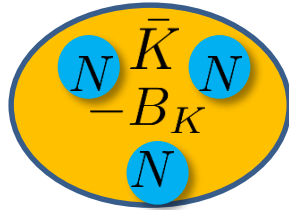
K.Miyahara, T.Hyodo, PRC 93, 015201 (2016)

- **Energy-dependent** $\bar{K}N$ single-channel potential
- Chiral SU(3) dynamics at NLO
- Pole energy: 1424 - 26i and 1381 - 81i MeV Y.Ikeda, T.Hyodo, W.Weise, NPA881 (2012) 98

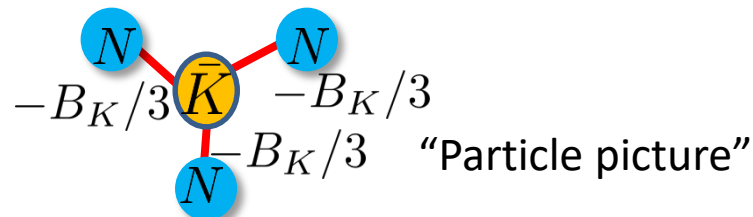
$\bar{K}N$ two-body energy in an N -body system are determined as:

$$\sqrt{s} = m_N + m_{\bar{K}} + \delta\sqrt{s} \quad , \quad -B_K \equiv \langle \Psi | H | \Psi \rangle - \langle \Psi | H_N | \Psi \rangle \quad ,$$

Type I: $\delta\sqrt{s} = -B_K \quad , \quad$ Type II: $\delta\sqrt{s} = -B_K / (N - 1) \quad , \quad \text{for } N\text{-body}$



“Field picture”



A. Dote, T. Hyodo, W. Weise, NPA804, 197 (2008).

Akaishi-Yamazaki (AY) potential

Akaishi, Yamazaki, PRC65, 04400(2002).

- **Energy-independent**
- Reproduce $\Lambda(1405)$ as $\bar{K}N$ quasi-bound state

Variational calculation for many-body quantum system

- Many-body wave function ψ has all information of the system
- Solve many-body Schoedinger equation
 $\Leftrightarrow$ Eigenvalue problem with Hamiltonian matrix
$$H\psi = E\psi$$
- Variational principle $\langle \psi | H | \psi \rangle = E \geq E_0$ (“Exact” energy)
(Equal holds if ψ is the “exact” solution)

Many degrees of freedom

- Expand ψ with several sets of basis functions
Correlated Gaussian + Global vectors

Explicitly correlated basis approach

Correlated Gaussian with two global vectors

Y. Suzuki, [W.H.](#), M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$\phi_{(L_1 L_2) L M_L}^{\pi}(A, u_1, u_2) = \exp(-\tilde{x} A x) [\mathcal{Y}_{L_1}(\tilde{u}_1 x) \mathcal{Y}_{L_2}(\tilde{u}_2 x)]_{L M_L}$$

x : any relative coordinates (cf. Jacobi)

$$\mathcal{Y}_{\ell}(r) = r^{\ell} Y_{\ell}(\hat{r})$$

$$\tilde{x} A x = \sum_{i,j=1}^{N-1} A_{ij} x_i \cdot x_j$$

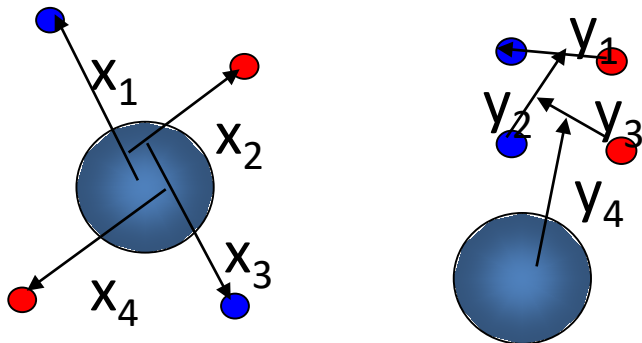
$$\tilde{u}_i x = \sum_{k=1}^{N-1} (u_i)_k x_k$$

Formulation for N-particle system
Analytical expression for matrix elements

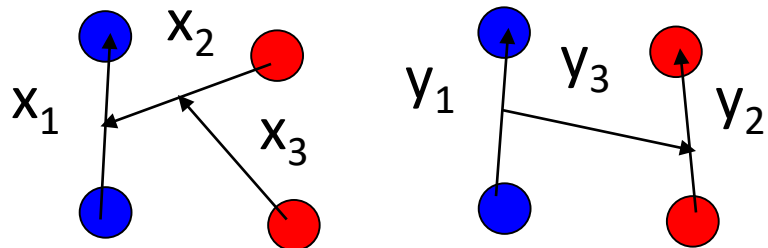
Functional form does not change under any coordinate transformation

$$y = T x \implies \tilde{y} B y = \tilde{x} \tilde{T} B T x \qquad \tilde{v} y = \tilde{T} v x$$

Shell and cluster structure



Rearrangement channels



See Review: J. Mitroy et al., Rev. Mod. Phys. 85, 693 (2013)

Basis optimization:

Stochastic Variational Method

Possibility of the stochastic optimization

1. increase the basis dimension one by one
2. set up an optimal basis by trial and error procedures
3. fine tune the chosen parameters until convergence

- 1. Generate** $(A_k^1, A_k^2, \dots, A_k^m)$ **randomly**
- 2. Get the eigenvalues** $(E_k^1, E_k^2, \dots, E_k^m)$
- 3. Select** A_k^n **corresponding to the lowest** E_k^n **and Include it in a basis set**
- 4. $k \rightarrow k+1$**

Y. Suzuki and K. Varga, Stochastic variational approach to quantum-mechanical few-body problems, LNP 54 (Springer, 1998).

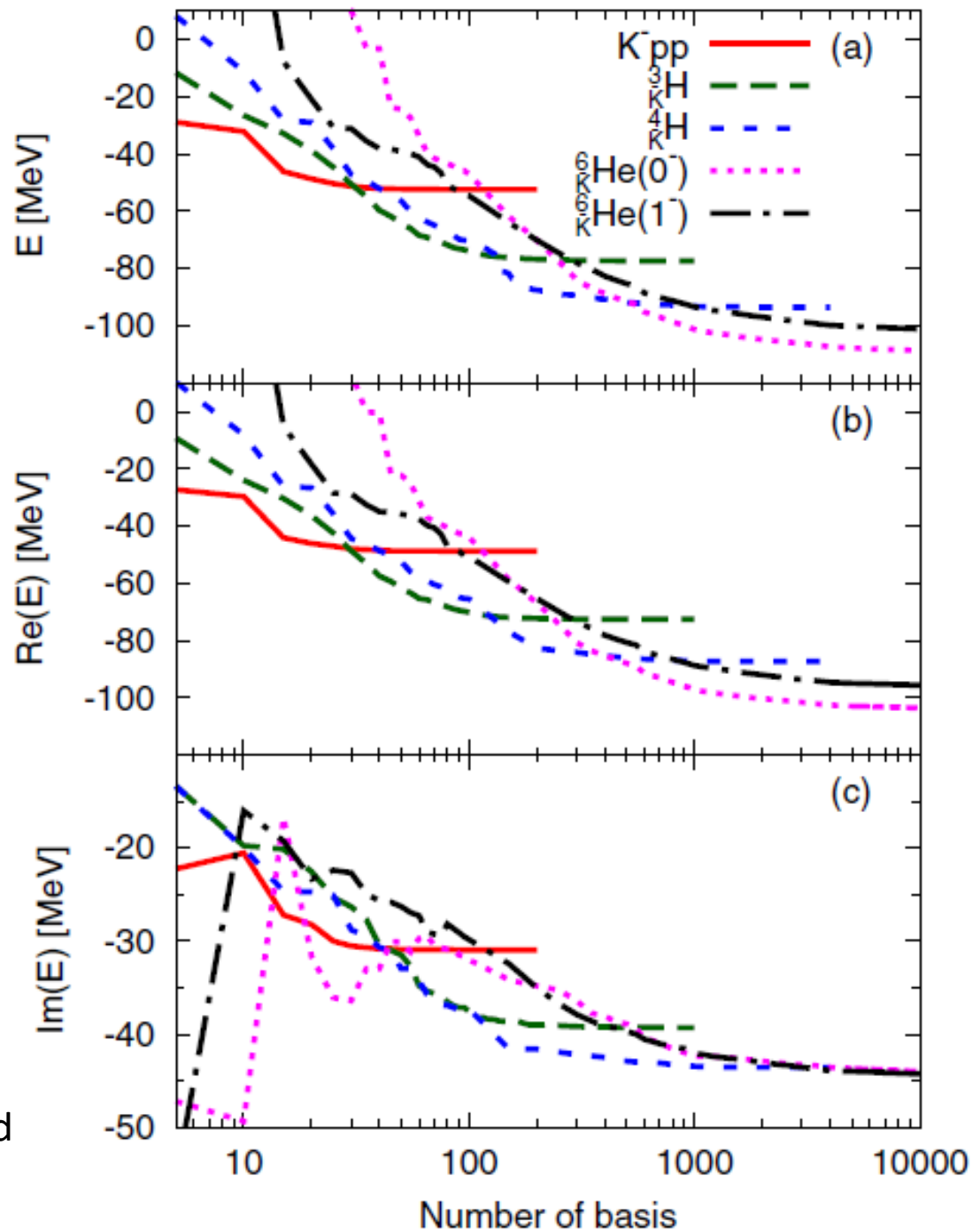
K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).

Energy curves

- Optimization only with a real part of the $\bar{K}N$ pot.
- Two-body $\bar{K}N$ energy is self-consistently determined
- AV4' NN pot. is employed

Full energy curves

Validity of this approach is confirmed
in the three-body (K-pp) system



Properties of K⁻pp

Model	Kyoto		AY
	Type I	Type II	
B (MeV)	27.9	26.1	48.7
Γ (MeV)	30.9	59.3	61.9
$\delta\sqrt{s}$ (MeV)	$-61.0 - i25.0$	$-30.2 - i23.7$	
P_{K^-}	0.65	0.65	0.64
$P_{\bar{K}^0}$	0.35	0.35	0.36
$\sqrt{\langle r_{NN}^2 \rangle}$ (fm)	2.16	2.07	1.84
$\sqrt{\langle r_{\bar{K}N}^2 \rangle}$ (fm)	1.80	1.73	1.55
$\sqrt{\langle r_N^2 \rangle}$ (fm)	1.12	1.08	0.958
$\sqrt{\langle r_{\bar{K}}^2 \rangle}$ (fm)	1.14	1.10	0.988

Kyoto $\bar{K}N$ pot.

Similar binding energies with
Types I and II $B \sim 27\text{-}28$ MeV

$\Gamma \sim 30\text{-}60$ MeV

AY pot.

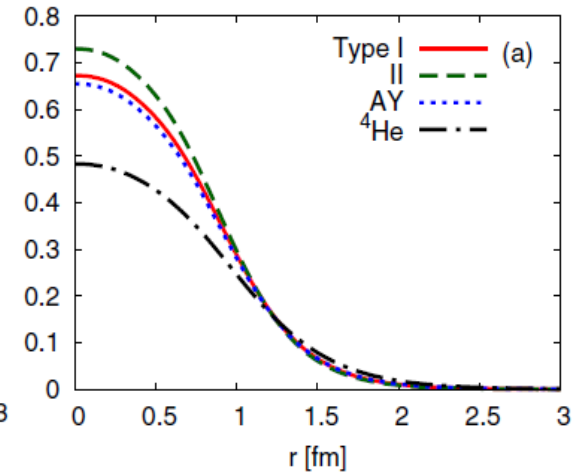
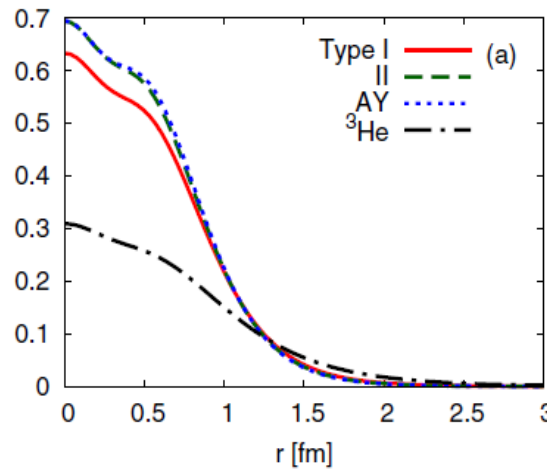
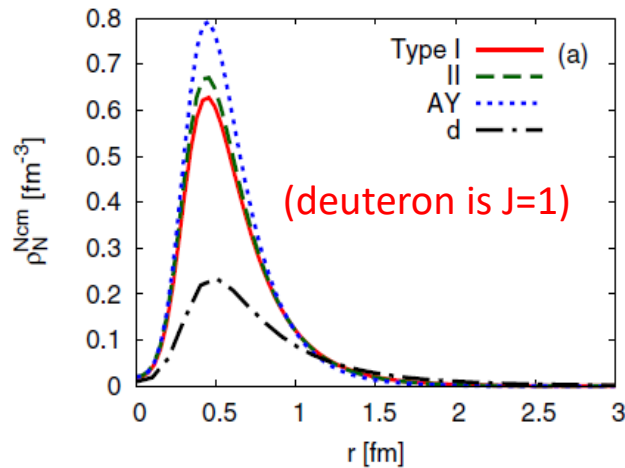
Deeper binding energy $\sim 49\text{MeV}$
 $\rightarrow$ Smaller rms radii

Nucleon Density distributions

$K^- pp$

${}^3\text{He}K^-$

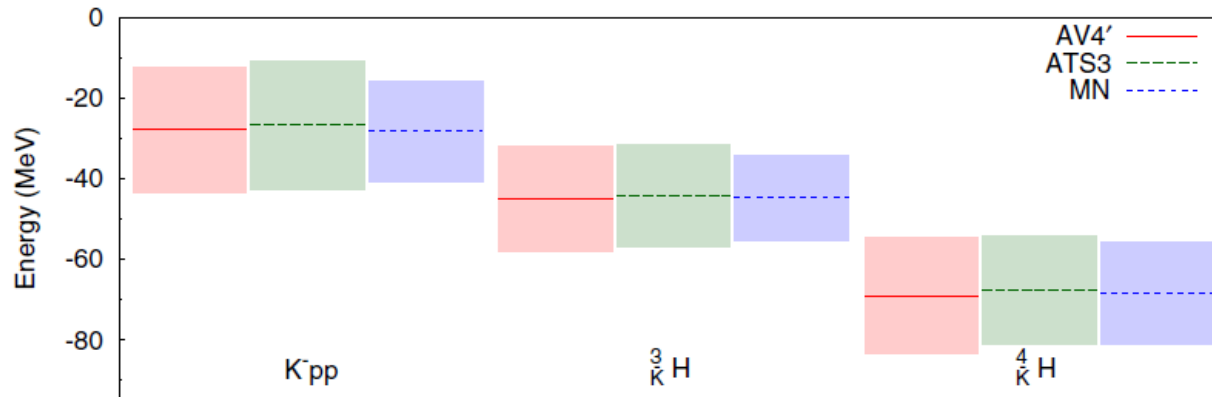
${}^4\text{He}K^-$



- Central nucleon density $\rho(0)$ is enhanced by kaon
- $\rho(0) \sim 0.7 \text{ fm}^{-3}$ at maximum, ~ 2 times higher than that without $\bar{K}$
(~ 4 times higher than saturation dens.)

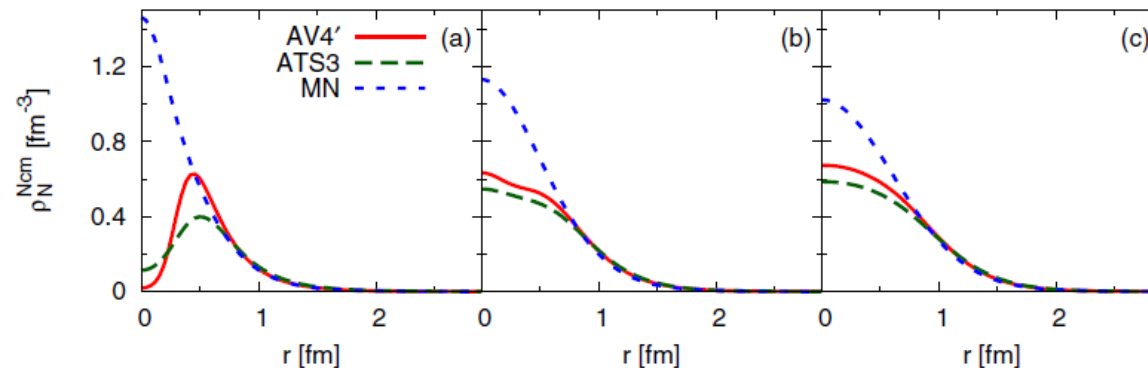
Interaction dependence

Binding energy and decay width with different NN potential models



Not sensitive to the NN interaction models

Nucleon density distributions



AV4' and ATS3 potential: strong short-range repulsion
MN: weak short-range repulsion

Structure of $\bar{K}NNNNNN$ with $J^\pi=0^-$ and 1^-

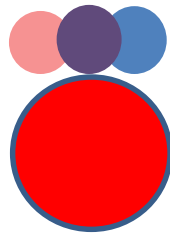
A=2

	NN	$\bar{K}NN$
J=0	unbound	Bound
J=1	bound (d)	unbound

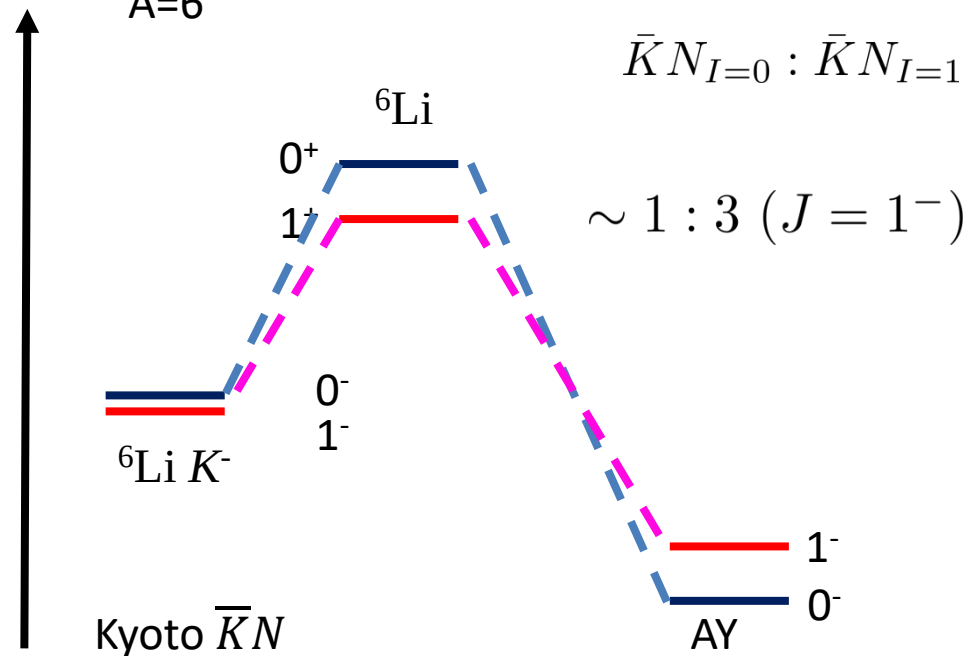
$\bar{K}N_{I=0} : \bar{K}N_{I=1}$

$\sim 3 : 1 (J = 0^-)$

$\sim 1 : 3 (J = 1^-)$



A=6



- $\bar{K}N$ interaction in $l=0$ is more attractive than in $l=1$, and $J=0$ state containing more $l=0$ component than $J=1$
 - Energy gain in $J=0$ is larger than $J=1$ channel
- AY potential in $l=0$ is strongly attractive
 - $J=0$ ground state

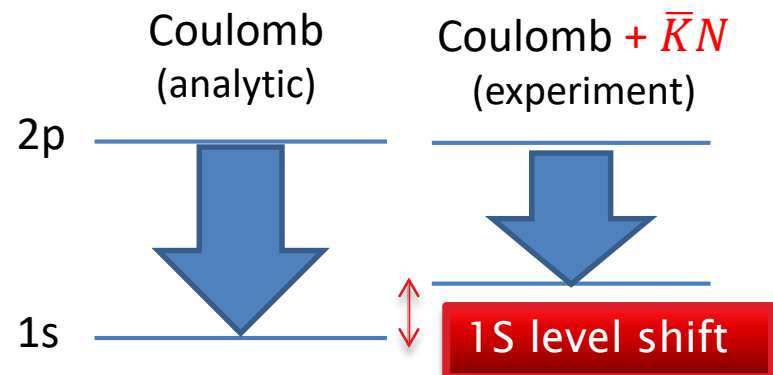
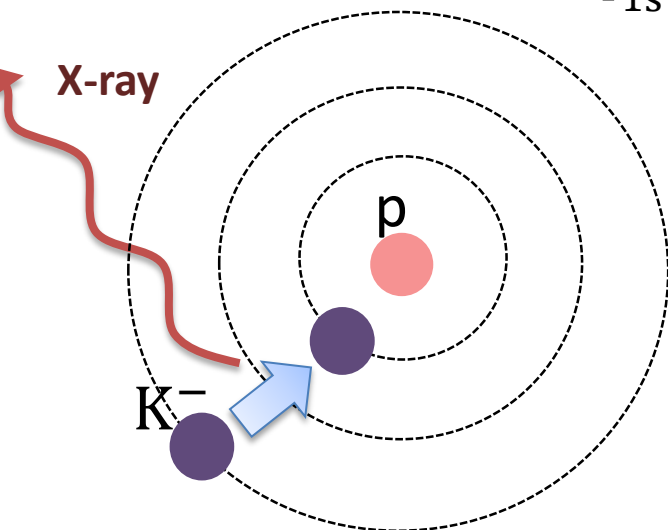
Kaonic hydrogen (atomic system)

- Bound mainly with Coulomb int. (like e^- in H)
- $\bar{K}N$ interaction induces “level shift”
- Precise measurement

(2011, SIDDHARTA experiment , DAΦNE) [Bazzi et al., NPA881 \(2012\)](#)

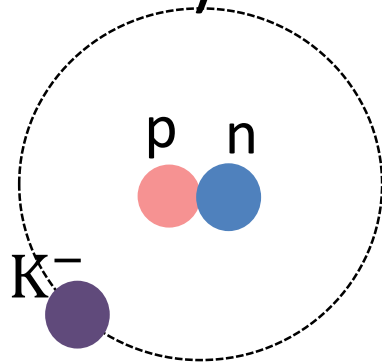
$$\epsilon_{1s} = 283 \pm 36(\text{stat}) \pm 6(\text{syst})\text{eV}$$

$$\Gamma_{1s} = 541 \pm 89(\text{stat}) \pm 22(\text{syst})\text{eV}$$



Constraint for $\bar{K}N$ interaction: Kaonic deuterium

- Isospin dependence of $\bar{K}N$ interactions
 - $I=0$: well determined by $\Lambda(1405)$ properties
 - $I=1$: constraint is weak only with kaonic hydrogen
- Precise kaonic deuterium data (Exp. and theor.) are highly desired



Kaonic deuterium

$$\begin{aligned} |K^- p\rangle &= |\uparrow\downarrow\rangle \\ &= |I = 0\rangle + |I = 1\rangle \end{aligned}$$

$$\begin{aligned} |K^- n\rangle &= |\uparrow\uparrow\rangle \\ &= |I = 1\rangle \end{aligned}$$

$$I=0:I=1=1:3$$

Three-body calculation for kaonic deuterium

Hamiltonian

$$H = T + V = \sum_{i=1}^3 \frac{\mathbf{p}_i^2}{2m_i} - T_{cm} + V^{NN} + V^{\bar{K}N} + V^{coul}$$

V^{NN} (Minnesota potential) [D. R. Thompson, M. Lemere and Y. C. Tang, NPA286 \(1977\)](#)

$V^{\bar{K}N}$ (Kyoto $\bar{K}N$ potential) [K.Miyahara, T.Hyodo, PRC93 \(2016\)](#)

- The Kyoto $\bar{K}N$ pot. simulates scattering amplitude calculated from NLO chiral SU(3) dynamics [Y.Ikeda, T.Hyodo, W.Weise, NPA881 \(2012\)](#)
 1. Scattering length extracted from the energy shift measured in SIDDHARTA experiment.
 2. Cross section of a $\bar{K}N$ two-body scattering
 3. Branching ratio of the $\bar{K}p$ decay

Three-body calculation

Variational Method

► Wave function

Correlated Gaussian basis

$$\Psi = \sum_{i=1}^N c_i \phi_i, \quad \phi = A_{NN} \{ e^{-\frac{1}{2} \tilde{x} A x} [[y_{L_1}(\tilde{u}x) y_{L_2}(\tilde{v}x)]_L \chi_S]_{JM} \eta_{TM_t} \}$$

A : 2×2 positive definite symmetric matrix

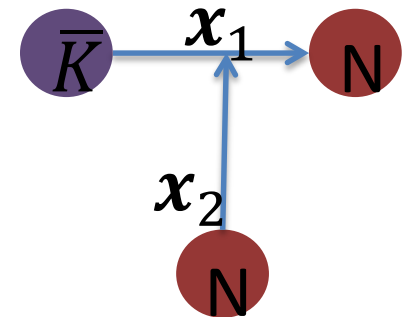
$$x = \{x_1, x_2\}, \quad \tilde{u}x = u_1 x_1 + u_2 x_2, \quad \tilde{v}x = v_1 x_1 + v_2 x_2$$

$$y_{lm}(\tilde{u}x) = (\tilde{u}x)^l Y_{lm}(\hat{\tilde{u}x})$$

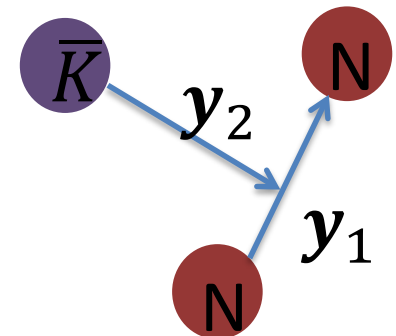
χ_{JM} : spin function, η_{TM_t} : isospin function

- Geometric progression for Gaussian fall-off parameters
 - Cover 0.1-500 fm
 - $L_1 + L_2 \leq 4$
 - About 8000 basis states
- Channel coupling, physical masses

$$|K^- pn\rangle = |\downarrow\uparrow\downarrow\rangle, \quad |\bar{K}^0 nn\rangle = |\uparrow\downarrow\downarrow\rangle$$



Configuration 1



Configuration 2

Precise calculation with huge number of nonorthogonal bases

Generalized eigenvalue problem

$$\sum_{j=1}^K (H_{ij} - EB_{ij})C_j = 0 \quad H_{ij} = \langle \Phi_i | H | \Phi_j \rangle \quad B_{ij} = \langle \Phi_i | \Phi_j \rangle$$

New orthonormal set $\phi_\mu = \frac{1}{\sqrt{\mu}} \sum_{i=1}^K c_i^{(\mu)} \Phi_i \quad \rightarrow \quad \sum_{j=1}^N (H_{ij} - EB_{ij})C_j = 0$

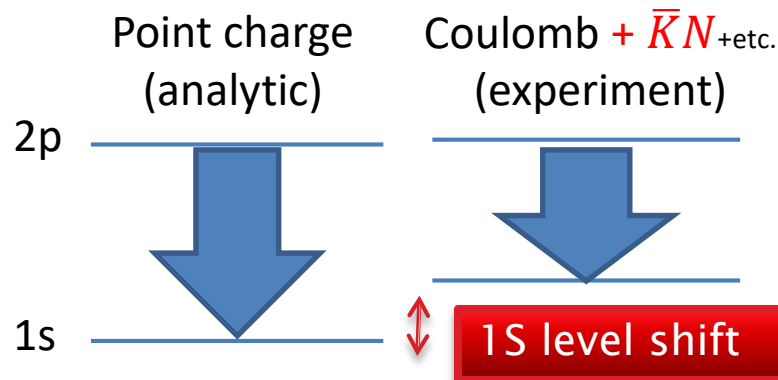
Cutoff parameter $\lambda_{\text{cut}} = \mu_{\text{max}}/\mu_{\text{min}}$

$\log_{10} \lambda_{\text{cut}}$	N	$\text{Re}[E] \text{ (MeV)}$
16	1677	-2.211689436
17	2194	-2.211722964
18	2377	-2.211732072
19	2511	-2.211735493
20	2621	-2.211737242
21	2721	-2.211737609
22	2806	-2.211737677
23	2879	-2.211737682

Energy levels of 1S, 2P, 2S states

	$E_{1S}(\text{keV})$	$E_{2P}(\text{keV})$	$E_{2S}(\text{keV})$
Coulomb	-10.398	-2.602	-2.600
Uniform charge (2-body)	-10.401	-2.602	-2.601
Point charge (2-body)	-10.406	-2.602	-2.602
Coulomb+ $\bar{K}N$	$-9.736 - i 0.508$	$-2.602 - i 0.000$	$-2.517 - i 0.067$

- No level shift of 2P state
- Transition energy can directly be used for the extraction of the 1S level shift



Level shift of kaonic deuterium

$$\Delta E - i\frac{\Gamma}{2} = (670 - i 508) \text{ eV},$$

Sensitivity of $I=1$ component

- ▶ $Re V^{\bar{K}N} = Re V_{I=0} + \beta \times Re V_{I=1}$
- ▶ Varying the strength of the real part of the KN potential **within the SIDDHARTA constraint** for the level shift of kaonic hydrogen
 - ▶ $\Delta E = 283 \pm 36(\text{stat}) \pm 6(\text{syst})\text{eV}$
 - ▶ $\Gamma = 541 \pm 89(\text{stat}) \pm 22(\text{syst})\text{eV}$

Energy shifts of kaonic hydrogen and deuterium

β	$K^- p$		$K^- d$	
	ΔE	Γ	ΔE	Γ
1.08	287	648	676	1020
1.00	283	607	670	1016
-0.17	310	430	506	980

~25% uncertainty
Possible constraint for $I=1$

Conclusions

Unified description of kaonic atom and nuclear systems

– Kaonic nucleus (3- to 7-body)

S. Ohnishi, WH, T. Hoshino, K. Miyahara, T. Hyodo,
Phys. Rev. C95, 065202 (2017)

- Central density is increased by ~ 2 times higher (4 times than ρ_0)
 - Soft NN interaction induces too high central densities
- Inverted spin-parity in the g.s, of ${}^6\text{Li } \bar{K}$ is predicted
 - Isospin dependence of $\bar{K}N$ interaction is essential

– Kaonic atom (3-body)

T. Hoshino, S. Ohnishi, WH, T. Hyodo, W. Weise, arXiv:1705.06857

- Prediction of the energy shift of the kaonic deuterium

$$\Delta E - i\frac{\Gamma}{2} = (670 - i 508) \text{ eV},$$

- $l=1$ component can be constrained if measurement is performed within 25% uncertainty

Planned exp. accuracy $\sim 5\text{-}10\%$

C. Curceanu *et al.*, Nucl. Phys. A914 251 (2013).

M. Iliescu *et al.*, J. Phys. Conf. Ser. 770, no. 1, 012034(2016).

J. Zmeskal *et al.*, Acta Phys. Polon. B 46, no. 1, 101 (2015).