Nuclear multipole responses with the time-dependent correlated Gaussian method

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INTRODUCTION (1/3)

☐ Background

Nuclear multipole responses

Nucleosysnthesis, Nuclear structure, Interactions …

Nuclear unbound states are essential.

Many-body unbound states are difficult to theoretically treated!

☐ Aim of this work

Description of **nuclear responses** w/o explicit **many-body unbound states**

INTRODUCTION (2/3)

☐ Strategy

Time-dependent method T. Kido *et. al*, Phys.Rev. C 53, 5 (1996)

- **No** explicit unbound states are needed.
- Nuclear responses are obtained from **time-propagated wave functions**.
- **Unified approach** to bound and unbound states

↑ Wave function

Correlated Gaussian (CG) basis

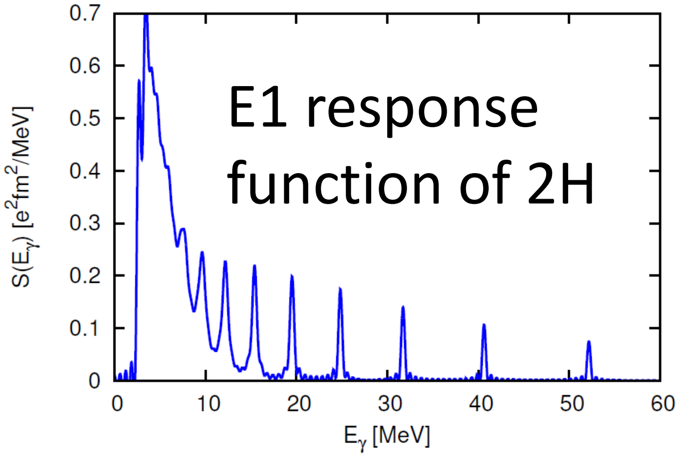
K. Varga, and Y. Suzuki, Phys. Rev. C 52, 2885 (1995)

- Many-body correlations are explicitly included.
- Application to many-body systems

INTRODUCTION (3/3)

Problem: Artificial reflection wave

Some artificial reflection waves appear in the model space boundaries.



New basis function

Time-dependent Correlated Gaussian basis

Reflection-free time propagation using explicitly time-dependent basis function K. Varga, Phys. Rev. E 85, 016705 (2012)

➡ **This idea is extended to many-body problem**

Outline

- Introduction
- **Formulation**
  - Time-dependent method
  - Correlated Gaussian method
  - Time-propagation with time-dependent basis
  - Time-dependent Correlated Gaussian
- **Results**
  - Simple tests by interaction-free time-propagation of 2- and 3- nucleon systems
- **Summary**

FORMURATION (1/6)

□ Time-dependent method

T. Kido *et. al*, Phys.Rev. C 53, 5 (1996)

Response function

$$S(E_\gamma) \equiv \sum_\mu \sum\!\!\!\!\!\!\int_\nu \left| \langle \phi_\nu | \hat{M}_{\lambda\mu} | \phi_0 \rangle \right|^2 \delta \left(E_\nu - E_0 - E_\gamma \right)$$

$\hat{M}$: Multipole operator

Unbound states ϕ_ν are needed.

Time-dependent form

$$S(E_\gamma) = \frac{1}{\pi\hbar} \sum_\mu \mathrm{Re} \left[\int_0^\infty dt \; \mathrm{e}^{i(E_\gamma + E_0 + i\varepsilon)t/\hbar} \underbrace{\langle \phi_0 | \hat{M}^\dagger_{\lambda\mu} \; \mathrm{e}^{i\hat{H}t/\hbar} \; \hat{M}_{\lambda\mu} | \phi_0 \rangle}_{\langle \psi(0) | \psi(t) \rangle} \right]$$

$$|\psi(t{=}0)\rangle \equiv \hat{M}_{\lambda\mu} | \phi_0 \rangle$$

NO explicit continuum states are needed!

①Calculation of the ground state wave function
②Time-propagation of the initial state
③Fourier transformation of $\langle \psi(0) | \psi(t) \rangle$

FORMURATION (2/6)

☐ Correlated Gaussian (CG) method

Basis function

K. Varga, and Y. Suzuki, Phys. Rev. C 52, 2885 (1995)

Basis expansion $\psi = \sum_{j=1}^{N} c_j \varphi_j$

$$\varphi_j = \hat{\mathcal{A}} \left\{ \exp\left(-\frac{1}{2} \tilde{\boldsymbol{x}} A_j \boldsymbol{x} \right) \mathcal{Y}_{LM} \left(\tilde{u}_j \boldsymbol{x} \right) \chi_j \eta \right\}$$

$$\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{A-1})$$

Jacobi coordinate

$$\boldsymbol{x}_i = (\boldsymbol{r}_1 + \cdots + \boldsymbol{r}_i)/i - \boldsymbol{r}_{i+1}$$

A_j : $(A-1)$-dimensional positive-definite Matrix

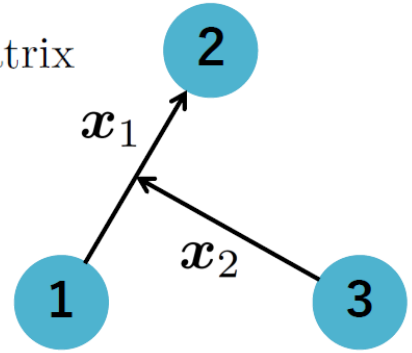
u_j : $(A-1)$-dimensional vector

χ_j : Spin wave function

η : Isospin wave function

$\mathcal{Y}_{LM}(\boldsymbol{r})$: Solidsphericalharmonics

$\mathcal{Y}_{LM} = |\boldsymbol{r}|^L Y_{LM}(\hat{\boldsymbol{r}})$



$$\tilde{\boldsymbol{x}} A \boldsymbol{x} = \sum_{j,k=1}^{A-1} A_{jk} \boldsymbol{x}_j \cdot \boldsymbol{x}_k$$

Non-diagonal components of A_j describe many-body correlations

FORMURATION (3/6)

☐ Time-propagation with the time-dependent basis function

K. Varga, Phys. Rev. E 85, 016705 (2012)

Basis expansion $\psi(t) = \sum_{j=1}^{N} c_j(t)\varphi_j(t)$

Time-dependent Schrödinger eq. (TDSE)

T : Kintetic term
V : Interaction term

$$\sum_{j=1}^{N}\left(i\hbar\frac{\partial c_j}{\partial t} - c_j V\right)\varphi_j(t) + \sum_{j=1}^{N} c_j(t)\left(i\hbar\frac{\partial \varphi_j}{\partial t} - T\varphi_j\right) = 0$$

Each basis is taken to satisfy **TDSE of free particles**

$$i\hbar\frac{\partial \varphi_j}{\partial t} - T\varphi_j = 0$$

$$\sum_{j=1}^{N}\left(i\hbar\frac{\partial c_j}{\partial t} - c_j V\right)\varphi_j(t) = 0$$

$\varphi_j(t)$ are propagated by **Kinetic** term.

$c_j(t)$ are governed by **Interaction**.

FORMURATION (4/6)

□ Time-dependent correlated Gaussian

Time-dependent basis for many-body problems

Generating function of CG

All matrix elements $\langle \varphi'_{L'M'}|\hat{O}|\varphi_{LM}\rangle$ are derived from $\langle g'|\hat{O}|g\rangle$.

$$g(A, \boldsymbol{s}, \boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x} + \tilde{\boldsymbol{s}}\boldsymbol{x}\right)$$

$$g(A, \boldsymbol{s}, \boldsymbol{x}, t) = f(t)\exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A(t)\boldsymbol{x} + \tilde{\boldsymbol{s}}(t)\boldsymbol{x}\right)$$

Time-dependencies of $f(t)$, $A(t)$, $\boldsymbol{s}(t)$ are dominated by

$$i\hbar\frac{\partial g(A, \boldsymbol{s}, \boldsymbol{x}, t)}{\partial t} - Tg(A, \boldsymbol{s}, \boldsymbol{x}, t) = 0$$

FORMURATION (5/6)

Kinetic term

$$T = \sum_{j=1}^{A} \frac{\hat{\boldsymbol{p}}_j^2}{2m} - T_{\mathrm{CM}} = \frac{1}{2}\tilde{\boldsymbol{\pi}}\Lambda\boldsymbol{\pi}$$

$$\tilde{\boldsymbol{\pi}} = (\boldsymbol{\pi}_1, \cdots, \boldsymbol{\pi}_{A-1})$$

$$\boldsymbol{\pi}_j = -i\hbar\frac{\partial}{\partial \boldsymbol{x}_j}$$

$\boldsymbol{x}_j$: Jacobi coordinates

T_{CM} : Kintetic energy of the center-of-mass

Λ : (A − 1)-dimensional matrix, Reduced mass

$$i\hbar\frac{\partial g(A, \boldsymbol{s}, \boldsymbol{x}, t)}{\partial t} - Tg(A, \boldsymbol{s}, \boldsymbol{x}, t) = 0$$

$$\Rightarrow \begin{cases} \frac{\partial}{\partial t}A(t) = -i\hbar A(t)\Lambda A(t) \\ \frac{\partial}{\partial t}\boldsymbol{s}(t) = -i\hbar A(t)\Lambda \boldsymbol{s}(t) \\ \frac{\partial}{\partial t}f(t) = -\frac{i\hbar}{2}\left(3\mathrm{Tr}(\Lambda A(t)) - \tilde{\boldsymbol{s}}(t)\Lambda\boldsymbol{s}(t)\right)f(t) \end{cases}$$

$f(t),\ A(t),\ \boldsymbol{s}(t)$ are analytically solved.

Matrix elements of TDCG are analytically obtained for N-body system!

FORMURATION (6/6)

☐ Summary of the formulation

Time-dependent method

Response functions are obtained from $\langle\psi(0)|\psi(t)\rangle$.

Time-dependent correlated Gaussian

Reflection-free time-propagation of N-body systems

Matrix elements are evaluated analytically.

- Description of **nuclear responses** w/o explicit **many-body continuum states**
- **Straightforwardly application** to arbitrary multipole and number of particles

RESULTS (1/4)

☐ Simple tests for TDCG

TDCG is a solution of interaction-free Hamiltonian.

Interaction-free time-propagation of ^{2}H and ^{3}He

☐ Initial state

Ground states ϕ_0 are obtained with Minnesota potential.

D.R. Thompson *et. al.*, Nucl. Phys. A286 (1977) 53-66

$$\psi(0) = \hat{D}_{10}\phi_0$$

$$\hat{D}_{10} = \sum_{p=1}^{A} \frac{1+\tau_{zp}}{2} e\sqrt{\frac{4\pi}{3}}\mathcal{Y}_{10}\left(\boldsymbol{r}_p - \boldsymbol{r}_{CM}\right)$$

τ_{zp}: z-component of isospin of p-th particle

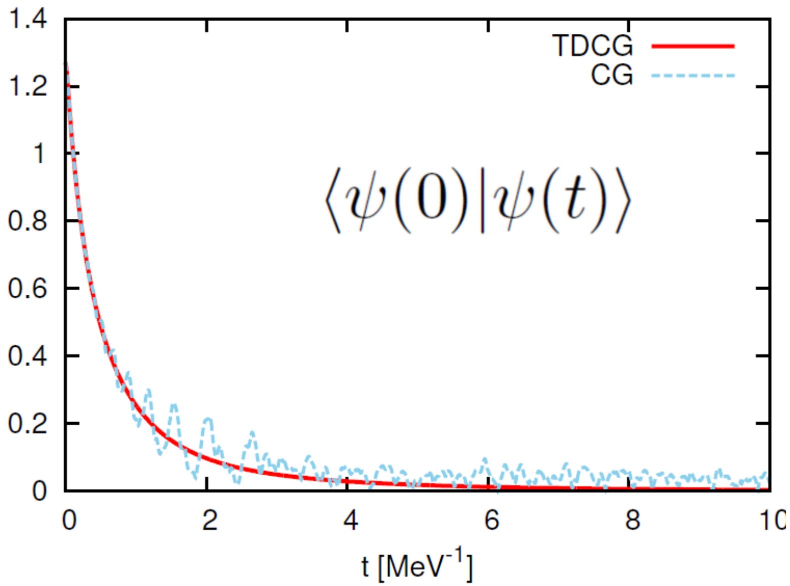
Initial states are propagated with interaction-free Hamiltonian.

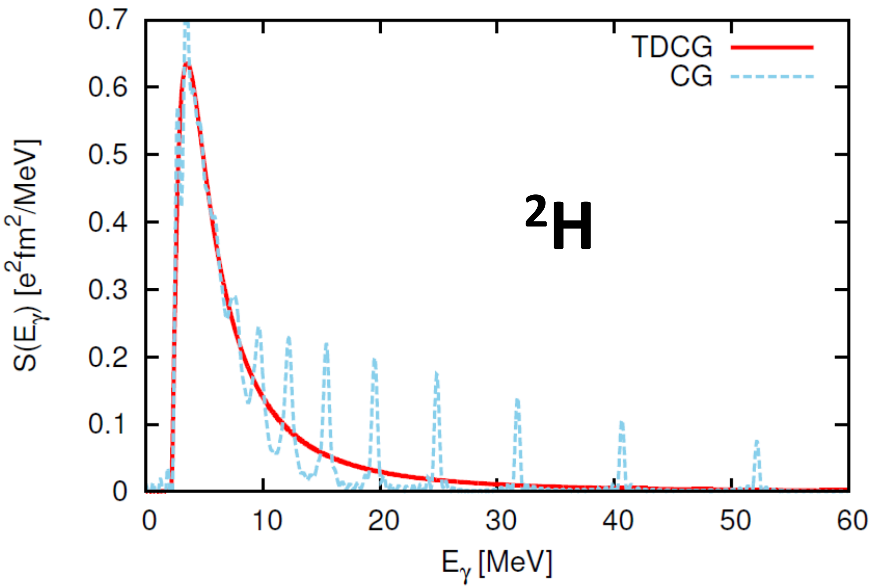
RESULTS (2/4)

□ Correlated Gauss v.s. Time-dependent CG

Problem: Artificial reflection wave

Some artificial reflection waves appear in the model space boundaries. → **RESOLVED!!**





Time-dependent correlated Gaussian provides reflection-free time-propagation.

RESULTS (3/4)

□ TDCG vs. Complex Scaling method

TDCG method is checked by a **comparison with a established method**, Complex scaling method.

□ Complex Scaling method

One of the method avoiding explicit construction of continuum states

N. Moiseyev, Phys. Rep. 302, 211-293 (1998)
S. Aoyama, *et .al.*, Prog. Theor. Phys. 116, 1 (2006)

$$U(\theta): \quad \boldsymbol{r}_j \to \boldsymbol{r}_j e^{i\theta}, \quad \boldsymbol{p}_j \to \boldsymbol{p}_j e^{-i\theta}$$

$$U(\theta)\hat{H}U^{-1}(\theta)\psi_\nu(\theta) = E_\nu \psi_\nu(\theta)$$

Unbound state are obtained as complex energy states.

Response function $$S(E_\gamma) = -\frac{1}{\pi}\sum_{\mu\nu} \mathrm{Im}\left[\frac{\langle \phi_\nu^*(\theta)|\hat{M}_{\lambda\mu}|\phi_0(\theta)\rangle^2}{E_\gamma - E_\nu(\theta) + i\varepsilon}\right]$$

RESULTS (4/4)

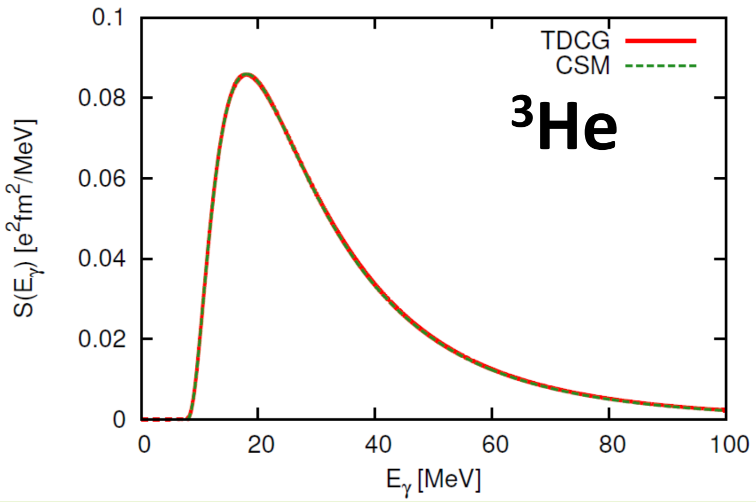
□ TDCG vs. Complex Scaling method

In CSM calculation,

- The wave functions are expanded by CG basis.
- Excite states are obtained with interaction-free H.

Must be consistent

Time-dependent method + interaction-free time-propagation with TDCG



Precise interaction-free time-propagations are obtained in ^{2}H and ^{3}He case.

Summary & Future work

Summary

- **Time-dependent method and basis expansion** are combined to investigate nuclear multipole responses.
- **Time-dependent correlated Gaussian (TDCG) basis** is developed for **application to many-body systems.**
- Matrix elements of TDCG are **analytically** obtained.
- TDCG provides **reflection-free time-propagation.**
- **Precision of interaction-free time-propagation is proved** by a comparison with Complex scaling method.

Future work

- More realistic case
  (ex. Final-state interaction is included)