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# Nucleon-nucleon interaction in covariant chiral effective field theory


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# OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary and perspectives

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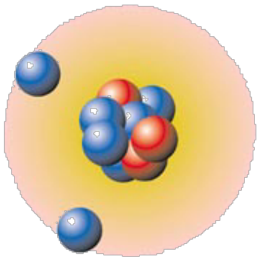
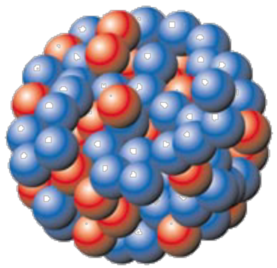
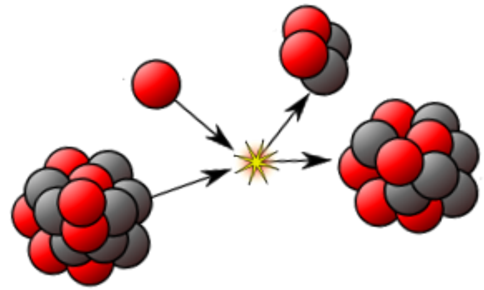
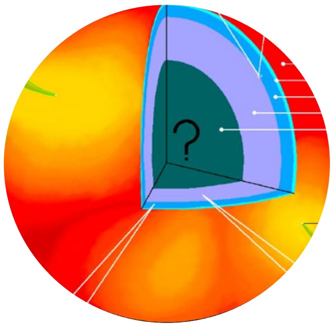
# Basic for all nuclear physics

## Precise understanding of the nuclear force



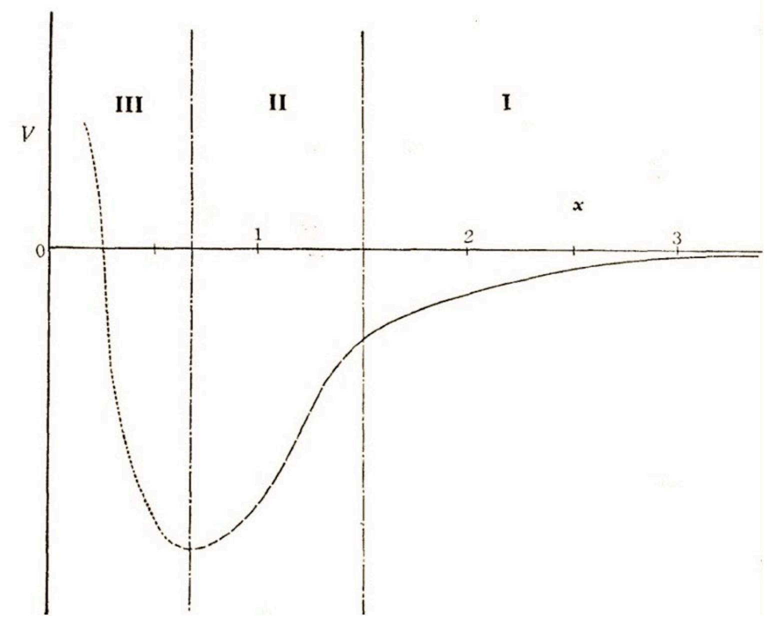







## Complexity of the nuclear force (vs. electromagnetic force)

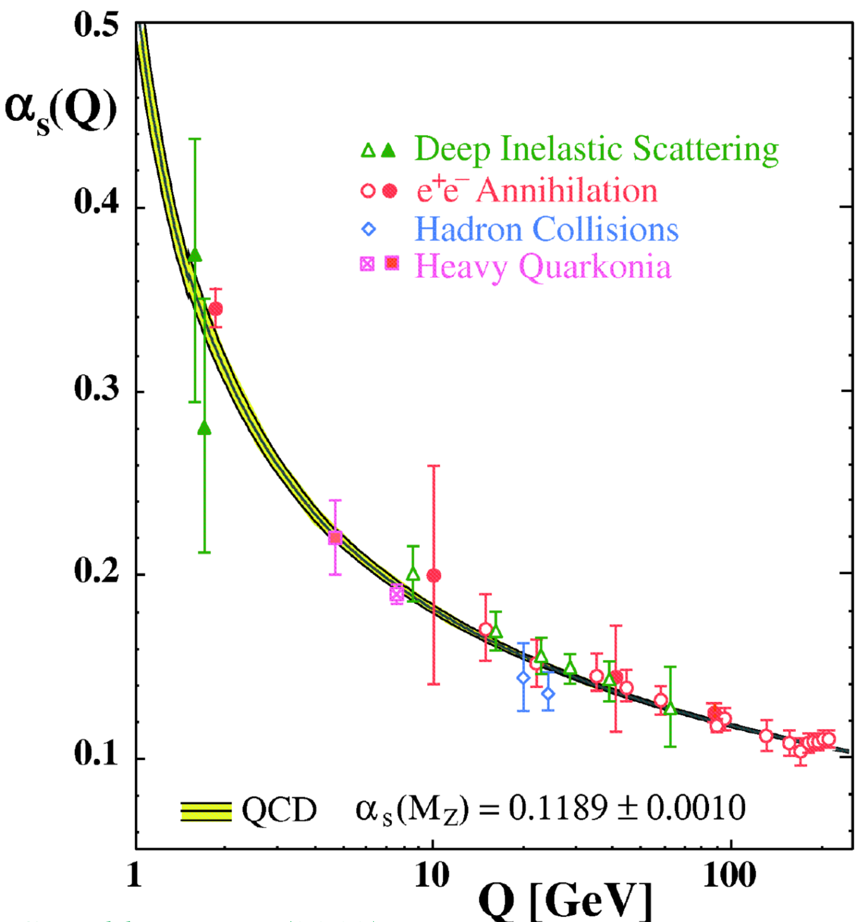
- Finite range
- Intermediate-range **attraction**
- Short-range **repulsion**-“hard core”
- Spin-dependent **non-central** force
  - Tensor interaction
  - Spin-orbit interaction
- Charge independent (approximate)



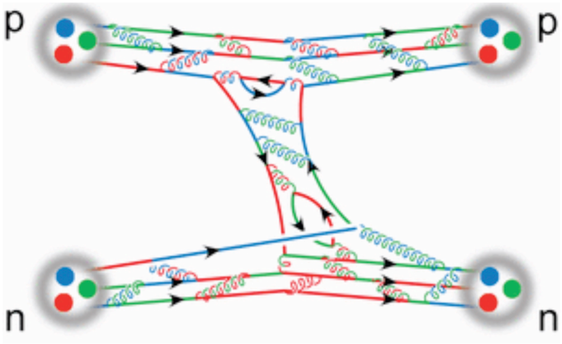

*M. Taketani, Suppl.PTP3(1956)1*

# Nuclear force (NF) from QCD

- **Residual quark-gluon strong interaction**
- **Understood from QCD**




*S.Bethke, PPNP(2013)*




At low-energy region

- Running coupling constant $\alpha_s \geq 1$
- **Nonperturbative QCD -- unsolvable**

⇨ {
**Phenomenological models**
**Lattice QCD simulation**
**Chiral effective field theory**

# NF from Chiral EFT

- Chiral effective field theory *S. Weinberg,Phys.A1979*
  - Effective field theory (EFT) of **low-energy QCD**
  - **Model independent** to study the nuclear force *S. Weinberg,PLB1990*
- Main advantages of chiral nuclear force
  - **Self-consistently include** many-body forces

$$V = V_{2N} + V_{3N} + \cdots + V_{iN} + \cdots$$

  - **Systematically improve** NF order by order

$$V_{iN} = V_{iN}^{\mathrm{LO}} + V_{iN}^{\mathrm{NLO}} + V_{iN}^{\mathrm{NNLO}} + \cdots$$

  - **Systematically estimate** theoretical uncertainties

$$|V_{iN}^{\mathrm{LO}}| > |V_{iN}^{\mathrm{NLO}}| > |V_{iN}^{\mathrm{NNLO}}| > \cdots$$

# Current status of chiral NF

## Nonrelativistic (NR) chiral NF

- NN interaction
  - up to NLO *U. van Kolck et al., PRL, PRC1992-94; N. Kaiser, NPA1997*
  - up to NNLO *U. van Kolck et al.,PRC1994; E. Epelbaum, et al.,NPA2000*
  - up to **N$^3$LO** *R. Machleidt et al., PRC2003; E. Epelbaum et al., NPA2005*
  - up to **N$^4$LO** *E. Epelbaum et al., PRL2015, D.R. Entem, et al., PRC2015*
  - up to **N$^5$LO** (dominant terms) *D.R. Entem, et al., PRC2015*
- 3N interaction
  - up to NNLO *U. van Kolck, PRC1994*
  - up to N$^3$LO *S. Ishikwas, et al, PRC2007; V. Bernard et al, PRC2007*
  - up to **N$^4$LO** *H. Krebs, et al., PRC2012-13*
- 4N interaction
  - up to N$^3$LO *E. Epelbaum, PLB 2006, EPJA 2007*

***P. F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339***
***E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773***
***R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1***

# Chiral NN potential is of high precision

| | Phenomenological forces | | | NR Chiral nuclear force | | | | |
|---|---|---|---|---|---|---|---|---|
| | **Reid93** | **AV18** | **CD-Bonn** | LO | NLO | NNLO | **N$^3$LO** | **N$^4$LO** |
| No. of para. | **50** | **40** | **38** | 2+2 | 9+2 | 9+2 | **24+2** | **24+3** |
| $\chi^2$/datum *np data 0-290 MeV* | **1.03** | **1.04** | **1.02** | 94 | 36.7 | 5.28 | **1.23**/ **1.27** | **1.14**/ **1.10** |

*P.Reinert's talk, Bochum-Juelich (2017)*

*D.Entem, et al., PRC96(2017)024004*

**Chiral force has been extensively applied in the studies of nuclear structure and reactions within the non-relativistic few-/many-body theories.** **e.g. K. Sekiguchi, U.-G. Meißner's talks**

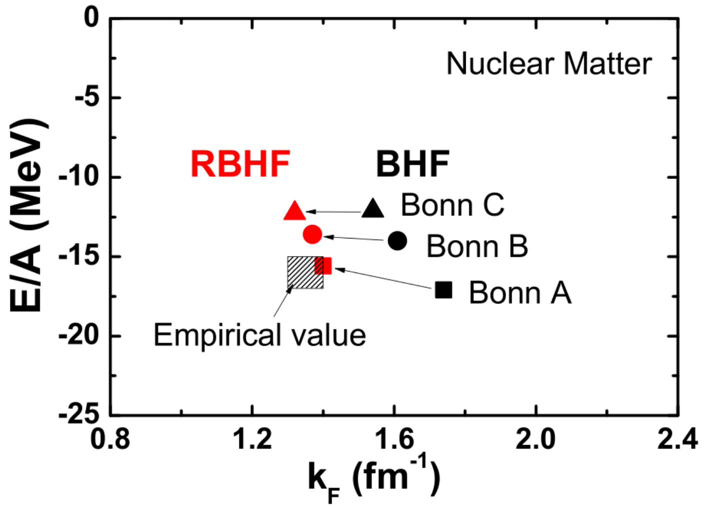
*E. Epelbaum, et al., PRL 106(2011) 192501, PRL109(2012)252501, PRL112(2014)102501; S. Elhatisari, et al., Nature 528 (2015) 111, arXiv:1702.05177; G. Hagen, et al., PRL109(2012)032502; H. Hergert, et al., PRL110(2013)24501; G.R. Jansen, et al., PRL113(2014)102501; S.K.Bogner, et al., PRL113(2014)142501; J.E. Lynn, et al., PRL113(2014)192501; V. Lapoux, et al., PRL117(2016)052501……*

# Motivation for the relativistic chiral force

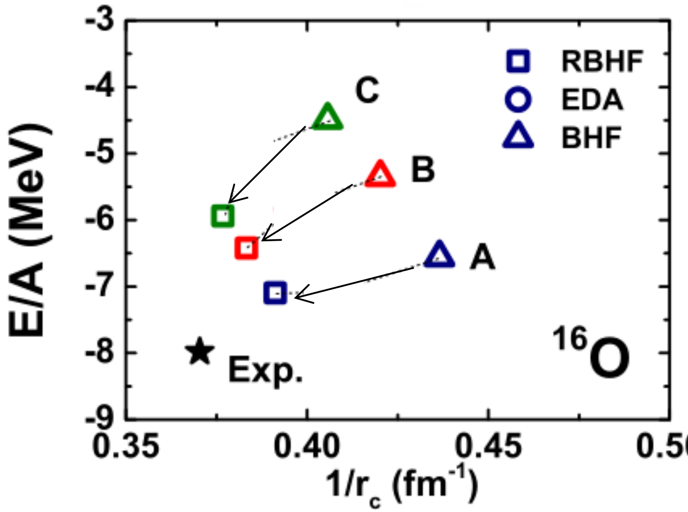
- The success of **covariant density functional theory (CDFT)** in the nuclear structure studies.

*P. Ring, PPNP (1996), D.Vretenar et al.,Phys.Rept.(2005), J. Meng, IRNP(2016)*

- Relativistic Brueckner-Hartree-Fock theory in nuclear matter and finite nuclei **(input: relativistic Bonn)**




*R. Brockmann & R. Machleidt, PRC(1990)*




*S.H. Shen, et al., CPL(2016), PRC(2017)*

**Relativistic nuclear force based on ChEFT is needed**

# In this work

**We extend covariant ChEFT to the nucleon-nucleon sector and construct a relativistic nuclear force up to leading order**

- Construct the interaction kernel in **covariant power counting**
  - Employ the Lorentz invariant chiral Lagrangains
  - Retain the complete form of Dirac spinor

$$u(\vec{p}, s) = N_p \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{\epsilon_p} \end{pmatrix} \chi_s, \quad N_p = \sqrt{\frac{\epsilon_p}{2M_N}}. \quad \begin{array}{l} E_p = \sqrt{M_N^2 + \vec{p}^2} \\ \epsilon_p = E_p + M_N \end{array}$$

  - Use naïve dimensional analysis to determine the chiral dimension

- Employ the 3D-reduced **Bethe-Salpeter** equation, such as **Kadyshevsky** equation, to re-sum the potential.

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  - NN potential concept
  - Relativistic chiral force up to LO
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# NN potential concept

- Often-thought as a nonrelativistic quantity
  - Appear in the **Schrödinger** equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(t,\boldsymbol{r}) + V(\boldsymbol{r})\Psi(t,\boldsymbol{r}) = i\hbar\frac{\partial}{\partial t}\Psi(t,\boldsymbol{r}).$$

  - (or) Appear in the **Lippmann-Schwinger** equation

$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int\frac{d\boldsymbol{k}}{(2\pi)^3}V(\boldsymbol{p}',\boldsymbol{k})\frac{m_N}{\boldsymbol{p}^2-\boldsymbol{k}^2+i\epsilon}T(\boldsymbol{k},\boldsymbol{p}).$$

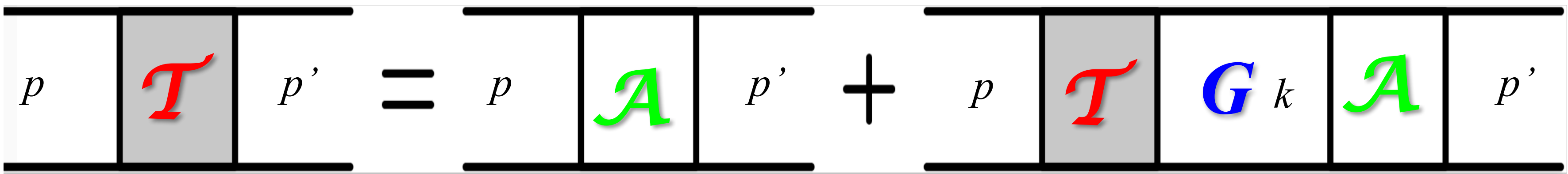
- Generalize the definition of NN potential
  - An interaction quantity appearing in a three-dimensional scattering equation can be referred as a **NN potential**.

⇨ **Relativistic potential**

*M.H. Partovi, E.L. Lomon, PRD2 (1970) 1999*
*K. Erkelenz, Phys.Rept. 13C(1974) 191*

# Bethe-Salpeter equation

- For the relativistic nucleon-nucleon scattering




$W = (\sqrt{s}/2, \mathbf{0})$

**Bethe-Salpeter equation with an operator form:**

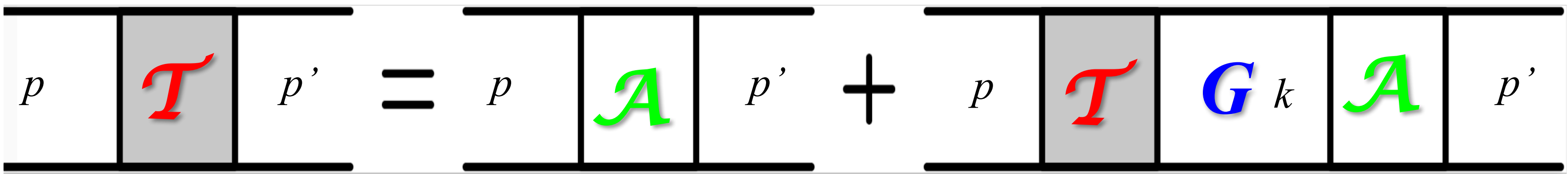
$$\mathcal{T}(p', p|W) = \mathcal{A}(p', p|W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p', p|W) G(k|W) T(k, p|W),$$

- $\mathcal{T}$: Invariant scattering amplitude
- $\mathcal{A}$: Interaction kernel (**sum all the irreducible diagrams**)
- G: Two-nucleon's Green function

$$G(k|W) = i \frac{1}{[\gamma^\mu (W+k)_\mu - m_N + i\epsilon]^{(1)} \, [\gamma^\mu (W-k)_\mu - m_N + i\epsilon]^{(2)}},$$

# Bethe-Salpeter equation

- For the relativistic nucleon-nucleon scattering

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**It is hard to solve the BS equation, one always perform the 3-dimensional reduction.**

# Reduction of BS equation

- Introduce a three dimensional Green function ***g***
  - Maintain the same elastic unitarity of ***G*** at physical region
  - We choose the Kadyshevsky propagator *V. Kadyshevsky, NPB (1968).*

$$\boxed{g = 2\pi \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\boldsymbol{k})\Lambda_+^{(2)}(-\boldsymbol{k})}{\sqrt{s} - 2E_k + i\epsilon} \delta[k_0 - (E_k - \frac{\sqrt{s}}{2})].}$$

- To replace ***G*** with ***g***, one can introduce the effective interaction kernel $\mathcal{V}$

$$\mathcal{T} = \mathcal{A} + \mathcal{A} G \mathcal{T}. \quad \left\{ \begin{array}{l} \mathcal{T} = \mathcal{V} + \mathcal{V}\, g\, \mathcal{T}. \\ \mathcal{V} = \mathcal{A} + \mathcal{A}\, (G - g)\, \mathcal{V}. \end{array} \right.$$

# Reduction of BS equation

- BS equation reduces to the Kadyshevsky equation:

$$
\begin{aligned}
\mathcal{T} &= \mathcal{V} + \mathcal{V}\, g\, \mathcal{T} \\
&= \mathcal{V} + \int \frac{d\boldsymbol{k}}{(2\pi)^3} \int \frac{dk_0}{2\pi}\, \mathcal{V} \times 2\pi \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\boldsymbol{k})\Lambda_+^{(2)}(-\boldsymbol{k})}{\sqrt{s} - 2E_k + i\epsilon} \delta[k_0 - (E_k - \frac{\sqrt{s}}{2})] \times \mathcal{T} \\
&= \mathcal{V} + \int \frac{d\boldsymbol{k}}{(2\pi)^3}\, \mathcal{V}\, \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\boldsymbol{k})\Lambda_+^{(2)}(-\boldsymbol{k})}{\sqrt{s} - 2E_k + i\epsilon}\, \mathcal{T}, \quad \text{with } k_0 = E_k - \frac{\sqrt{s}}{2}.
\end{aligned}
$$

- Sandwiched by Dirac spinors:

$$
\boxed{T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3}\, V(\boldsymbol{p}', \boldsymbol{k})\, \frac{m_N^2}{2E_k^2} \frac{1}{E_p - E_k + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p}),}
$$

*V. Kadyshevsky, NPB (1968).*

- Relativistic potential definition:

$$
\begin{aligned}
V(\boldsymbol{p}', \boldsymbol{p}) \;=\; & \bar{u}(\boldsymbol{p}', s_1)\bar{u}(-\boldsymbol{p}', s_2) \times \\
& \mathcal{V}(p_0' = E_{p'} - \sqrt{s}/2, \boldsymbol{p}'; p_0 = E_p - \sqrt{s}/2, \boldsymbol{p}|W) \;\times u(\boldsymbol{p}, s_1)u(\boldsymbol{p}', s_2).
\end{aligned}
$$

# Calculate potential in ChEFT

- To obtain the potential

$$V(\boldsymbol{p}', \boldsymbol{p}) = \bar{u}_1 \bar{u}_2\, \mathcal{V}(p, p')\, u_1 u_2.$$

- Solve the iterated equation perturbatively

$$\boxed{\mathcal{V} = \mathcal{A} + \mathcal{A}(G - g)\mathcal{V}.}$$

$$\begin{aligned}\mathcal{V}^{(2)} &= \mathcal{A}^{(2)},\\ \mathcal{V}^{(4)} &= \mathcal{A}^{(4)} + \mathcal{A}^{(2)}(G-g)\mathcal{A}^{(2)},\\ \ldots. &\quad \ldots.\end{aligned}$$

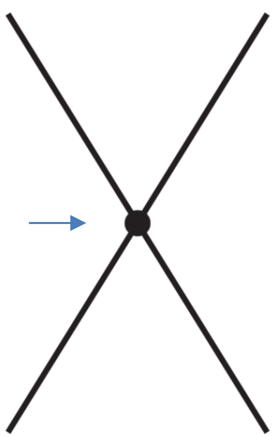
- Interaction kernel, $\mathcal{A}$, can be calculated by using covariant chiral perturbation theory order by order.

# Relativistic chiral NF up to LO

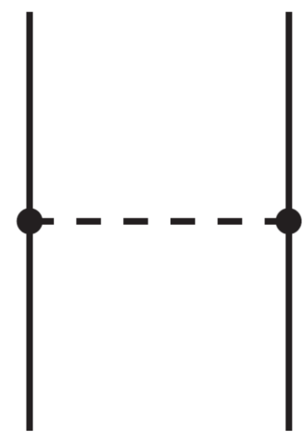
$$V_{2N}^{\mathrm{LO}} = \bar{u}_1 \bar{u}_2 \, \mathcal{A}^{\mathrm{LO}} \, u_1 u_2$$

$$= \bar{u}_1 \bar{u}_2 \, \left( \mathcal{A}_{\mathrm{CTP}} + \mathcal{A}_{\mathrm{OPEP}} \right) \, u_1 u_2 .$$

$\mathcal{L}_{NN}^{(0)}$



$\mathcal{L}_{\pi N}^{(1)}$



# Covariant chiral Lagrangians

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)}.$$

- Pion-pion interaction:

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + (U+U^\dagger) m_\pi^2 \rangle.$$

$$U = 1 + i\frac{\Phi}{f_\pi} - \ldots$$
$$\Phi = \tau_\sigma \pi^\sigma$$
$$f_\pi = 92.4 \text{ MeV}$$

- Pion-nucleon interaction:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}(i\not{\partial} - M_N)\Psi + \frac{g_A}{2}\bar{\Psi}\gamma^\mu\gamma^5 u_\mu \Psi.$$

$$u_\mu = -\frac{1}{f_\pi}\partial_\mu \Phi + \ldots$$
$$\Psi = (p, n)^\dagger$$
$$g_A = 1.26$$

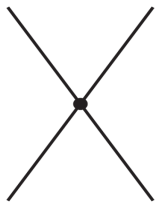
- Nucleon-nucleon interaction: *H.Polinder, et al.,NPA(2006)*

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2}\left[ C_S(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + C_A(\bar{\Psi}\gamma_5\Psi)(\bar{\Psi}\gamma_5\Psi) + C_V(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) + C_{AV}(\bar{\Psi}\gamma_5\gamma_\mu\Psi)(\bar{\Psi}\gamma_5\gamma^\mu\Psi) + C_T(\bar{\Psi}\sigma_{\mu\nu}\Psi)(\bar{\Psi}\sigma^{\mu\nu}\Psi). \right]$$

5 unknown low-energy constants (LECs)

# Relativistic chiral potential at LO

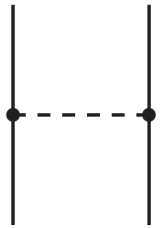
- Contact potential (momentum space):



$$\begin{aligned} V_{\mathrm{CTP}} &= C_S(\bar{u}_2 u_2)(\bar{u}_1 u_1) + C_A(\bar{u}_2 \gamma_5 u_2)(\bar{u}_1 \gamma_5 u_1) \\ &\quad + C_V(\bar{u}_2 \gamma_\mu u_2)(\bar{u}_1 \gamma^\mu u_1) + C_{AV}(\bar{u}_2 \gamma_\mu \gamma_5 u_2)(\bar{u}_1 \gamma^\mu \gamma_5 u_1) \\ &\quad + C_T(\bar{u}_2 \sigma_{\mu\nu} u_2)(\bar{u}_1 \sigma_{\mu\nu} u_1). \end{aligned}$$

- One-pion-exchange potential (momentum space):

$$V_{\mathrm{OPEP}} = -\frac{g_A^2}{4 f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\bar{u}_1 \gamma^\mu \gamma_5 q_\mu u_1)(\bar{u}_2 \gamma^\nu \gamma_5 q_\nu u_2)}{(E_{p'} - E_p)^2 - \boldsymbol{q}^2 - m_\pi^2}.$$



Retardation effect is included

- In the static limit ($m_N \rightarrow$ infinity), the NR results can be recovered

$$V^{\mathrm{NonRel.}} = \underbrace{\boxed{(C_S + C_V)}}_{C_S^{\mathrm{HB}}} - \underbrace{\boxed{(C_{AV} - 2C_T)}}_{C_T^{\mathrm{HB}}} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{4 f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q} \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{\boldsymbol{q} + m_\pi^2 + i\epsilon} + \mathcal{O}(\frac{1}{M_N}).$$

*S. Weinberg, PLB1990*

# Scattering equation and Phase shifts

- Perform the partial wave projection, one can obtain the Kadyshevesky equation in |LSJ> basis

$$T^{SJ}_{L',L}(\boldsymbol{p}',\boldsymbol{p}) = V^{SJ}_{L',L}(\boldsymbol{p}',\boldsymbol{p}) + \sum_{L''}\int_0^{+\infty}\frac{\boldsymbol{k}^2 dk}{(2\pi)^3} V^{SJ}_{L',L}(\boldsymbol{p}',\boldsymbol{k})\frac{M_N^2}{2(\boldsymbol{k}^2+M_N^2)}\frac{1}{\sqrt{\boldsymbol{p}^2+M_N^2}-\sqrt{\boldsymbol{k}^2+M_N^2}+i\epsilon}T^{SJ}_{L'',L}(\boldsymbol{k},\boldsymbol{p}).$$

*V. Kadyshevsky, NPB (1968).*

- Cutoff renormalization for scattering equation
  - Potential regularized by an **exponential regulator function**

$$V(\boldsymbol{p}',\boldsymbol{p}) \to V(\boldsymbol{p}',\boldsymbol{p})\exp[-(|\boldsymbol{p}'|/\Lambda)^{2n}-(|\boldsymbol{p}|/\Lambda)^{2n}]. \quad n=2$$

*E.Epelbaum et al., NPA(2000)*

- On-shell $\boldsymbol{S}$ matrix and phase shift $\boldsymbol{\delta}$

$$S^{SJ}_{L'L} = \delta_{L'L} - \frac{i}{8\pi^2}\frac{M_N^2|\boldsymbol{p}|}{E_p}T^{SJ}_{L'L}. \qquad S=\exp(2i\boldsymbol{\delta})$$

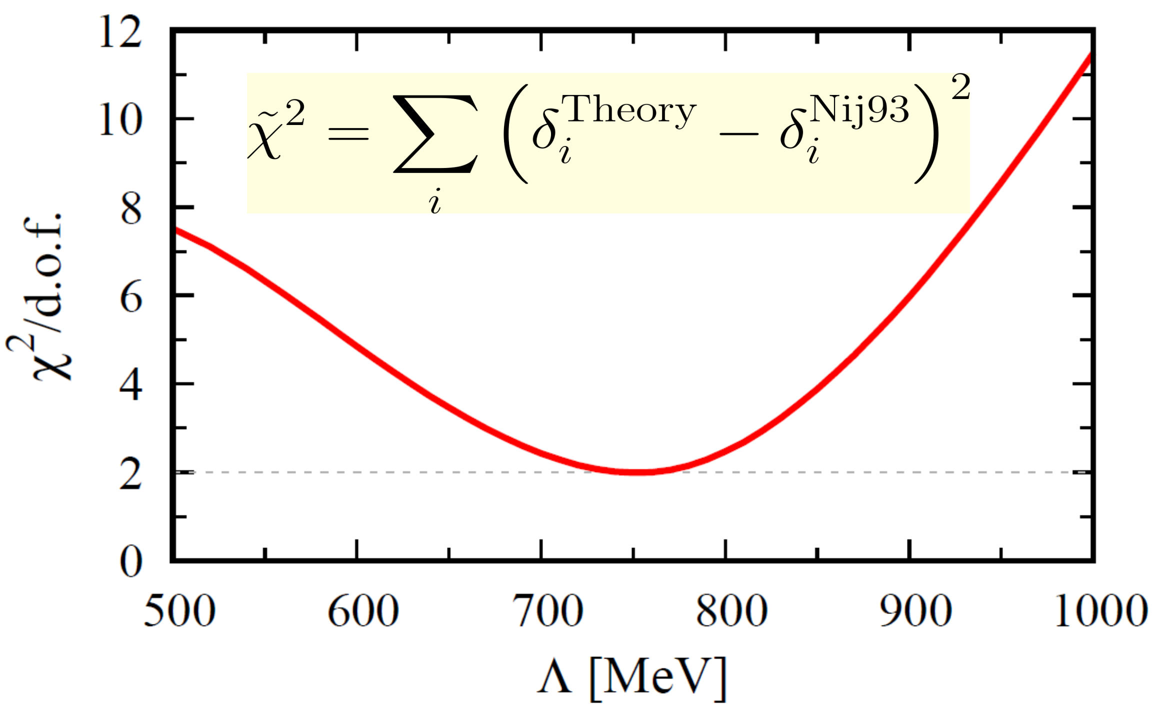
**Couple channel: Stapp parameterization**

# Numerical details

- 5 LECs $C_{S,A,V,AV,T}$ are determined by fitting
  - **NPWA**: ***p-n*** scattering phase shifts of Nijmegen 93

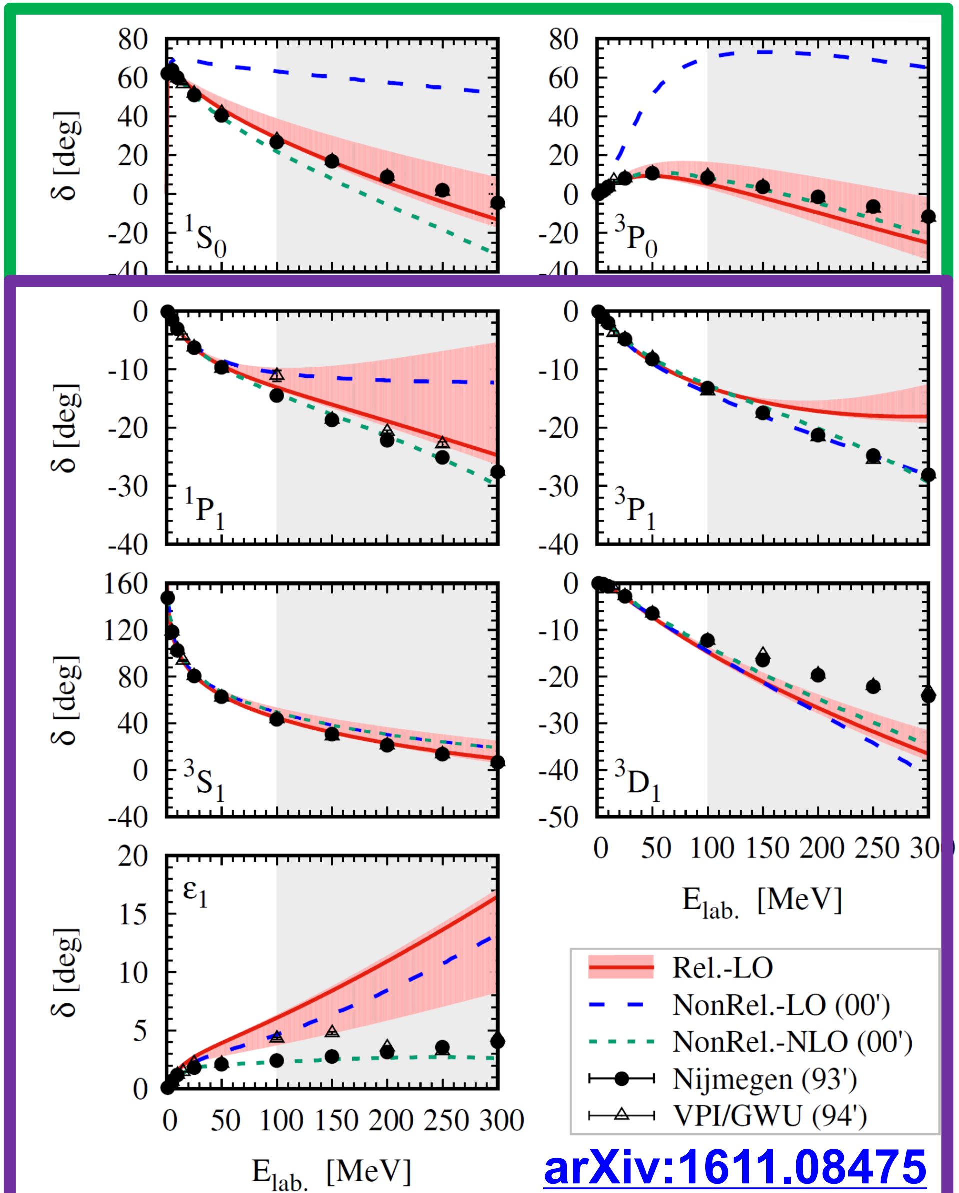
    *V. Stoks et al., PRC48(1993)792*
  - **7** partial waves: ***J=0, 1*** $^1S_0,\ ^3P_0,\ ^1P_1,\ ^3P_1,\ ^3D_1,\ ^3S_1,\ \epsilon_1$
  - **42** data points: 6 data points for each partial wave

    $(E_{\rm lab} = 1,\ 5,\ 10,\ 25,\ 50,\ 100\ \text{MeV})$




| LECs | Values [$10^4$ GeV$^{-2}$] |
|---|---|
| $C_S$ | -0.125 |
| $C_A$ | 0.040 |
| $C_V$ | 0.248 |
| $C_{AV}$ | 0.221 |
| $C_T$ | 0.059 |

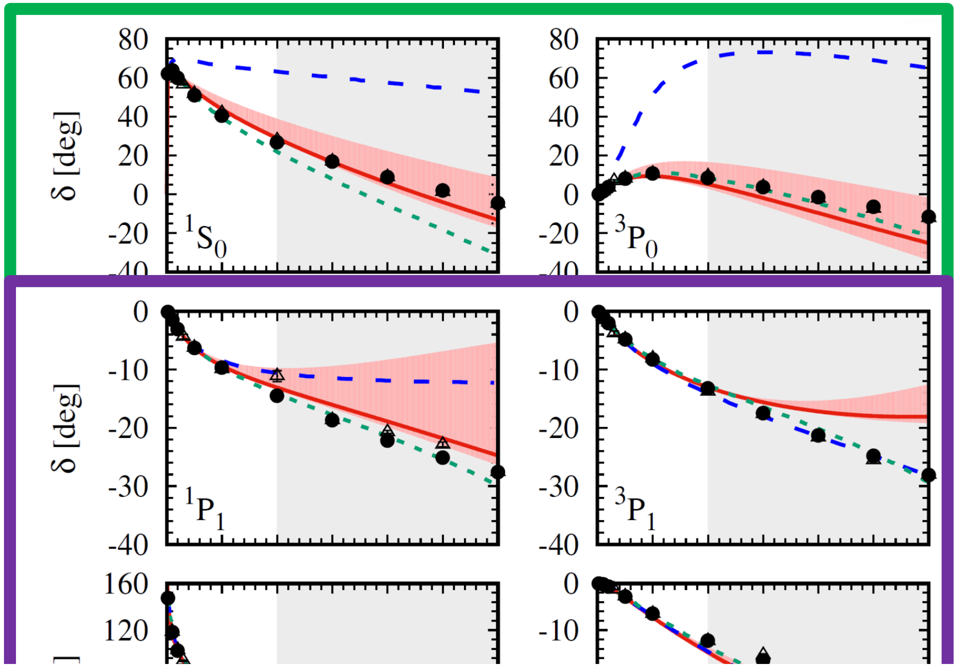
# Description of J=0, 1 partial waves




arXiv:1611.08475

- Red variation bands: **cutoff 500~1000 MeV**
- **Improve description** of $^1S_0$, $^3P_0$ phase shifts
- **Quantitatively similar** to the nonrelativistic case for J=1 partial waves

# Description of J=0, 1 partial waves



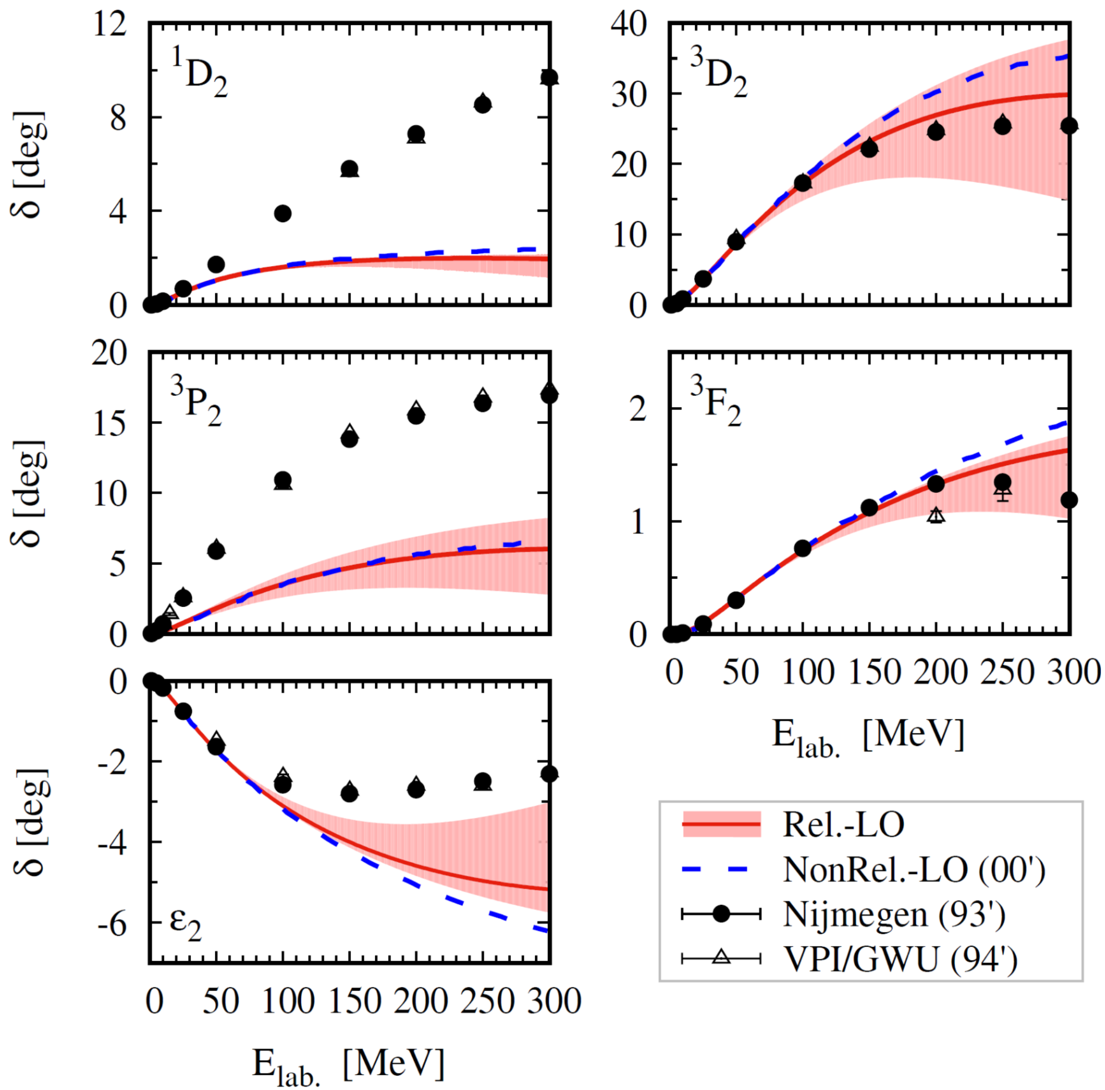

- Red variation bands: **cutoff 500~1000 MeV**
- **Improve description** of $^1S_0$, $^3P_0$ phase shifts

| | Relativistic Chiral NF | Non-relativistic Chiral NF | |
|---|---|---|---|
| Chiral order | LO | LO* | NLO* |
| No. of LECs | **5** | 2 | **9** |
| **$\chi^2$/d.o.f.** | **2.0~11.8** | **147.9** | **~2.5** |

*E. Epelbaum, et. al., NPA(2000)

# Higher partial waves

**Only OPEP contributes**




- The relativistic results are almost **the same** as the non-relativistic case.
- Relativistic correction of OPEP is small !

# 1S0 wave phenomena

- Interesting phenomena of 1S0 wave
  - Large variance of phase shift from 60 to -10 (zero point: $k_0=340.5$ MeV)
  - Virtual bound state at very low-energy region (pole: $-i10$ MeV)
  - Significantly large scattering length ($a=-23.7$ fm)

**Energy scales smaller than chiral symmetry breaking scale**

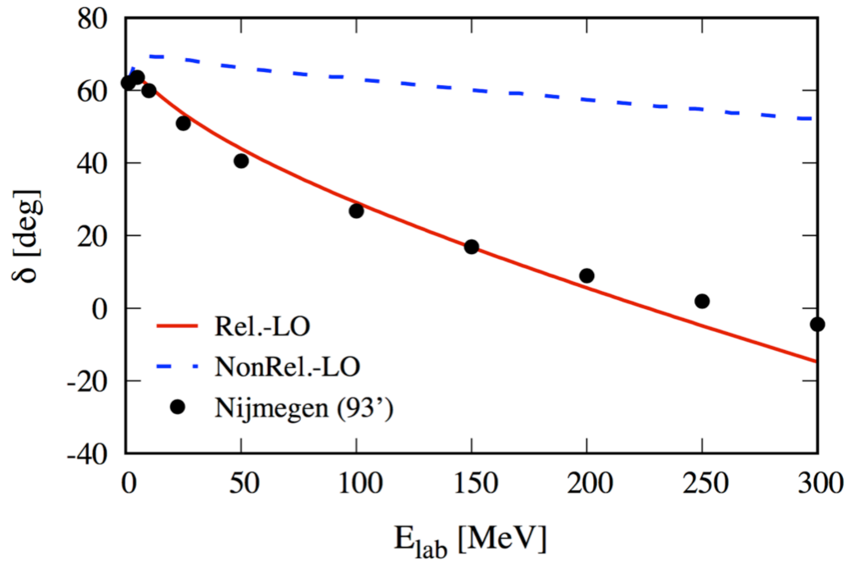
⇨ The 1S0 phenomena **should be roughly reproduced simultaneously** at the **lowest order** of chiral nuclear force

*Bira van Kolck, et al., 1704.08524.*

**However, the NR chiral force at LO cannot achieve such description.**

# 1S0 in relativistic chiral force (LO)

- Rather good description of phase shift :




- Predicted results: (**reproduced simultaneously**)

| | Nijmegen PWA [60, 61] | Global-Fit |
|---|---|---|
| $\Lambda$ [MeV] | – | 750 |
| scattering length $a$ [fm] | $-23.7$ | $-20.3$ |
| effective range $r$ [fm] | 2.70 | 2.45 |
| virtual pole position $i\gamma$ [MeV] | $-i10$ | $-i9.2$ |
| zero phase shift position $k_0$ [MeV] | 340.5 | 326.5 |

*XLR, Li-Sheng Geng, et al., in preparation*

# Summary and perspectives

- **We performed an exploratory study to construct the relativistic nuclear force up to leading order in covariant ChEFT**
  - Relativistic chiral force can **improve the description of $^1S_0$ and $^3P_0$** phase shifts at LO
  - For the phase shifts of partial waves with high angular momenta (J>=1), the relativistic results are **quantitatively similar to** the nonrelativistic counter parts.

- **We are now working on the NLO studies**
  - Calculate the two-pion exchange potentials (**almost finished**)
  - Construct the contact Lagrangians with two derivatives
  - Expect to achieve a better description of phase shifts ⇨ **Stay tuned**

Thank you very much
for your attention!

# Back up slides

# Hint at a more efficient formulation

## $V_{1S0}$: $1/m_N$ expansion

$$V_{1S0} = 4\pi\left[C_{1S0} + (C_{1S0}+\hat{C}_{1S0})\left(\frac{\vec{p}^2+\vec{p}'^2}{4M_N^2}+\cdots\right)\right]$$
$$+\frac{\pi g_A^2}{2f_\pi^2}\int_{-1}^{1}\frac{dz}{\vec{q}^2+m_\pi^2}\left[\vec{q}^2-\left(\frac{(\vec{p}^2-\vec{p}'^2)^2}{4M_N^2}+\cdots\right)\right].$$

- Relativistic corrections are suppressed
- One has to be careful with **the new contact term, the momentum dependent term**, which is desired to achieve a reasonable description of the phase shifts of 1S0 channel.

*J. Soto et al., PRC(2008), B. Long, PRC (2013)*

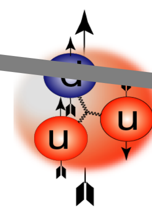
# Motivation for the relativistic formulation

- Relativistic effects in nuclear physics
  - **Kinematical effect**: safely neglected or perturbatively treated

    *NR approximation:* $$\sqrt{p^2 + m_N^2} = m_N\sqrt{1 + 0.102}$$

  - **Dynamical effect**: nucleon spin, spin-orbit splitting, anti-nucleon …

    *NR approximation:*




$$\cancel{f(r)\boldsymbol{S}\cdot\boldsymbol{L}}$$

- **Relativistic (dynamical) effects are important**
  - Nuclear system:
    - **Covariant density functional theory (CDFT)**

      *P. Ring, PPNP (1996),*
      *D.Vretenar et al.,Phys.Rept.(2005), J. Meng, IRNP(2016)*
  - One-nucleon system:
    - **Covariant ChEFT with extended-on-mass-shell (EOMS) scheme**

      *J. Gegelia,PRD(1999), T. Fuchs,PRD(2003)*

# Errors and correlation matrix

| | $C_S$ | $C_A$ | $C_V$ | $C_{AV}$ | $C_T$ |
|---|---|---|---|---|---|
| $C_S$ | **1.00** | 0.21 | -0.93 | -0.58 | -0.39 |
| $C_A$ | 0.23 | 1.00 | -0.15 | 0.45 | 0.21 |
| $C_V$ | **-0.93** | -0.15 | 1.00 | 0.77 | 0.69 |
| $C_{AV}$ | -0.57 | 0.45 | **0.77** | 1.00 | 0.89 |
| $C_T$ | -0.39 | 0.21 | 0.69 | **0.89** | 1.00 |

# Only two LECs fit:

$$V_{\rm CTP}^{\rm NonRel.} = (C_S + C_V) - (C_{AV} - 2C_T)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \mathcal{O}(\frac{1}{M_N}).$$

- Take CS and CAV as free parameters
- Best fit result:
  - chi^2/d.o.f. = **84.5**

| | **Relativistic Chiral NF** | **Non-relativistic Chiral NF** | |
|---|---|---|---|
| Chiral order | LO | LO | NLO* |
| No. of LECs | **5** | 2 | **9** |
| **$\chi^2$/d.o.f.** | **2.0-11.0** | **147.9** | **~2.5** |

| Tlab [MeV] | 1 | 50 | 100 | 150 | 200 | 250 | 300 |
|---|---|---|---|---|---|---|---|
| Pcm [MeV] | 21.67 | 153.22 | 216.68 | 265.38 | 306.43 | 342.60 | 375.30 |
| Vcm | 0.023 **c** | 0.16 **c** | 0.23 **c** | 0.28 **c** | 0.33 **c** | 0.36 **c** | 0.40 **c** |
| E_corr(2n) [MeV] | 0.25 | 12.5 | 25 | 37.5 | 50 | 62.5 | 75 |

$$p_{cm} = \sqrt{\frac{m_N T_{\mathrm{lab}}}{2}} \qquad V_{\mathrm{cm}} = \frac{p_{\mathrm{cm}}}{m_N} c$$

$$E_T^{\mathrm{corr}} = \frac{p_{\mathrm{cm}}^2}{2m_N}$$