Phase-equivalently transformed nucleon-nucleon forces in nucleon-deuteron elastic scattering

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Outline

Motivation:

- Phase-equivalent transformations
- Daejeon16 potential

Method

- Faddeev formalism for 3N processes

Results

- Nucleon-deuteron elastic scattering

Summary

Motivation

- One motivation comes from the Daejeon16 NN potential which uses unitary transformation to minimize the role of 3NF.
- Without 3N force (NCSM) calculations are simpler and faster, larger model spaces become available; hence predictions are more reliable.
- The sensitivity of 3N observables to unitary transformations is interesting by itself.
- Such study is important to understand the role of induced 3NFs.

Motivation

- Nuclear structure calculations require substantial amount of computer resources.
- Various ideas are introduced to facilitate calculations:
- SRG methods transform nondiagonal part of NN interaction to low-momentum domain
- SRG transformations, as other unitary transformations, applied to many-body Hamiltonian introduce „induced many-body forces".

- W.N.Polyzou, W.Glöckle, Few-Body Syst. 9 (1990) 97. (3N system)

$$H = H_0 + \sum_{i,j} V_{ij} \quad \rightarrow \quad H' = H_0 + \sum_{i,j} V'_{ij} + V_{ijk}$$

where V_{ij} and V'_{ij} are phase-equivalent.

- H and H' gives the same values of 3N observables.

Daejeon16

- Idea for the Daejeon16: the unitary transformation can be used to reduce a role of 3NF

$$H = H_0 + V_{2N} + V_{3N}^{genuine}$$

$$\xrightarrow{SRG} \quad H' = H_0 + V'_{2N} + V_{3N}^{genuine} + V_{3N}^{induced(SRG)}$$

$$\xrightarrow{PET} \quad H'' = H_0 + V''_{2N} + V_{3N}^{genuine} + V_{3N}^{induced(SRG)} + V_{3N}^{induced(PET)}$$

- ? Is it possible to find such unitary transformation that

$$\xrightarrow{?\,PET} \quad H'' = H_0 + V''_{2N} + \underbrace{V_{3N}^{genuine} + V_{3N}^{induced(SRG)} + V_{3N}^{induced(PET)}}_{=0\,(\approx 0)}$$

- Daejeon16 is NN interaction minimizing effects of 3NF in 3N system.

Daejeon16

A.M.Shirokov, I.J.Shin, Y.Kim, M.Sosonkina, P.Maris, J.P.Vary, Phys. Lett. B761 (2016) 87

- Idaho χEFT N3LO NN force from D.R.Entem, R.Machleidt, (2003)
- SRG-evolution with λ=1.5 fm^{-1} to soften the interaction
- Phase equivalent transformation (PET) to describe ground states and some exited states of nuclei up to A=16 without 3NF
- PET is done in HO base, mixes (two) main HO components in each partial wave – one parameter in each partial wave
- PET mixing angles are:

wave	$^1S_0$	$^3S_1$-$^3D_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$-$^3F_2$	$^3D_2$
angle [deg]	-2.997	+4.461	+5.507	+1.785	+4.299	-2.031	+7.833

- SRG and PET do not affect the description of NN phase shifts and the deuteron binding energy provided by the Idaho force.
- Daejeon16 provides good description of light nuclei (without 3NF) and allows for fast nuclear structure calculations.

PET - technicalities

- Yu.A.Lurie, A.M.Shirokov, Annals of Physics 312 (2004) 284

$$H\Psi(r) = E\Psi(r)$$

$$\Psi(r) = \sum_{k=0}^{\infty} C_k |k\rangle$$ e.g. HO basis

$$\sum_{k'=0}^{\infty} \langle k|H|k'\rangle C_{k'} = EC_k$$

$$[H'] = [U^+][H][U]$$

$$[U] = \begin{pmatrix} [U_0] & 0 \\ 0 & 1 \end{pmatrix} \qquad [U_0] = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma) \end{bmatrix}$$

we can mix
any of two (three, …)
HO components

$$H' = T + V_{PET}$$

$$[V_{PET}] = [V] + [U^+][H][U] - [H]$$

Formalism for 3N scattering

More details e.g. in:
W.Glöckle et al.,
Phys. Rept. 274 (1996) 107

- Nonrelativistic formalism, momentum space
- 2N:

  Schrödinger equation,

  Lippmann-Schwinger equation for the t-matrix

  (interaction + free propagation)

$$t(E) = V + VG_0(E)V + VG_0VG_0(E)V + \dots$$

$$G_0(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E - H_0 + i\varepsilon}$$

- 3N: Faddeev equation

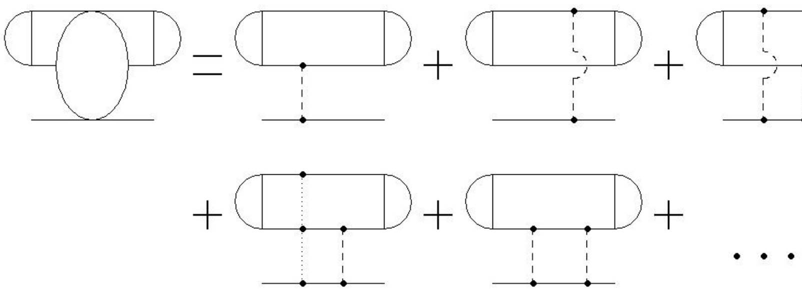
$$T = tP\phi + (1 + tG_0)V_{123}^{(1)}(1+P)\phi + tPG_0T + (1 + tG_0)V_{123}^{(1)}(1+P)G_0T$$

Transition amplitudes

$$U = PG_0^{-1} + V_{123}^{(1)}(1+P)\phi +$$

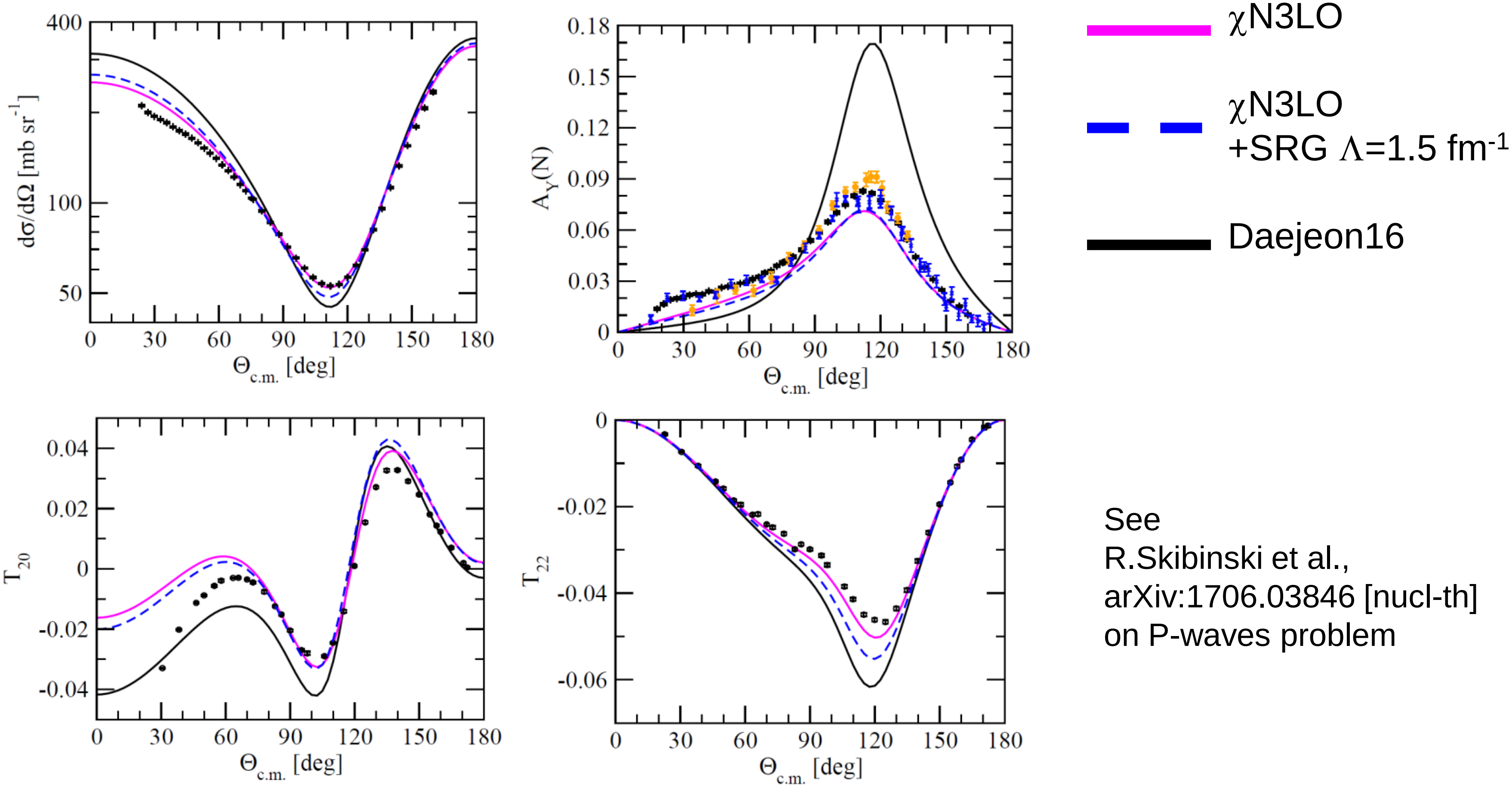
$$+ PT + V_{123}^{(1)}(1+P)G_0T$$

$$U_0 = (1+P)T$$



Evolution from χN3LO to Daejeon16

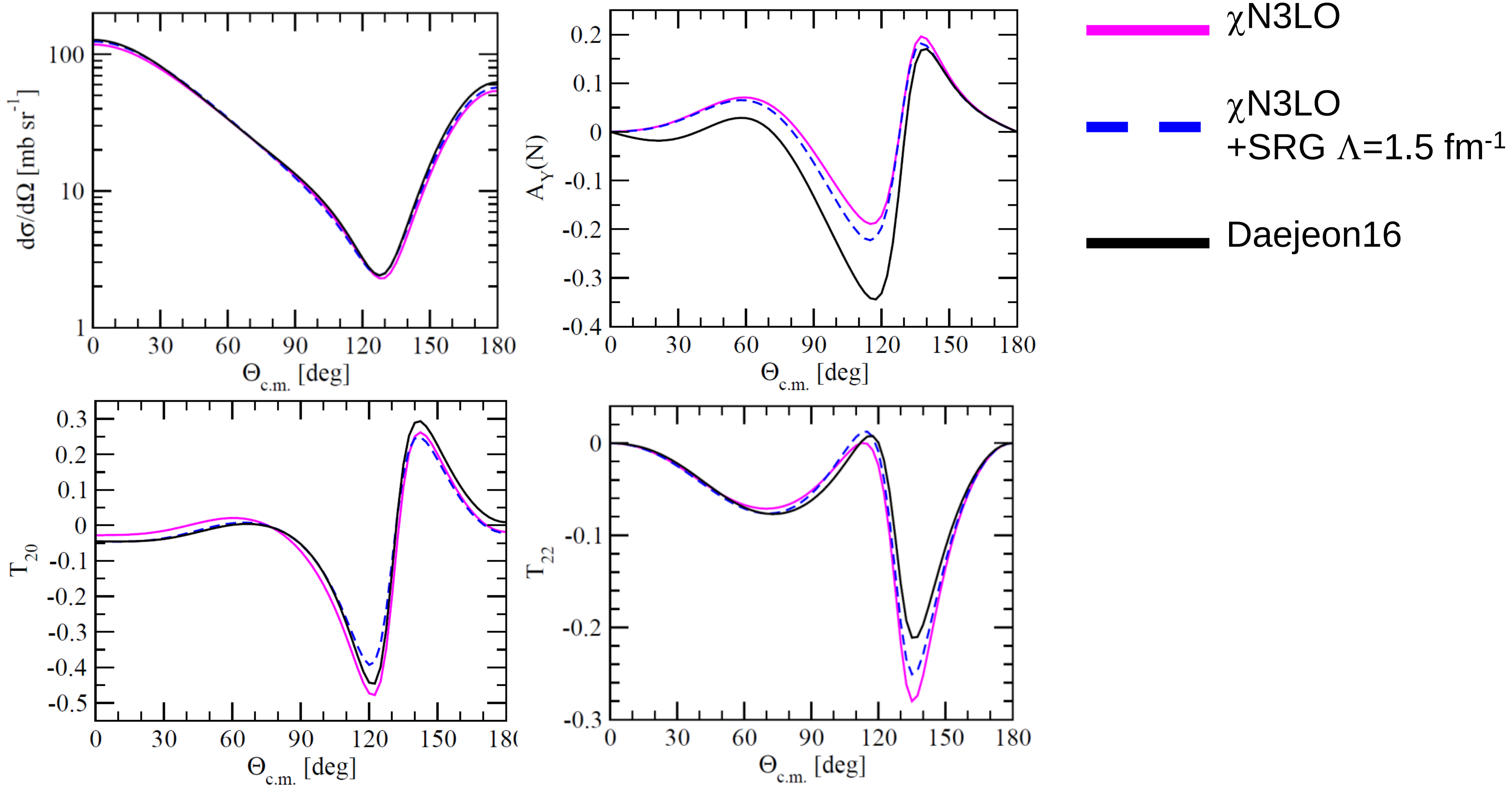
- E=5 MeV



See
R.Skibinski et al.,
arXiv:1706.03846 [nucl-th]
on P-waves problem

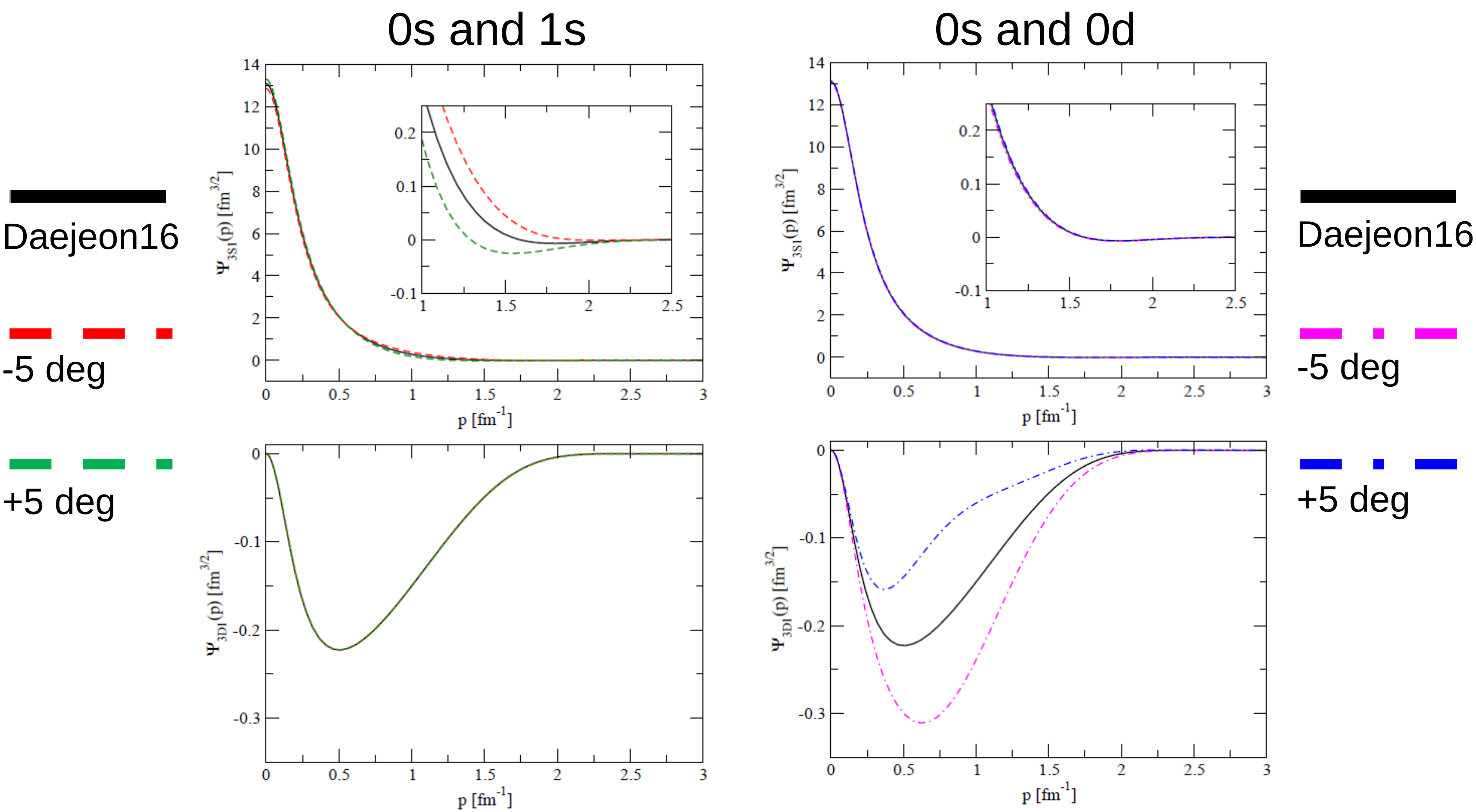
Evolution from χN3LO to Daejeon16

- E=25 MeV

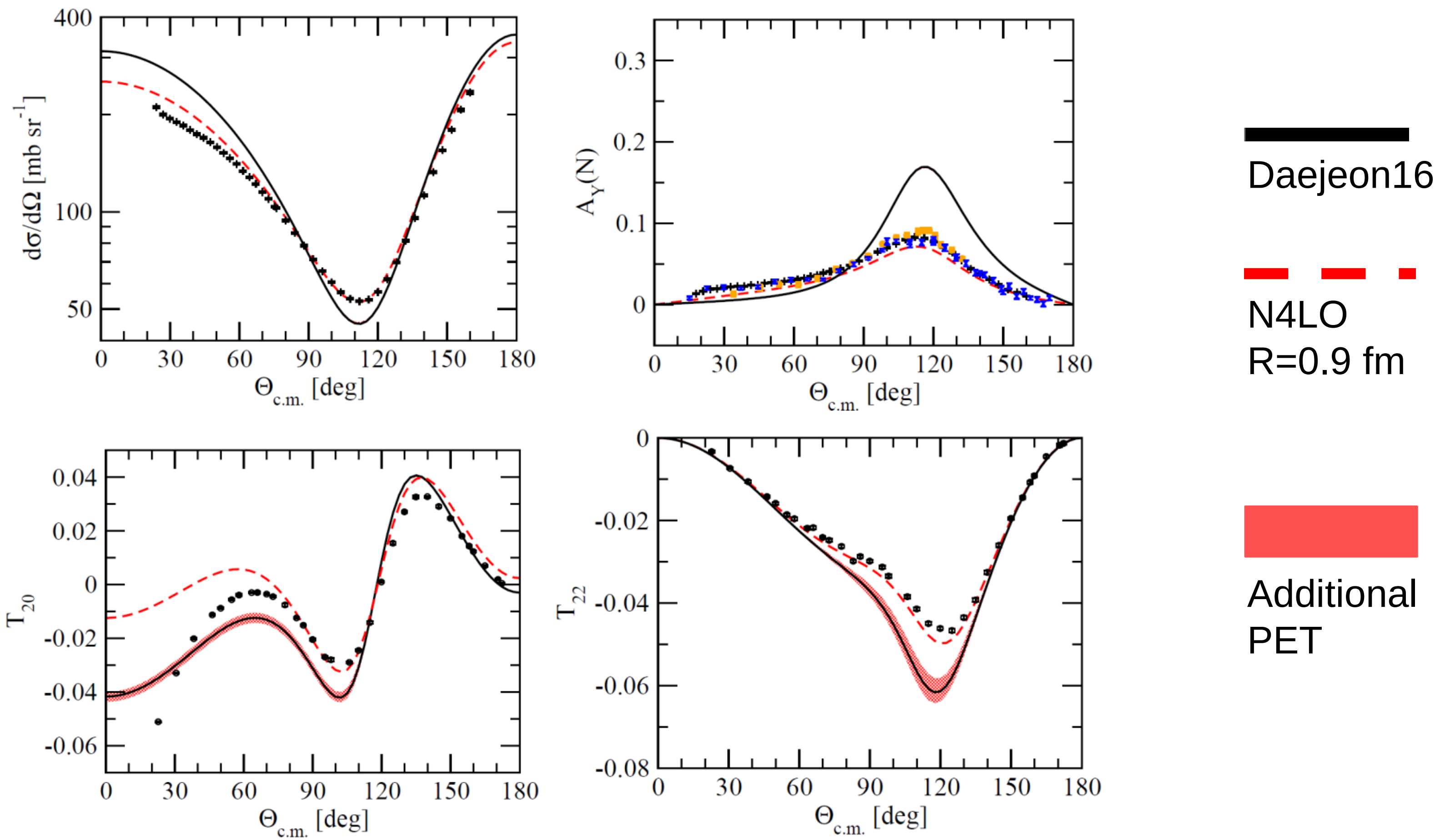


Additional PET - deuteron wave function

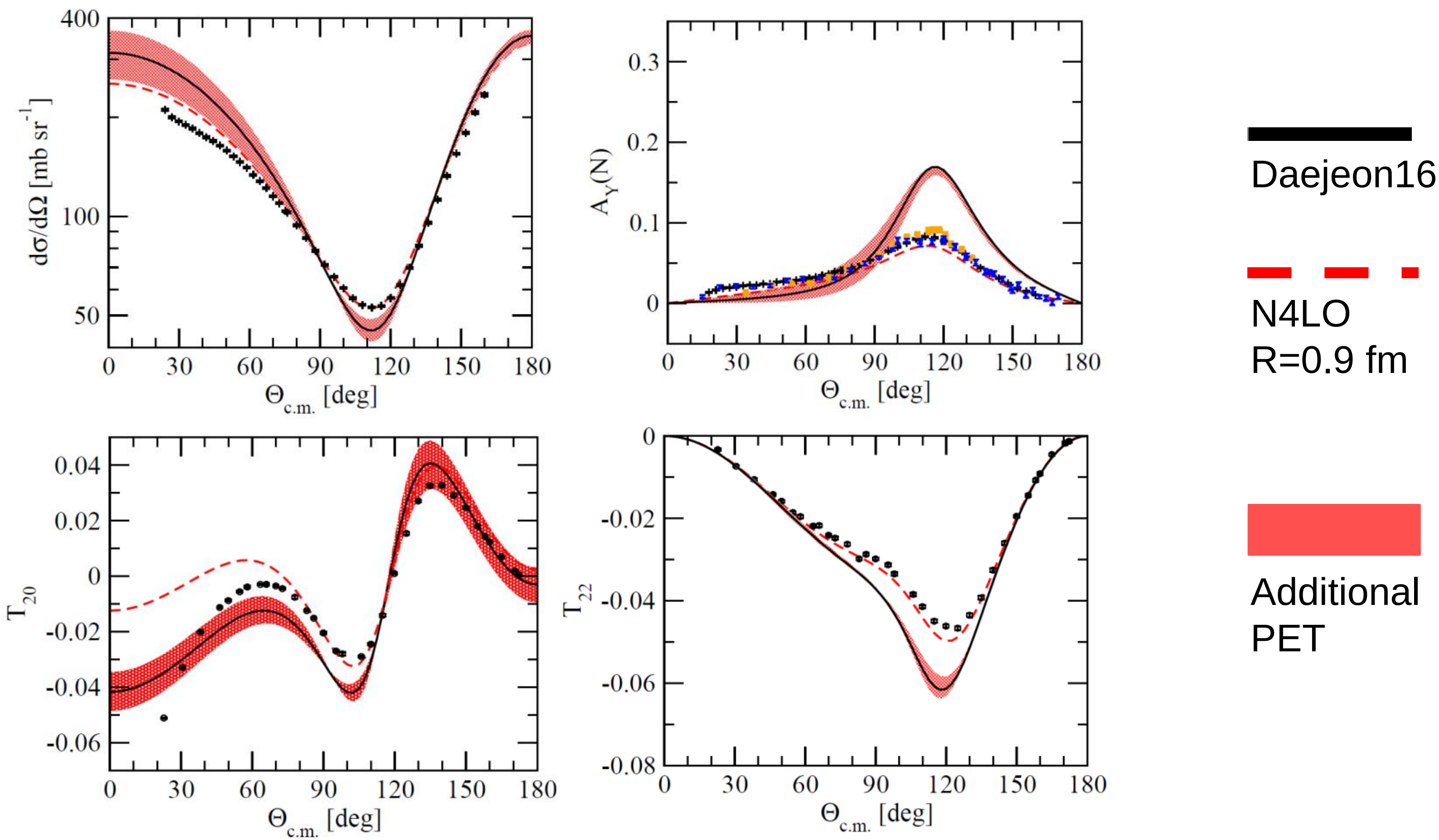
- Binding energy remains unchanged -2.2247 MeV



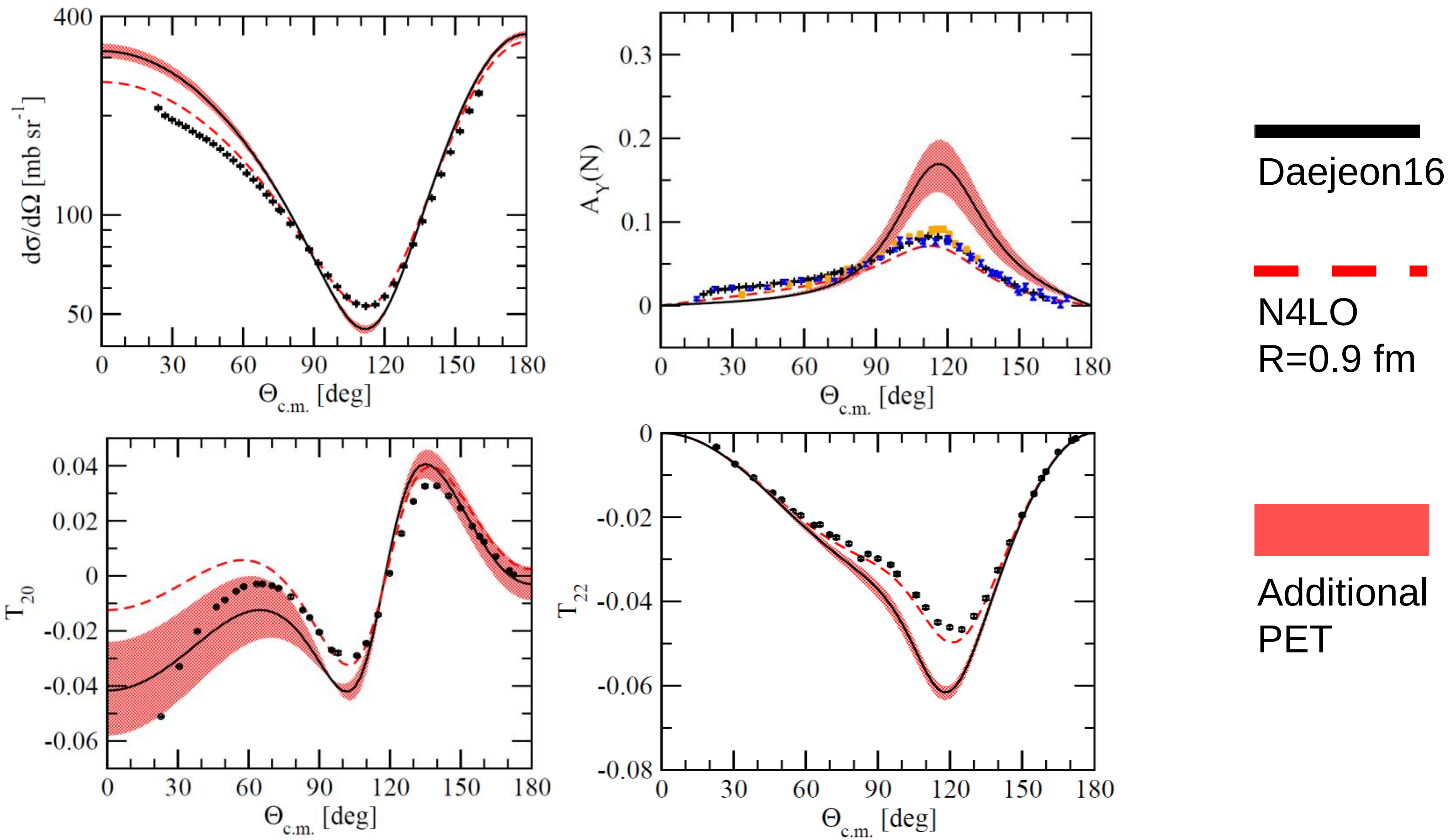
Elastic Nd scattering at E=5 MeV
Additional PET in $^1S_0$ in range -5° ÷ +5°



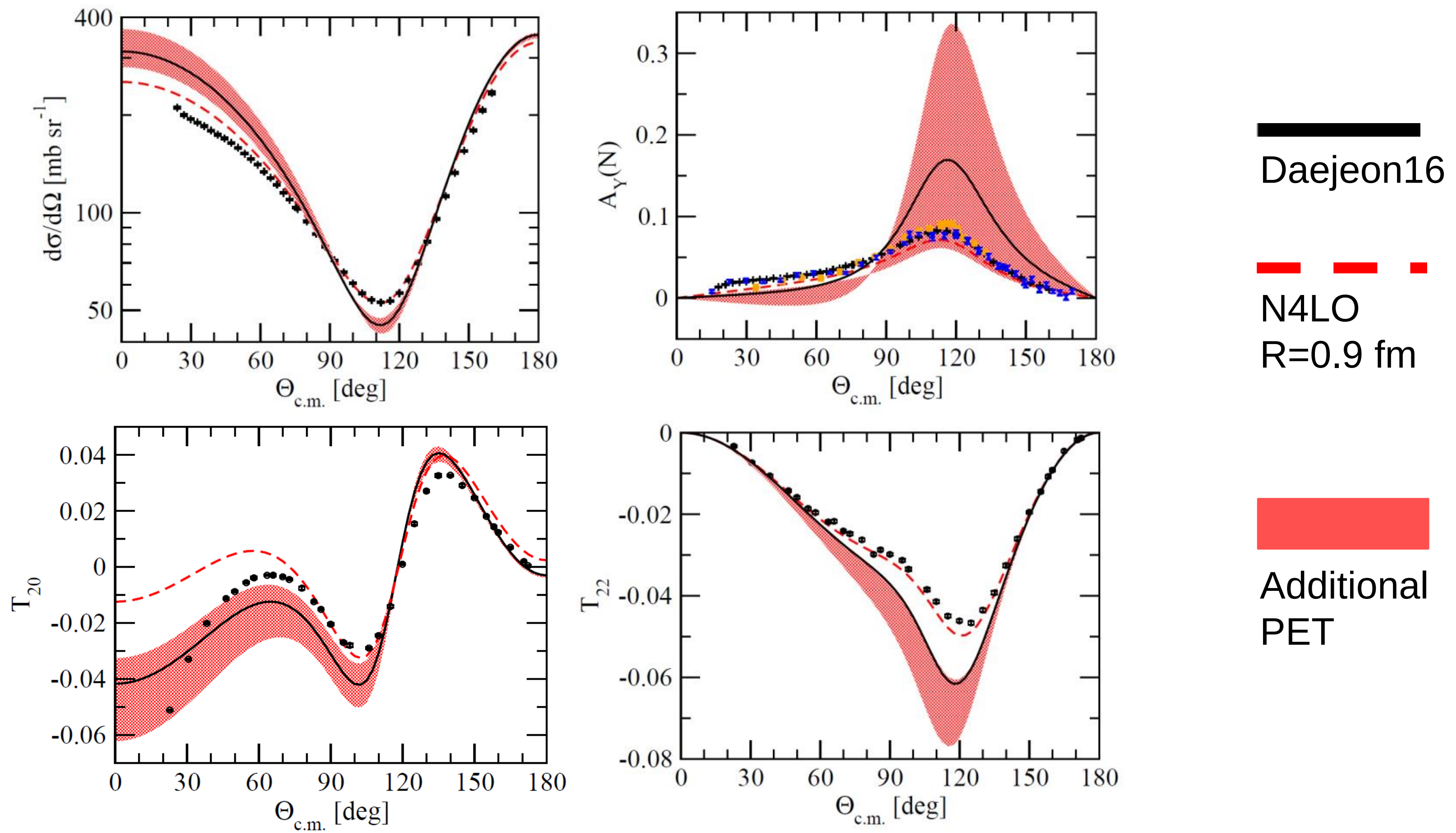
Elastic Nd scattering at E=5 MeV
Additional PET in $^3S_1$ in range $-5^o \div +5^o$



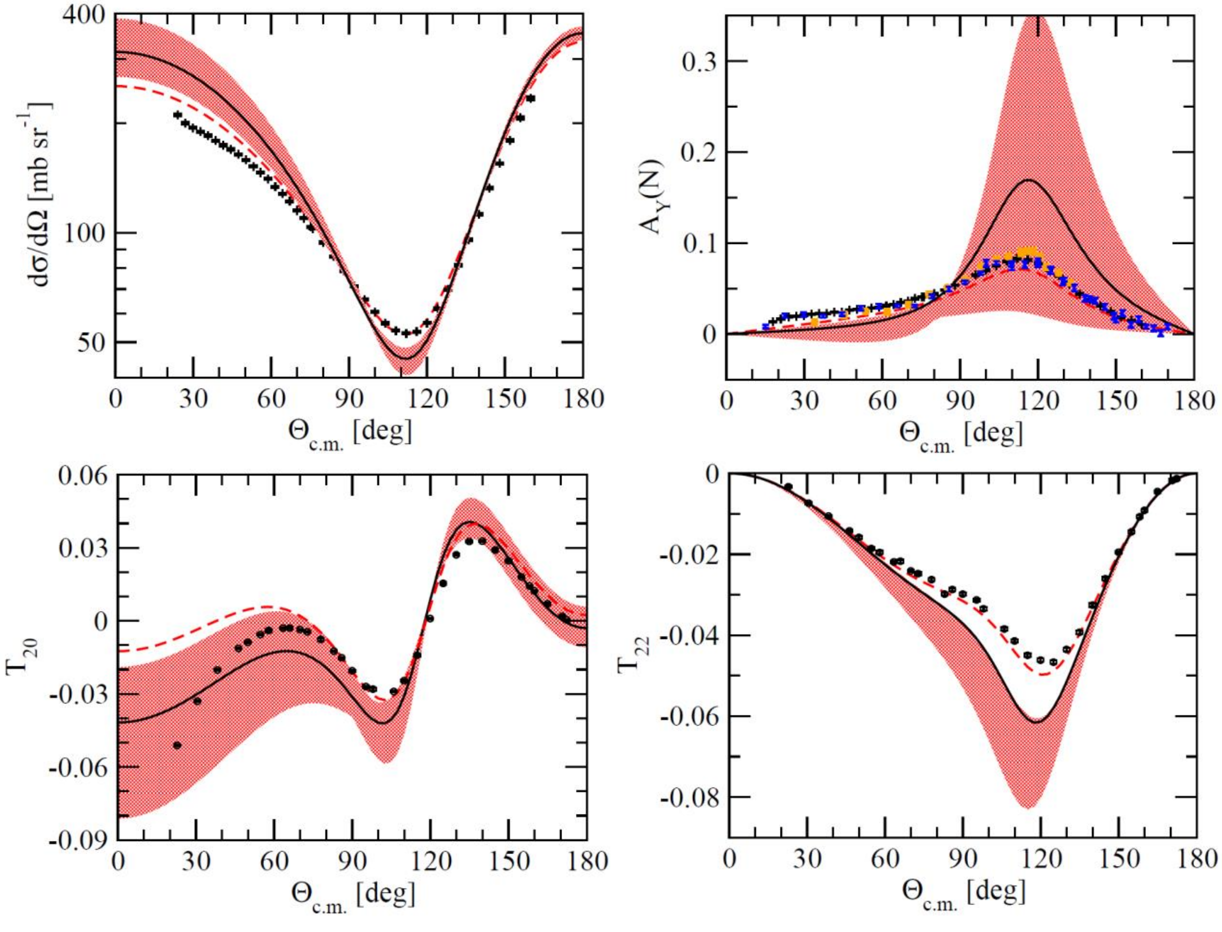
Elastic Nd scattering at E=5 MeV
Additional PET in $^3P_1$ in range $-5^o \div +5^o$



Elastic Nd scattering at E=5 MeV
Additional PET in $^3P_2$ in range $-5^o \div +5^o$



Elastic Nd scattering at E=5 MeV
Additional PET in $^3P_1$ and $^3P_2$ in range -5° ÷ +5°



Daejeon16

N4LO
R=0.9 fm

Additional
PET

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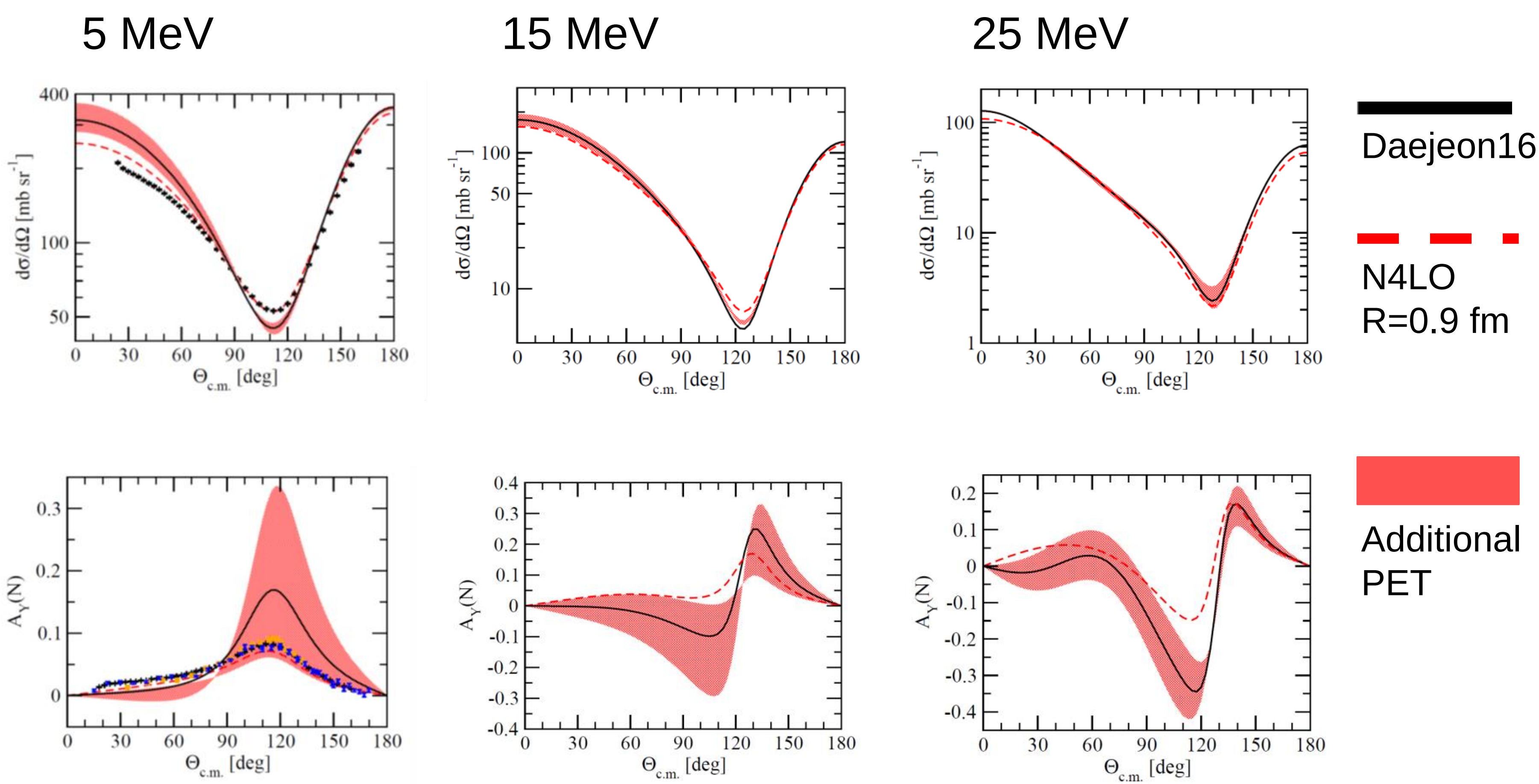
Energy dependence of $d\sigma/d\Omega$ and $A_Y(N)$
Additional PET in $^3P_2$ in range $-5^o \div +5^o$



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DET-PET (deuteron equivalent transformations)

A.M.Shirokov, V.A.Kulikov, A.I.Mazur, J.P.Vary, P.Maris, Phys. Rev. C85 (2012) 034004

- In general it is possible to construct PET preserving any state.
- In the simplest case (unitary transformation of rank 2) the input parameters are the state to be preserved |d⟩, the mixing angle and four basis (HO) states.

- Technically, to build the unitary operator one uses states orthogonal to chosen state |d⟩
- For U^0 of rank 2 we define a_1 and a_2 as a linear combination of a finite numer of HO basis states |k>, e.g.

$$U^0 = \sum_{i,j\le 2} \left|a_i\right\rangle U^0_{ij} \left\langle a_j\right| \qquad \left\langle a_i \middle| d\right\rangle = 0 \qquad i \le 2$$

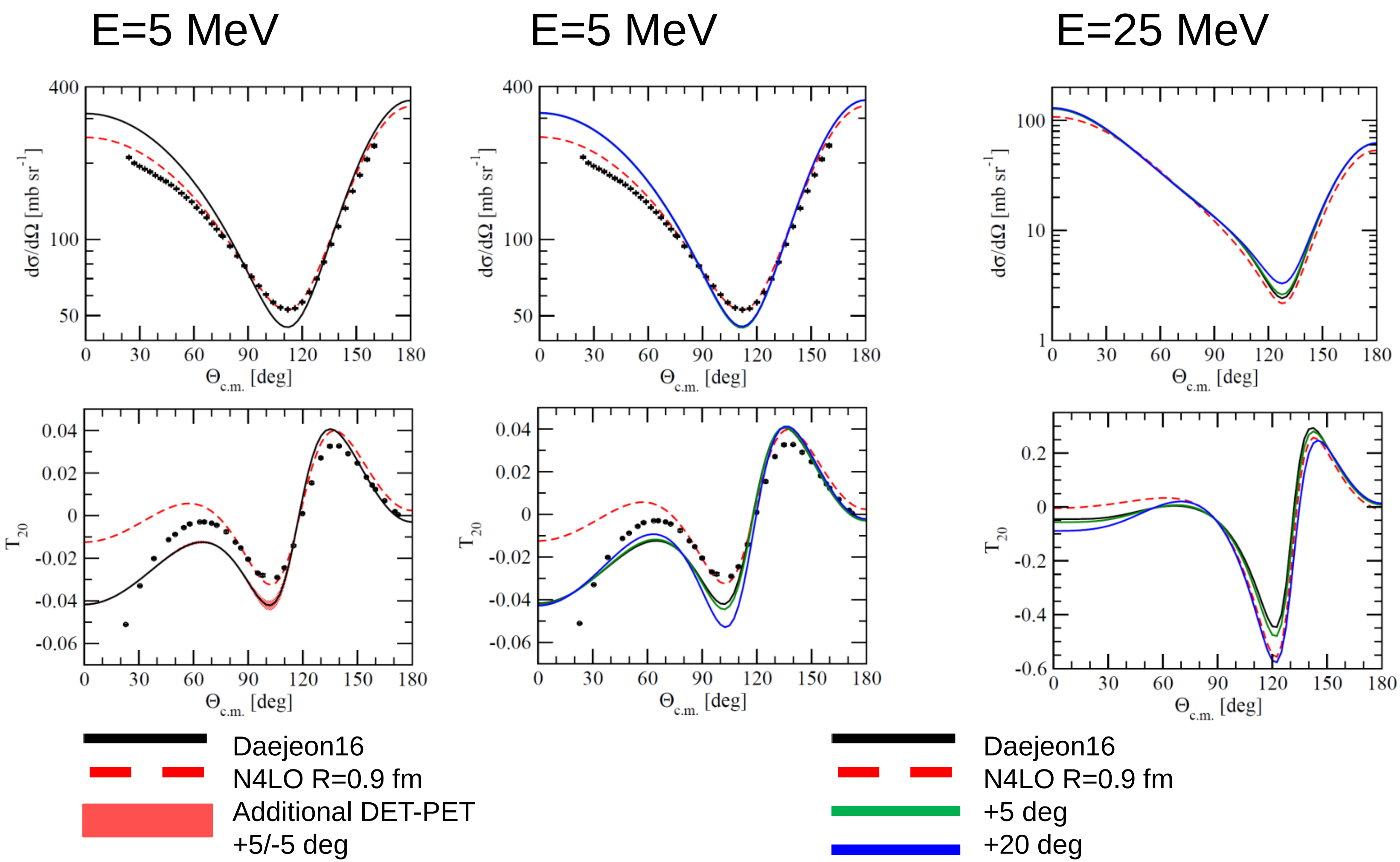
$$\left|a_1\right\rangle = a_1^n \left|k_n\right\rangle + a_1^m \left|k_m\right\rangle$$

$$\left|a_2\right\rangle = a_2^l \left|k_l\right\rangle + a_2^p \left|k_p\right\rangle$$

- Formulas for a^i_j parameters are given in Phys. Rev. C85 (2012) 034004 as functions of expansions parameters of state |d⟩ in HO (|k⟩) basis.

- We chose preserving the deuteron wave function in addition to NN phase shift and the deuteron binding energy.
- Observables in many nucleon systems are not preserved

Elastic Nd scattering $d\sigma/d\Omega$ and T_{20} at 5 MeV and 25 MeV
Additional DET-PET in $^3S_1$ with $+5^o$ or $+20^o$



Summary and Outlook

1. Daejeon16 force which works well for nuclei, gives reasonably good description of Nd scattering, but, as the JISP16, requires improvement of its P-waves.

2. Phase equivalent transformations have been used to change the NN force to study subsequent changes of predictions for 3N observables.

3. We find a big sensitivity of Nd scattering observables when using NN force transformed with PET or DET-PET (and neglecting induced 3NF). This can be used to minimize effects of 3NFs but requires simultaneous transformations in various partial waves.

4. Various observables behave in a different way under PET transformations (e.g. T_{20} and T_{22}).

5. If many-body observables are used to fix parameters of NN interaction it is necessary to include scattering observables – used up to now nuclear structure observables are not enough sensitive e.g. to P-waves.

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Thank you for your attention !

JISP16 NN force

A.M.Shirokov et al., Phys. Lett. B644 (2007) 33

Origins in J-matrix inverse scattering approach (A.M.Shirokov et al., Phys. Rev. C70 (2004) 044005) and in the next step is modified by phase-equivalent transformation to achieve reasonable description of many-body systems.

Fitted to the binding energies of some nuclei with A≤16 and low energy states of ^{6}Li

Works very well for nuclear structure calculations (in NCSM) and quite well for nuclear matter.

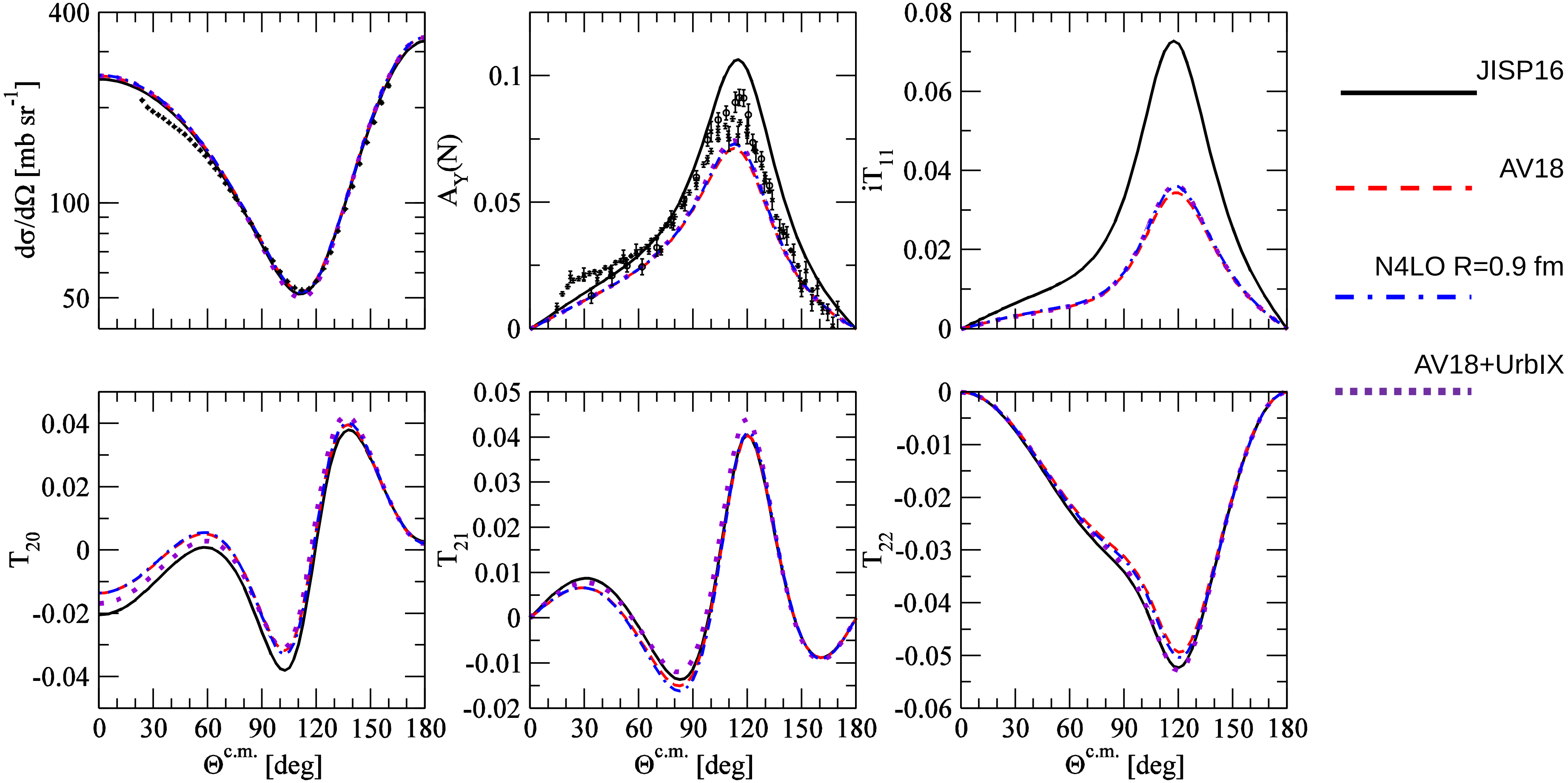
No three-nucleon force is required to describe binding energies and spectra of light nuclei.

Provides faster convergence of nuclear structure calculations than realistic potentials.

Given as a matrix in the harmonic oscillator basis (easy transformation to the momentum space).

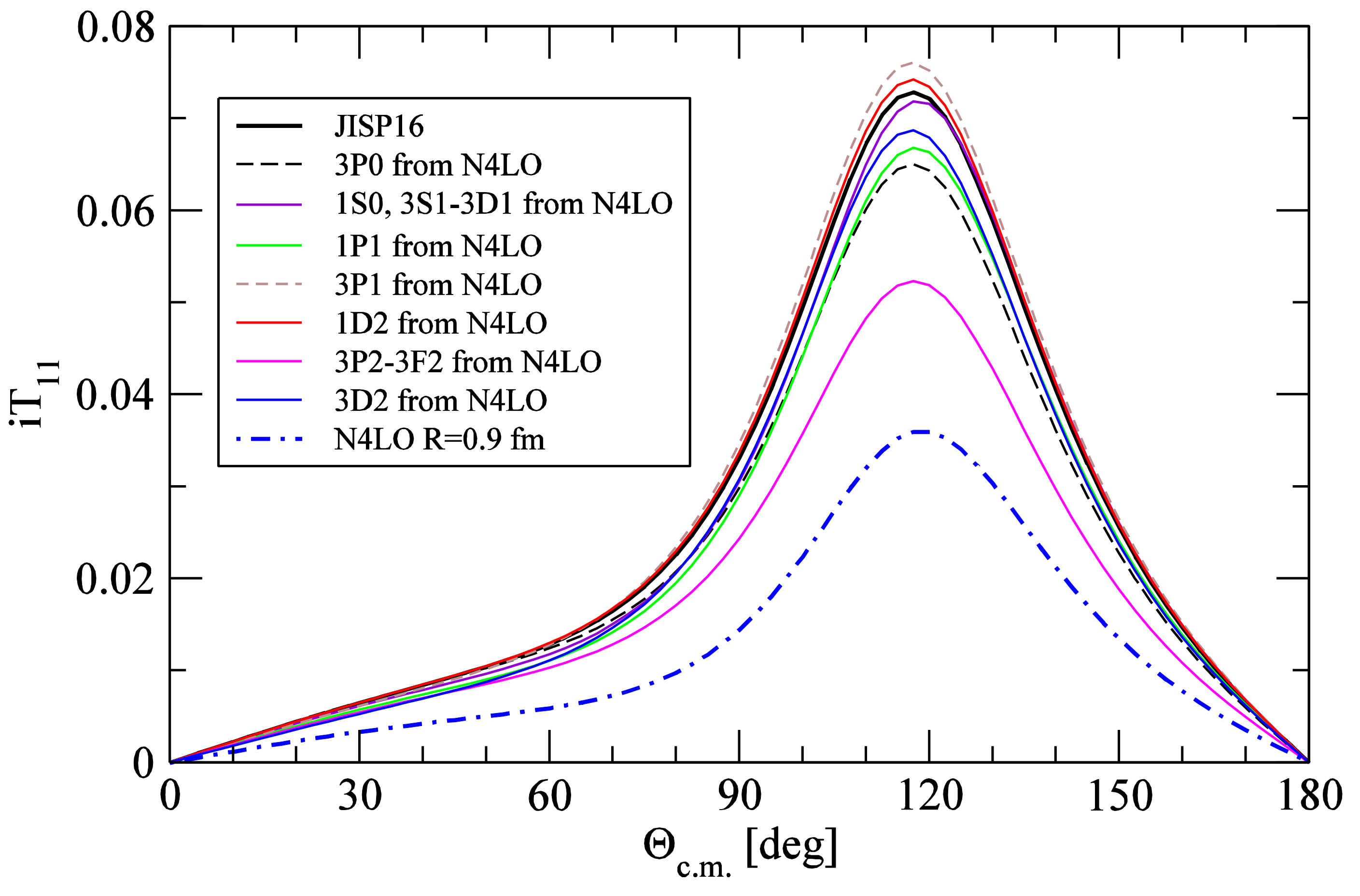
Neutron-deuteron scattering at E=5 MeV

All predictions obtained with states up to j=4, J=25/2 and only neutron-proton force



iT_{11} at E=5 MeV

All predictions obtained with states up to j=4, J=25/2 and only neutron-proton force



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