Quasi-bound state in the $\overline{K}NNN$ system

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The 7th Asia-Pacific Conference on Few-Body Problems in Physics (Aug. 25 – 30, 2017, Guilin, China)

On the way to the quasi-bound state in the $\bar{K}NNN$ system

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K^-pp quasi-bound state – interest to antikaonic nuclei

Theory

Prediction of the existence of deep and narrow K^-pp bound state

T. Yamazaki and Y. Akaishi, Phys. Lett. B535 (2002) 70:

$$E_B = -48 \text{ MeV}, \Gamma = 61 \text{ MeV}$$

Many theoretical calculations, different models and inputs (Faddeev, variational calculations, FCA):

$$E_B \sim -14 - 80 \text{ MeV}, \quad \Gamma \sim 40 - 110 \text{ MeV}$$

- agree only on the fact that the quasi-bound state in K^-pp exists

Experiment

FINUDA collaboration: E_B = -115 MeV, Γ = 67 MeV

M. Agnello et. al., Phys. Rev. Lett. 94 (2005) 212303

DISTO collaboration: E_B = -103 MeV, Γ = 118 MeV

T. Yamazaki et al. Phys. Rev. Lett. 104, (2010)

A series of Faddeev calculations with coupled $\bar{K}NN - \pi\Sigma N$ channels

NVS, J.Révai

•Three-body pole positions and widths of the quasi-bound states in the K^-pp and K^-K^-p systems were evaluated

•It was demonstrated that there is no quasi-bound state in the K^-d system (caused by the strong interaction)

•Near-threshold elastic K^-d amplitudes were calculated (including the K^-d sc.l)

•The three-body K^-d amplitudes were used for an approximate calculation of the *1s* level shift and width of (anti)kaonic deuterium

Antikaon-nucleon potentials used:

- Phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

- reproducing SIDDHARTA data on kaonic hydrogen *1s* level shift and width together with the scattering K^-p data with the same level of accuracy

Three-body Faddeev equations in Alt-Grassberger-Sandhas form

$$U_{\alpha\beta}(z) = (1-\delta_{\alpha\beta})(G_0(z))^{-1} + \sum_{\gamma=1}^{3}(1-\delta_{\alpha\gamma})T_\gamma(z)\,G_0(z)\,U_{\gamma\beta}(z), \qquad \alpha,\beta = 1,2,3$$

$U_{\alpha\beta}(z)$ - 3-body transition operators $\beta+(\alpha\gamma)\to\alpha+(\beta\gamma)$
$G_0(z)$ - free Green function
$T_\alpha(z)$ - 2-body T-marix

A separable potential leading to a separable T-marix

$$V_\alpha = \lambda_\alpha |g_\alpha\rangle\langle g_\alpha| \Rightarrow T_\alpha(z) = |g_\alpha\rangle\,\tau_\alpha(z)\,\langle g_\alpha|$$

allows to write the three-body equations in the form

$$X_{\alpha\beta}(z) = Z_{\alpha\beta}(z) + \sum_{\gamma=1}^{3} Z_{\alpha\gamma}(z)\,\tau_\gamma(z)\,X_{\gamma\beta}(z)$$

with $X_{\alpha\beta}(z) = \langle g_\alpha | G_0(z)\,U_{\alpha\beta}(z)\,G_0(z) | g_\beta\rangle$, $Z_{\alpha\beta}(z) = (1-\delta_{\alpha\beta})\langle g_\alpha | G_0(z) | g_\beta\rangle$

Four-body Alt-Grassberger-Sandhas equations

P. Grassberger, W. Sandhas, Nucl. Rev. B2 (1967) 181

$$U^{\sigma\rho}_{\alpha\beta}(z) = \left(1-\delta_{\sigma\rho}\right)\delta_{\alpha\beta}G_0^{-1}(z)\,T_\alpha^{-1}(z)G_0^{-1}(z) + \\ + \sum_{\tau,\gamma}\left(1-\delta_{\sigma\tau}\right)U^{\tau}_{\alpha\gamma}(z)G_0(z)\,T_\gamma(z)G_0(z)U^{\tau\rho}_{\gamma\beta}(z)$$

$U^{\sigma\rho}_{\alpha\beta}(z)$ - 4-body transition operators
$U^{\tau}_{\alpha\beta}(z)$ - 3-body transition operators
$G_0(z)$ - free Green function
$T_\alpha(z)$ - 2-body *T*-marix

A separable potential → separable T-marix → four-body equations:

$$\overline{U}^{\sigma\rho}(z) = \left(1-\delta_{\sigma\rho}\right)\left(\overline{G_0}(z)\right)^{-1} + \sum_{\tau}\left(1-\delta_{\sigma\tau}\right)\overline{T}^{\tau}(z)\,\overline{G_0}(z)\,\overline{U}^{\tau\rho}(\mathrm{z}),$$

with $\overline{U}^{\sigma\rho}_{\alpha\beta}(\mathrm{z}) = \left\langle g_\alpha \,|\, G_0(z)\,U^{\sigma\rho}_{\alpha\beta}(z)\,G_0(z) \,|\, g_\beta\right\rangle$

$\overline{T}^{\tau}_{\alpha\beta}(z) = \left\langle g_\alpha \,|\, G_0(z)\,U^{\tau}_{\alpha\beta}(z)\,G_0(z) \,|\, g_\beta\right\rangle$

and $\overline{(G_0)}_{\alpha\beta}(z) = \delta_{\alpha\beta}\tau_\alpha(z)$

Four-body AGS equations for separable potentials

A.Casel, H.Haberzettl, W. Sandhas, Phys. Rev. C25 (1982) 1738

$$\overline{U}^{\sigma\rho}(z) = \left(1-\delta_{\sigma\rho}\right)\left(\overline{G_0}(z)\right)^{-1} + \sum_{\tau}\left(1-\delta_{\sigma\tau}\right)\overline{T}^{\tau}(z)\,\overline{G_0}(z)\,\overline{U}^{\tau\rho}(\mathrm{z})$$

look similar to the three-body AGS equations in the general form.
Separable form of the “effective potentials” → separable “T”-matrix:

$$\overline{T^{\tau}_{\alpha\beta}}(z) = \left|\overline{g}^{\tau}_{\alpha}\right\rangle \overline{\tau}^{\tau}_{\alpha\beta}(\mathrm{z}) \left\langle \overline{g}^{\tau}_{\beta}\right|$$

allows to write the four-body equations in the form

$$\overline{X}^{\sigma\rho}(z) = \overline{Z}^{\sigma\rho}(z) + \sum_{\tau}\overline{Z}^{\sigma\tau}(\mathrm{z})\,\overline{\tau}^{\tau}(\mathrm{z})\,\overline{X}^{\tau\rho}(z)$$

with
$$\overline{X}^{\sigma\rho}(z) = \left\langle \overline{g}^{\sigma} \mid \overline{G_0}\,\overline{U}^{\sigma\rho}(z)\,\overline{G_0} \mid \overline{g}^{\rho}\right\rangle,$$
$$\overline{Z}^{\sigma\rho}(z) = \left(1-\delta_{\sigma\rho}\right)\left\langle \overline{g}^{\sigma} \mid \overline{G_0}(z) \mid \overline{g}^{\rho}\right\rangle$$

4-body equations for the $\bar{K}NNN$ system

$$\bar{X}_1 = \bar{Z}_{12}\bar{\tau}_2\bar{X}_2 + \bar{Z}_{13}\bar{\tau}_3\bar{X}_3$$
$$\bar{X}_2 = \bar{Z}_{21} + \bar{Z}_{21}\bar{\tau}_1\bar{X}_1 + \bar{Z}_{22}\bar{\tau}_2\bar{X}_2 + \bar{Z}_{23}\bar{\tau}_3\bar{X}_3$$
$$\bar{X}_3 = \bar{Z}_{31} + \bar{Z}_{31}\bar{\tau}_1\bar{X}_1 + \bar{Z}_{32}\bar{\tau}_2\bar{X}_2$$

Two types of partitions: 3+1 and 2+2:

$$\left|\bar{K} + (NNN)\right\rangle$$
$$\left|N + (\bar{K}NN)\right\rangle$$
$$\left|(\bar{K}N) + (NN)\right\rangle$$

Initial channel $\left|\bar{K} + (NNN)\right\rangle$ is fixed

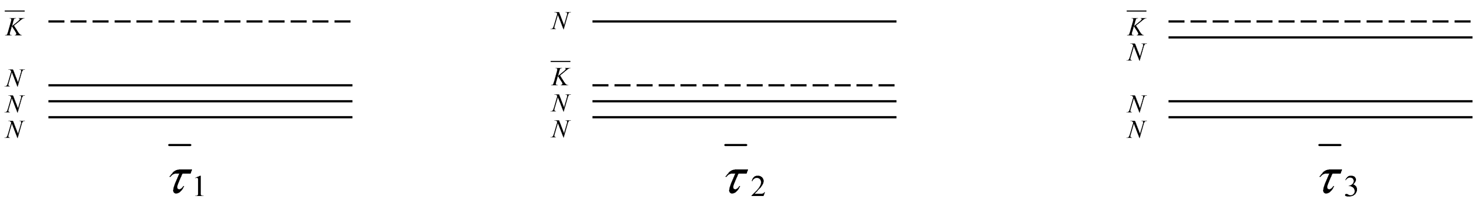
The channels:

channel 1: $\left|\bar{K} + (N_1N_2N_3)\right\rangle$

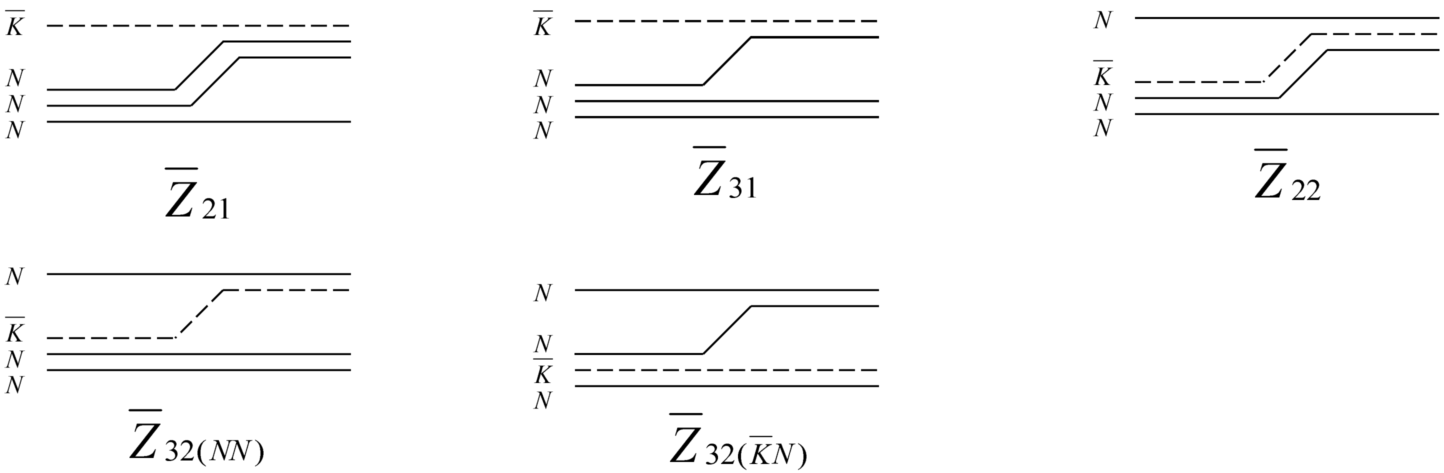
channel 2_1 : $\left|N_1 + (\bar{K}N_2N_3)\right\rangle$, 2_2 : $\left|N_2 + (\bar{K}N_3N_1)\right\rangle$, 2_3 : $\left|N_3 + (\bar{K}N_1N_2)\right\rangle$

channel 3_1 : $\left|(\bar{K}N_1) + (N_2N_3)\right\rangle$, 3_2 : $\left|(\bar{K}N_2) + (N_3N_1)\right\rangle$, 3_3 : $\left|(\bar{K}N_3) + (N_1N_2)\right\rangle$

Propagators τ:



Z operators:



Separabelization of potentials - Hilbert-Schmidt expansion

Lippmann-Schwinger equation:

$$T(p,p';z)=V(p,p')+4\pi\int_0^\infty \frac{V(p,p'')T(p'',p';z)}{z-p''^2/(2\mu)}\,p''^2\,dp''$$

Separable potential leads to the separable *T*-matrix

$$V(p,p')=-\sum_{n=1}^{\infty}\lambda_n g_n(p)g_n(p') \;\Rightarrow\; T(p,p';z)=-\sum_{n=1}^{\infty}\frac{\lambda_n}{1-\lambda_n}g_n(p)g_n(p')$$

were the eigenvalues λ_n and eigenfunctions $g_n(p)$ are found from

$$g_n(p)=\frac{1}{\lambda_n}4\pi\int_0^\infty \frac{V(p,p'')g_n(p'')}{z-p''^2/(2\mu)}\,p''^2\,dp''$$

with normalization condition $$4\pi\int_0^\infty \frac{g_n(p'')g_{n'}(p'')}{z-p''^2/(2\mu)}\,p''^2\,dp''=-\delta_{nn'}$$

Separabelization of "potentials" - Hilbert-Schmidt expansion

Three-body AGS equations:

$$X_{\alpha\beta}(p,p';z)=Z_{\alpha\beta}(p,p';z)+\sum_{\gamma=1}^{3}4\pi\int_0^\infty Z_{\alpha\gamma}(p,p'';z)\tau_\gamma(p'';z)X_{\gamma\beta}(p'',p';z)p''^2\,dp''$$

Separable "potential" leads to the separable "*T*-matrix"

$$Z_{\alpha\beta}(p,p';z)=-\sum_{n=1}^{\infty}\lambda_n g_{n\alpha}(p)g_{n\beta}(p') \;\Rightarrow\; X_{\alpha\beta}(p,p';z)=-\sum_{n=1}^{\infty}\frac{\lambda_n}{1-\lambda_n}g_{n\alpha}(p)g_{n\beta}(p')$$

were the eigenvalues λ_n and eigenfunctions $g_{n\alpha}(p)$ are found from

$$g_{n\alpha}(p)=\frac{1}{\lambda_n}\sum_{\gamma=1}^{3}4\pi\int_0^\infty Z_{\alpha\gamma}(p,p'';z)\tau_\gamma(p'';z)g_{n\gamma}(p'',p';z)p''^2\,dp''$$

with normalization condition

$$\sum_{\gamma=1}^{3}4\pi\int_0^\infty g_{n\gamma}(p,p'';z)\tau_\gamma(p'';z)g_{n'\gamma}(p'',p';z)p''^2\,dp''=-\delta_{nn'}$$

Z functions: momentum, isospin and spin parts

$$\overline{Z}_{\alpha\beta} = Z_{\alpha\beta}(p, p'; z)\ {}_I Z_{\alpha\beta, I_\alpha I_\beta}\ {}_S Z_{\alpha\beta, S_\alpha S_\beta}$$

$K^- ppn:\ I^{(4)} = 0, S^{(4)} = 1/2,$ orbital momentum $L^{(4)} = 0$

Three nucleons - antisymmetrization

$$\overline{X}_1 = \sum_{n2=1}^{3} \overline{Z}_{12_{n2}} \overline{\tau}_{2_{n2}} \overline{X}_{2_{n2}} + \sum_{n3=1}^{3} \overline{Z}_{13_{n3}} \overline{\tau}_{3_{n3}} \overline{X}_{3_{n3}}$$

$$\overline{X}_{2_{n2'}} = \overline{Z}_{21} + \overline{Z}_{2_{n2'}1} \overline{\tau}_1 \overline{X}_1 + \sum_{n2=1}^{3} \overline{Z}_{2_{n2'}2_{n2}} \overline{\tau}_{2_{n2}} \overline{X}_{2_{n2}} + \sum_{n3=1}^{3} \overline{Z}_{2_{n2'}3_{n3}} \overline{\tau}_{3_{n3}} \overline{X}_{3_{n3}}$$

$$\overline{X}_{3_{n3'}} = \overline{Z}_{3_{n3'}1} + \overline{Z}_{3_{n3'}1} \overline{\tau}_1 \overline{X}_1 + \sum_{n2=1}^{3} \overline{Z}_{3_{n3'}2_{n2}} \overline{\tau}_{2_{n2}} \overline{X}_{2_{n2}}$$

where *n1,n2, n3* – indices of the particular nucleons

$\bar{K}N$ interaction with coupled $\pi\Sigma$ and $\pi\Lambda$ channels

Potentials were fitted to the experimental data

- $1s$ level shift and width of kaonic hydrogen (by SIDDHARTA)

$$\Delta_{1s}^{SIDD} = -283 \pm 36 \pm 6 \text{ eV}, \quad \Gamma_{1s}^{SIDD} = 541 \pm 89 \pm 22 \text{ eV}$$

- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n
- ◦ $\Lambda(1405)$ with one- or two-pole structure

$$M_{\Lambda(1405)}^{PDG} = 1405.1_{-1.0}^{+1.3} \text{ MeV}, \quad \Gamma_{\Lambda(1405)}^{PDG} = 50.5 \pm 2.0 \text{ MeV}$$

The "exact optical" versions of the:

- Phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

constructed and used for the three-body calculations

Two-term *NN* potential

reproduces: Argonne V18 *NN* phase shifts (with sign change)

Solution of the four-body equations:

$$\overline{X}_{\alpha} = \overline{Z}_{\alpha\beta} + \sum_{\gamma=1}^{3} \overline{Z}_{\alpha\gamma}\, \overline{\tau}_{\gamma}\, \overline{X}_{\gamma},\ \alpha = 1,2,3;\ \beta = 1$$

1. Calculate separable 3-body "T-matrices": evaluate eigenvalues and eigenfunctions for the $\bar{K}NN$ and NNN subsystems
2. Evaluate momentum, isospin and spin parts of the 4-body "potentials" Z
3. Solve the homogeneus system of 4-body equations in the complex plane

Solution of the four-body equations:

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1. Calculate separable 3-body “T-matrices”: evaluate eigenvalues and eigenfunctions for the $\bar{K}NN$ and NNN subsystems
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Results will be soon