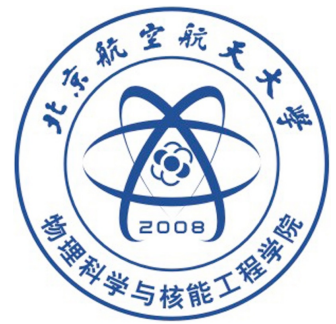




APFB2017
August 25-30, 2017, Guilin, China



Recent progress in covariant baryon chiral perturbation theory

Li-Sheng Geng (耿立升)

**School of Physics and Nuclear Energy Engineering,
Beihang University, Beijing, China**

Xiu-Lei Ren, LSG, Jie Meng. Phys.Rev. D91 (2015) 051502

Xiu-Lei Ren, L. Alvarez-Ruso, LSG, T. Ledwig, Jie Meng, M. J. Vicente Vacas .Phys.Lett. B766 (2017) 325

Outline

❖ Introduction

- *Essence of chiral perturbation theory*
- *Power counting breaking problem and solutions*

❖ Recent progress

- **Determination of octet baryon sigma terms**
- *Relativistic nucleon-nucleon interaction* Xiu-Lei Ren, Oral 5
- *Baryon magnetic moments at NNLO* Yang Xiao, Oral 10

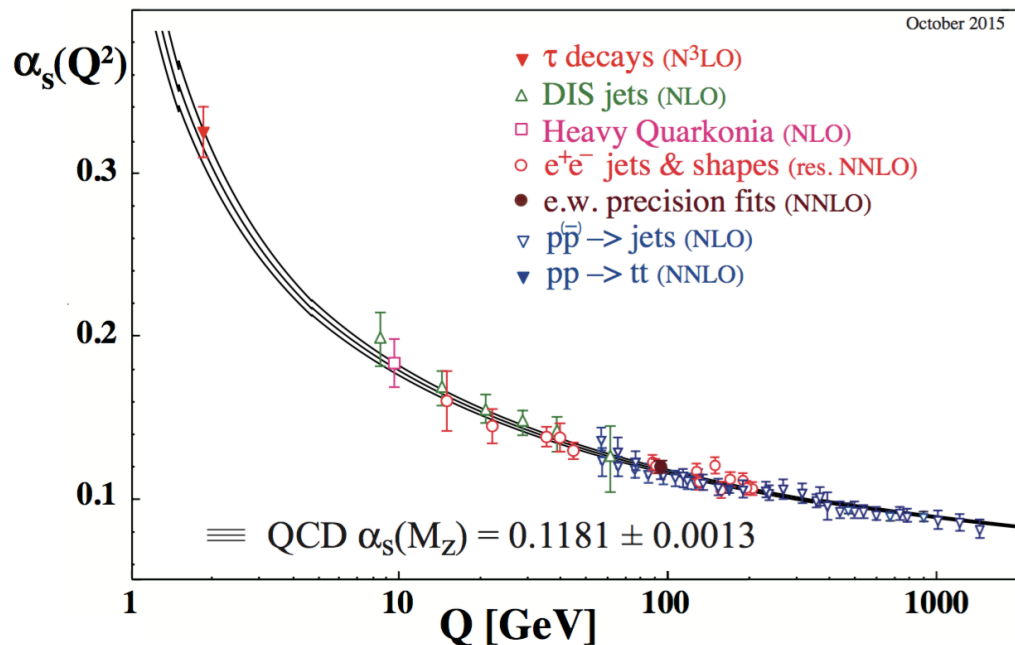
❖ Summary

QCD—theory of the strong interaction

- Simple: four parameters
- **Difficult to be applied to solve low-energy strong interaction physics**

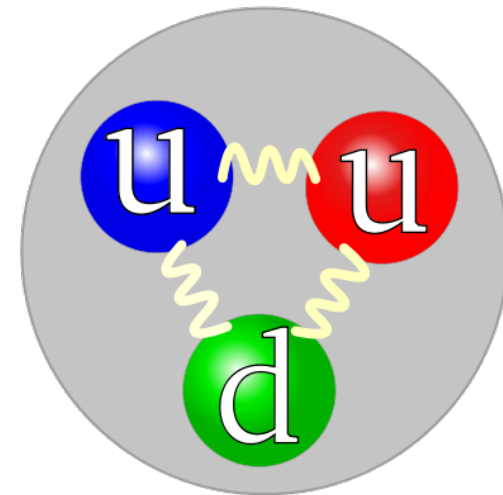
$$\begin{aligned}
 L_{QCD} = & -\frac{1}{4}(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu)(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) + \sum_f \bar{q}_f^\alpha (i\gamma^\mu \partial_\mu - m_f) q_f^\alpha \\
 & + g_s G_a^\mu \sum_f \bar{q}_f^\alpha \gamma^\mu \left(\frac{\lambda^a}{2} \right)_{\alpha\beta} q_f^\beta \\
 & - \frac{g_s^2}{2} f^{abc} (\partial^\mu G_a^\nu - \partial^\nu G_a^\mu) G_\mu^b G_\nu^c - \frac{g_s^2}{4} f^{abc} f_{ade} G_b^\mu G_\nu^c G_\mu^d G_\nu^e
 \end{aligned}$$

Asymptotic freedom



Nobel prize in physics 2004

Color confinement



Clay Mathematics Institute seven million dollar problems, 2000

Chiral perturbation theory

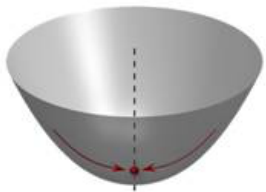


Chiral symmetry : QCD Lagrangian invariant under

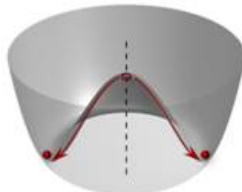
1979

$$SU(3)_L \times SU(3)_R$$

Unbroken Symmetry



Broken Symmetry



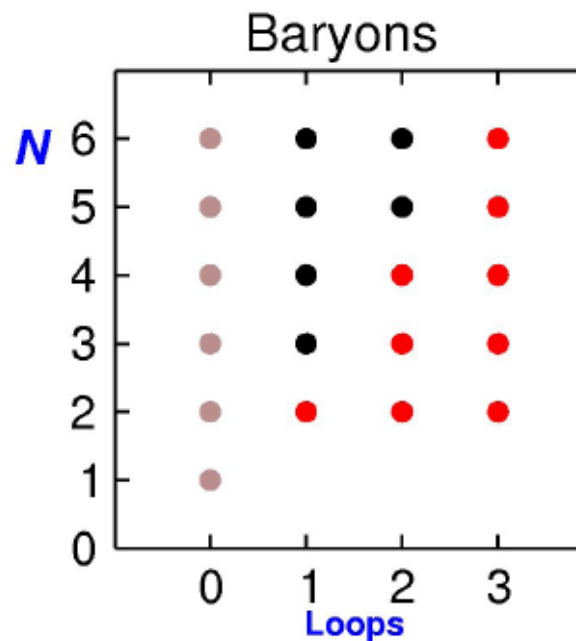
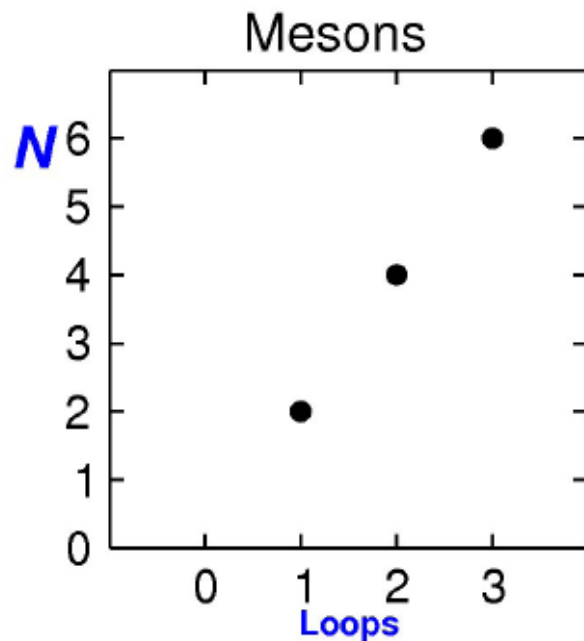
- **Spontaneously** broken: pseudoscalar nonet
- **Explicitly** broken: small mass, perturbative

In short

- Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons
- Perturbative formulation of low energy QCD in powers of the external momenta and the light quark masses, instead of the running coupling constant

Power-counting-breaking (PCB) in the one-baryon sector

- ChPT very successful in the study of Nambu-Goldstone boson self-interactions. (at least in SU(2))
- In the one-baryon sector, things become problematic because of the **nonzero (large)** baryon mass in the chiral limit, which leads to the fact that high-order loops contribute to lower-order results, i.e., **a systematic power counting is lost!**

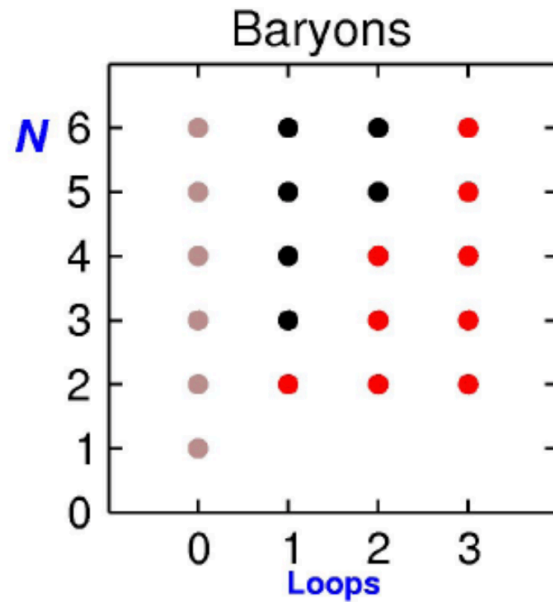


red dots denote
possible
PCB terms (pion-
nucleon scattering)

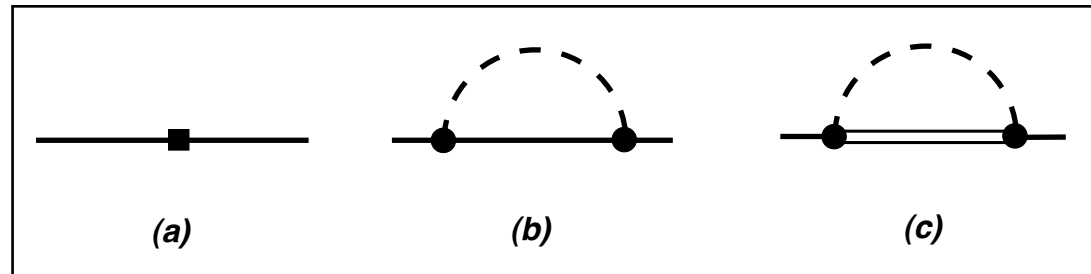
*J. Gasser et al.,
NPB 307, 779(1988)*

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.$$

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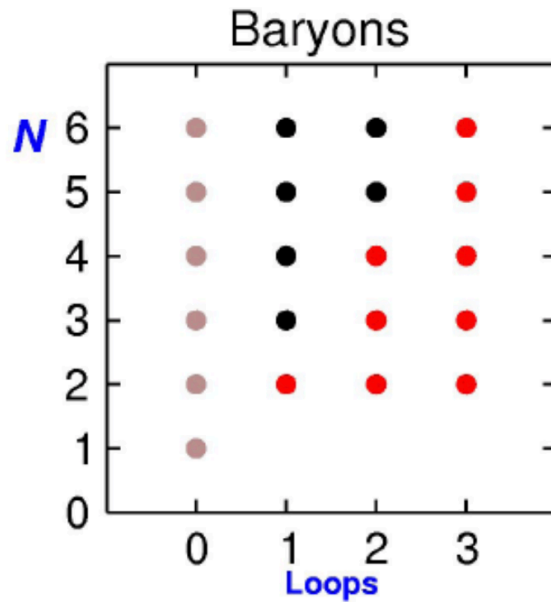


Nucleon mass up to $O(p^3)$

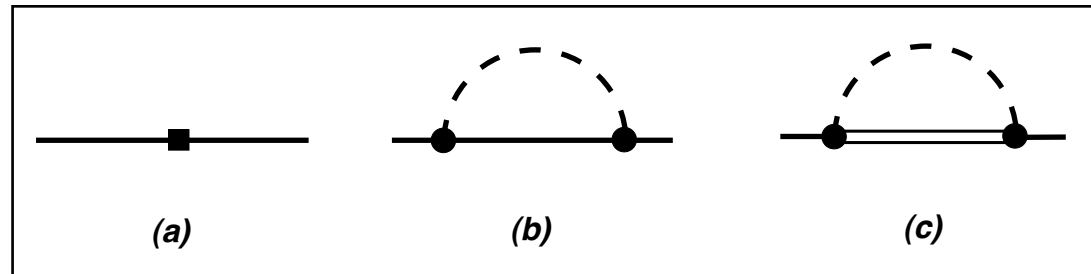


$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k k V_k$$



Nucleon mass up to $O(p^3)$



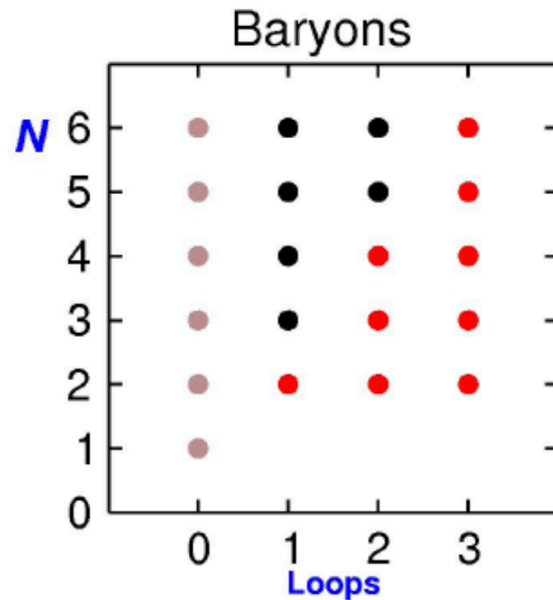
$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

Naively
(no PCB)

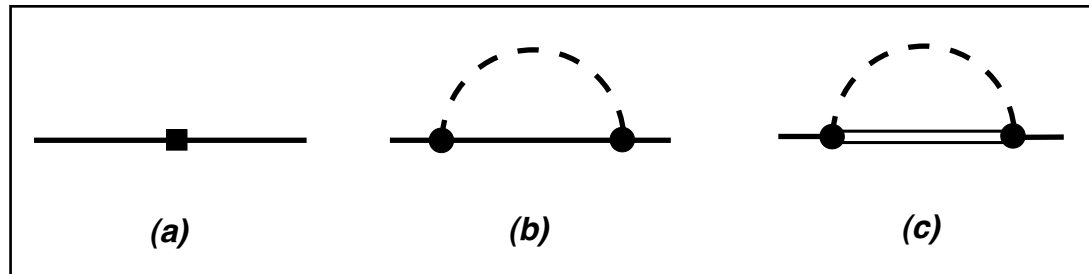
$$M_N = M_0 + bm_\pi^2 + \text{loop}$$

$$\text{loop}(= cm_\pi^3 + \dots)$$

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k$$



Nucleon mass up to $O(p^3)$

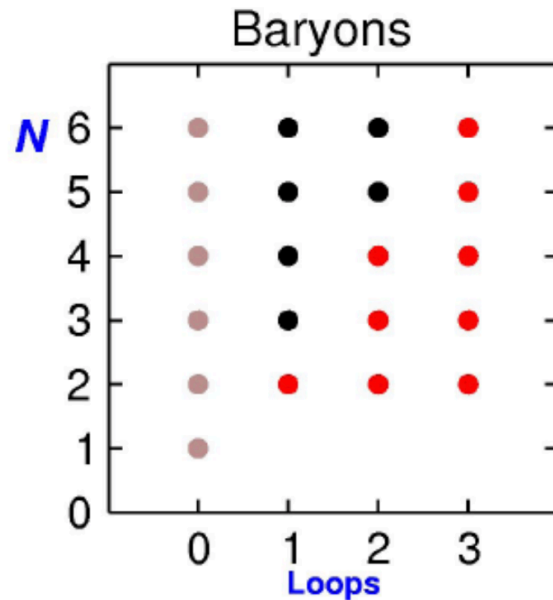


$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

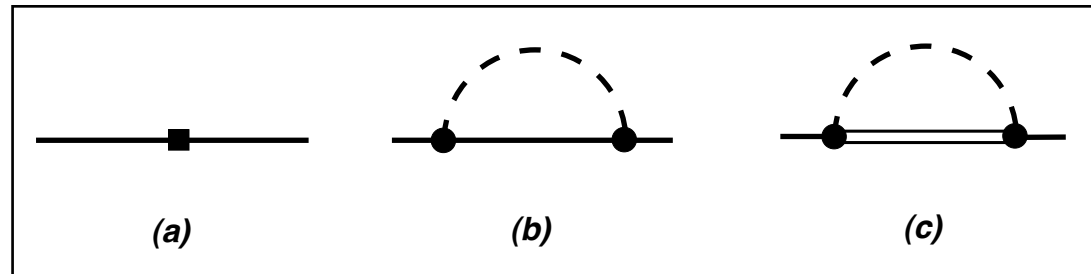
Naively
(no PCB) $M_N = M_0 + bm_\pi^2 + \text{loop}$
 $\text{loop}(= cm_\pi^3 + \dots)$

However $\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots$

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.$$



Nucleon mass up to $O(p^3)$



$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

Naively
(no PCB)

$$M_N = M_0 + bm_\pi^2 + \text{loop}$$

$$\text{loop}(= cm_\pi^3 + \dots)$$

However

$$\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots$$

No need to calculate, simply recall that $M_0 \sim O(p^0)$

Power-counting-restoration methods

- **Heavy baryon ChPT:** baryons treated “semi-relativistically” (Jenkins et al., 1991, Bernard et al., 1992)
- **Relativistic baryon ChPT:** removing power counting breaking terms but retaining higher order corrections, thus keeping relativity
 - **Infrared baryon ChPT:** Becher & Leutwyler 1999
 - **Extended on mass shell (EOMS) scheme:** Gegelia 1999, Fuchs 2003

HB & Infrared

- **HB:** a simultaneous expansion in terms of external momenta and $1/m_B$
- **Infrared:** separating the full Feynman integral into a regular part and finite part

$$H = \frac{1}{ab} = \int_0^1 dz \frac{1}{[(1-z)a + zb]^2} \equiv I + R = \int_0^\infty \dots dz - \int_1^\infty \dots dz$$

HB & Infrared

- **HB:** a simultaneous expansion in terms of external momenta and $1/m_B$
- **Infrared:** separating the full Feynman integral into a regular part and finite part

$$H = \text{Infrared}$$

Extended-on-Mass-Shell (EOMS)

- “Drop” the PCB terms

$$\boxed{\text{tree} = M_0 + b m_\pi^2} \quad + \quad \boxed{\text{loop} = a M_0^3 + b' M_0 m_\pi^2 + c m_\pi^3 + \dots}$$

$$\Downarrow a = 0; b' = 0$$

$$\boxed{M_N = M_0 + b m_\pi^2 + c m_\pi^3 + \dots \quad (\mathcal{O}(p^3))}$$

Extended-on-Mass-Shell (EOMS)

- **“Drop” the PCB terms**

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$$\boxed{M_N = M_0 + b m_\pi^2 + cm_\pi^3 + \dots \quad (\mathcal{O}(p^3))}$$

- **Equivalent to redefinition of the LECs**

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots}$$

$$\Downarrow M_0^r = M_0(1 + aM_0^2); b^r = b^0 + b'M_0$$

$$\boxed{M_N = M_0^r + b^r m_\pi^2 + cm_\pi^3 + \dots \quad (\mathcal{O}(p^3))}$$

Extended-on-Mass-Shell (EOMS)

- “Drop” the PCB terms

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots}$$

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- Equivalent to redefinition of the LECs

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots}$$

ChPT contains all possible terms allowed by symmetries, therefore whatever analytical terms come out from a loop amplitude, they must have a corresponding LEC

HB vs. Infrared vs. EOMS

- Heavy baryon (HB) ChPT
 - non-relativistic
 - breaks analyticity of loop amplitudes
 - converges slowly (particularly in three-flavor sector)
 - strict PC and simple non-analytical results
- Infrared BChPT
 - breaks analyticity of loop amplitudes
 - converges slowly (particularly in three-flavor sector)
 - analytical terms the same as HBChPT
- Extended-on-mass-shell (EOMS) BChPT
 - satisfies all symmetry and analyticity constraints
 - converges relatively faster--an appealing feature

Some successful applications of covariant BChPT (in the 3f sector)

❖ Magnetic moments

PRL101:222002,2008; PLB676:63,2009; PRD80:034027,2009

❖ Masses and sigma terms

PRD82:074504,2010; PRD84:074024,2011; JHEP12:073,2012;
PRD 87:074001,2013; PRD89:054034,2014 ; EPJC74:2754,2014 ;
PRD91:051502,2015; Phys.Lett. B766:325, (2017)

❖ Vector form factors (couplings)

PRD79:094022,2009; PRD89:113007,2014

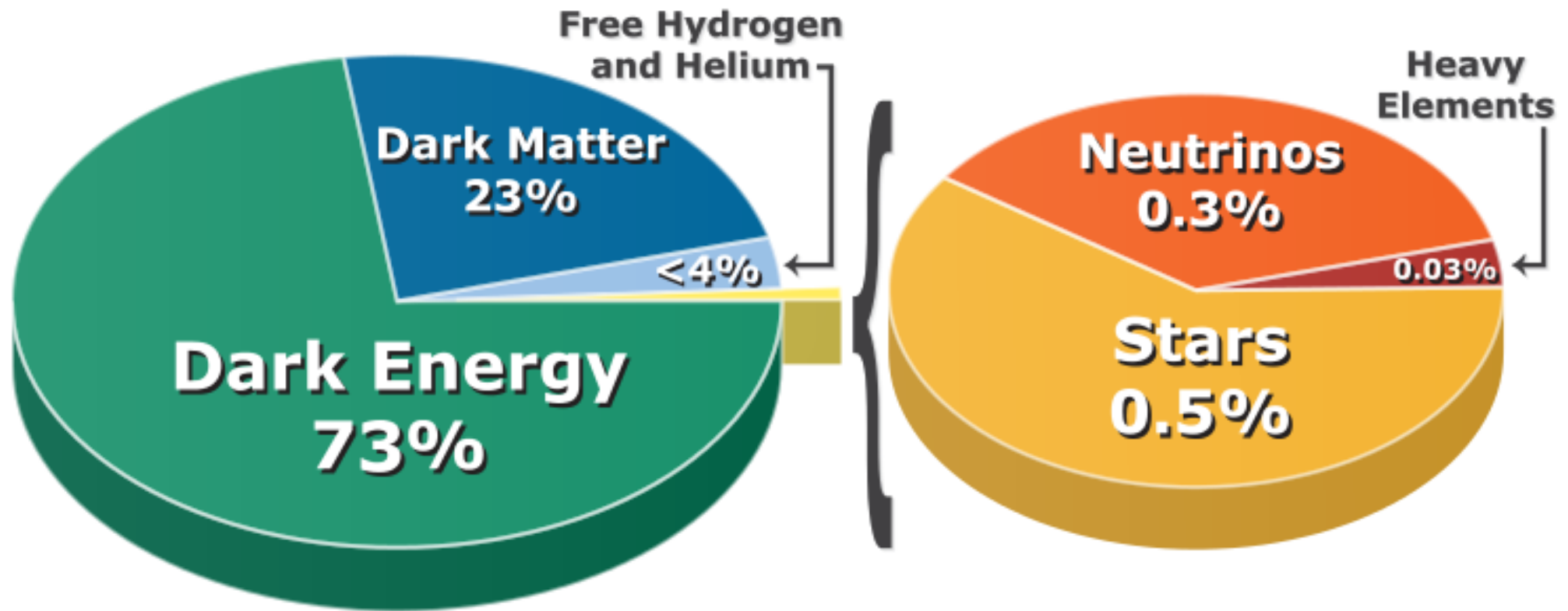
❖ Axial form factors (couplings)

PRD78:014011,2008; PRD90:054502,2014

Motivation: why sigma terms

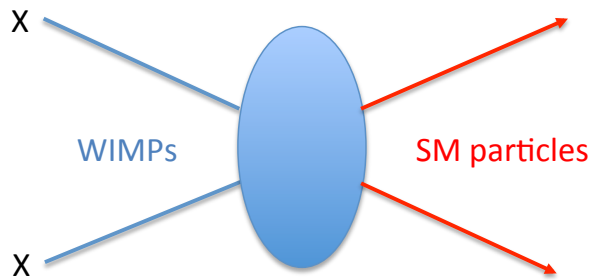
Energy/matter composition of the universe

Plank: 1303.5062

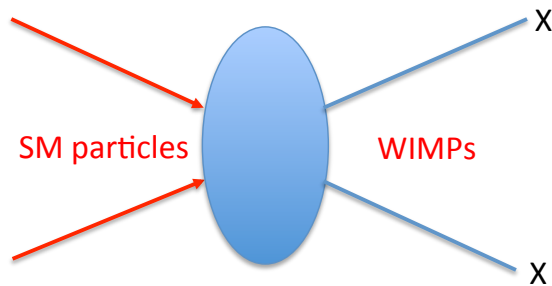
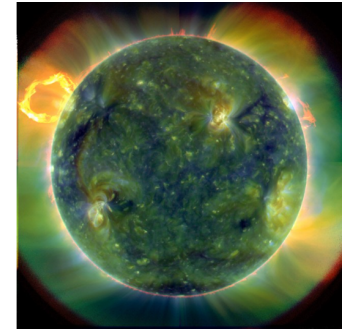


- **Weakly Interacting Massive Particles (WIMPS)**
e.g., Neutralino in MSSM.

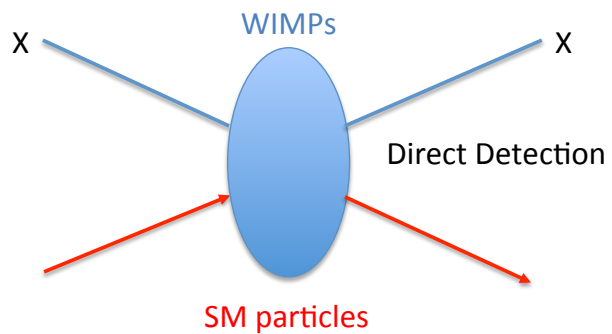
Particle searches for WIMPs



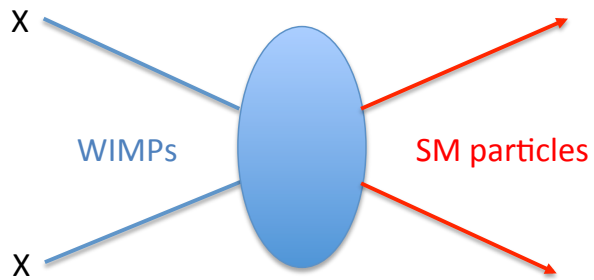
Indirect Detection



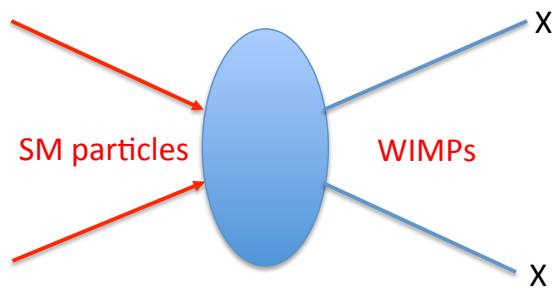
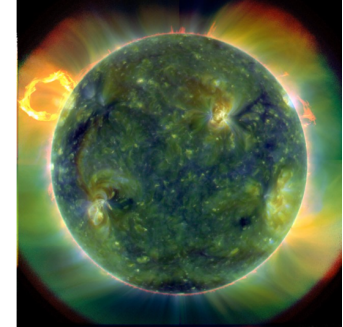
Collider Searches



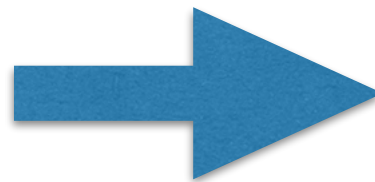
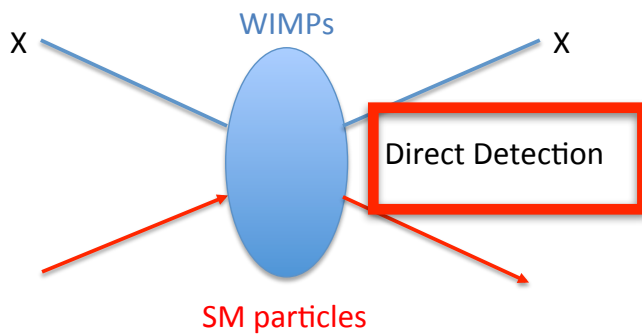
Particle searches for WIMPs



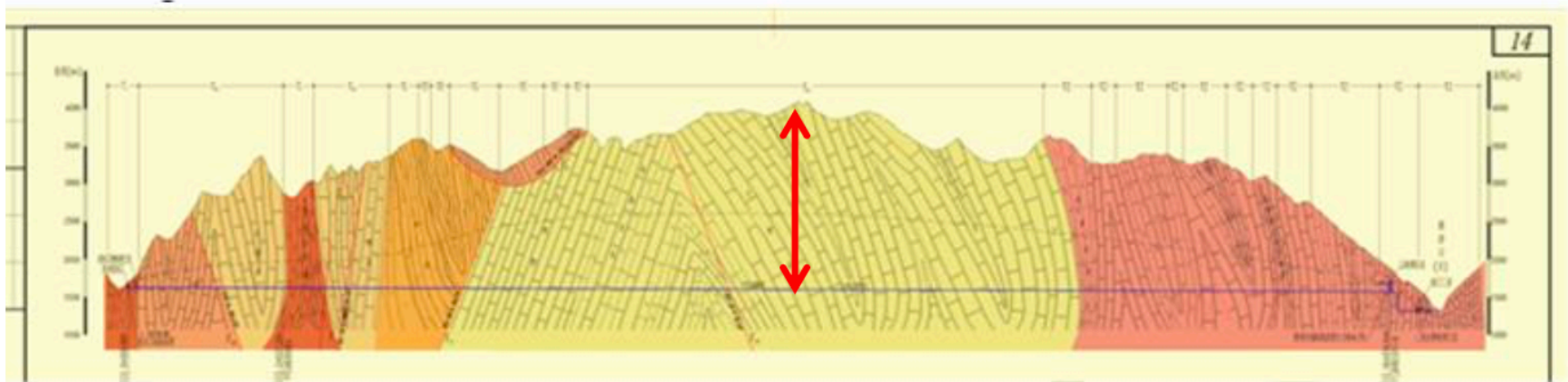
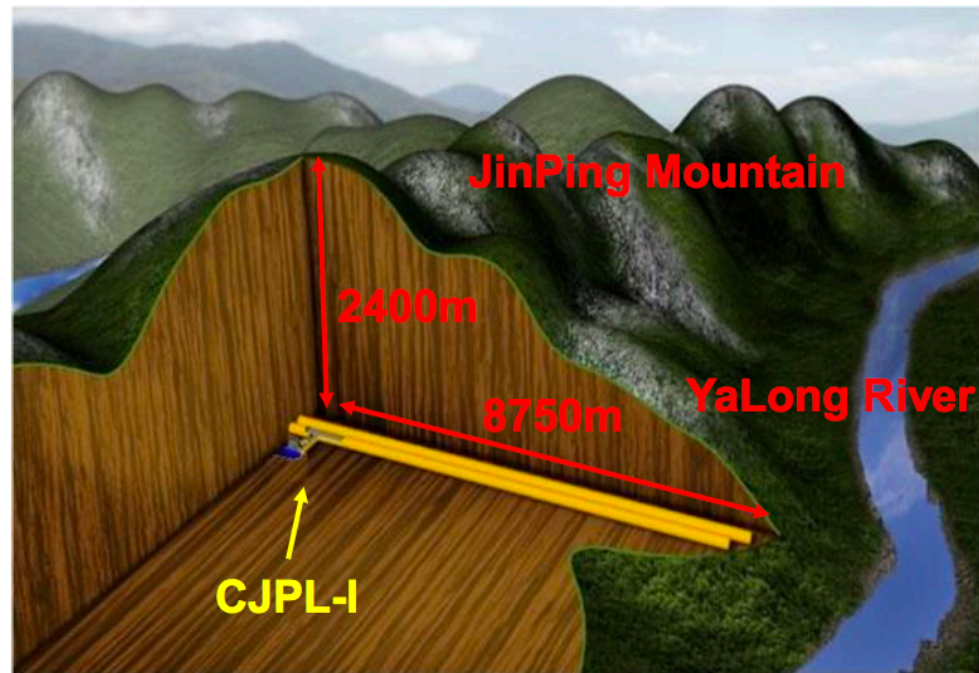
Indirect Detection



Collider Searches



Direct dark matter detection in China



China JinPing Laboratory (CPJL): deepest and largest underground lab. in the world; PandaX and CDEX

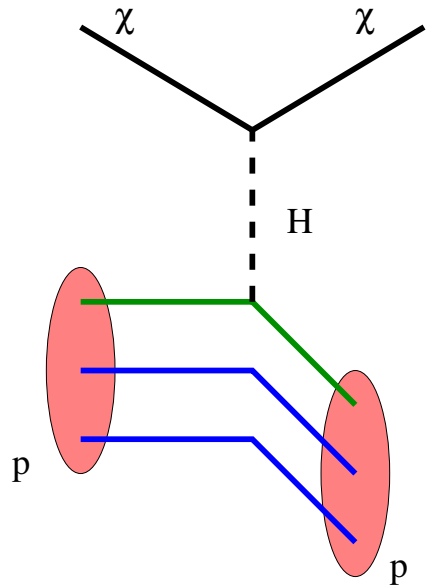
Indirect dark matter detection in China

DAMPE (DARk Matter Particle Explore), Wukong (悟空), launched on December 17, 2015



Spin-independent neutralino-nucleon scattering

MSSM



$$\mathcal{L}_{int} = \lambda_N \bar{n} n \bar{\chi} \chi \rightarrow \mathcal{L}_{int} = \lambda_q \bar{q} q \bar{\chi} \chi$$

$$\lambda_N \longrightarrow \sum_{q=1}^6 f_q^N \lambda_q$$

Spin indep. WIMP-N X-section

$$\sigma_{SI} = \frac{4M^2}{\pi} [Z f_P + (A - Z) f_N]$$

with

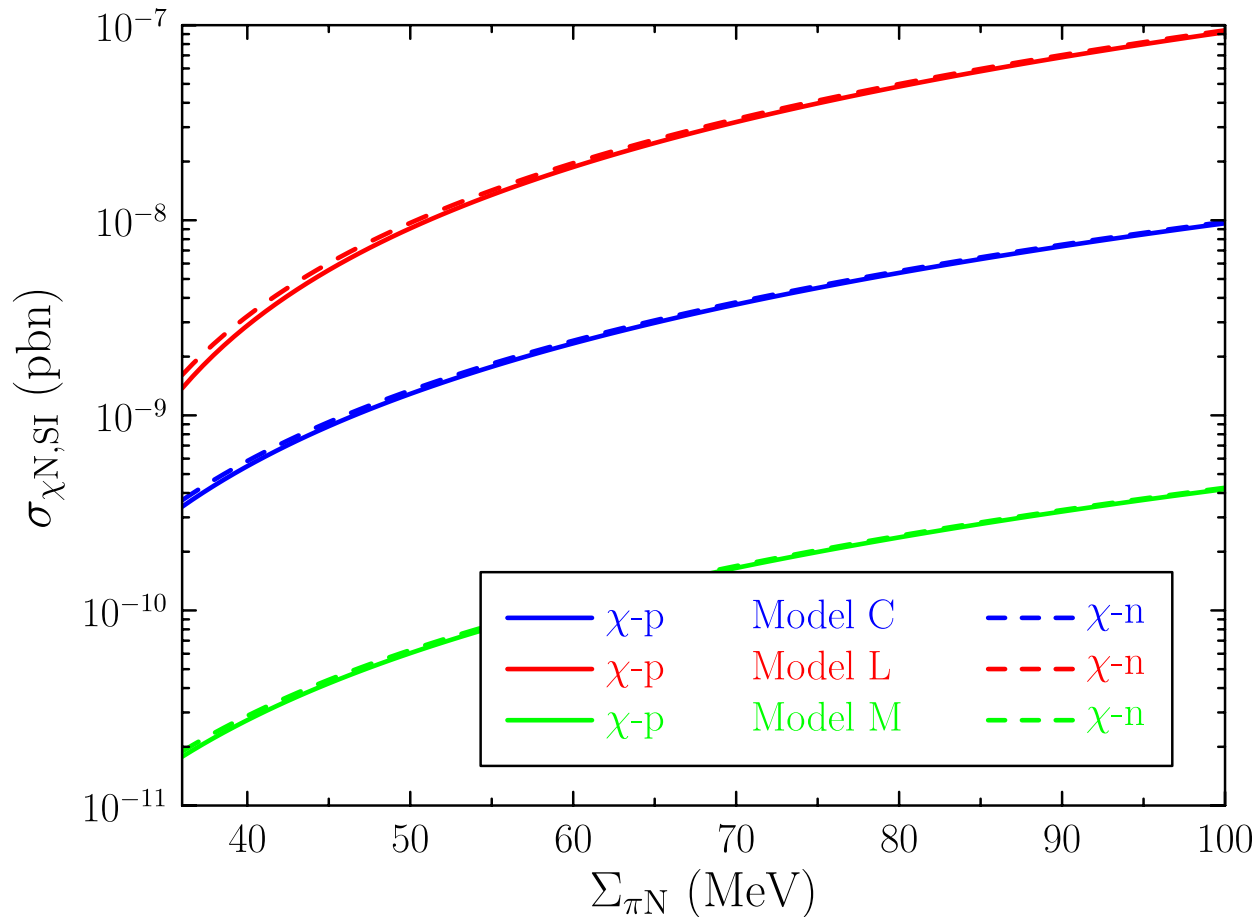
$$\frac{f_N}{M_N} = \sum_q f_q^N \frac{\lambda_q}{m_q}$$

pion- and strangeness sigma terms

$$f_{ud}^N M_N = \sigma_{\pi N} = m_q \langle N | u \bar{u} + d \bar{d} | N \rangle$$

$$f_s^N M_N = \sigma_{sN} / 2 = m_s \langle N | s \bar{s} | N \rangle$$

Strong dependence on the strangeness sigma term



$$\sigma_{\chi p, \text{SI}} \sim (\Sigma_{\pi N} - \sigma_0)^2.$$

$$\sigma_{sN} \propto \Sigma_{\pi N} - 36$$

Quark-flavor structure of the proton

- Naive quark model—minimal quark contents

$$|p\rangle = |uud\rangle$$

- **In reality,** $|p\rangle = |uud\rangle(1 + |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$

- to the **spin**

- deep-inelastic lepton scattering

- to the **electromagnetic formfactors**

- parity-violating electron-proton scattering

- to the **mass**

- scalar strangeness content, cannot be measured directly $\langle N | s\bar{s} | N \rangle$

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Determination of the scalar (strangeness) content of the nucleon (baryons) using ChPT

How to determine sigma terms

Pion-nucleon sigma term

- Experimentally, [the pion-nucleon sigma term](#) can be inferred from pion-nucleon scattering data at Cheng-Dashen point

$$(s = u = m_N^2, t = 2M_\pi^2)$$

$$\sigma_{\pi N} = 45 \pm 8 \text{ MeV}$$

J. Gasser et al., PLB253,252

$$\sigma_{\pi N} = (59.1 \pm 1.9 \pm 3.0) \text{ MeV}$$

Hoferichter et al., PRL115, 092301

$$\sigma_{\pi N} = 59(7) \text{ MeV}$$

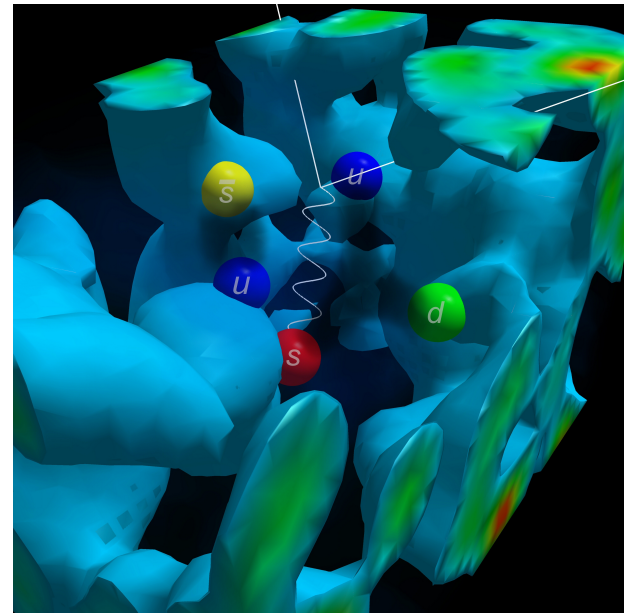
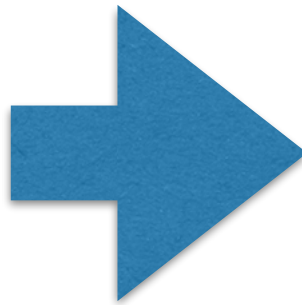
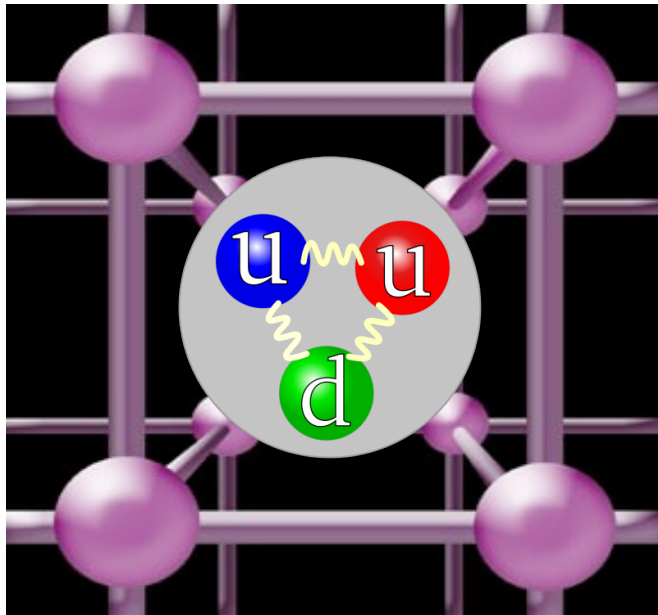
Alarcon et al., PRD 85, 051503(R)

$$\sigma_{\pi N} = 45 \pm 6 \text{ MeV}$$

Chen et al., PRD 87, 054019

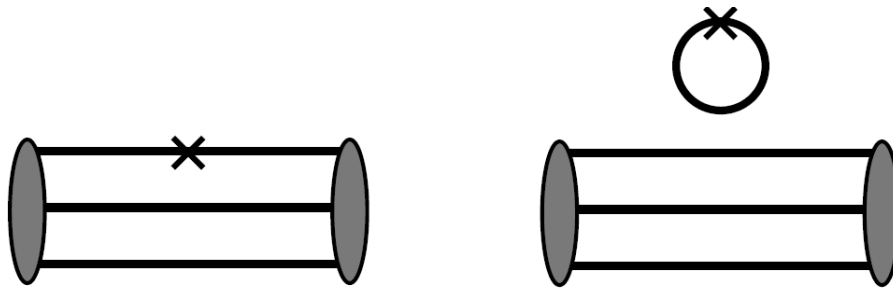
Strangeness sigma term

- Because of lack of kaon-nucleon scattering data, the **strangeness-sigma** term cannot be obtained this way
- **Lattice QCD** might be our hope to predict it from first principles



LQCD determination of sigma terms

- **Direct method**—calculates the 3-point connected and disconnect diagrams



- JLQCD, PRD83,114506 (2011)
- R. Babich *et al.*, PRD85,054510 (2012)
- QCDSF, PRD85, 054502 (2012)
- ETMC, JHEP 1208,037(2012)
- M. Engelhardt *et al.*, PRD86, 114510 (2012)
- JLQCD, PRD87, 034509 (2013)
- ETMC, PRL 116, 252001 (2016)
- RQCD, PRD 93, 094504 (2016)
- χ QCD, PRD94, 054503 (2016)

- **Spectrum method**-calculates the baryon masses, and relates the sigma terms to their quark mass dependence via the **Feynman Hellmann** theorem *R. Young, Plenary 01*

$$\sigma_{\pi B} = m_l \langle B(p) | \bar{u}u + \bar{d}d | B(p) \rangle = m_l \frac{\partial M_B}{\partial m_l}$$

$$\sigma_{sB} = m_s \langle B(p) | \bar{s}s | B(p) \rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

- JLQCD, PRD83,114506 (2011)
- R. Babich *et al.*, PRD85,054510 (2012)
- QCDSF, PRD85, 054502 (2012)
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- M. Engelhardt *et al.*, PRD86, 114510 (2012)
- JLQCD, PRD87, 034509 (2013)
- BMW, PRL 116, 172001 (2016).

Our aim

- To apply the **Feynman-Hellmann** theorem to predict the baryon sigma terms using the **covariant (EOMS) baryon chiral perturbation theory**

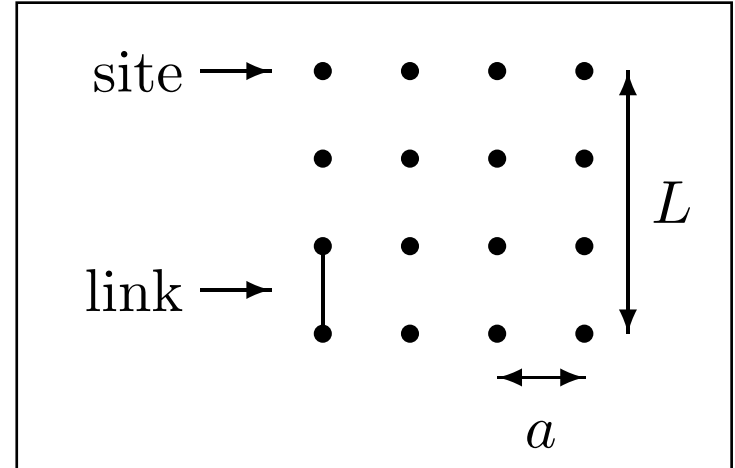
$$\begin{aligned}\sigma_{\pi B} &= m_l \langle B(p) | \bar{u}u + \bar{d}d | B(p) \rangle = m_l \frac{\partial M_B}{\partial m_l} \\ \sigma_{sB} &= m_s \langle B(p) | \bar{s}s | B(p) \rangle = m_s \frac{\partial M_B}{\partial m_s}.\end{aligned}$$

- To fix the unknown low-energy constants of BChPT, we **rely on the IQCD simulations** of baryon masses

LQCD parameters and simulation costs

- light quark masses: m_u/m_d
- lattice spacing: a
- lattice volume: $V=L^4$

$$\text{cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_\pi^2 a}$$



- To reduce cost: employ larger than **physical light quark masses, finite lattice spacing and volume.**
- To obtain **physical** quantities, multiple extrapolations are needed

Multiple extrapolations

- **Chiral extrapolations:** light quark masses to their physical values

$$m_q \rightarrow m_q(\text{Phys.})$$

- **Finite volume corrections:** infinite space-time

$$L \rightarrow \infty$$

- **Continuum extrapolations:** zero lattice spacing

$$a \rightarrow 0$$

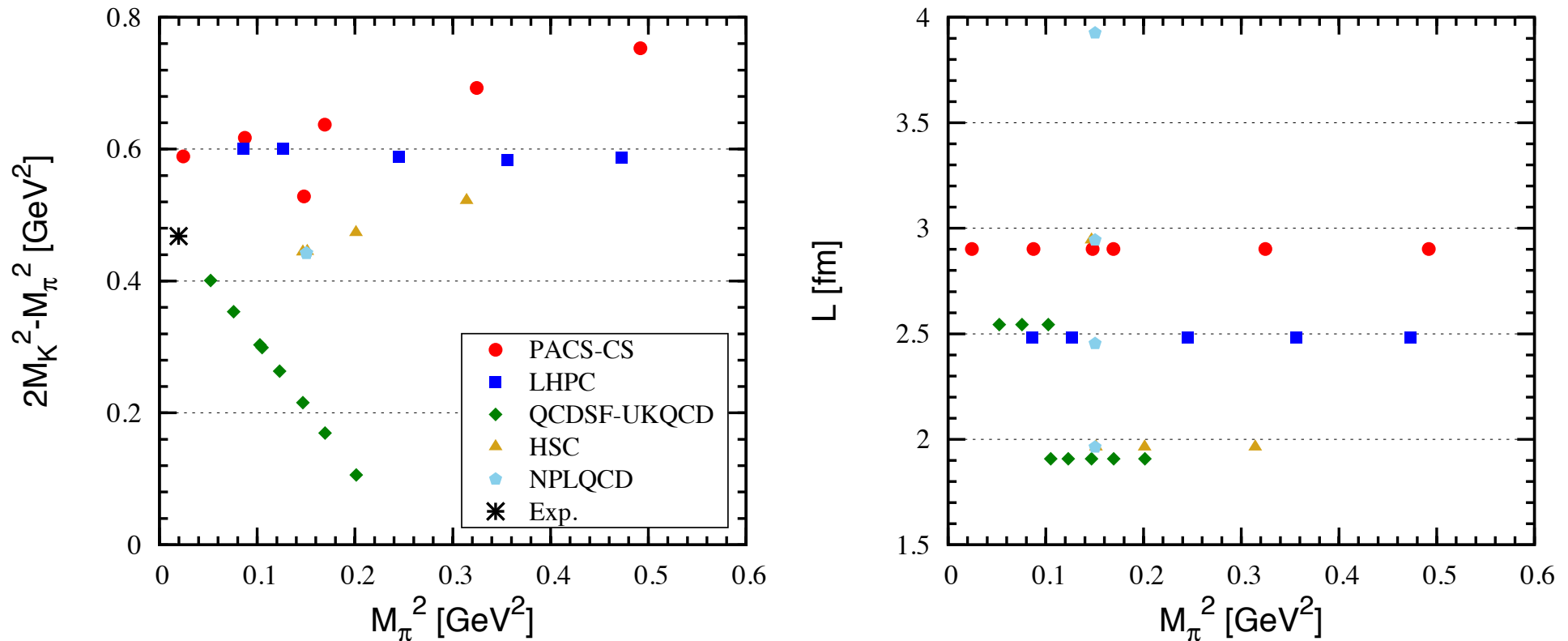
Two key factors for a reliable determination of sigma terms

- A reliable formulation of ChPT, which **not only** can well describe the LQCD data, **but also** needs to satisfy all symmetry and analyticity constraints

Our choice: the EOMS BChPT

- Lattice QCD simulations of baryon masses at **various** quark masses, volumes, and lattice spacings, and with various fermion/gauge actions

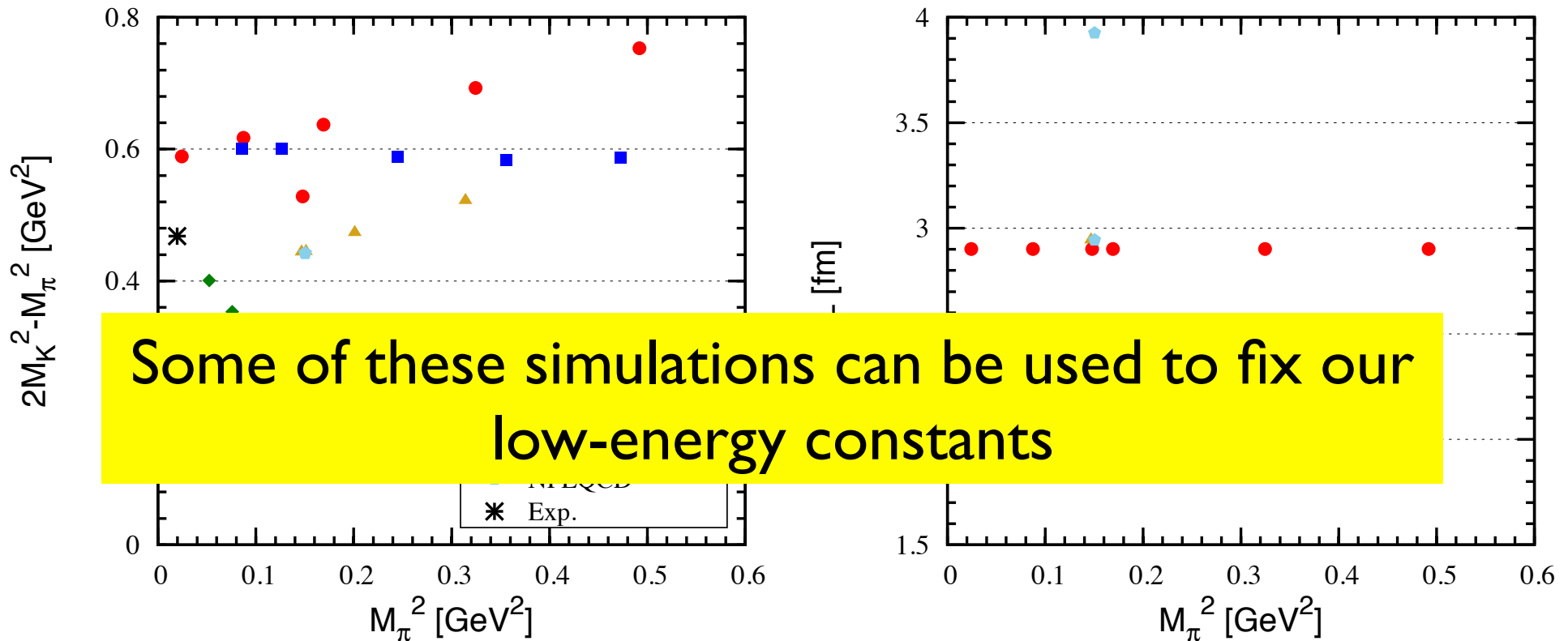
Landscape of latest (before 2014) 2+1f LQCD simulations of g.s. octet baryon



To obtain g.s. baryon masses in the physical world

- Extrapolate to the continuum: $a \rightarrow 0$
- Extrapolate to physical light quark masses: $m_q \rightarrow m_q(\text{Phys.})$
- Extrapolate to infinite space-time: $L \rightarrow \infty$

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A careful selection of LQCD data

- All $n_f=2+1$ LQCD simulations
 - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD, BWM
 - **BMW**—not publicly available
 - HSC/NPLQCD—Low statistics/single combination of quark masses
- We take **PACS-CS, LHPC, QCDSF-UKQCD**

$$\begin{array}{l} m_\pi < 500 \text{ MeV} \\ m_\phi L > 4 \end{array}$$

An accurate determination of baryon sigma terms

- **Scale setting:** mass independent (given by the LQCD simulations or self-consistently determined) vs. mass dependent (r_0 , r_1 , X_π)
- **Isospin breaking effects:** better constrain the LQCD LECs—consistent with the latest BMW study [Science 347 (2015) 1452]
- **Theoretical uncertainties caused by truncating chiral expansions:** NNLO vs. N³LO; EOMS vs. FRR

Systematic study of the LQCD data with the EOMS BChPT

- NNLO EOMS BChPT study of the PACS-CS and LHPC data: [Camalich, LSG, Vacas, PRD82\(2010\)074504](#)
- Finite volume corrections: [LSG, Ren, Camalich, Weise, PRD84\(2011\)074024](#);
- First systematic study of all publically available LQCD data: [Ren, LSG, Camalich, Meng, Toki, JHEP12\(2012\)073](#);
- Effects of virtual decuplet baryons: [Ren, LSG, Meng, Toki, PRD87\(2013\)074001](#)
- Continuum extrapolations: [Ren, LSG, Meng, Eur.Phys.J. C74:2754,2014](#)

Systematic study of the LQCD data with the EOMS BChPT

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- Effects of virtual decuplet baryons: *Ren, LSG, Meng, Toki, PRD87(2013)074001*
- Continuum extrapolations: *Ren, LSG, Meng, Eur.Phys.J. C74:2754,2014*

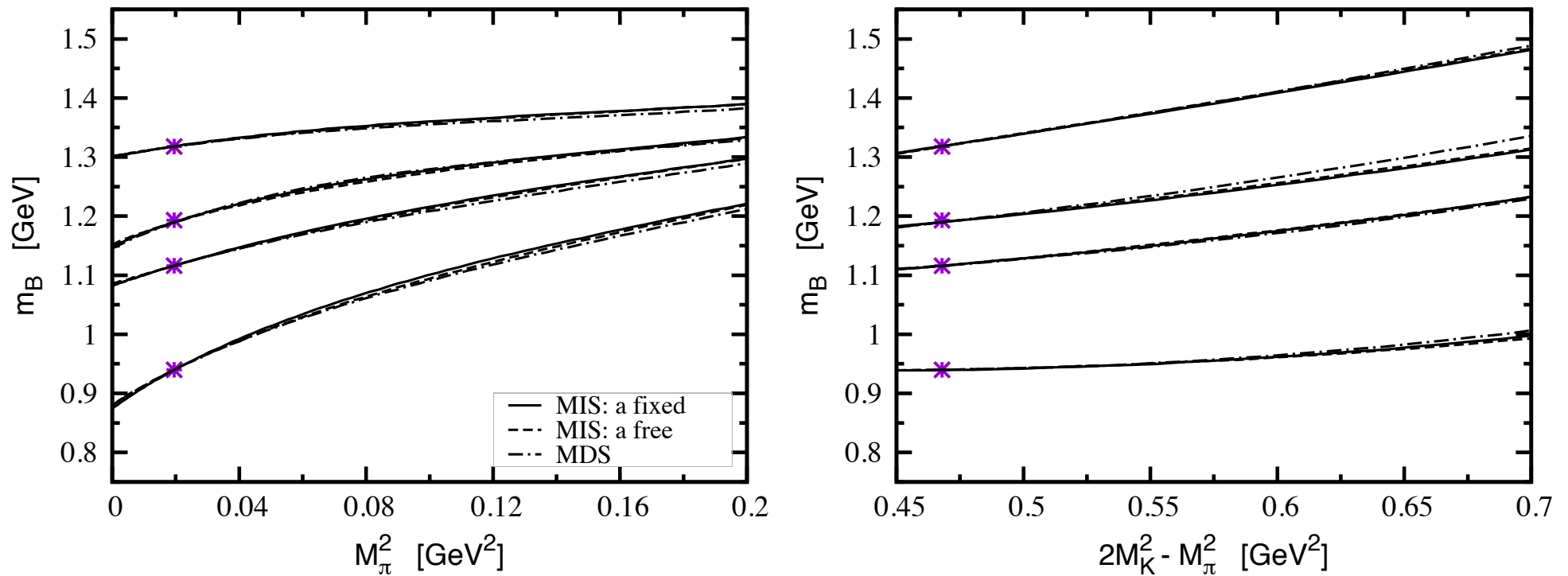
The EOMS BChPT can be trusted to predict
the baryon sigma terms

Three different fits at N³LO

	MIS		MDS
	a fixed	a free	
m_0 [MeV]	884(11)	877(10)	887(10)
b_0 [GeV ⁻¹]	-0.998(2)	-0.967(6)	-0.911(10)
b_D [GeV ⁻¹]	0.179(5)	0.188(7)	0.039(15)
b_F [GeV ⁻¹]	-0.390(17)	-0.367(21)	-0.343(37)
b_1 [GeV ⁻¹]	0.351(9)	0.348(4)	-0.070(23)
b_2 [GeV ⁻¹]	0.582(55)	0.486(11)	0.567(75)
b_3 [GeV ⁻¹]	-0.827(107)	-0.699(169)	-0.553(214)
b_4 [GeV ⁻¹]	-0.732(27)	-0.966(8)	-1.30(4)
b_5 [GeV ⁻²]	-0.476(30)	-0.347(17)	-0.513(89)
b_6 [GeV ⁻²]	0.165(158)	0.166(173)	-0.0397(1574)
b_7 [GeV ⁻²]	-1.10(11)	-0.915(26)	-1.27(8)
b_8 [GeV ⁻²]	-1.84(4)	-1.13(7)	0.192(30)
d_1 [GeV ⁻³]	0.0327(79)	0.0314(72)	0.0623(116)
d_2 [GeV ⁻³]	0.313(26)	0.269(42)	0.325(54)
d_3 [GeV ⁻³]	-0.0346(87)	-0.0199(81)	-0.0879(136)
d_4 [GeV ⁻³]	0.271(30)	0.230(24)	0.365(23)
d_5 [GeV ⁻³]	-0.350(28)	-0.302(50)	-0.326(66)
d_7 [GeV ⁻³]	-0.435(10)	-0.352(8)	-0.322(7)
d_8 [GeV ⁻³]	-0.566(24)	-0.456(30)	-0.459(33)
$\chi^2/\text{d.o.f.}$	0.87	0.88	0.53

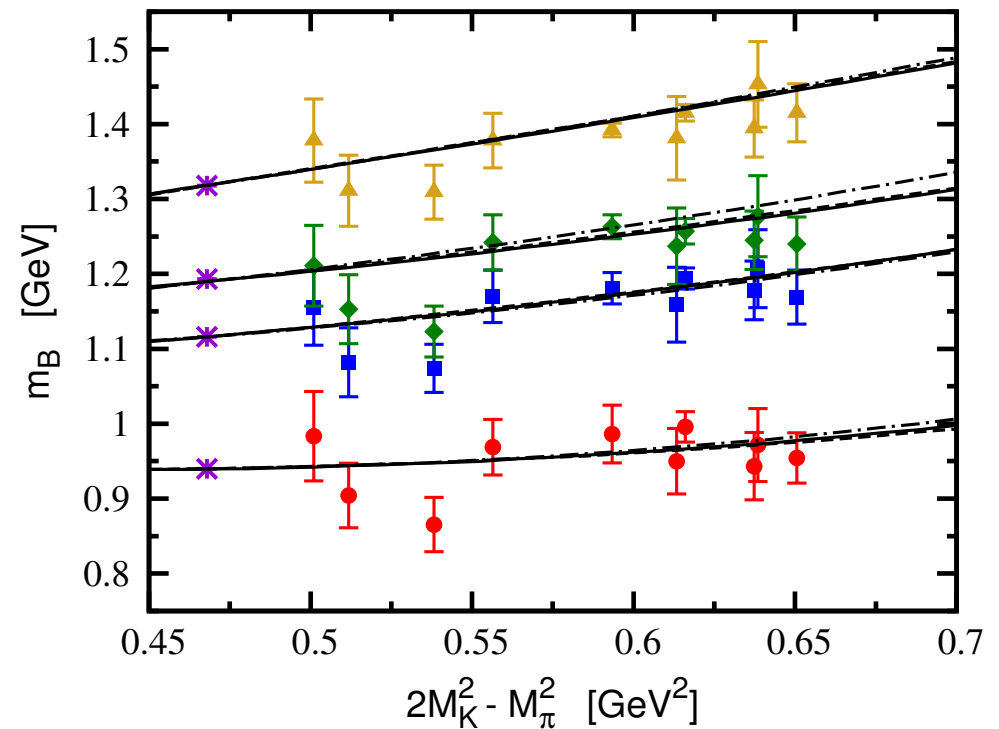
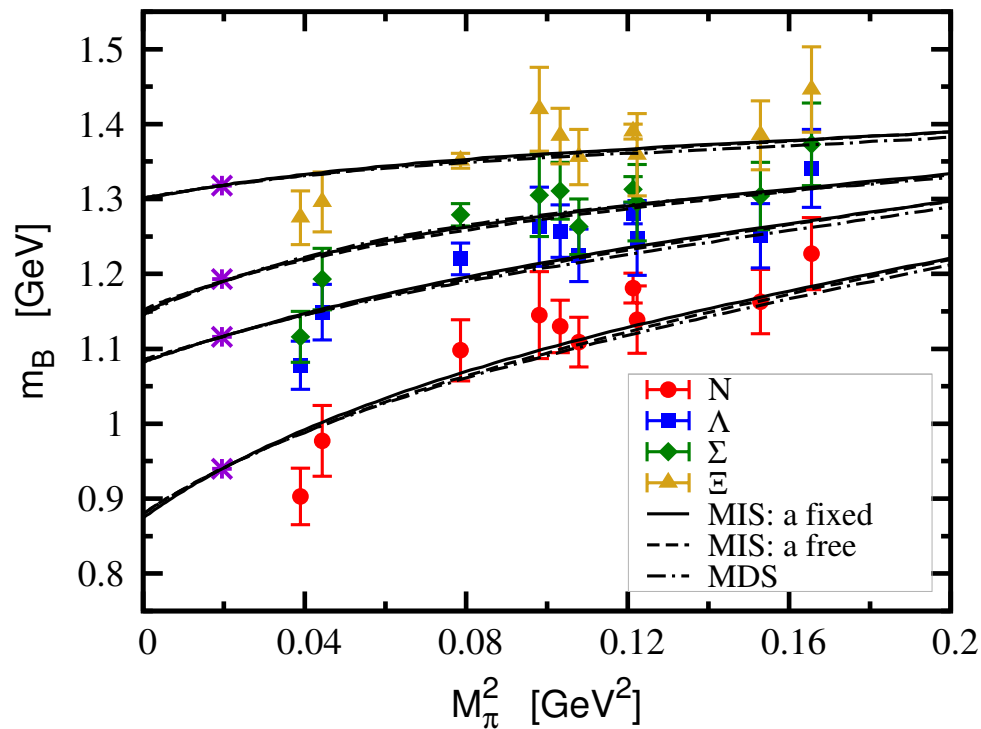
- **Mass independent**
 - Lattice spacing a **fixed** to the published value
 - Lattice spacing a determined **self-consistently**
- **Mass dependent**
 - r_0 for PACS-CS
 - r_1 for LHPC
 - X_π for QCDSF-UKQCD

Evolution of baryon masses with u/d and s quark masses



Only central values are shown!

In comparison with the BMW13 data

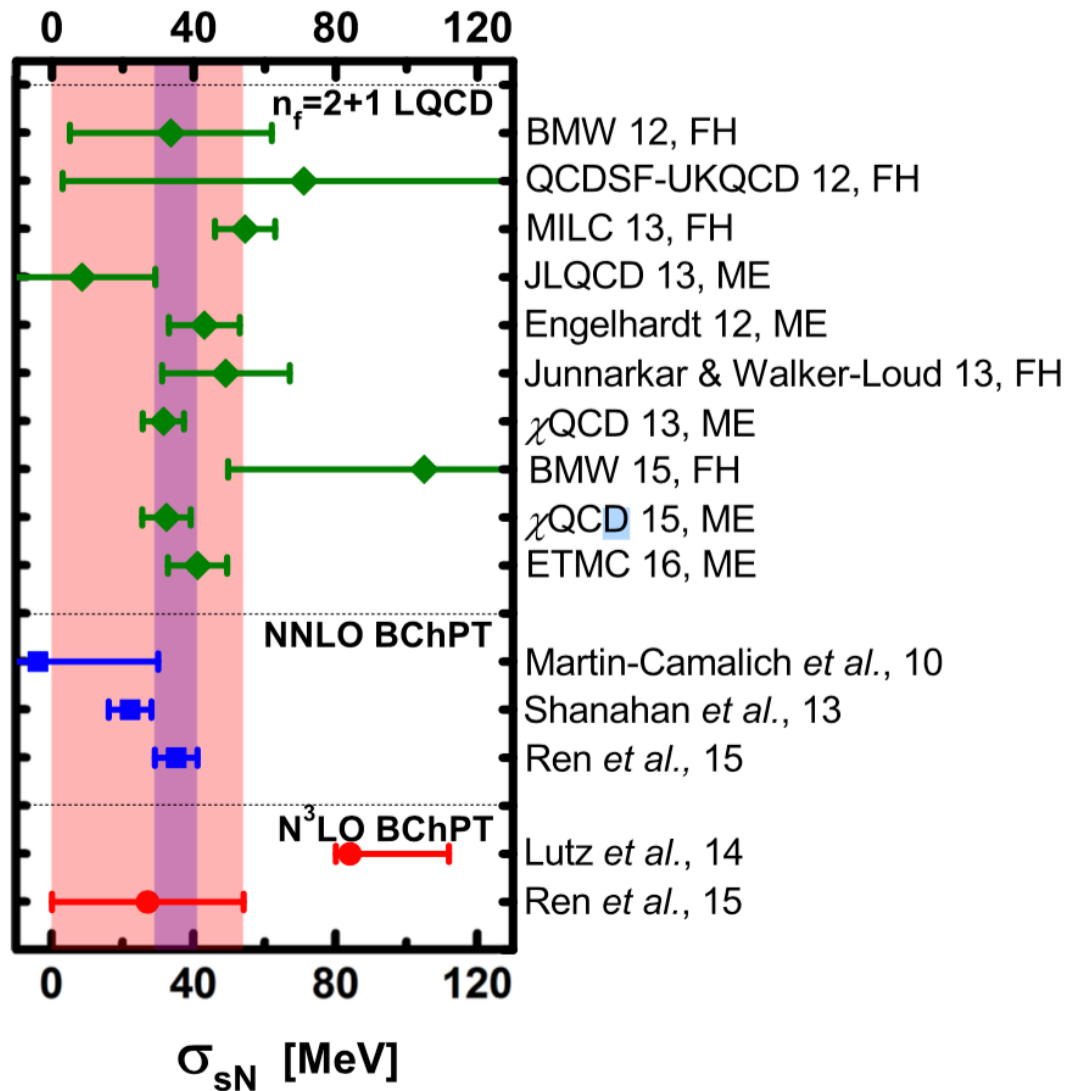


Baryon sigma terms from N³LO BChPT

	MIS		MDS
	a fixed	a free	
$\sigma_{\pi N}$	<u>55(1)(4)</u>	54(1)	51(2)
$\sigma_{\pi \Lambda}$	32(1)(2)	32(1)	30(2)
$\sigma_{\pi \Sigma}$	34(1)(3)	33(1)	37(2)
$\sigma_{\pi \Xi}$	16(1)(2)	18(2)	15(3)
σ_{sN}	<u>27(27)(4)</u>	23(19)	26(21)
$\sigma_{s\Lambda}$	185(24)(17)	192(15)	168(14)
$\sigma_{s\Sigma}$	210(26)(42)	216(16)	252(15)
$\sigma_{s\Xi}$	333(25)(13)	346(15)	340(13)

- All three scale-setting methods yield **similar** baryon sigma terms

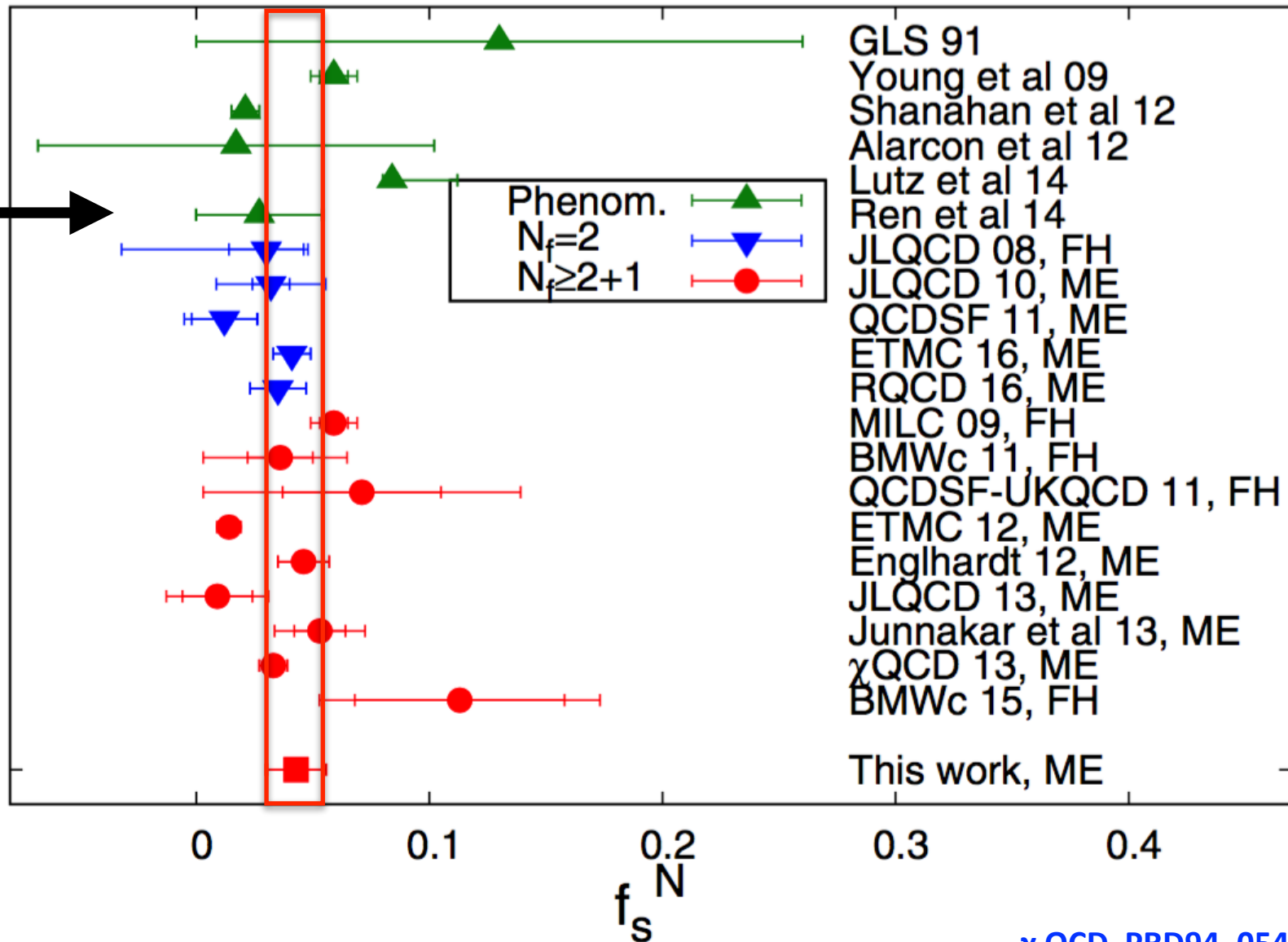
Comparison with other studies



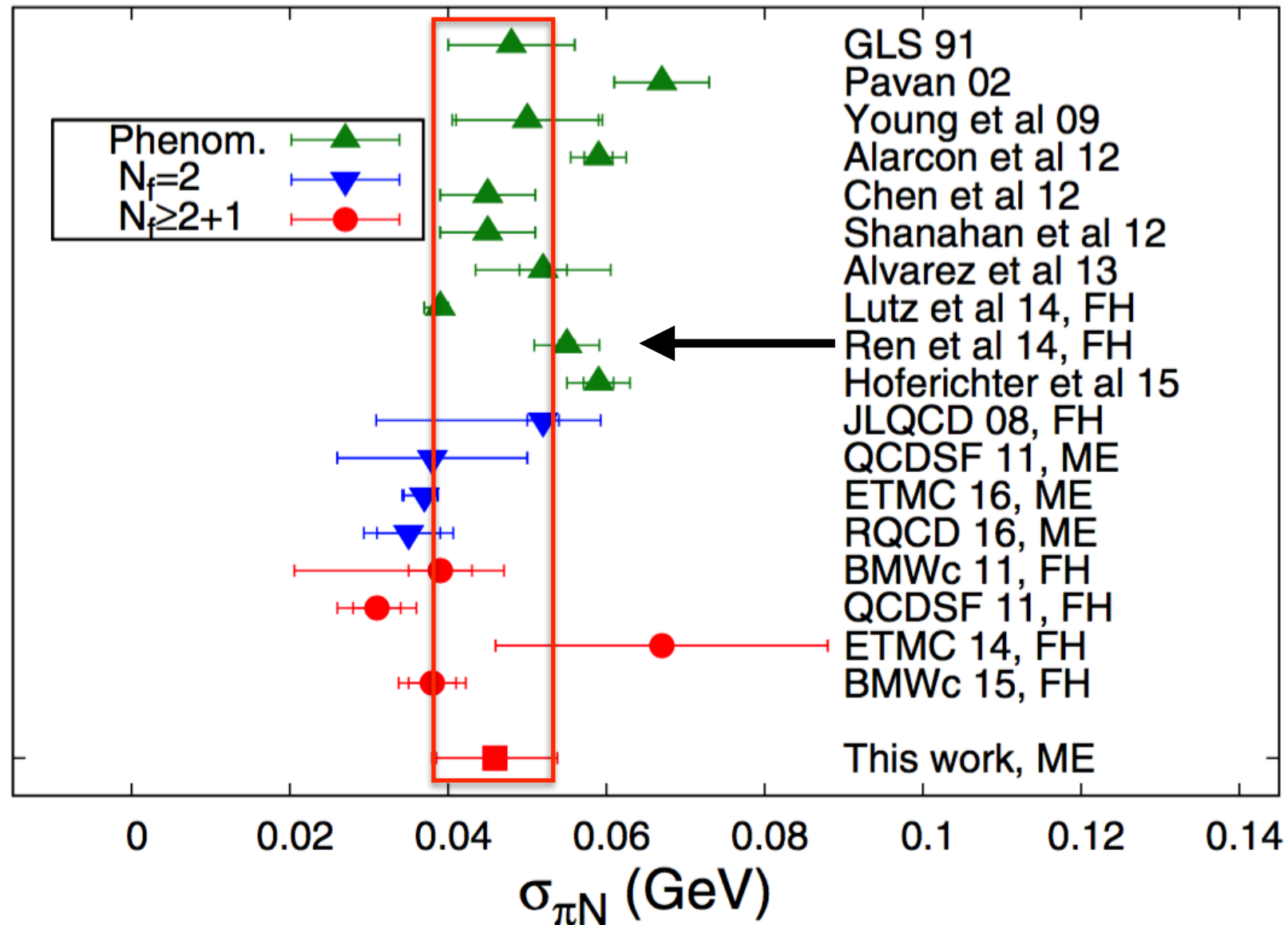
- **Consistent** with most recent LQCD studies and those of NNLO ChPT, e.g., that of Young and Shanahan
- **Uncertainties** at N^3 LO substantially **larger**, because of the extra LECs

Nucleon Strangeness Sigma Term

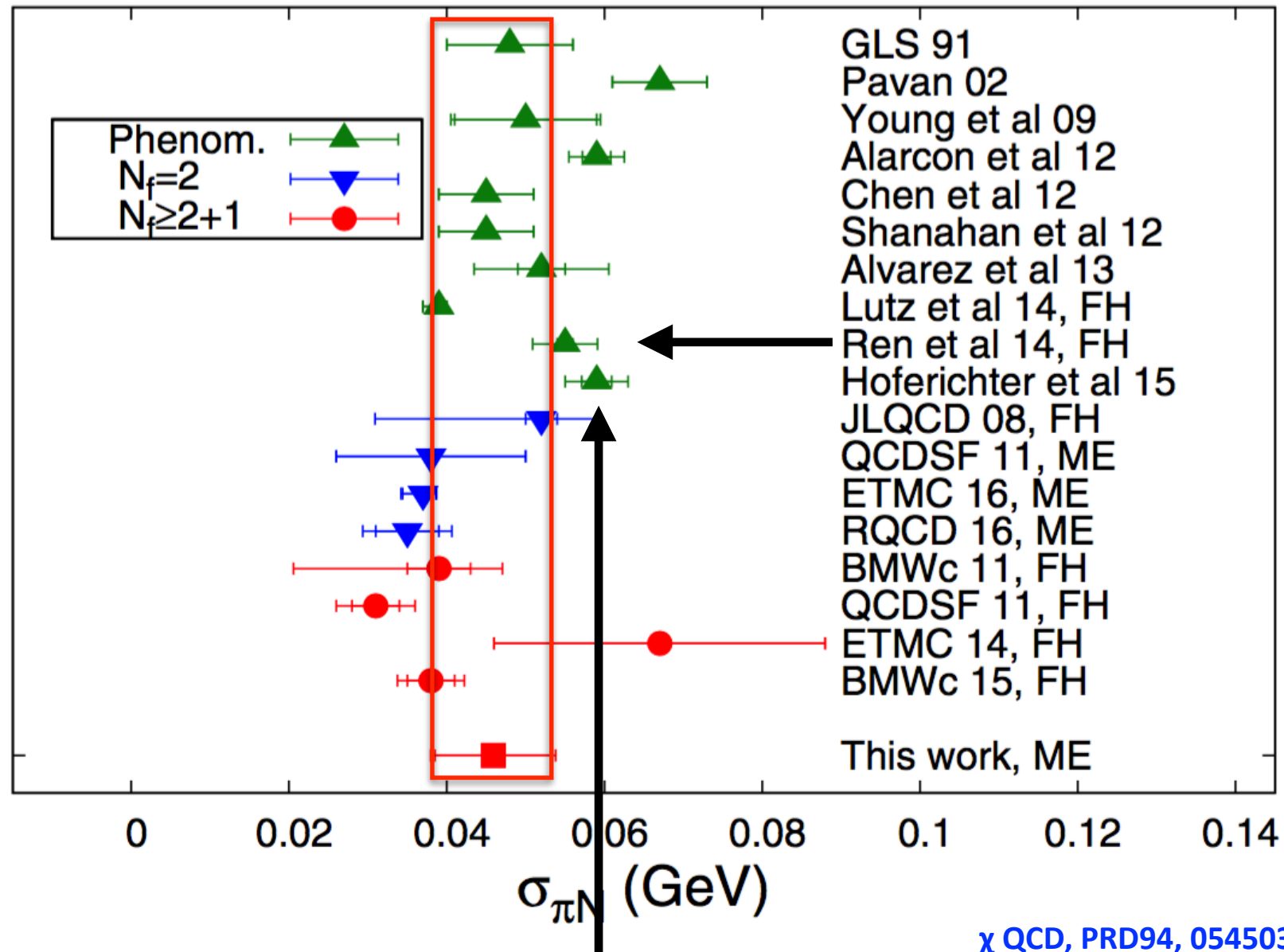
Strangeness-nucleon sigma term



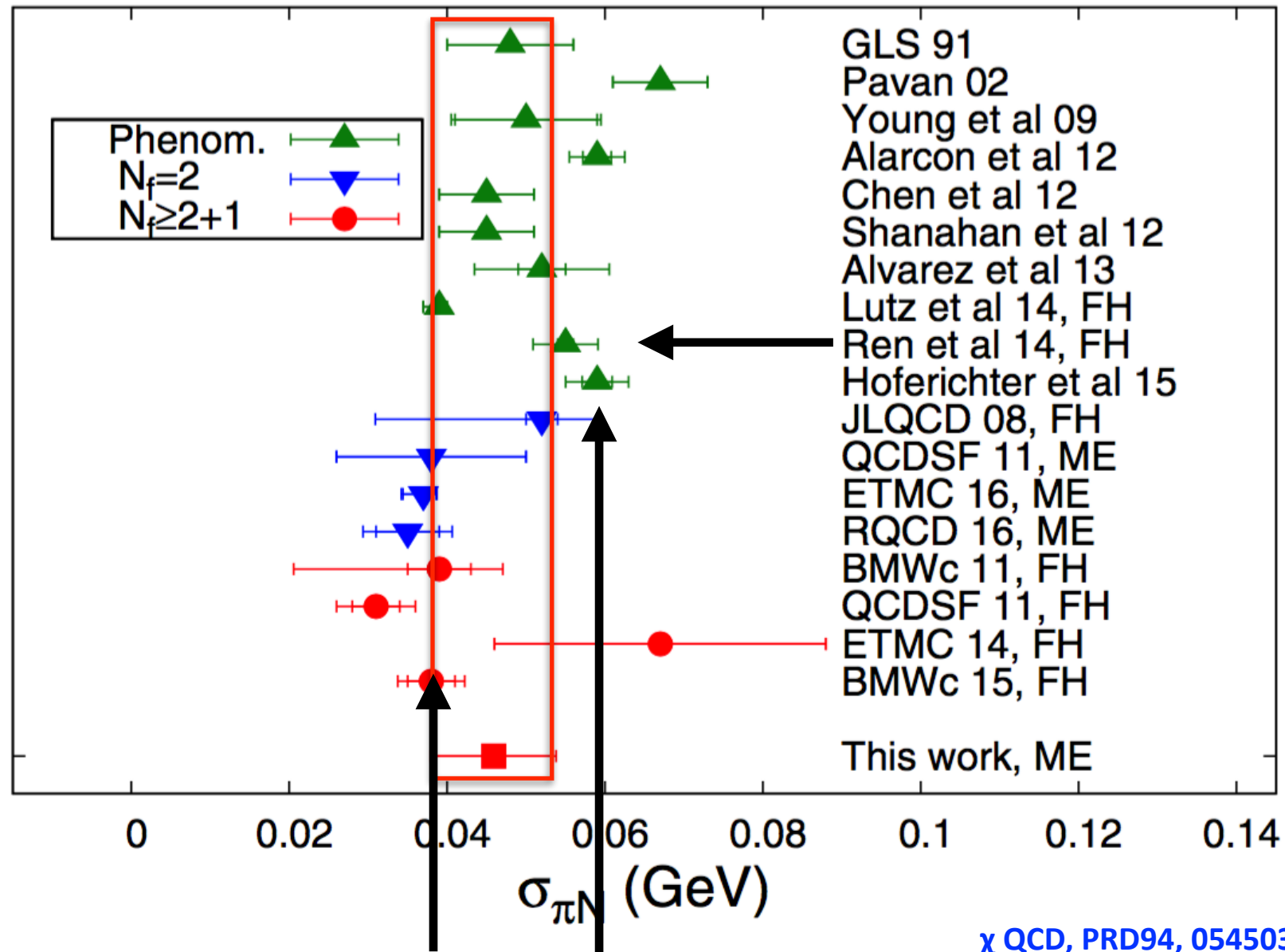
Pi-N sigma term still controversial



Pi-N sigma term still controversial



Pi-N sigma term still controversial



A latest determination of pi-N sigma term

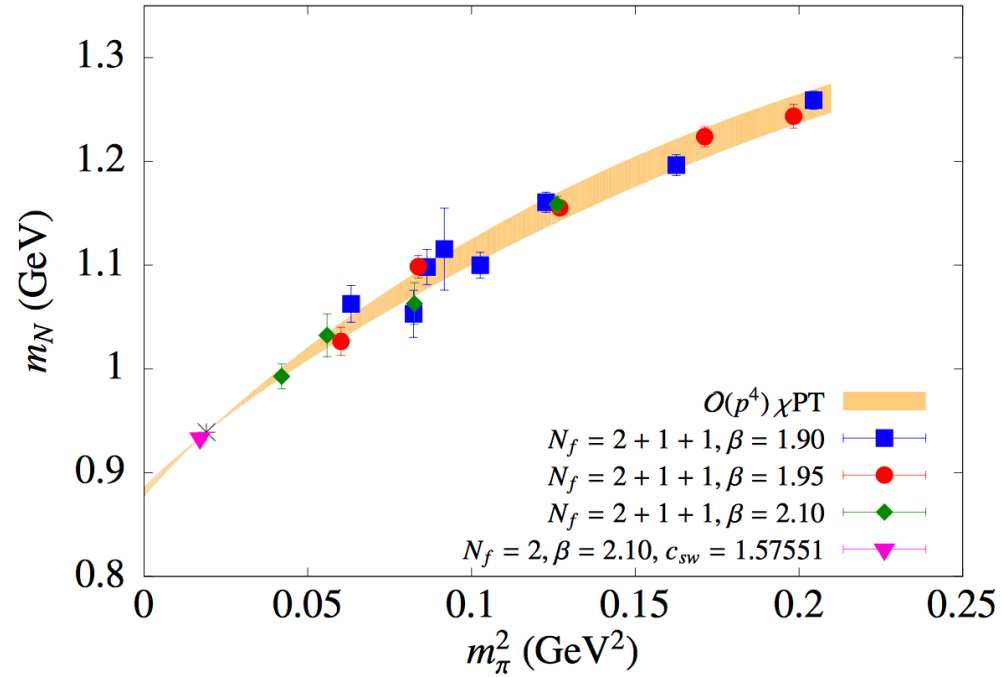
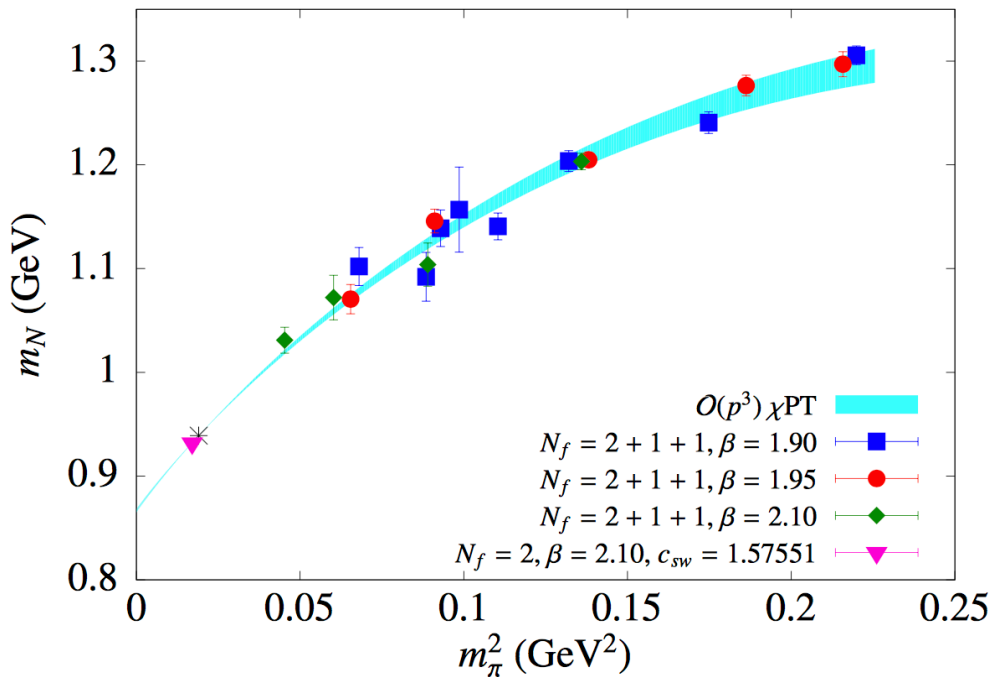
1704.02647

Low-lying baryon masses using $N_f = 2$ twisted mass clover-improved fermions directly at the physical point

C. Alexandrou^(a,b), C. Kallidonis^(b)

^(a) Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus

^(b) Computation-based Science and Technology Research Center, The Cyprus Institute, 20 Kavafi Str., Nicosia 2121, Cyprus



$$\sigma_{\pi N} = 64.9 \pm 1.5 \pm 13.2 \text{ MeV}$$

Further checks

- We have studied **most relevant lattice artifacts**: chiral extrapolation, finite volume effects, finite lattice spacing effects, effects of heavier virtual states, and used all publicly available **LQCD data**
- What else is still **missing**?— an explicit check on the validity of SU(3) baryon chiral perturbation theory
 - large kaon mass leads to concerns about convergence of SU(3) BChPT, as seen in early failures of heavy-baryon BChPT and infrared BChPT

Prominent examples

- Octet baryon magnetic moments and chiral extrapolation of nucleon magnetic moments
 - V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.
 - LSG, J. Martin Camalich , L. Alvarez-Ruso, M.J. Vicente Vacas, Phys.Rev.Lett.101:222002,2008
- lattice QCD baryon masses at leading one-loop order in HBChPT
 - LHPC (A. Walker-Loud et al.), Phys.Rev.D79:054502, 2009.
 - PACS-CS (K.-I. Ishikawa), Phys.Rev.D80:054502, 2009.
 - J. Martin Camalich, LSG, V. Vacas, PRD82(2010)074504

The application of the EOMS formulation seems to remove or at least **alleviate the problem**

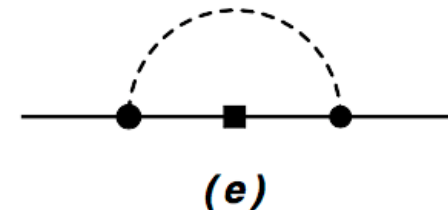
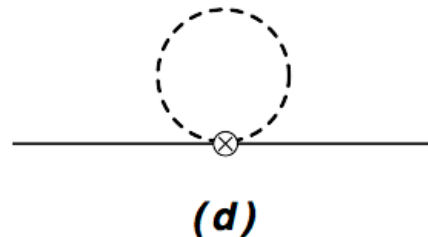
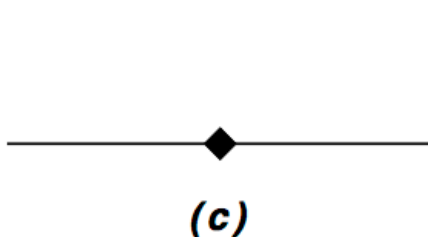
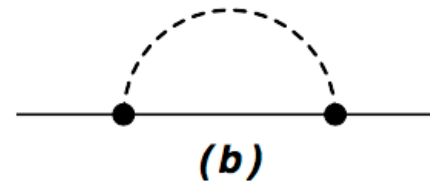
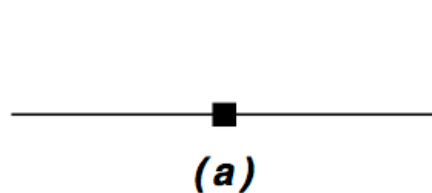
Matching SU(3) to SU(2)

- Take the strange quark mass as a heavy scale and perform an expansion in terms of $m_{u/d}/m_s$ of the SU(3) results and compare them with the results of the SU(2) study
 - Alvarez-Ruso, Ledwig, Camalich, and Vicente-Vacas, PRD88, 054507 (2013)
- An earlier study similar in spirit, but with no quantitative analysis, tried to constrain the SU(3) LECs with SU(2) inputs
 - M. Frink and U.-G. Meissner, JHEP 0407, 028 (2004)

The procedure

- In SU(3) up to $O(p^4)$

$$\begin{aligned}
 M_N^{\text{SU}(3)} &= m_0 + m_N^{(2)} + m_N^{(3)} + m_N^{(4)} \\
 &= m_0 + \xi_{N\pi}^{(a)} m_\pi^2 + \xi_{NK}^{(a)} m_K^2 + \xi_{N\pi}^{(c)} m_\pi^4 + \xi_{NK}^{(c)} m_K^4 + \xi_{N\pi K}^{(c)} m_\pi^2 m_K^2 \\
 &\quad + \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi, K, \eta} \left[\xi_{N\phi}^{(b)} H_N^{(b)} + \xi_{N\phi}^{(d)} H_N^{(d)} + \sum_{B=N, \Lambda, \Sigma} \xi_{NB\phi}^{(e)} H_{NB}^{(e)} \right]
 \end{aligned}$$



The procedure

- Isolate the strange quark contribution, introducing a fictitious meson

$$m_{s\bar{s}}^2 = 2B_0 m_s$$

- Leading-order ChPT

$$m_K^2 = \frac{1}{2}(m_\pi^2 + m_{s\bar{s}}^2), \quad m_\eta^2 = \frac{1}{3}(m_\pi^2 + 2m_{s\bar{s}}^2)$$

- Expand the kaon and eta contributions in terms of

$$m_\pi / m_{s\bar{s}}$$

$$\Sigma_{K, \eta}^{(i)} = A_{K, \eta}^{(i)} + B_{K, \eta}^{(i)} m_\pi^2 + C_{K, \eta}^{(i)} m_\pi^4 + \mathcal{O}\left(\frac{m_\pi}{m_{s\bar{s}}}\right)^5$$

- **SU(2) equivalent** nucleon mass

$$M_N = m_0^{\text{eff}} - 4c_1^{\text{eff}} m_\pi^2 + \alpha^{\text{eff}} m_\pi^4 + \beta^{\text{eff}} m_\pi^4 \log \frac{\mu^2}{m_\pi^2} \\ + \frac{1}{(4\pi F_\phi)^2} \frac{3}{2} (D + F)^2 \left[H_N^{(b)}(m_0, m_\pi) + \frac{1}{2} H_N^{(e)}(m_0, m_\pi, \Delta m_N, \mu) \right]$$

- To be compared with **the SU(2) result**

Alvarez-Ruso, Ledwig, Camalich, and Vicente-Vacas, PRD88, 054507 (2013)

$$M_N^{\text{SU}(2)} = M_0 - 4c_1 m_\pi^2 + \frac{1}{2} \alpha m_\pi^4 \\ + \frac{1}{(4\pi f_\pi)^2} \frac{3}{8} [2(-8c_1 + c_2 + 4c_3) + c_2] m_\pi^4 - \frac{1}{(4\pi f_\pi)^2} \frac{3}{4} (8c_1 - c_2 - 4c_3) m_\pi^4 \log \frac{\mu^2}{m_\pi^2} \\ + \frac{1}{(4\pi f_\pi)^2} \frac{3}{2} g_A^2 \left[H_N^{(b)}(M_0, m_\pi) + \frac{1}{2} H_N^{(e)}(M_0, m_\pi, (-4c_1 m_\pi^2), \mu) \right],$$

- **SU(2) equivalent** nucleon mass

$$M_N = m_0^{\text{eff}} - 4c_1^{\text{eff}} m_\pi^2 + \alpha^{\text{eff}} m_\pi^4 + \beta^{\text{eff}} m_\pi^4 \log \frac{\mu^2}{m_\pi^2} \\ + \frac{1}{(4\pi F_\phi)^2} \frac{3}{2} (D + F)^2 \left[H_N^{(b)}(m_0, m_\pi) + \frac{1}{2} H_N^{(e)}(m_0, m_\pi, \Delta m_N, \mu) \right]$$

- To be compared with **the rewritten SU(2) results**

$$M_N^{\text{SU}(2)} = M_0 - 4c_1 m_\pi^2 + \alpha^{\text{SU}(2)} m_\pi^4 + \beta^{\text{SU}(2)} m_\pi^4 \log \frac{\mu^2}{m_\pi^2} \\ + \frac{1}{(4\pi f_\pi)^2} \frac{3}{2} g_A^2 \left[H_N^{(b)}(M_0, m_\pi) + \frac{1}{2} H_N^{(e)}(M_0, m_\pi, (-4c_1 m_\pi^2), \mu) \right]$$

$$\alpha^{\text{SU}(2)} = \frac{1}{2} \alpha - \frac{1}{(4\pi f_\pi)^2} \frac{3}{4} \left[(8c_1 - c_2 - 4c_3) - \frac{1}{2} c_2 \right]$$

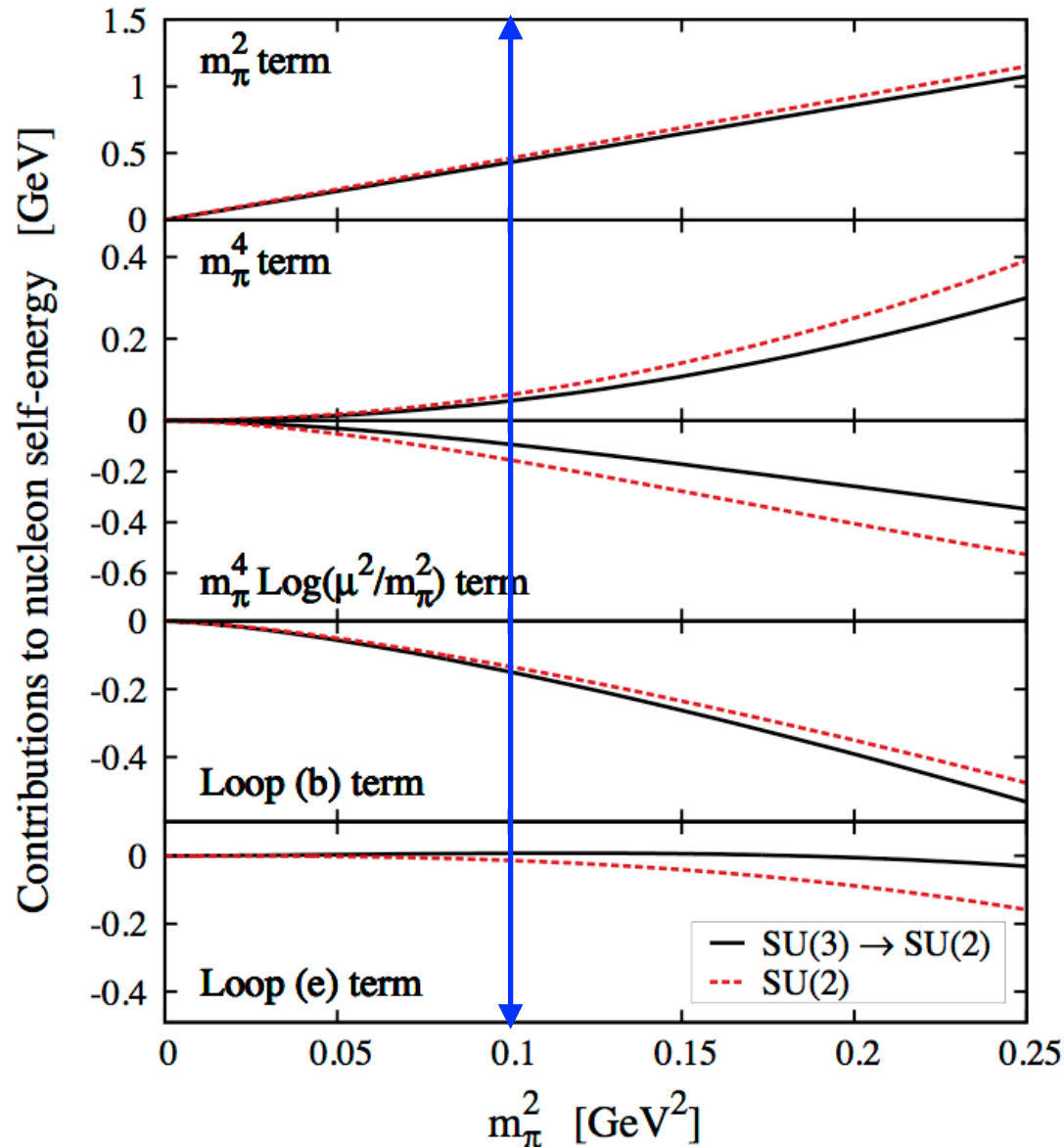
$$\beta^{\text{SU}(2)} = -\frac{3}{4(4\pi f_\pi)^2} (8c_1 - c_2 - 4c_3).$$

Comparison of effective parameters

SU(3)→SU(2)	SU(2)
😊 $m_0^{\text{eff}} = 875(10) \text{ MeV}$	$M_0 = 870(3) \text{ MeV}$
😊 $c_1^{\text{eff}} = -1.07(4) \text{ GeV}^{-1}$	$c_1 = -1.15(3) \text{ GeV}^{-1}$
😬 $\alpha^{\text{eff}} = 4.81(9) \text{ GeV}^{-3}$	$\alpha^{\text{SU}(2)} = 6.27(1.98) \text{ GeV}^{-3}$
😬 $\beta^{\text{eff}} = -4.02(20) \text{ GeV}^{-3}$	$\beta^{\text{SU}(2)} = -7.62(93) \text{ GeV}^{-3}$

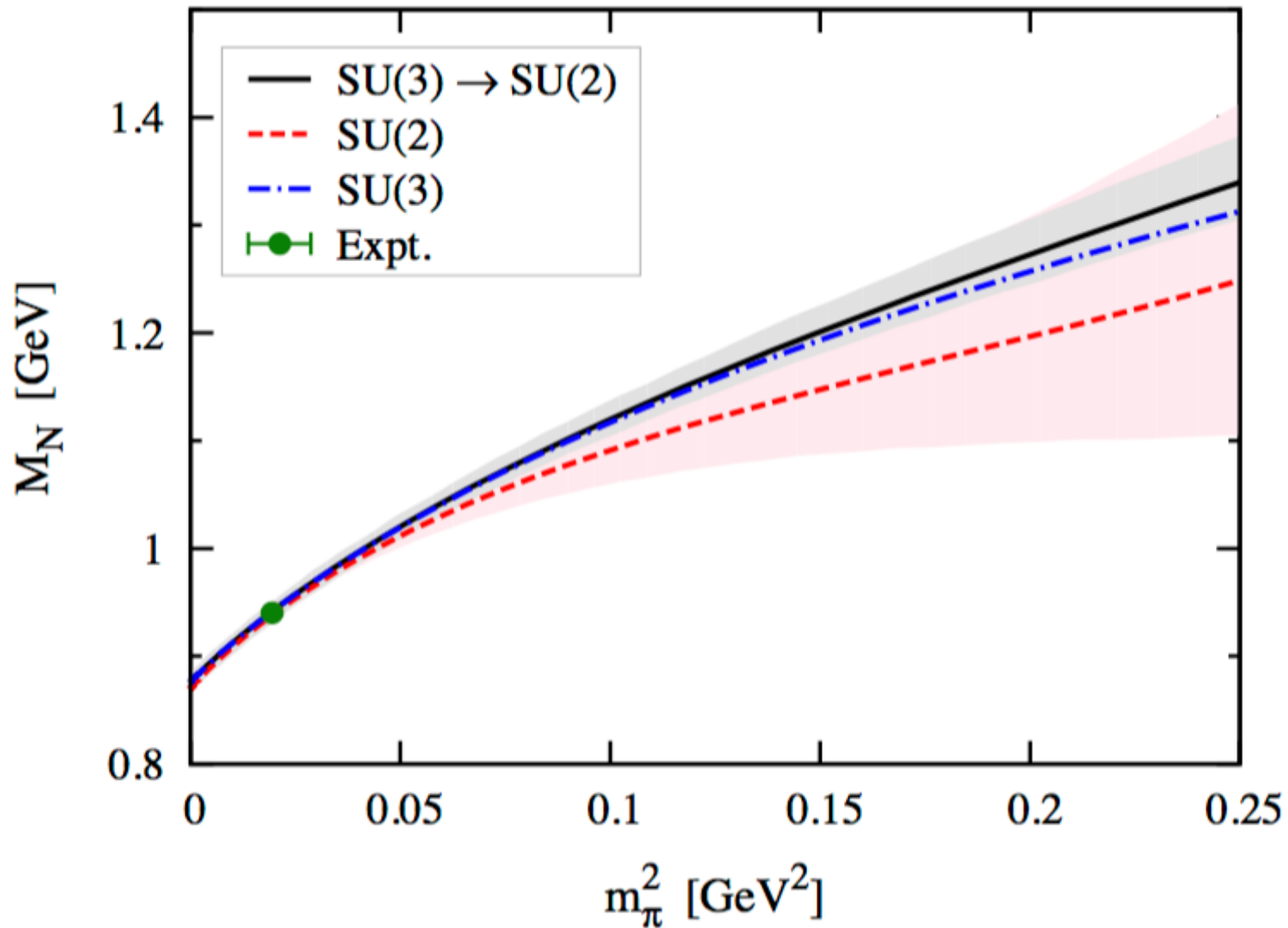
- *SU(3): Ren, Geng, Meng, PRD91 (2015) 051502*
- *SU(2): Alvarez-Ruso, Ledwig, Camalich, and Vicente-Vacas, PRD88, 054507 (2013)*

Decomposition of different contributions

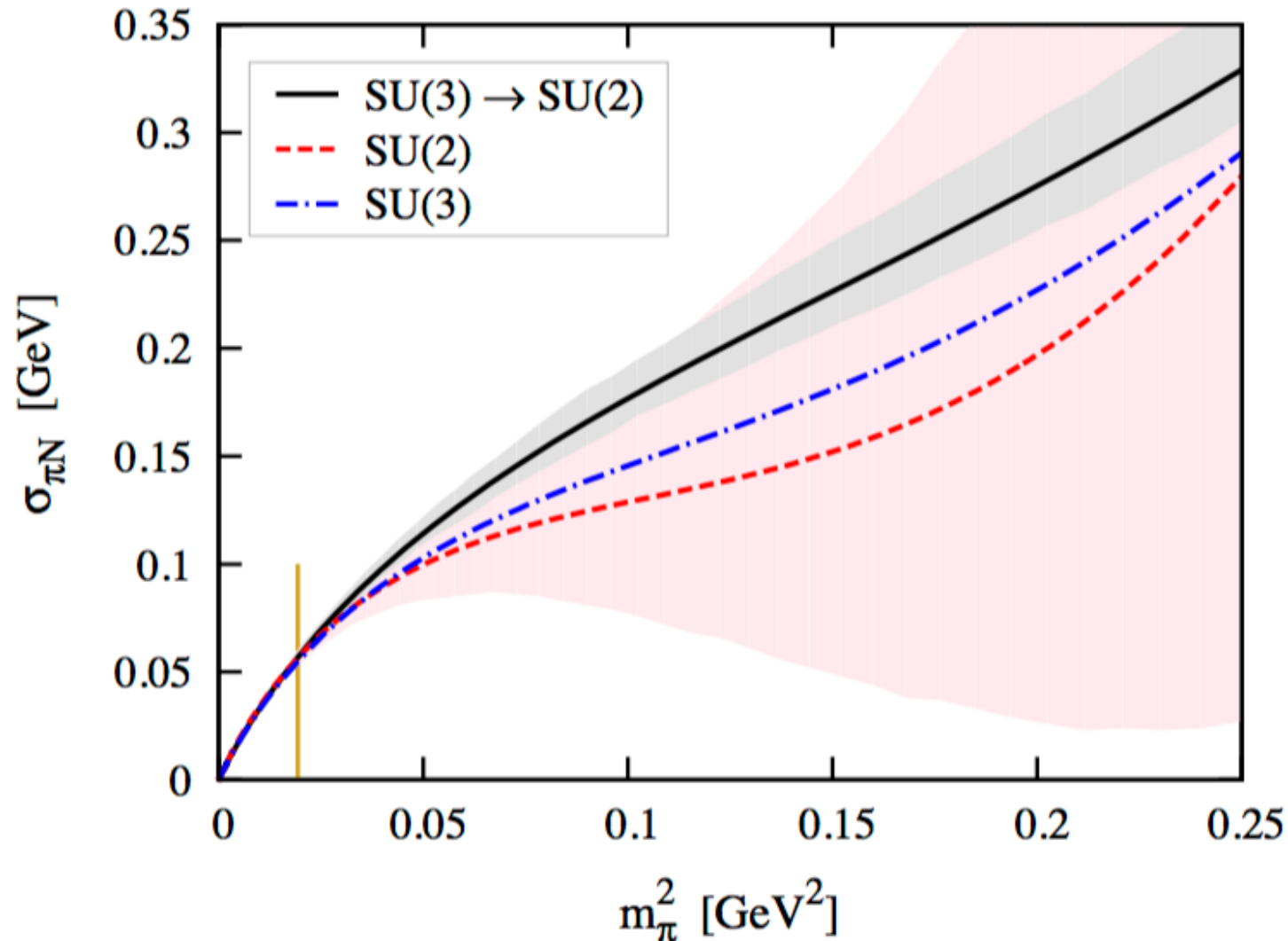


agree at small
 $m_\pi < 300 \text{ MeV}$
differ at larger m_π

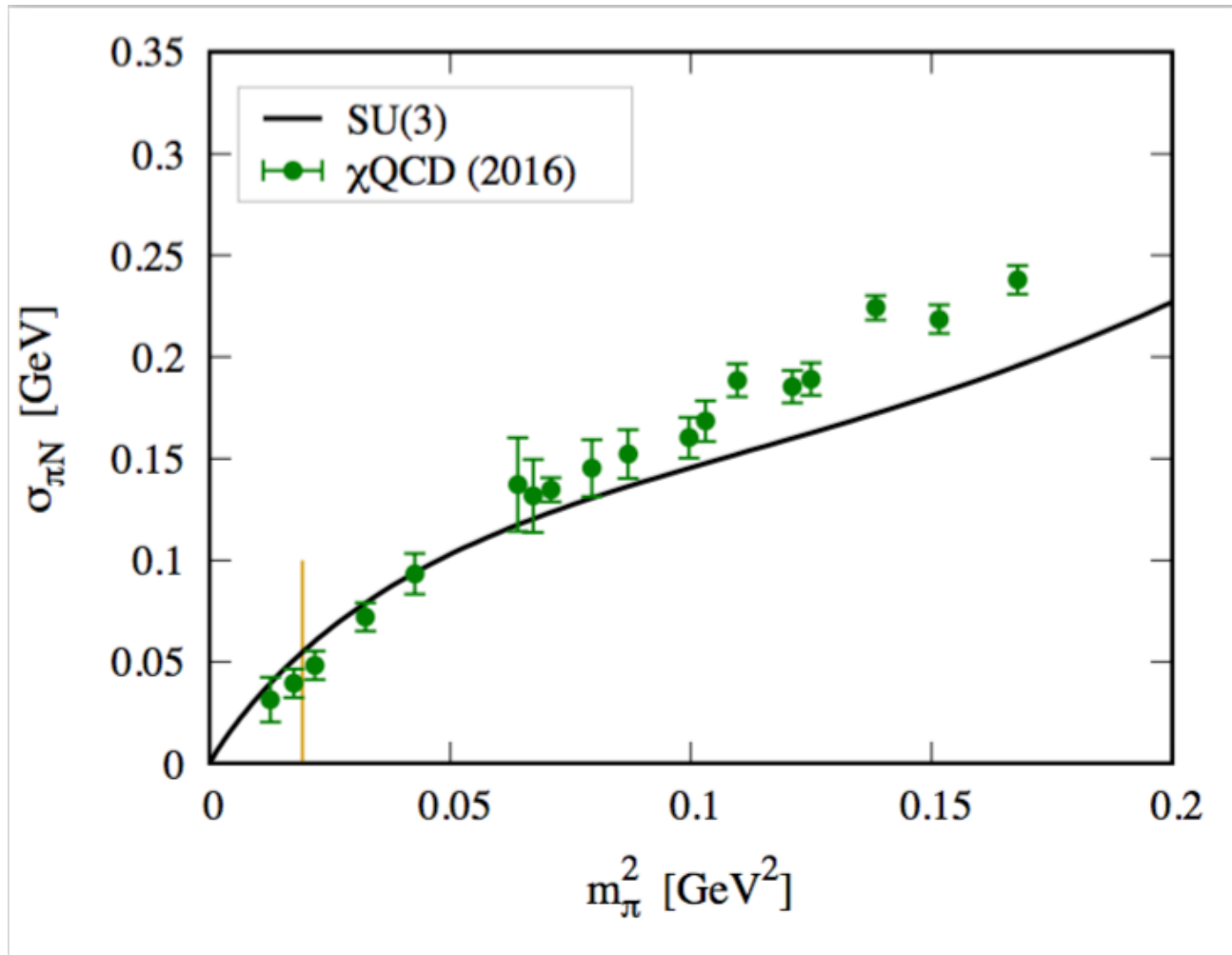
Chiral extrapolation of nucleon mass



Light quark dependence of pion-nucleon sigma term



In comparison with χ QCD16



Summary

- ❖ Explained how the **baryon sigma terms** (particularly those of the nucleon) are **related** to dark matter direct searches and the quark-flavor structure of the nucleon.
- ❖ Shown how a combination of lattice QCD simulations and baryon chiral perturbation theory allows us to make a **reliable prediction** of these terms.
- ❖ It is to be stressed that we have taken into account **as much as possible lattice artifacts** and checked the validity of SU(3) BChP
- ❖ We should bear in mind, however, that our results **are tied to the reliability and accuracy** of the available IQCD simulations

**Thank you very much
for your attention!**

Scale-setting effects on the determination of baryon sigma terms

arXiv:1301.3231

P.E. Shanahan, A.W. Thomas and R.D. Young*

- Lattice-scale setting
 - PACS-CS data with **mass independent** scale-setting:

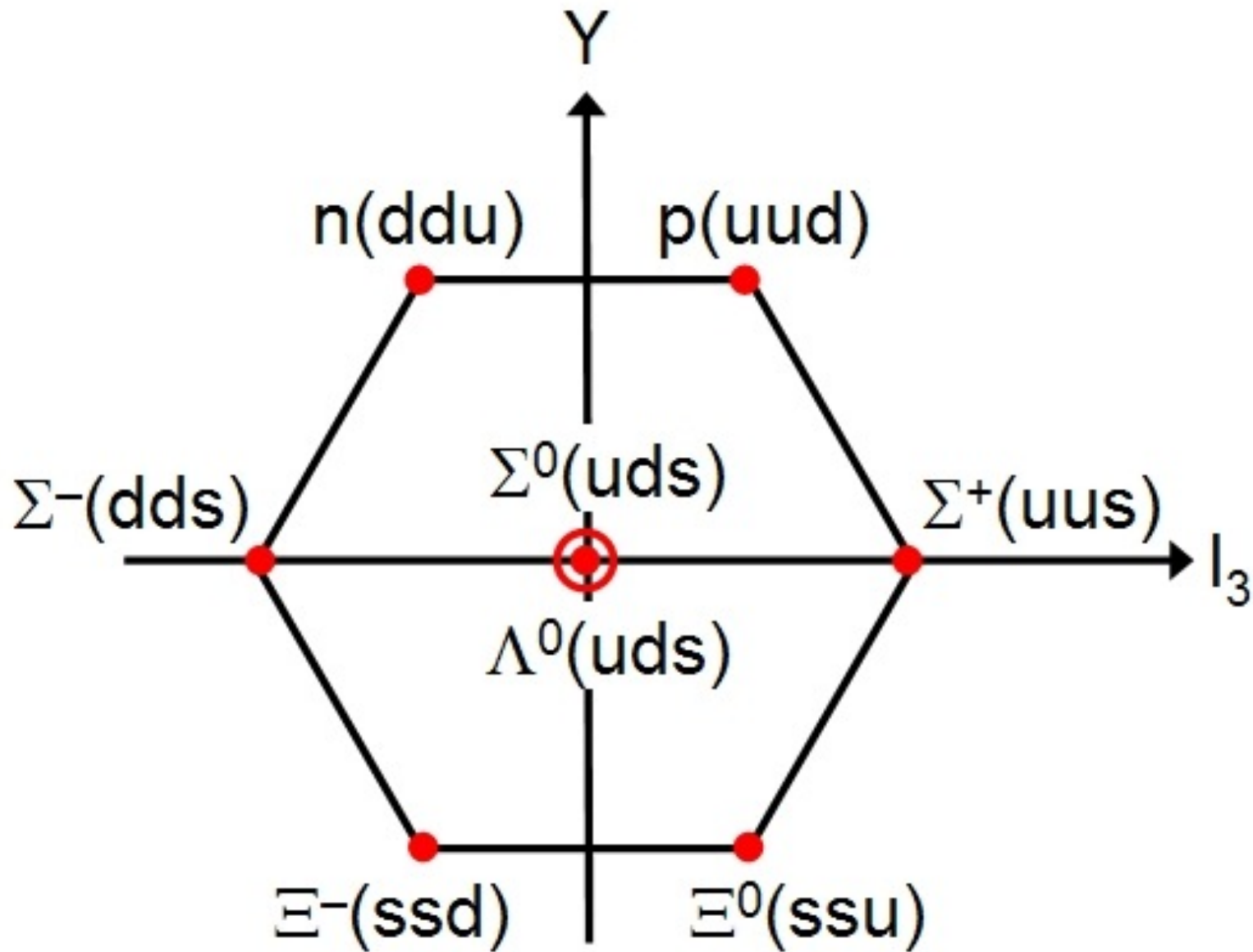
$$\sigma_{sN} = 59 \pm 7 \text{ (MeV)}$$

- PACS data with **mass dependent** (r_0) scale-setting:

$$\sigma_{sN} = 21 \pm 6 \text{ (MeV)}$$

- Whether other LQCD data will show the same trend?

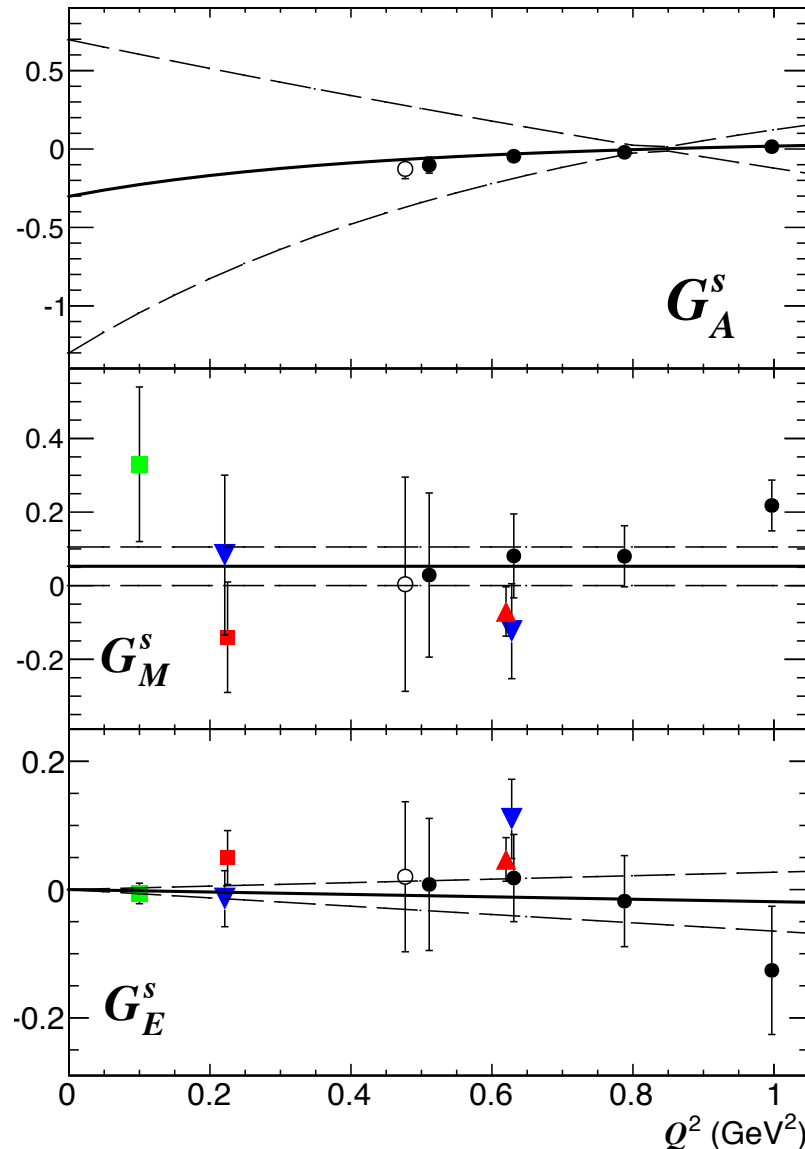
Quark-flavor structure of octet baryons



Naive quark model

Global fit of strangeness vector and axial vector form factors of nucleon

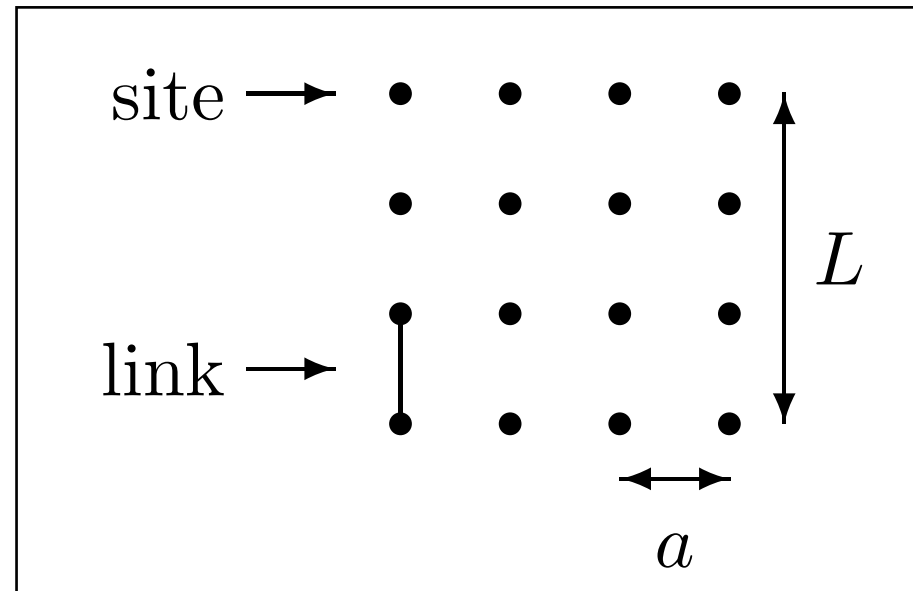
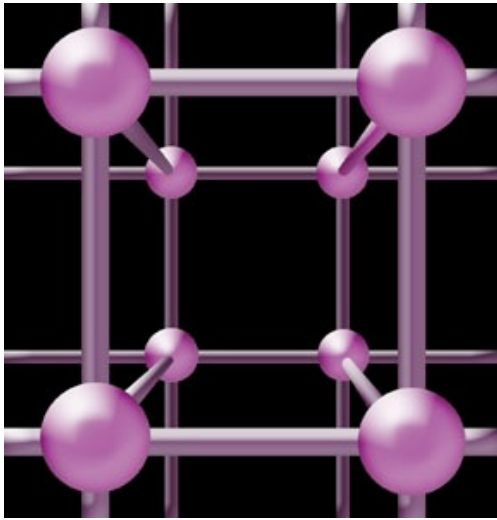
arXiv:1308.5694



Parameter	Fit value
ρ_s	-0.071 ± 0.096
μ_s	0.053 ± 0.029
ΔS	-0.30 ± 0.42
Λ_A	1.1 ± 1.1
S_A	0.36 ± 0.50

The electric and magnetic form factors are consistent with **zero, but not** the axial-vector form factor

Lattice QCD



Basic idea: discretize space-time and solve non-perturbative strong interaction physics in a finite hypercube, utilizing monte carlo sampling techniques