Lattice Study of Exotic Hadrons

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Outline

I. Introduction

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IV Summary

I. Introduction

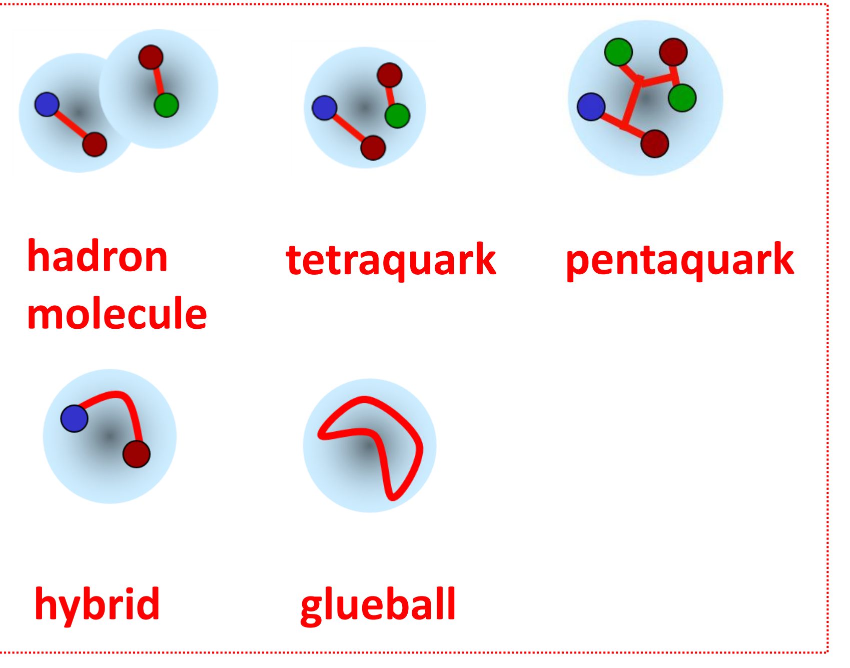
In naïve quark model, hadrons are sorted into two categories:

$\bar{q}q$ mesons and qqq baryons

Extented constituent picture:
multi-quark states
$\bar{q}q\bar{q}q$ mesons
$qqqq\bar{q}$ baryons

Gluons can be also the building blocks of hadrons:

$\bar{q}qg$ mesons (hybrid)
ggg mesons (glueball)



Exotic hadrons

On the other hand, many XYZ particle have been discovered in recent years

- Experimental hadron spectroscopy --- XYZ particles

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi)\ldots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5) LHCb [1003] (> 10)	2005	Ok
				$B \to K(\gamma \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2) LHCb [1003] (4.4)	2008	NC!
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2) T. Xiao *et al.* [CLEO data] [1009] (>5)	2013	Ok
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

Brambilla et al., arXiv:1404.2723

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment ($\#\sigma$)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9),	2009	NC!
					LHCb [1062] (1.4), CMS [1063] (>5)		
					D0 [1064] (3.1)		
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11)	2005	Ok
					Belle [1008, 1056] (15), BES III [1007] (np)		
				$e^+e^- \to (f_0(980)J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0),	2011	NC!
					CMS [1063] (>3), D0 [1064] (np)		
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4)	2007	Ok
					LHCb [1077] (13.9)		
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

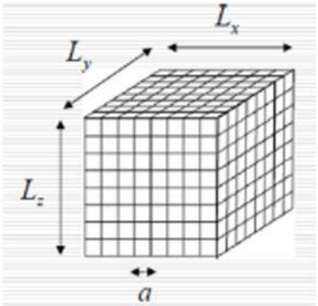
The lattice formulation of QCD---Lattice QCD

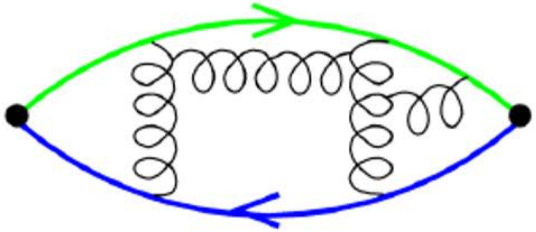
$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \; e^{-S}$$

$$S = S_{gauge} + S_{quarks} = \int d^4x \, \left(\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) - \textstyle\sum_i \log(\mathrm{Det} M_i)$$

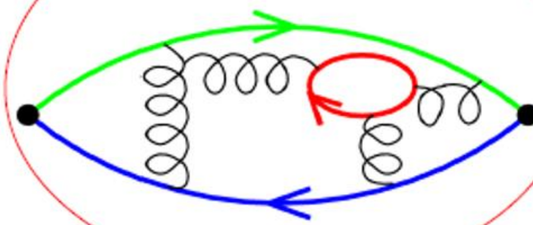
$$Z = \int \mathcal{D}A_\mu \, \det M \; e^{\int d^4x \, \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right)} .$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S} \, .$$





(A) Quenched QCD: quark loops neglected



(B) Full QCD

Dominated in the present era

The methods for the hadron spectroscoapy in lattice QCD

- Interpolation field operators --- starting point for a meson (-like) system with given J^{PC} and flavor quantrum numbers:

$$\mathcal{O}_i: \quad \bar{q}_1 \Gamma q_2 \quad [\bar{q}_1 \Gamma_1 q\,][\bar{q}\Gamma_2 q_2\,] \quad \left[q_1^T \Gamma_1 q\right]\left[\bar{q}\Gamma_2 \bar{q}_2^T\,\right], \ldots\ldots$$

- Two-point functions --- Observables

$$\mathcal{C}_{ij}(t) = \left\langle 0 \middle| \mathcal{O}_i(t)\mathcal{O}_j^{+}(0) \middle| 0 \right\rangle$$
$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle \left\langle n \middle| \mathcal{O}_j^{+} \middle| 0 \right\rangle e^{-E_n t}$$

In principle, all the physical states with the same quantum numbers $|n\rangle$ contribute to the two point functions $\mathcal{C}_{ij}(t)$ as the eigenstates of the QCD Hamiltonian with the energy eigenvalue E_n:

- " one-particle state" : $E_n = m_n$
- " two-particle state" : $E_n = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + \vec{p}^2} + \Delta E,\ \vec{p} = \frac{2\pi}{L}\vec{n}$
- ……

Comparison of the hadron spectra

Euclidean spacetime lattice

One particle states

Multiple particle states
with discrete relative spatial
Momentum (scattering
States in a finite volume

All the energies are
Discretized.

Minkowski continuum spacetime

Stable particles

Bound states of hadrons

Resonances

Continuum scattering states

Luescher's Relation:

$$E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$$

$$\tan\delta(p) = \frac{\sqrt{\pi}\ p\ L}{2\ \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

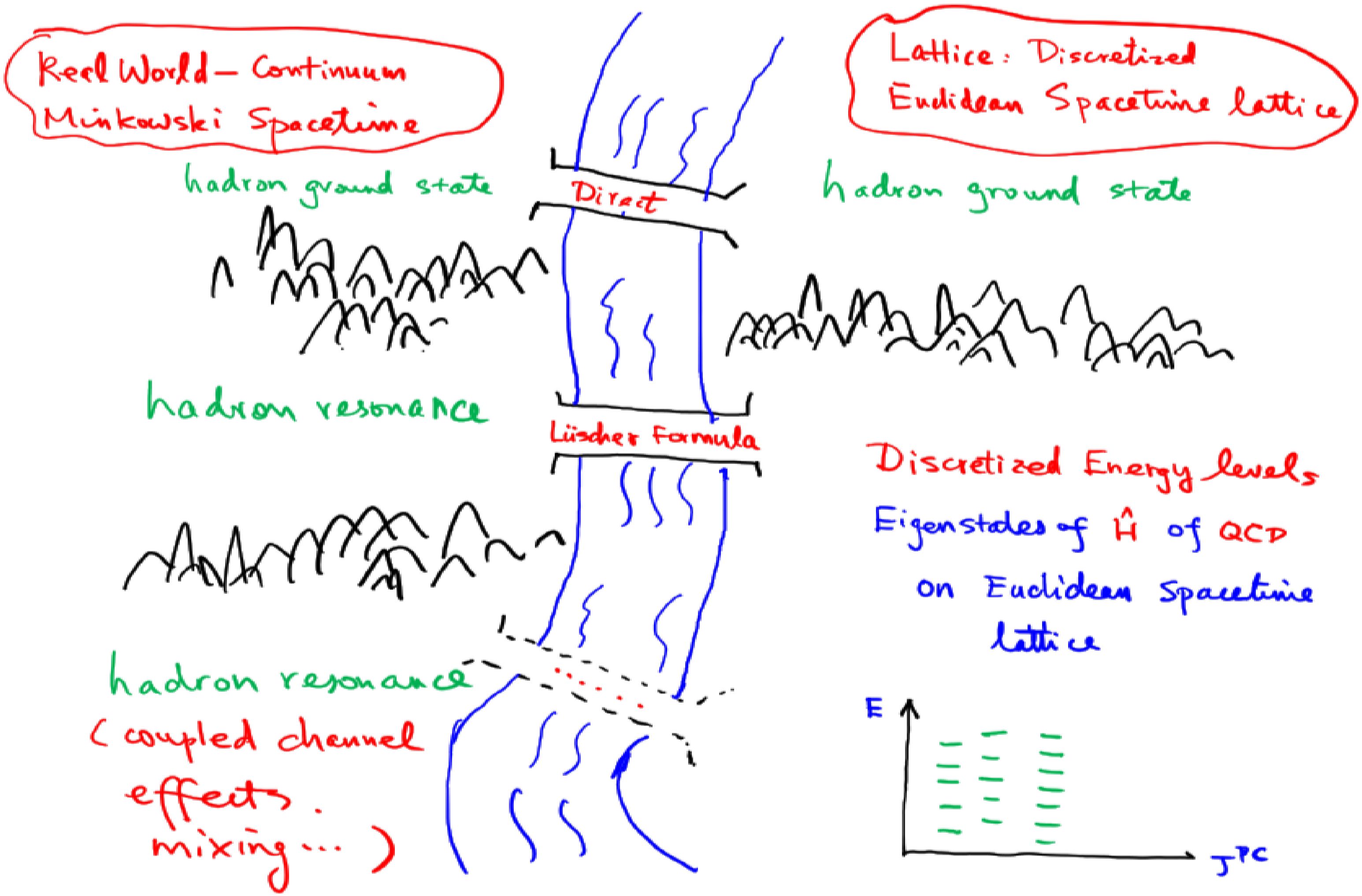
Resonances

$$\left\{ \begin{array}{l} T(p) = \dfrac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\,\Gamma(p)} = \dfrac{1}{\cot\delta(p) - i} \\ \Gamma(p) = g^2\,\dfrac{p^{2l+1}}{s}\,, \quad \dfrac{p^{2l+1}}{\sqrt{s}}\cot\delta(p) = \dfrac{1}{g^2}(m_R^2 - s) \end{array} \right.$$

Bound states

$$\left\{ \begin{array}{l} p\cot(\delta_0(p)) = \dfrac{1}{a_0} + \dfrac{1}{2} r_0 p^2\,, \quad -|p_B| = \dfrac{1}{a_0} - \dfrac{1}{2} r_0 |p_B|^2 \\ T = \dfrac{\;}{\cot(\delta_l(p_B)) - i} = \infty \\ m_B = E_{H_1}(p_B) + E_{H_2}(p_B)\,,\ p_B = i|p_B| \end{array} \right.$$

Present status of lattice QCD study on hadron spectroscopy

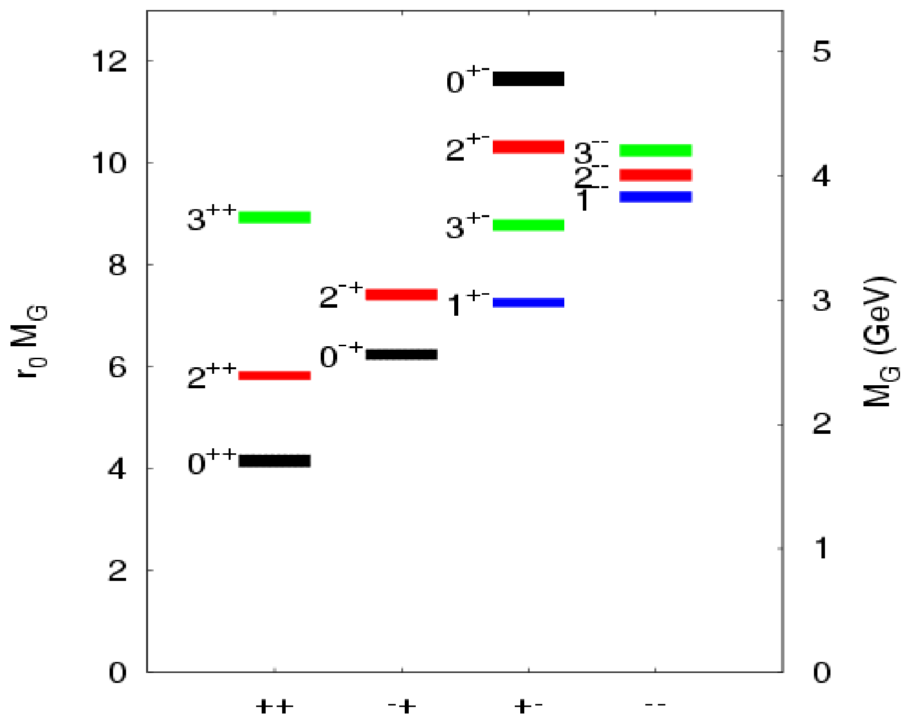


II. Glueballs on the lattice

I). Glueball mass spectrum

- **Quenched LQCD predicts glueball spectrum**

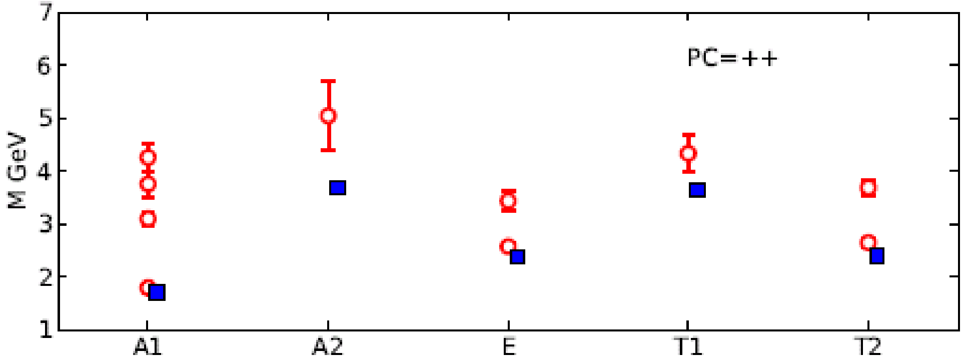
  Lowest-lying glueballs have masses in the range 1~3GeV



J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al, Phys. Rev. D 73, 014516 (2006)

- **Glueball masses from 2+1 flavor dynamical lattice QCD study, which confirm the prediction of the quenched lattice QCD.**
  [E.Gregory et al, JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)]



Open circles are full-QCD results, and the filled squares are from quenched lattice QCD studies

- **Recent results from Nf=2 dynamical lattice QCD on anisotropic lattices**
  [W. Sun et al (CLQCD), arXiv:1702.08174(hep-lat)]

	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1397(25)	2367(35)	2559(50)
	650	1480(52)	2380(61)	2605(52)
$N_f = 2+1$ [13]	360	1795(60)	2620(50)	—
quenched [8]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [9]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

[9] C. Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999
[8] Y. Chen et al, Phys. Rev. D 73, 014516, 2006
[13] E. Gregory et al., JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)

- **Interesting observations in the pseudoscalar channel**

	$P(x)$	$q(x)$	O_G
$N_f = 0$	——	2563(34)MeV A.Chowdhury, PRD91(2015)	2590(40)(130)MeV Y.Chen, PRD73(2006)
$N_f = 2$	768(24)MeV C.Urbach, Lattice2017	890(38)MeV this work $m_\pi = 650$MeV	2605(52)MeV this work $m_\pi = 650$MeV
$N_f = 2+1$	947(142)MeV N.Christ, PRL105(2010)	1019(119)MeV JLQCD, PRD92(2015)	——
$N_f = 2+1+1$	1006(54)(38)MeV C.Michael, PRL111(2013)	——	——

- P(x): $\bar{\psi}\gamma_5\psi$
- q(x): topological charge density
- O_G: glueball operators

$$\boxed{\phi_\alpha^{A_1^{-+}}(\mathbf{x},t) \propto \epsilon_{ijk} Tr B_i(\mathbf{x},t) D_j B_k(\mathbf{x},t) + O(a_s^2)}$$

$U_A(1)$ **Anomaly**

$$\partial_\mu A^\mu(x) = 2mP(x) - \frac{N_f}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma} Tr F_{\mu\nu} F_{\rho\sigma}$$

Topological charge density

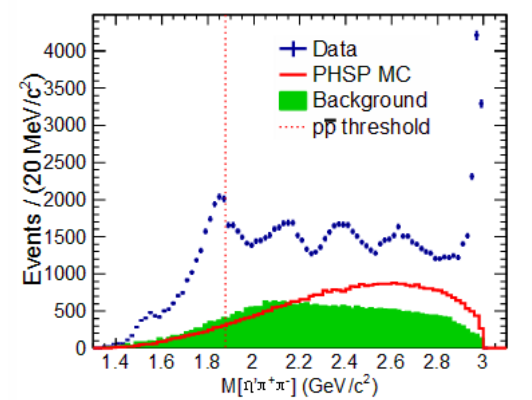
$$q(x) = -\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma} Tr F^{\mu\nu}(x) F^{\rho\sigma}(x)$$

TABLE I. Mass, width, $\mathcal{B}(J/\psi \to \gamma X \to \gamma\phi\phi)$ (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

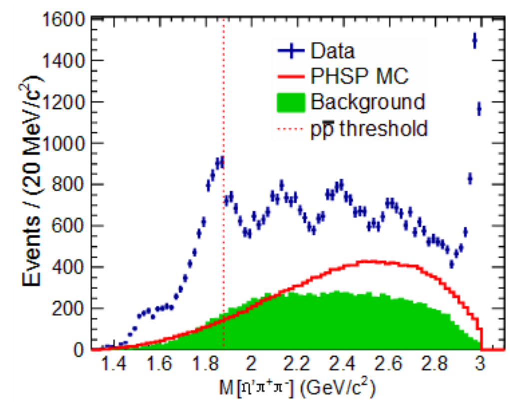
Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.($\times 10^{-4}$)	Sig.
$\eta(2225)$	2216^{+4+21}_{-5-11}	185^{+12+43}_{-14-17}	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	$28\ \sigma$
$\eta(2100)$	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	$22\ \sigma$
$X(2500)$	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	$8.8\ \sigma$
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	$24\ \sigma$
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	$9.5\ \sigma$
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	$6.4\ \sigma$
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	$11\ \sigma$
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	$6.8\ \sigma$

BESIII, PRD93(2016)112011

$J/\psi \to \gamma\eta'\pi^+\pi^-$



$\eta' \to \gamma\pi^+\pi^-$



$\eta' \to \eta(\to \gamma\gamma)\pi^+\pi^-$

BESIII, PRL117(2016)042002
arXiv:1603.09653

II). The production rates of glueballs in the J/psi radiative decays

- **Radiative decay width:**

$$\Gamma(i \to \gamma f) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_i^2} \frac{1}{2J_i+1} \times \sum_{r_i, r_j, r_\gamma} \left| M_{r_i, r_j, r_\gamma} \right|^2,$$

- **Transition amplitudes:**

$$M_{r_i, r_f, r_\gamma} = \epsilon_\mu^*(\vec{q}, r_\gamma) \langle f(\vec{p}_f, r_f) | j_{\rm em}^\mu(0) | i(\vec{p}_i, r_i) \rangle$$

- **Multipole decomposition:**

$$\langle f(\vec{p}_f, r_f) | j_{\rm em}^\mu(0) | i(\vec{p}_i, r_i) \rangle = \sum_k \alpha_k^\mu(p_i, p_f) F_k(Q^2).$$

- **Decay width expressed in terms of the form factors**

$$\Gamma(i \to \gamma f) \propto \sum_k F_k^2(0).$$

- **So the major task is to calculate the matrix elements, which can be derived from the three-point functions**

$$\Gamma^{(3)\mu i}(\vec{p}_f, \vec{q}; t_f, t) = \frac{1}{T} \sum_{\tau=0}^{T-1} \sum_{\vec{y}} e^{+i\vec{q}\cdot\vec{y}} \left\langle O_G(\vec{p}_f, t_f + \tau) j^\mu(\vec{y}, t+\tau) O_{J/\psi}^{i,+}(\tau) \right\rangle$$

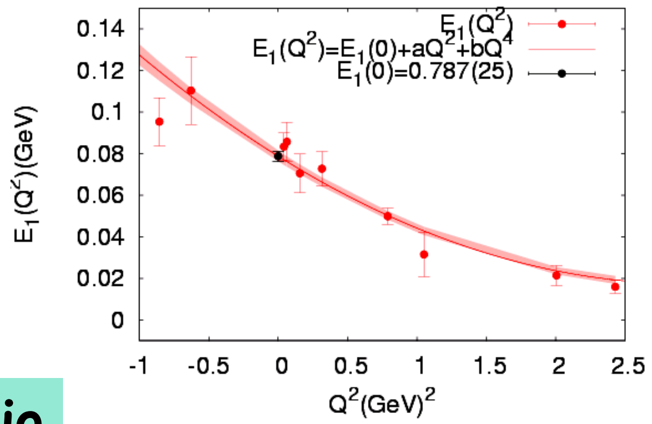
A). J/psi radiatively decaying to the scalar glueball

(L.Gui, et al. (CLQCD Collaboration), Phys. Rev. Lett. 110, 021601 (2013))

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2$$

Interpolated on-shell form factor E1(0) and its continuum limit

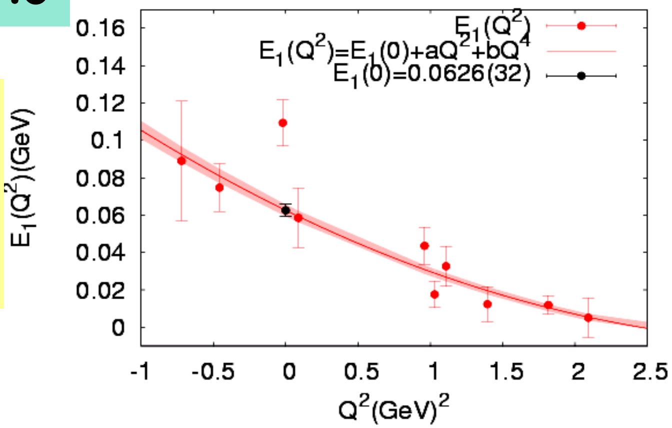
β	M_G(GeV)	$Z_V^{(s)}(a)$	$E_1(0,a)$ (GeV)	Γ(keV)
2.4	1.360(9)	1.39(2)	0.0787(25)	-
2.8	1.537(7)	1.11(1)	0.0626(32)	-
∞	1.710(90) [3]	-	0.0536(57)	0.35(8)



The predicted width and the branch ratio

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$



Lattice prediction:

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$

Experimental results for J/psi radiatively decaying to scalars

C. Amsler et al. (Particle Data Group), Phy. Rev. D 86, 010001 (2012)

$$J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi \qquad (1.01\pm 0.32)\times 10^{-4}$$

$$Br(f_0(1500)\to\pi\pi) = (34.9\pm 2.3)\% \quad \Rightarrow Br(J/\psi \to \gamma f_0(1500)) = 2.9\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K} \qquad (8.5^{+1.2}_{-0.9})\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi \qquad (4.0\pm 1.0)\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega \qquad (3.1\pm 1.0)\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \qquad > (1.5\pm 0.3)\times 10^{-3}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta \qquad (2.35^{+1.27}_{-0.77})\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1500) \to \gamma\eta\eta \qquad (1.65^{+0.57}_{-1.50})\times 10^{-4}$$

BESIII results (PRD87, 092009)

Using $Br(f_0(1710)\to K\underline{K})=0.36 \quad \Rightarrow \quad Br(J/\psi\to\gamma f_0(1710)) = 2.4\times 10^{-3}$

$Br(f_0(1710)\to \pi\pi) = 0.15 \quad \Rightarrow \quad Br(J/\psi\to\gamma f_0(1710)) = 2.7\times 10^{-3}$

This result support f0(1710) as the candidate for the scalar glueball

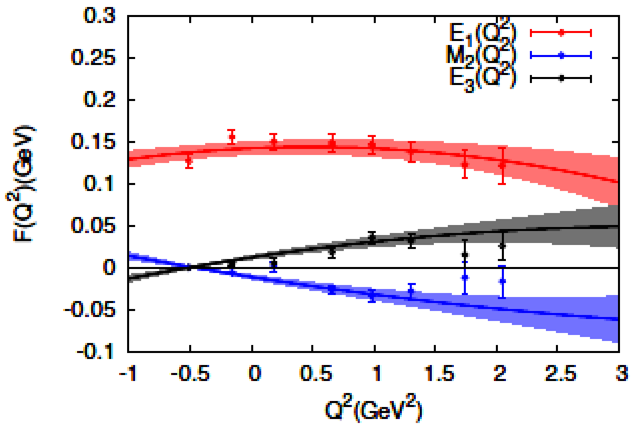
B). J/psi radiatively decaying to the tensor glueball

(Y.B. Yang ,et al .(CLQCD Collaboration), Phys. Rev. Lett. 111, 091601 (2013))

$$\Gamma(J/\psi \to \gamma G_{2^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left[|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2\right]$$

- **The form factors we obtained from the lattice QCD**

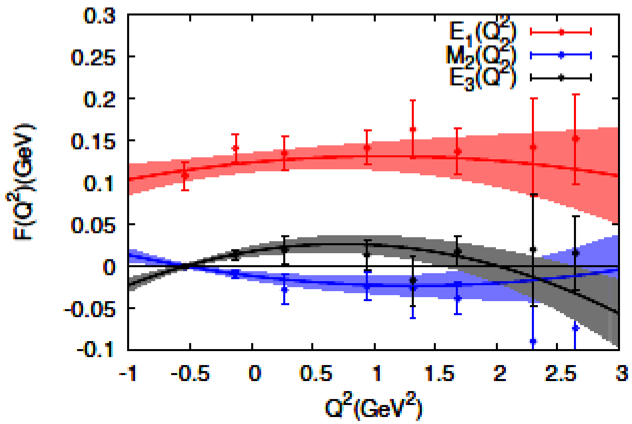
β	M_T (GeV)	E_1 (GeV)	M_2 (GeV)	E_3 (GeV)
2.4	2.360(20)	0.142(07)	−0.012(2)	0.012(2)
2.8	2.367(25)	0.125(10)	−0.011(4)	0.019(6)
∞	2.372(28)	0.114(12)	−0.011(5)	0.023(8)



- **We also carry out a similar lattice study on the tensor glueball production rate in J/psi radiative decay.**

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)\,keV$$

$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$



- **BESIII new results for** $J/\psi \to \gamma\phi\phi$

TABLE I. Mass, width, $\mathcal{B}(J/\psi \to \gamma X \to \gamma\phi\phi)$ (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.($\times 10^{-4}$)	Sig.
$\eta(2225)$	$2216^{+4}_{-5}{}^{+21}_{-11}$	$185^{+12}_{-14}{}^{+43}_{-17}$	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28 σ
$\eta(2100)$	$2050^{+30}_{-24}{}^{+75}_{-26}$	$250^{+36}_{-30}{}^{+181}_{-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22 σ
$X(2500)$	$2470^{+15}_{-19}{}^{+101}_{-23}$	$230^{+64}_{-35}{}^{+56}_{-33}$	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8 σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24 σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5 σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4 σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11 σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8 σ

- **BESIII new results for** $J/\psi \to \gamma\eta\eta$

(M. Ablikim et al. (BES Collaboration), Phys. Rev. D 87, 092009 (2013) (arXiv:1301.0053)

Resonance	Mass(MeV/c^2)	Width(MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	8.2 σ
$f_0(1710)$	$1759\pm6^{+14}_{-25}$	$172\pm10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	25.0 σ
$f_0(2100)$	$2081\pm13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	13.9 σ
$f_2'(1525)$	$1513\pm5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	11.0 σ
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	6.4 σ
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	7.6 σ

In this analysis, the best fit favors the presence of f2(2340) with a mass of 2362(30)MeV and a width of 334(60) MeV. No evident narrow peak around 2.2GeV over the broad bump is observed in the eta-eta mass specturm.

Comments on the lattice results on glueballs:

1. Glueballs are well-defined objects in SU(3) pure gauge theory.
2. For the lattice studies with dynamical quarks, the masses of the lowest-lying glueballs are in agreement with the quenched results. No substantial unquenched effects have been observed yet.
3. However, the systematics uncertainties, such as the chiral limit, and the continuum limit extrapolation have not been performed.
4. In full-QCD, glueballs must mix strongly with conventional $q\bar{q}$ mesons. On the other hand, since the decay channels open, the situation would be much more complicated.
5. There is still a long way to go to settle the problem of glueball sector from lattice QCD study.

III. XYZ particles on the lattice

1. Zc States

Zc(3900) : first observed as a structure in $J/\psi\pi^+$ invariant mass spectrum, its "mass" is close to the $D\overline{D^*}$ threshold

Zc(4025) : first observed as a structure in $h_c\pi^+$ invariant mass spectrum, its "mass" is close to the $D^*\overline{D^*}$ threshold.

Lattice studies from three aspects:

a. $D\overline{D}$ **scattering (**Y. Chen et al., PRD89(2014)094506, PRD92(2015)054507 **)**
b. Spectroscopy study (S. Prelovsek et al., PRD91(2015)014504)
c. Potential matrix and scattering amplitudes
(Y. Ikeda et al (HAL Collab.), PRL117(2016) 242001)

a. Scattering:

Calculation procedure:

i) The masses of D mesons

ii) The energies of DD*bar system

iii) Define the scattering momenta of D mesons in the DD*bar system

$$E_{1\cdot 2}(\mathbf{k}) = \sqrt{m_1^2 + \bar{\mathbf{k}}^2} + \sqrt{m_2^2 + \bar{\mathbf{k}}^2} \qquad q^2 = \bar{\mathbf{k}}^2 L^2/(2\pi)^2$$

iv) Use the Leuscher formular to get the scattering phase shift

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} Z_{00}(1; q^2)$$

v) For near threshold scattering, one can use the effective range expansion to parameterize the phase shift versus q.

$$k^{2l+1} \cot \delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \cdots$$

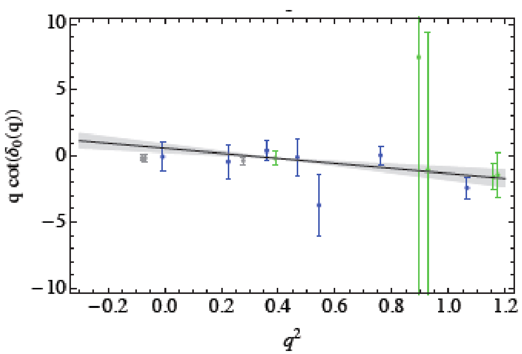
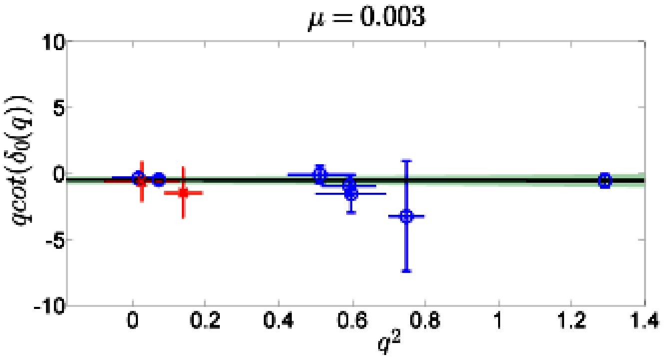


TABLE VI. The values for a_0 and r_0 in physical units obtained from the numbers for the correlated fit in Table IV.

	$\mu = 0.003$	$\mu = 0.006$	$\mu = 0.008$
a_0[fm]	−0.67(1)	−2.1(1)	−0.51(7)
r_0[fm]	−0.78(3)	−0.27(7)	0.82(27)



	$\mu = 0.003$	$\mu = 0.006$	$\mu = 0.008$
a_0[fm]	$-0.76^{+0.14}_{-0.21}$	$-0.86^{+0.22}_{-0.22}$	$-0.59^{+0.19}_{-0.25}$
r_0[fm]	$-0.0022^{+0.18}_{-0.19}$	$-0.14^{+0.15}_{-0.18}$	$0.64^{+0.50}_{-0.51}$

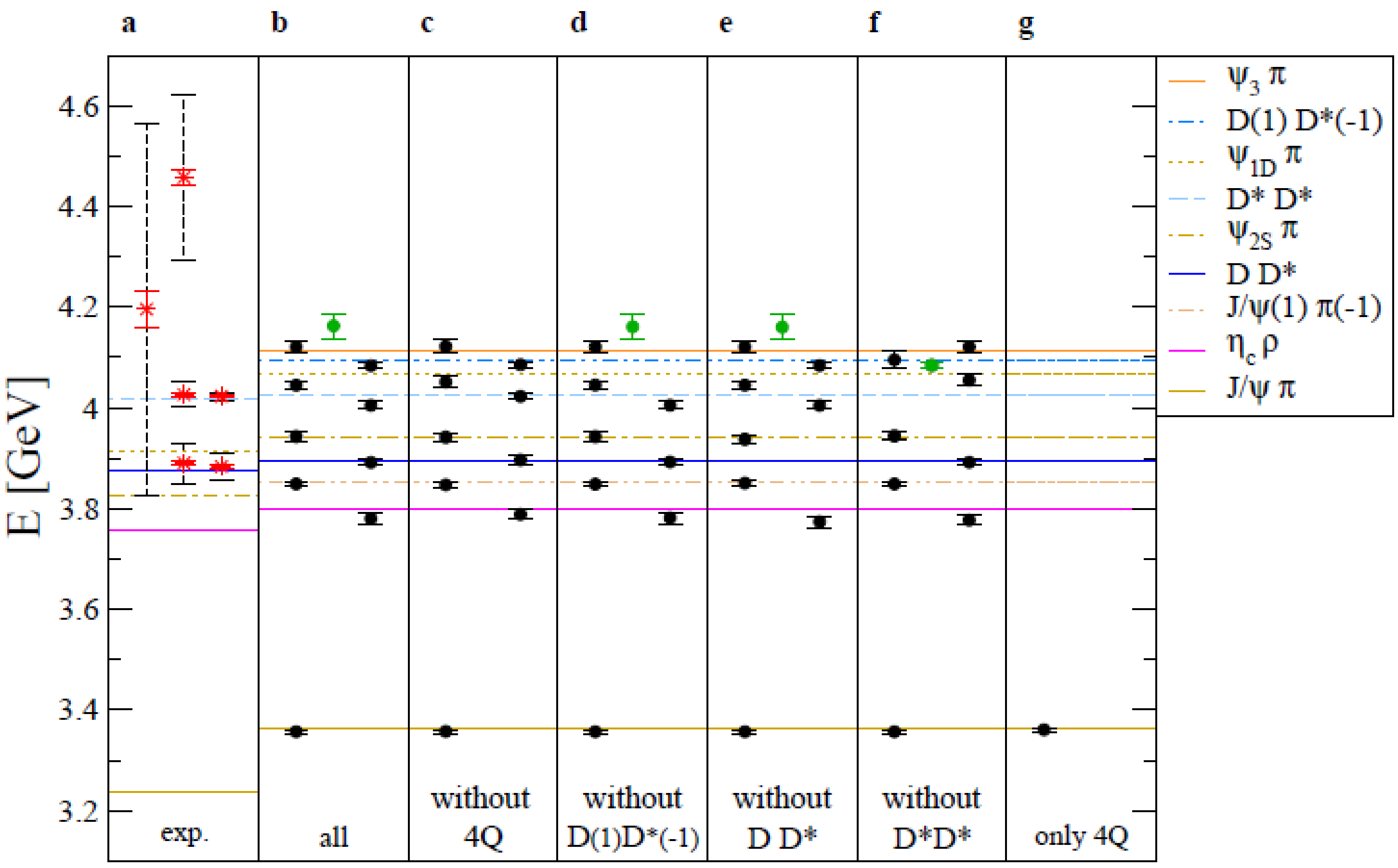
In the J^P=1^+ channel, the scattering lengths are negative, indicating a weak repulsive interaction between D(D*) and D*bar. These results does not support a bound state in this channel. However, since the pion mass is still much higher than the physical pion mass, we cannot rule out the possible appearance of a bound state. A more systematic lattice study is demanding.

b. Spectroscopy study on Zc states

S. Prelovsek et al., Phys. Rev. D 91, 014504 (2015) arXiv:1405.7623(hep-lat)

- Spectroscopic study
- Quite a lot of two-particle operators and tetraquark operators are involved

$$\begin{aligned}
\mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0)\ \bar{d}\gamma_5 u(0)\,, \qquad (4)\\
\mathcal{O}^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k)\ \bar{d}\gamma_5 u(-e_k)\,,\\
O^{\psi(2)\pi(-2)} &= \sum_{|u_k|^2=2} \bar{c}\gamma_i c(u_k)\ \bar{d}\gamma_5 u(-u_k)\,,\\
\mathcal{O}^{\eta_c(0)\rho(0)} &= \bar{c}\gamma_5 c(0)\ \bar{d}\gamma_i u(0)\,,\\
\mathcal{O}_1^{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0)\ \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,\\
\mathcal{O}^{D^*(0)D^*(0)} &= \epsilon_{ijk}\ \bar{c}\gamma_j u(0)\ \bar{d}\gamma_k c(0)\,,\\
\mathcal{O}_1^{4q} &\propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5 \bar{d}_c\ c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i \bar{d}_c\ c_{b'}\gamma_5 C u_{c'})\,,\\
\mathcal{O}_2^{4q} &\propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C \bar{d}_c\ c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5 \bar{d}_c\ c_{b'} C u_{c'})\ ,
\end{aligned}$$



All the scatering states below 4.2 GeV are obtained.

They concluded that No convincing exotic Zc state is observed.

c. On the structure of Zc(3900) from lattice QCD

(Y. Ikeda et al (HAL Collab.), PRL117(2016) 242001)

a) **BS wave functions of two-meson systems are extracted from lattice QCD.**

$$C^{\alpha\beta}(\vec{r},t) \equiv \sum_{\vec{x}} \langle 0|\, \phi_1^{\alpha}(\vec{x}+\vec{r},t)\phi_2^{\alpha}(\vec{x},t)\overline{\mathcal{J}}^{\beta}\, |0\rangle \,/\, \sqrt{Z_1^{\alpha} Z_2^{\alpha}}$$

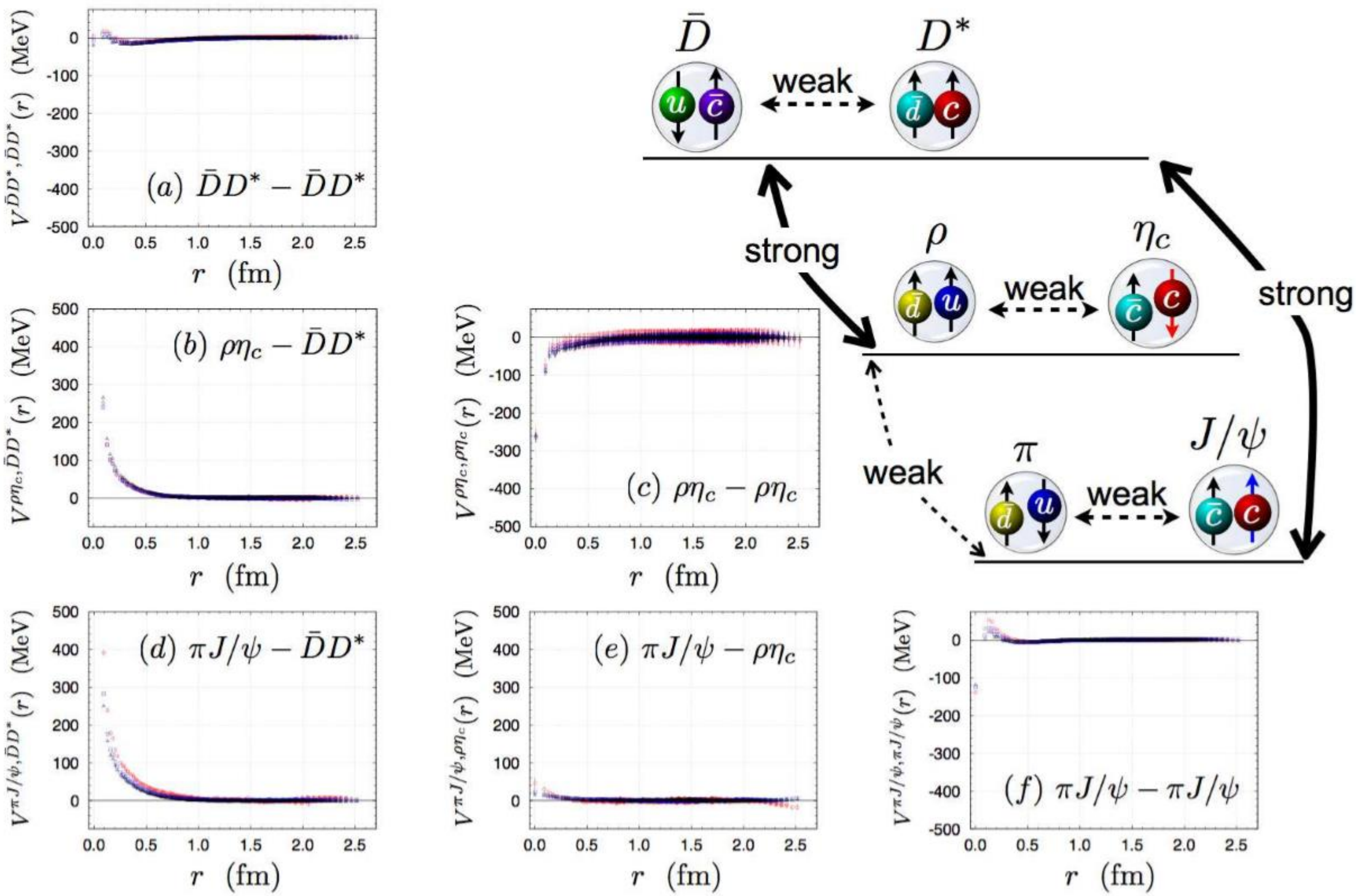
b) **Subsequently, the interaction potentials are obtained.**

$$\left(-\frac{\partial}{\partial t} - H_0^{\alpha}\right) R^{\alpha\beta}(\vec{r},t) = \sum_{\gamma} \Delta^{\alpha\gamma} \int d\vec{r'}\, U^{\alpha\gamma}(\vec{r},\vec{r'}) R^{\gamma\beta}(\vec{r'},t)$$

$$R^{\alpha\beta}(\vec{r},t) \equiv C^{\alpha\beta}(\vec{r},t) e^{(m_1^{\alpha}+m_2^{\alpha})t} \qquad H_0^{\alpha} = -\nabla^2/2\mu^{\alpha}$$

c) **Couple channel effects of eta_c rho, J/psi pi and DD*bar is considered.**

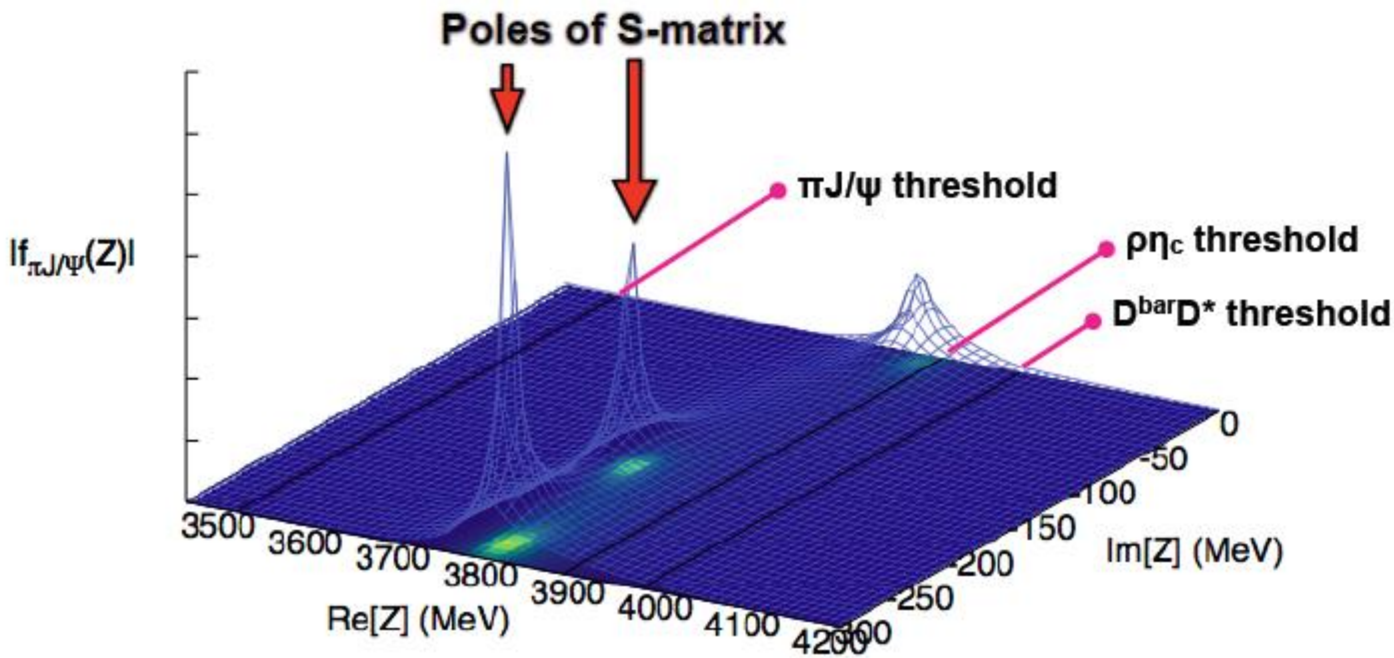
d) **They conclude the near DD*bar threshold enhancement is due to the J/psi pi and DD*bar coupling.**



(a) $\bar{D}D^* - \bar{D}D^*$
(b) $\rho\eta_c - \bar{D}D^*$
(c) $\rho\eta_c - \rho\eta_c$
(d) $\pi J/\psi - \bar{D}D^*$
(e) $\pi J/\psi - \rho\eta_c$
(f) $\pi J/\psi - \pi J/\psi$
$V^{\bar{D}D^*,\bar{D}D^*}(r)$ (MeV)
$V^{\rho\eta_c,\bar{D}D^*}(r)$ (MeV)
$V^{\rho\eta_c,\rho\eta_c}(r)$ (MeV)
$V^{\pi J/\psi,\bar{D}D^*}(r)$ (MeV)
$V^{\pi J/\psi,\rho\eta_c}(r)$ (MeV)
$V^{\pi J/\psi,\pi J/\psi}(r)$ (MeV)
r (fm)
$\bar{D}$
D^*
ρ
η_c
π
J/ψ
weak
strong

Pole search ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)

✤ <u>input : LQCD potential matrix @ m_π=410MeV</u>



✓ "Virtual (shadow)" poles on the most adjacent complex energy plane for $Z_c(3900)$ energy region are found

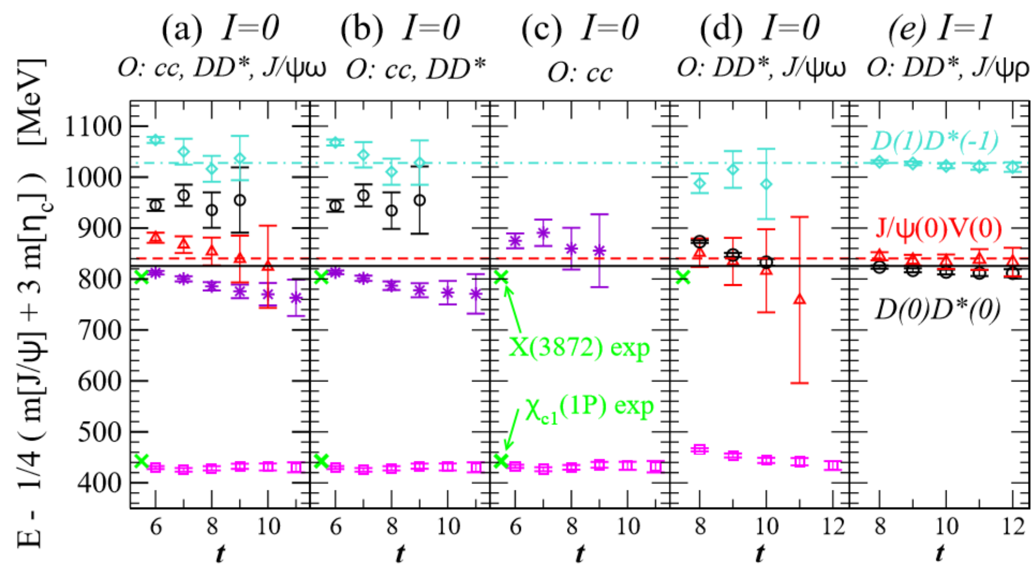
✓ These poles contribute to threshold enhancement of amplitude

2. X(3872)

S. Prelovsek & L. Leskovec, PRL111(2013)192001

Operators:

$$
\begin{aligned}
O^{\bar{c}c}_{1-8} &= \bar{c}\hat{M}_i c(0) \qquad \text{(only } I=0\text{)} \qquad (2)\\
O_1^{DD^*} &= [\bar{c}\gamma_5 u(0)\ \bar{u}\gamma_i c(0) - \bar{c}\gamma_i u(0)\ \bar{u}\gamma_5 c(0)] + f_I\{u\to d\}\\
O_2^{DD^*} &= [\bar{c}\gamma_5\gamma_t u(0)\ \bar{u}\gamma_i\gamma_t c(0) - \bar{c}\gamma_i\gamma_t u(0)\ \bar{u}\gamma_5\gamma_t c(0)]\\
&\quad + f_I\ \{u\to d\}\\
O_3^{DD^*} &= \sum_{e_k=\pm e_{x,y,z}} [\bar{c}\gamma_5 u(e_k)\ \bar{u}\gamma_i c(-e_k) - \bar{c}\gamma_i u(e_k)\ \bar{u}\gamma_5 c(-e_k)]\\
&\quad + f_I\ \{u\to d\}\\
O_1^{J/\psi V} &= \epsilon_{ijk}\ \bar{c}\gamma_j c(0)\ [\bar{u}\gamma_k u(0) + f_I\ \bar{d}\gamma_k d(0)]\\
O_2^{J/\psi V} &= \epsilon_{ijk}\ \bar{c}\gamma_j\gamma_t c(0)\ [\bar{u}\gamma_k\gamma_t u(0) + f_I\ \bar{d}\gamma_k\gamma_t d(0)]\ ,
\end{aligned}
$$



$$
p\cdot\cot\delta(p) = \frac{2\ Z_{00}(1;q^2)}{\sqrt{\pi}L}\ , \qquad q^2 \equiv \left(\frac{L}{2\pi}\right)^2 p^2
$$

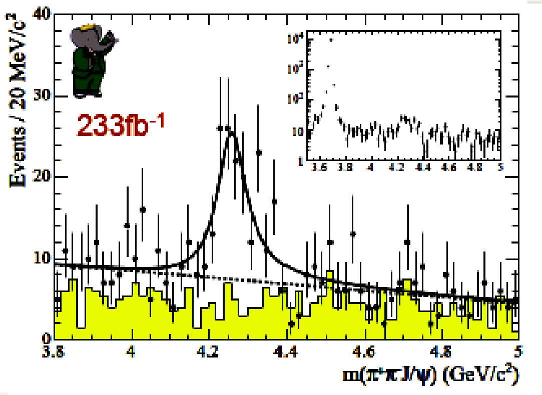
$$
p\cot\delta(p) = \frac{1}{a_0^{DD*}} + \frac{1}{2}r_0^{DD*}p^2\ .
$$

$$
a_0^{DD^*} = -1.7\pm0.4\ \text{fm}\ , \quad r_0^{DD^*} = 0.5\pm0.1\ \text{fm}\ .
$$

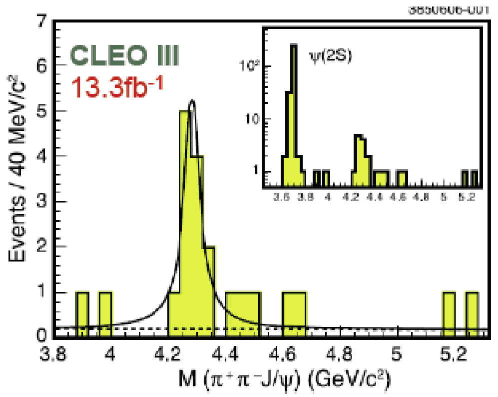
$$
\cot\delta(p_{BS}) = i.
$$

$X(3872)$	$m_X - \frac{1}{4}(m_{\eta_c}+3m_{J/\psi})$	$m_X-(m_{D^0}+m_{D^{0*}})$
lat$^{L\to\infty}$	815 ± 7 MeV	-11 ± 7 MeV
exp	804 ± 1 MeV	-0.14 ± 0.22 MeV

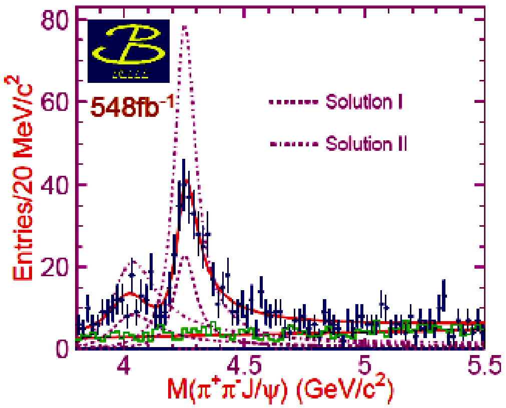
3. Y(4260) relevant study from quenched lattice QCD



BaBar,PRL95,142001(05)



CLEO-III,PRD74,
091104(R)(2006)



Belle,PRL99,182004(07)

$$e^+e^- \to \gamma_{ISR} X$$
$$X \to J/\psi\pi^+\pi^-$$
$$J/\psi \to l^+l^-$$

1. Observed in the initial state radiation process

2. The resonance parameter (PDG2012)

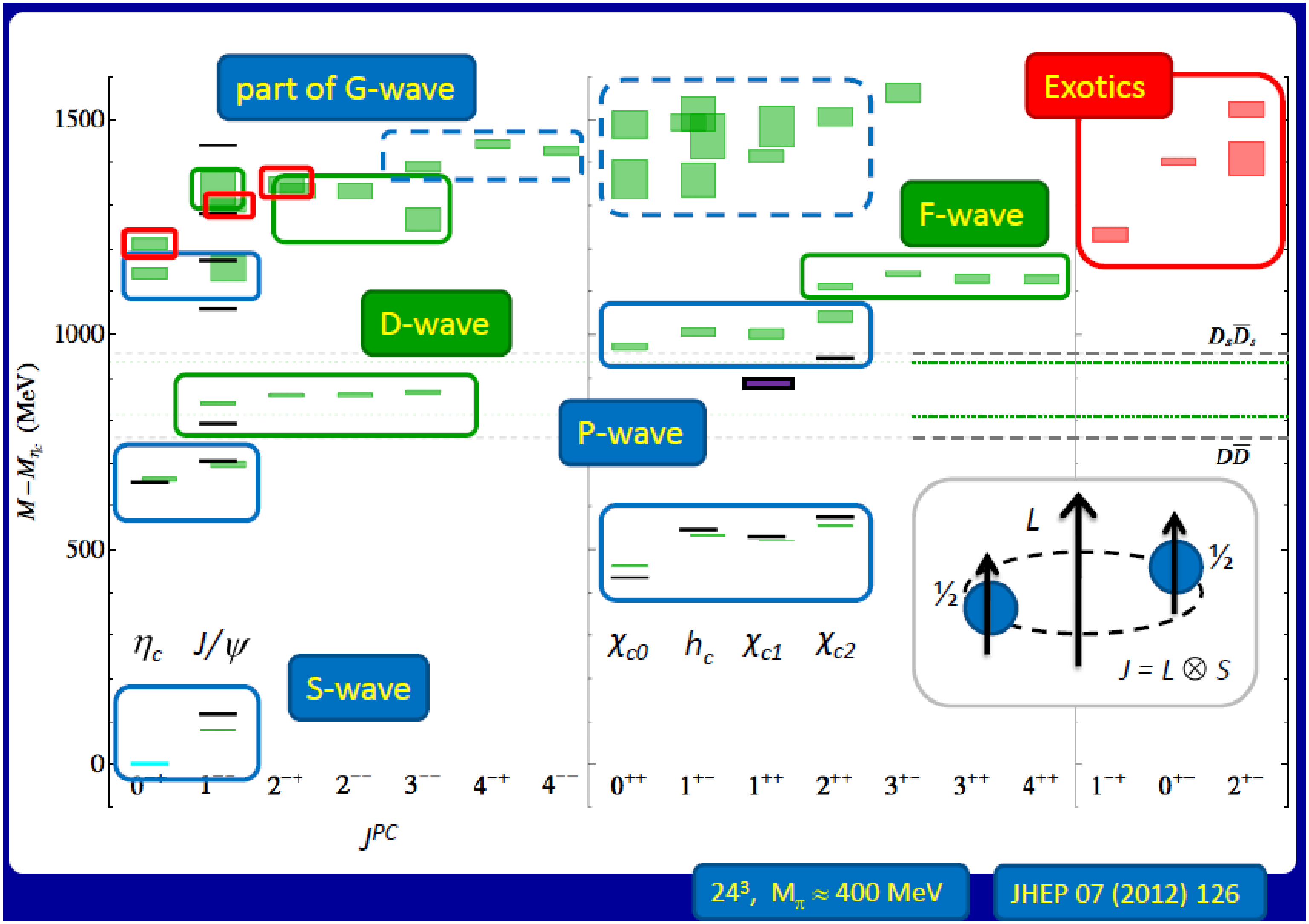
$$M_X = 4263(8)MeV$$

$$\Gamma_X = 95(14)MeV$$

3. The leptonic decay width

$$\boxed{\Gamma(Y(4260) \to e^+e^-)\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8eV}$$

Latest charmonium spectrum from lattice QCD



A. 1– hybrid-like interpolation field operator

Y. Chen, W. Chiu et al., Chin. Phys. C40, 081002 (2016)

$$\bar{\psi}^a \gamma_5 \psi^b B_i^{ab}$$

Nonrelativistic decomposition

$$q = e^{\frac{\gamma \cdot D}{2m}} \begin{pmatrix} \psi \\ \chi \end{pmatrix} = \left[1 + \frac{\gamma \cdot D}{2m} + \frac{\gamma \cdot \vec{D}\, \gamma \cdot D}{8m^2} O(1/m^3)\right] \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$
$$= \begin{pmatrix} \psi \\ \chi \end{pmatrix} + \frac{i}{2m} \begin{pmatrix} -\sigma \cdot \vec{D}\chi \\ \sigma \cdot \vec{D}\psi \end{pmatrix} + \frac{(\vec{D}^2 + \sigma \cdot B)}{8m^2} \begin{pmatrix} \psi \\ \chi \end{pmatrix} + O(1/m^3),$$
$$\bar{q} = \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} e^{-\frac{\gamma \cdot \overleftarrow{D}}{2m}} = \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} + \frac{i}{2m} \begin{pmatrix} \chi^\dagger \sigma \cdot \overleftarrow{D}^\dagger & \psi^\dagger \sigma \cdot \overleftarrow{D}^\dagger \end{pmatrix}$$
$$+ \frac{(\overleftarrow{D}^2 + \sigma \cdot B)}{8m^2} \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} + O(1/m^3),$$

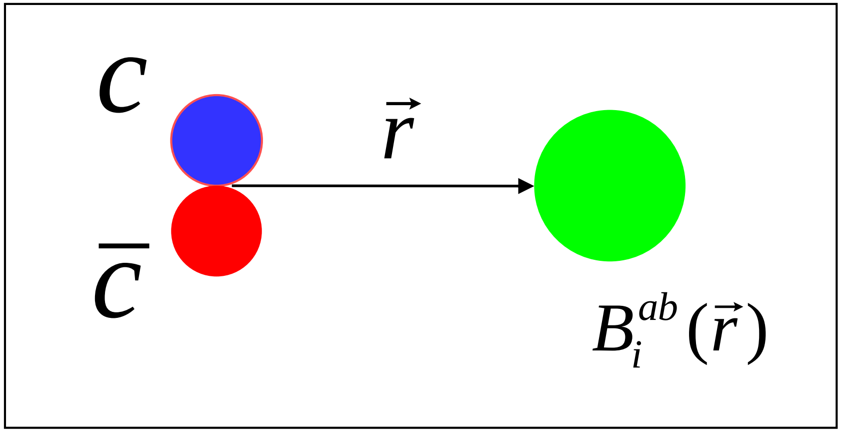
0^{-+}	$\bar{\psi}\gamma_5\psi$	$\chi^+\varphi$
1^{--}	$\bar{\psi}\gamma_i\psi$	$\chi^+\sigma_i\varphi$
$1^{--}{}_H$	$\bar{\psi}^a\gamma_5\psi^b B_i^{ab}$	$\chi^{a+}\varphi^b B_i^{ab}$

$$O_i^{(H)} \equiv \bar{c}^a \gamma_5 c^b B_i^{ab} \to \chi^{a\dagger} \phi^b B_i^{ab} + O(\frac{1}{m_c}),$$ → **c-cbar spin singlet**

$$O_i^{(M)} \equiv \bar{c}^a \gamma_i c^a \to \chi^{a\dagger} \sigma_i \phi^a + O(\frac{1}{m_c}).$$ → **c-cbar spin triplet**

B. Spatially extended interpolation field operator for the vector charmonium-like state

In the Coulomb gauge, $$O(\vec{r}) = (\bar{c}^{a}\gamma_5 c^{b})(0)B_i^{ab}(\vec{r})$$



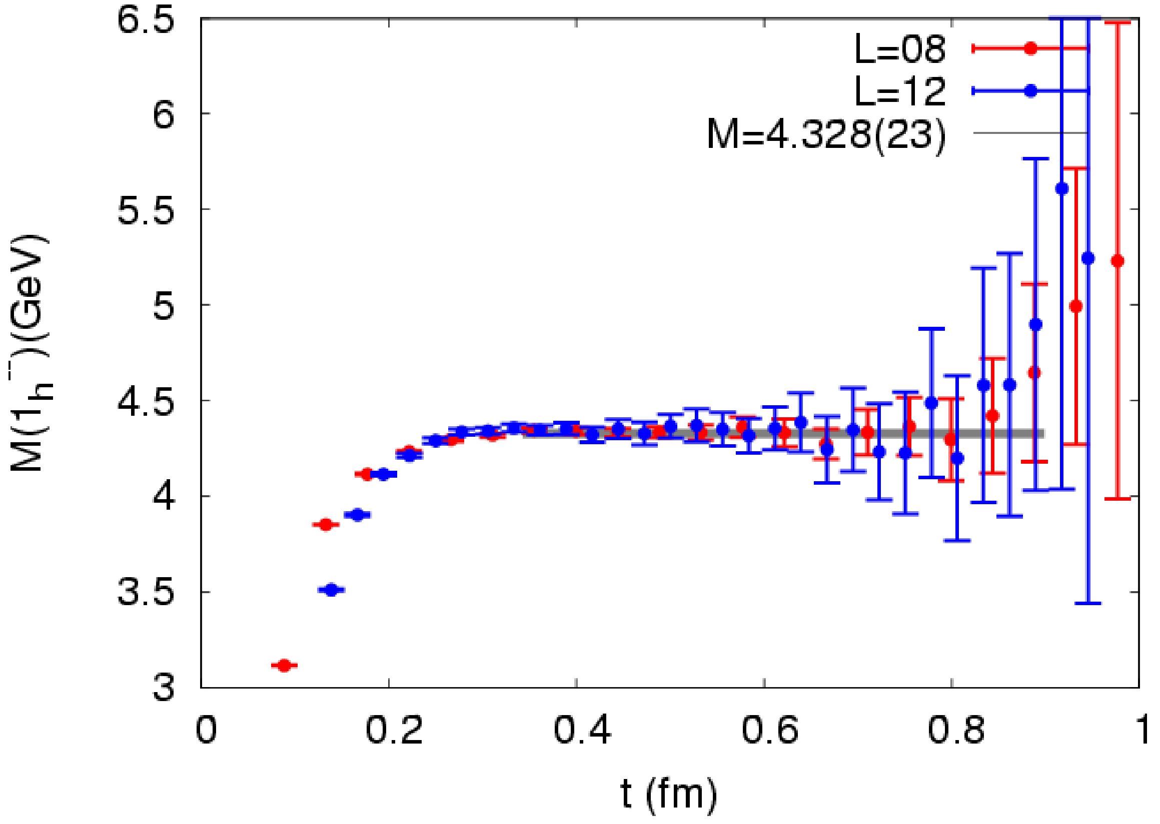
This is equivalent to giave a c-cbar center of mass motion, which describes the recoil of the c-cbar against additional degrees of freedom.

Intuitively, the coupling of this kind of operators to conventional vector charmonia can be suppressed from two aspects:

a) spin states of the c-cbar (spin flipping is suppressed by the heavy quark mass.

b) center-of-mass motion (to the leading order of NR, there is no cneter-of-mass motions for conventional charmonia.)

This kind of operator does couple strongly to a state with mass near 4.3 GeV, as seen in the effective mass plateau



IV) The leptonic decay width of the exotic vector charnomium

1. The leptonic decay width of this exotic vector chamonium is an important quantity, which can shed light on the nature of Y(4260).

$$\boxed{\Gamma(Y(4260)\to e^+e^-)\Gamma(Y(4260)\to J/\psi\pi^+\pi^-)/\Gamma_{tot}=5.8eV}$$

2. The leptonic decay constant of the exotic state can be calculated directly in lattice QCD.

The decay constant of a vector meson is defined as

$$\boxed{\langle 0|\bar{q}\gamma_\mu q|V(\vec{p},r)\rangle = m_V f_V \varepsilon_\mu(\vec{p},r)}$$

where the matrix element on the left can be derived by calculate the two point function

$$\sum_{\vec{x}}\langle 0|\bar{q}\gamma_\mu q(\vec{x},t)O^{(w)}(0)|0\rangle = \sum_{i,r}\frac{1}{2M_i}\langle 0|\bar{q}\gamma_\mu q|V_i,r\rangle\langle V_i,r|O^{(w)}|0\rangle e^{-M_i t}$$

Using the formula $$\Gamma(V_{c\bar{c}} \to e^+e^-) = \frac{16\pi}{27}\alpha_{\rm QED}^2 \frac{f_V^2}{M_V}$$

One can predict $$\Gamma(X \to e^+e^-) < 40\,\mathrm{eV}.$$

$$\boxed{\Gamma(Y(4260) \to e^+e^-)\underbrace{\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot}}_{>\ 10\%} = 5.8eV}$$

“Halo charmonium”:

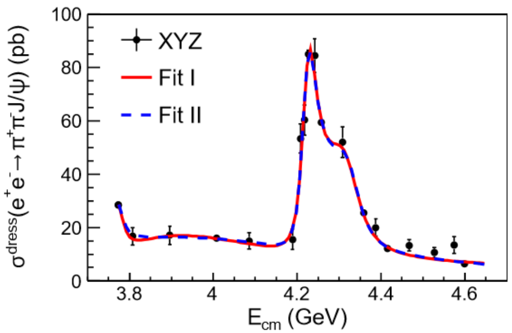
A relatively localized kernal of color octet ccbar surrounded by a gluonic cloud.

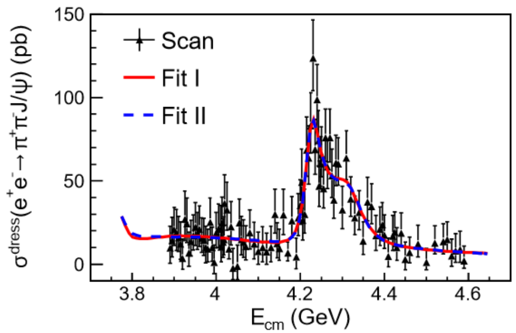
The glounic cloud can be easily hadronized into light hadrons by emitting or absorbing a soft gluon.

Consequently halo charmonium has large branching ratios to decay into a conventional charmoium by emitting light hadrons.

$e^+e^- \to J/\psi\pi^+\pi^-$

BESIII, PRL118(2017)092001 [arXiv: 1611.01317(hep-ex)]

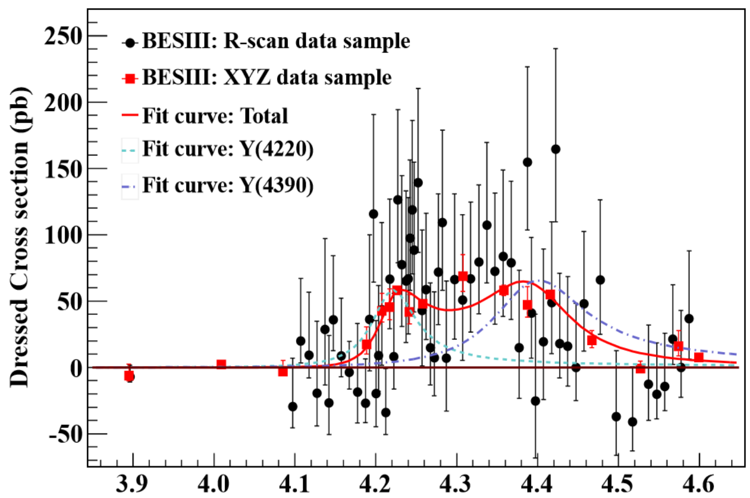




Parameters	Fit result
$M(R_1)$	$3812.6^{+61.9}_{-96.6}$ $(\cdots)$
$\Gamma_{tot}(R_1)$	$476.9^{+78.4}_{-64.8}$ $(\cdots)$
$M(R_2)$	4222.0 ± 3.1 (4220.9 ± 2.9)
$\Gamma_{tot}(R_2)$	44.1 ± 4.3 (44.1 ± 3.8)
$M(R_3)$	4320.0 ± 10.4 (4326.8 ± 10.0)
$\Gamma_{tot}(R_3)$	$101.4^{+25.3}_{-19.7}$ $(98.2^{+25.4}_{-19.6})$

Parameters	Solution I	Solution II	Solution III	Solution IV
$\Gamma_{e^+e^-}\mathcal{B}[\psi(3770) \to \pi^+\pi^- J/\psi]$	0.5 ± 0.1 (0.4 ± 0.1)			
$\Gamma_{e^+e^-}\mathcal{B}(R_1 \to \pi^+\pi^- J/\psi)$	$8.8^{+1.5}_{-2.2}$ $(\cdots)$	$6.8^{+1.1}_{-1.5}$ $(\cdots)$	$7.2^{+0.9}_{-1.5}$ $(\cdots)$	$5.6^{+0.6}_{-1.0}$ $(\cdots)$
$\Gamma_{e^+e^-}\mathcal{B}(R_2 \to \pi^+\pi^- J/\psi)$	13.3 ± 1.4 (12.0 ± 1.0)	9.2 ± 0.7 (8.9 ± 0.6)	2.3 ± 0.6 (2.1 ± 0.4)	1.6 ± 0.4 (1.5 ± 0.3)
$\Gamma_{e^+e^-}\mathcal{B}(R_3 \to \pi^+\pi^- J/\psi)$	21.1 ± 3.9 (17.9 ± 3.3)	$1.7^{+0.8}_{-0.6}$ $(1.1^{+0.5}_{-0.4})$	$13.3^{+2.3}_{-1.8}$ $(12.4^{+1.9}_{-1.7})$	$1.1^{+0.4}_{-0.3}$ (0.8 ± 0.3)
ϕ_1	-58 ± 11 (-33 ± 8)	-116^{+9}_{-10} (-81^{+7}_{-8})	65^{+24}_{-20} (81^{+16}_{-14})	8 ± 13 (33 ± 9)
ϕ_2	-156 ± 5 (-132 ± 3)	68 ± 24 (107 ± 20)	-115^{+11}_{-9} (-95^{+6}_{-5})	110 ± 16 (144 ± 14)

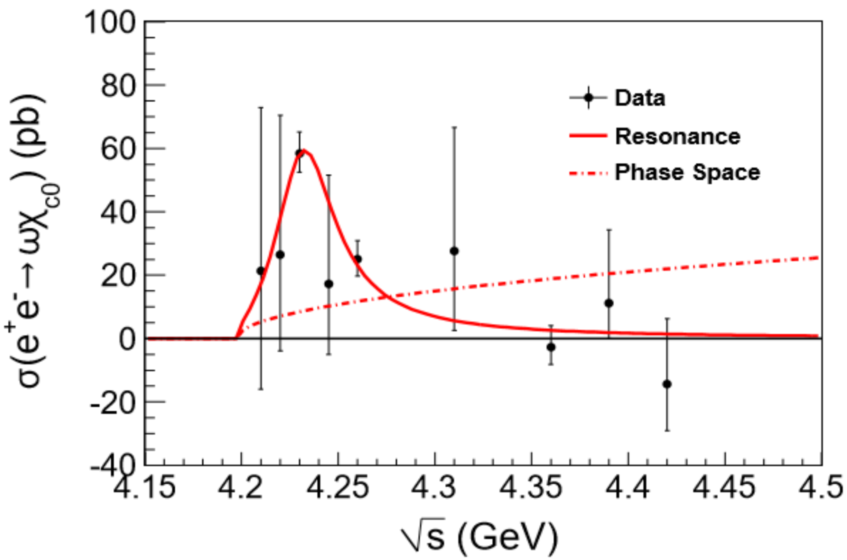
$$e^+e^- \to h_c\pi^+\pi^-$$



BESIII, PRL118(2017)092002
arXiv: 1610.07044(hep-ex)

$$M = (4218.4 \pm 4.0 \pm 0.9)\ \mathrm{MeV}/c^2$$
$$\Gamma = (66.0 \pm 9.0 \pm 0.4)\ \mathrm{MeV}$$
$$\Gamma^{el} = (4.6 \pm 4.1 \pm 0.8)\ \mathrm{eV}$$

$$e^+e^- \to \chi_{c0}\omega$$



$$M = (4230 \pm 8)\ \mathrm{MeV}/c^2$$
$$\Gamma_t = (38 \pm 12)\ \mathrm{MeV}$$
$$\Gamma_{ee}\mathcal{B}(\omega\chi_{c0}) = (2.7 \pm 0.5)\ \mathrm{eV}$$

$J/\psi\pi^+\pi^-$ **mode: relative S-wave between** J/ψ **and** $\pi^+\pi^-$

$\chi_{c0}\omega$ **mode: relative S-wave between** χ_{c0} **and** ω

$h_c\pi^+\pi^-$ **mode: relative P-wave between** h_c **and** $\pi^+\pi^-$

The $c\bar{c}$ **in the halo charmonium is spin singlet (S=0),**

$J/\psi\pi^+\pi^-$ **mode:** J/ψ **(S=1), spin flipping,** m_c **suppressed,**
no refugal barrier

$\chi_{c0}\omega$ **mode:** χ_{c0} **(S=1), spin flipping,** m_c **suppressed,**
no refugal barrier

$h_c\pi^+\pi^-$ **mode:** h_c **(S=0), no spin flipping,**
but suppressed by the refugal barrier .

In this picture, it is understandable that the above three modes have similar cross section at $\sqrt{s} \sim 4.22\ GeV$

IV. Summary

- Glueball spectrum has been investigated both from quenched lattice QCD and full-QCD lattice study. There are not large unquenched effects observed. Apart from the spectrum, lattice QCD can also provide useful theoretical information to the production properties of glueballs.

- XYZ particles are also hot topics in lattice QCD study, but the results are preliminary.

- There are still many difficulties for lattice QCD to study exotic hadrons, from both the theoretical tools and numerical calculations.

Thanks!