Masses of singly and doubly heavy baryons within the SU(3) chiral quark-soliton model

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- Motivation
- Chiral quark-soliton model
- Mass splittings of the singly heavy baryons
- Masses of the singly heavy baryons
- Masses of the doubly heavy baryons
The masses of light baryons are well known in the ground state and were described very well in the chiral quark-soliton model.

We want to extend the model to explain the mass spectra of heavy baryons.

As \( m_Q \to \infty \), heavy quark spin is conserved, which leads to the fact that the light-quark spin is also conserved.

The soliton and a heavy quark are decoupled and the heavy quark plays a role of the static color source.

C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.
The masses of charmed baryons are well known. Thus we will first check the validity of the present approach in the charmed sector.

On the other hand, some of bottom baryon masses are unknown. In this talk, we will show how the mass of $\Omega_{b}^{*-}$ is predicted.

C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.

\[ M_{\Xi^{+}_{cc}} = (3519 \pm 1) \text{ MeV} \]

R. Aaij et al. [LHCb Collaboration], arXiv:1707.01621 [hep-ex]:

\[ M_{\Xi^{++}_{cc}} = (3621.4 \pm 0.72 \pm 0.27 \pm 0.14) \text{ MeV} \]

- The mass of the \( \Omega^{+}_{cc} \) is unknown. Recently, the mass of the \( \Xi^{++}_{cc} \) was found. Using this data, we can predict that of \( \Omega^{+}_{cc} \).
Five $\Omega_c^0$s were announced by LHCb Coll.

CHIRAL QUARK-SOLITON MODEL

- Effective chiral action

\[ S_{\text{eff}} = -N_c \text{Tr} \ln(i\gamma_\mu \partial^\mu + i\hat{m} + iMU\gamma^5) \]

\[ \hat{m} = \text{diag}(m_u, m_d, m_s) \]

- Hedgehog Ansatz

\[ U_c = e^{i\gamma_5 n^a \tau^a \pi(r)} \]

- Self-Consistent soliton solution

\[ \frac{\delta}{\delta U} \left[ N_c E_{\text{val}} + E_{\text{sea}} \right] \bigg|_{U_c} = 0, \quad M_{\text{cl}} \approx 1300 \text{ MeV} \]

Zero-mode quantization

\[ U(r) = \begin{pmatrix} U_c(r) & 0 \\ 0 & 1 \end{pmatrix}. \]

Witten’s embedding

\[ \vec{J} + \vec{T} = 0 \]

Slowly rotating soliton

Collective Hamiltonian

\[ H = H_{\text{sym}} + H_{\text{sb}}^{(1)} \]

\[ H_{\text{sym}} = M_{\text{cl}} + \frac{1}{2I_1} \sum_{i=1}^{3} \hat{j}_i^2 + \frac{1}{2I_2} \sum_{a=4}^{7} \hat{j}_a^2, \]

\[ H_{\text{sb}}^{(1)} = \alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D_{8i}^{(8)} \hat{J}_i, \]

---

Change of the pion mean fields for Singly heavy baryon

The soliton mass formula is changed only for the valence part in the singly heavy baryon sector.

\[ M_{\text{sol}} = (N_c - 1)E_{\text{val}} + E_{\text{sea}} \approx 1100 \text{ MeV}. \]

Change of the pion mean fields for Singly heavy baryon

- The moment of inertias and dynamical parameters are also changed

<table>
<thead>
<tr>
<th>Light baryon</th>
<th>This work</th>
<th>Ref [14]</th>
<th>Singly heavy baryon</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{I}_1$ [fm]</td>
<td>1.108</td>
<td>1.230 ± 0.002</td>
<td>$I_1$ [fm]</td>
<td>0.844</td>
</tr>
<tr>
<td>$\tilde{I}_2$ [fm]</td>
<td>0.529</td>
<td>0.420 ± 0.006</td>
<td>$I_2$ [fm]</td>
<td>0.404</td>
</tr>
<tr>
<td>$\tilde{K}_1$ [fm]</td>
<td>0.428</td>
<td>-</td>
<td>$K_1$ [fm]</td>
<td>0.286</td>
</tr>
<tr>
<td>$\tilde{K}_2$ [fm]</td>
<td>0.272</td>
<td>-</td>
<td>$K_2$ [fm]</td>
<td>0.181</td>
</tr>
<tr>
<td>$\tilde{N}_0$ [fm]</td>
<td>0.457</td>
<td>-</td>
<td>$N_0$ [fm]</td>
<td>0.499</td>
</tr>
<tr>
<td>$\tilde{N}_1$ [fm]</td>
<td>0.410</td>
<td>-</td>
<td>$N_1$ [fm]</td>
<td>0.380</td>
</tr>
<tr>
<td>$\tilde{N}_2$ [fm]</td>
<td>0.323</td>
<td>-</td>
<td>$N_2$ [fm]</td>
<td>0.286</td>
</tr>
<tr>
<td>$\tilde{\Sigma}_{\pi N}$ [MeV]</td>
<td>43.7</td>
<td>36.4 ± 3.9</td>
<td>$\Sigma_{\pi N}$ [MeV]</td>
<td>40.0</td>
</tr>
<tr>
<td>$\tilde{\alpha}$ [MeV]</td>
<td>-392.0</td>
<td>-262.9 ± 5.9</td>
<td>$\alpha$ [MeV]</td>
<td>-326.3</td>
</tr>
<tr>
<td>$\tilde{\beta}$ [MeV]</td>
<td>-99.3</td>
<td>-144.3 ± 3.2</td>
<td>$\beta$ [MeV]</td>
<td>-77.8</td>
</tr>
<tr>
<td>$\tilde{\gamma}$ [MeV]</td>
<td>-49.4</td>
<td>-104.2 ± 2.4</td>
<td>$\gamma$ [MeV]</td>
<td>-37.8</td>
</tr>
<tr>
<td>$M_{sol}$ [MeV]</td>
<td>1296.1</td>
<td>-</td>
<td>$M_{sol}$ [MeV]</td>
<td>1099.9</td>
</tr>
</tbody>
</table>

**ROTATIONAL ENERGY**

\[ \bar{M}_{R,Q} = \begin{cases} M_{\text{cl}} & \text{Light baryon} \\ M_{\text{cl}} = M_{\text{sol}} + m_Q & \text{Singly heavy baryon} \end{cases} \]

\[ \bar{M}_{R,Q} = M_{\text{cl}} + \frac{1}{2I_1} J(J + 1) + \frac{1}{2I_2} [C_2(p, q) - J(J + 1)] - \frac{3}{8I_2} \bar{Y}^2. \]

\[ \psi_B^{(R)}(J'_3, J; A) = \sum_{m_3 = -1/2}^{1/2} \sqrt{\text{dim}(p, q)}(-1)^{-\frac{3}{2} + J_3} C_{J_Q m_3 J - J_3} ^ {J'_3} \left[ \langle Y, T, T_3 | D^{(8)}(A) | \bar{Y}, J, -J_3 \rangle \right]^* \chi m_3, \]

- The rotational energy yields the energy difference between the \( \bar{3} \) and the 6.
  
  \( \bar{3} \to (p, q) = (0, 1) \to J = 0 \) antitriplet
  
  \( 6 \to (p, q) = (2, 0) \to J = 1 \) sextet.

\[ \bar{M}_{6,c} - \bar{M}_{\bar{3},c} = \frac{1}{I_1}. \]

HYPERFINE MASS SPLITTING

- We have to introduce the hyperfine interaction to lift the degeneracy in the sextet representations.

\[ H_{hf} = \frac{2}{3} \frac{\kappa}{m_Q M_{\text{sol}}} \vec{J} \cdot \vec{J}_Q, \]

\[ M^{(hf)}_{6_{1/2},Q} = -\frac{2\kappa}{3m_Q M_{\text{sol}}}, \]

\[ M^{(hf)}_{6_{3/2},Q} = \frac{\kappa}{3m_Q M_{\text{sol}}}, \]

- The hyperfine mass splittings are determined by using center values of the sextet masses [1]

\[ \frac{\kappa}{m_c M_{\text{sol}}} = (68.1 \pm 1.1) \text{ MeV} \]

\[ \frac{\kappa}{m_b M_{\text{sol}}} = (20.3 \pm 1.0) \text{ MeV} \]

---

SU(3) SYMMETRY BREAKING

$$H_{sb}^{(1)} = +\alpha D^{(8)}_{88} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D^{(8)}_{8i} \hat{J}_i,$$

$$\alpha = \left( -\frac{\Sigma_{\pi N}}{3m_0} + \frac{K_2}{I_2} \bar{Y} \right) m_s, \quad \beta = -\frac{K_2}{I_2} m_s, \quad \gamma = 2 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s.$$

**Mass correction**

$$M_{B,\mathcal{R}}^{(1)} = \langle B, \mathcal{R} | H_{sb}^{(1)} | B, \mathcal{R} \rangle = Y \delta_{\mathcal{R}}$$

$$\delta_3 = \frac{3}{8} \alpha + \beta, \quad \delta_6 = \frac{3}{20} \alpha + \beta - \frac{3}{10} \gamma.$$
MASS SPLITTING OF THE SINGLY HEAVY BARYON

LIGHT BARYON  
(1st order mass correction)

\[ m_s = 210 \text{MeV} \]

CHARMED BARYON (1st order mass correction)

$m_s = 177 \text{MeV}$
MASS SPLITTING OF THE SINGLY HEAVY BARYON

BOTTOM BARYON (1st order mass correction)

\[ m_s = 167 \text{MeV} \]

\[ m_s = 167 \text{MeV} \]
LIGHT BARYON (2nd order mass correction)

CHARMED BARYON (2nd order mass correction)

\[ m_s = 174 \text{ MeV} \]
BOTTOM BARYON (2nd order mass correction)

\[ m_s = 166 \text{ MeV} \]

<table>
<thead>
<tr>
<th>perturbation</th>
<th>( m_s \text{[MeV]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>light baryon</td>
<td>210 202</td>
</tr>
<tr>
<td>charmed baryon</td>
<td>177 174</td>
</tr>
<tr>
<td>bottom baryon</td>
<td>167 166</td>
</tr>
</tbody>
</table>
CHARMED BARYON

- The heavy baryon masses are calculated by using the determined strange current quark mass.
- The soliton mass is overestimated. Thus, instead of using it, we employ the experimental center mass, so we predict the all masses of $\bar{3}$ and $6$.

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>$m_s$ [MeV]</th>
<th>1st</th>
<th>2nd</th>
</tr>
</thead>
<tbody>
<tr>
<td>light baryon</td>
<td>210</td>
<td>202</td>
<td></td>
</tr>
<tr>
<td>charmed baryon</td>
<td>177</td>
<td>174</td>
<td></td>
</tr>
<tr>
<td>bottom baryon</td>
<td>167</td>
<td>166</td>
<td></td>
</tr>
</tbody>
</table>

The results obtained are in good agreement with the experiment result!

Especially, the mass of $\Omega_b^*$ is predicted in the chiral quark-soliton model.

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>$m_s$ [MeV]</th>
<th>$\bar{m}_s$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>light baryon</td>
<td>210</td>
<td>202</td>
</tr>
<tr>
<td>charmed baryon</td>
<td>177</td>
<td>174</td>
</tr>
<tr>
<td>bottom baryon</td>
<td>167</td>
<td>166</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B_c$</th>
<th>Experiment [MeV]</th>
<th>ref. [14] [MeV]</th>
<th>Perturbative 1st order</th>
<th>Perturbative 2nd order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3_{1/2}^b$ Λ$_b$</td>
<td>5619.5 ± 0.2</td>
<td>5599.3 ± 2.4</td>
<td>5607.1</td>
<td>5615.1</td>
</tr>
<tr>
<td>$3_{1/2}^b$ Ξ$_b$</td>
<td>5793.1 ± 0.7</td>
<td>5803.1 ± 1.2</td>
<td>5799.3</td>
<td>5795.3</td>
</tr>
<tr>
<td>$6_{1/2}^b$ Σ$_b$</td>
<td>5813.4 ± 1.3</td>
<td>5804.3 ± 2.1</td>
<td>5821.0</td>
<td>5815.3</td>
</tr>
<tr>
<td>$6_{1/2}^b$ Ξ$_b$</td>
<td>5935.0 ± 0.05</td>
<td>5939.5 ± 1.5</td>
<td>5931.8</td>
<td>5933.0</td>
</tr>
<tr>
<td>$6_{1/2}^b$ Ω$_b$</td>
<td>6048.0 ± 1.9</td>
<td>6074.7 ± 4.5</td>
<td>6042.6</td>
<td>6046.4</td>
</tr>
<tr>
<td>$6_{3/2}^b$ Σ$_b$</td>
<td>5833.6 ± 1.3</td>
<td>5824.6 ± 2.3</td>
<td>5840.9</td>
<td>5835.5</td>
</tr>
<tr>
<td>$6_{3/2}^b$ Ξ$_b$</td>
<td>5955.3 ± 0.1</td>
<td>5959.8 ± 1.2</td>
<td>5951.7</td>
<td>5953.2</td>
</tr>
<tr>
<td>$6_{3/2}^b$ Ω$_b$</td>
<td>-</td>
<td>6095.0 ± 4.1</td>
<td>6062.5</td>
<td>6066.6</td>
</tr>
</tbody>
</table>

Our prediction: $M_{\Omega_b^*} = 6066.6$ MeV

HEAVY PENTAQUARK


<table>
<thead>
<tr>
<th>Resonance</th>
<th>Mass (MeV)</th>
<th>Width (MeV)</th>
<th>Yield (Expected Rate)</th>
<th>Significance ($N_\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_c(3000)^0$</td>
<td>$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$</td>
<td>$4.5 \pm 0.6 \pm 0.3$</td>
<td>$1300 \pm 100 \pm 80$</td>
<td>$20.4$</td>
</tr>
<tr>
<td>$\Omega_c(3050)^0$</td>
<td>$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$</td>
<td>$0.8 \pm 0.2 \pm 0.1$</td>
<td>$970 \pm 60 \pm 20$</td>
<td>$20.4$</td>
</tr>
<tr>
<td>$\Omega_c(3066)^0$</td>
<td>$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$</td>
<td>$3.5 \pm 0.4 \pm 0.2$</td>
<td>$1740 \pm 100 \pm 50$</td>
<td>$23.9$</td>
</tr>
<tr>
<td>$\Omega_c(3090)^0$</td>
<td>$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$</td>
<td>$8.7 \pm 1.0 \pm 0.8$</td>
<td>$2000 \pm 140 \pm 130$</td>
<td>$21.1$</td>
</tr>
<tr>
<td>$\Omega_c(3119)^0$</td>
<td>$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$</td>
<td>$1.1 \pm 0.8 \pm 0.4$</td>
<td>$480 \pm 70 \pm 30$</td>
<td>$10.4$</td>
</tr>
</tbody>
</table>

$< 1.2$ MeV, 95% CL

| $\Omega_c(3188)^0$ | $3188 \pm 5 \pm 13$ | $60 \pm 15 \pm 11$ | $1670 \pm 450 \pm 360$ |

$< 2.6$ MeV, 95% CL

HEAVY PENTAQUARK

Prof. Hyun-Chul Kim - Possible existence of charmed exotics

The hyperfine mass splitting is determined in the charmed baryon sector.

\[ \frac{\kappa}{m_c M_{\text{sol}}} = (68.1 \pm 1.1) \text{ MeV} \]

[Masses of the singly heavy baryons]

**HEAVY PENTAQUARK**

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[Diagram of baryon states]

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[Diagram of baryon states]

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[Diagram of baryon states]
### MASSES OF THE SINGLY HEAVY BARYONS

<table>
<thead>
<tr>
<th>Perturbative</th>
<th>1st order</th>
<th>2nd order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_s$</td>
<td>177</td>
<td>174</td>
</tr>
<tr>
<td>$M_{B^c_{5c}}$</td>
<td>2738.1</td>
<td>2751.7</td>
</tr>
<tr>
<td>$M_{\Sigma^c_{5c}}$</td>
<td>2827.9</td>
<td>2861.0</td>
</tr>
<tr>
<td>$M_{\Lambda^c_{5c}}$</td>
<td>2849.1</td>
<td>2859.9</td>
</tr>
<tr>
<td>$M_{\Xi^{3/2}_{5c}}$</td>
<td>2917.7</td>
<td>2923.3</td>
</tr>
<tr>
<td>$M_{\Xi_{5c}}$</td>
<td>2949.6</td>
<td>2964.3</td>
</tr>
<tr>
<td>$M_{\Omega_c}$</td>
<td>[Input]</td>
<td></td>
</tr>
<tr>
<td>$M_{B^c_{5c}}$</td>
<td>2808.1</td>
<td>2820.7</td>
</tr>
<tr>
<td>$M_{\Sigma^c_{5c}}$</td>
<td>2897.9</td>
<td>2930.0</td>
</tr>
<tr>
<td>$M_{\Lambda^c_{5c}}$</td>
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<td>2928.9</td>
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<tr>
<td>$M_{\Xi^{3/2}_{5c}}$</td>
<td>2987.7</td>
<td>2992.3</td>
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<tr>
<td>$M_{\Xi_{5c}}$</td>
<td>3019.6</td>
<td>3033.7</td>
</tr>
<tr>
<td>$M_{\Omega_c}$</td>
<td>[Input]</td>
<td></td>
</tr>
</tbody>
</table>

**DOUBLY CHARMED BARYON**

- The pion mean fields of the doubly heavy baryon is modified \( N_c \) to the \( N_c - 2 \)

\[
M_{BQ_1Q_2}^{(1)} = Y \delta_R.
\]

\[
\delta_3 = \frac{3}{16} \alpha' + \beta' - \frac{9}{32} \gamma'.
\]

Our prediction

\[
M_{\Omega_{cc}} = 3732.5 \text{ MeV}
\]
We investigated the mass splitting of the lowest-lying heavy baryons in a pion mean-field approach.

We examined the dependence of the strange current quark mass on systems of heavy-quark baryons.

We predicted the masses of particles $\Omega_b^*$ and $\Omega_{cc}^+$ by using determined strange current quark masses and also the whole heavy pentaquark members.

We also discussed the masses of the $\Omega_c^0$ recently found by the LHCb Collaboration.

Decay widths of the heavy baryons are under investigation in the chiral quark-soliton model.
THANK YOU VERY MUCH!