The Effective Interaction Induced by Instantons in QCD

Baiyang Zhang

Institute of Modern Physics, Chinese Academy of Sciences

9th Workshop on Hadron physics in China and Opportunities Worldwide

In Collaboration with
N. Kochelev (JINR), A. Radzhabov (MISDCT), P.M. Zhang (IMP, CAS)

The QCD Vacua

QCD vacuum is not an empty space but fill with fluctuations.
Due to the gauge invariance, QCD vacuum comes in different topological structures.

Basic properties of instanton

- Instantons are solution to equation of motion (EoM) in Euclidean space,
- Instantons corresponds to field configurations that gives finite Euclidean action in infinite spacetime,
- Every field configuration with instanton number n can not be smoothly deformed into other configurations with different instanton number n' where $n \neq n'$, i.e. instanton solutions are stable.

QCD instantons connect infinity degenerate vacua with each other, complementing the perturbative description.

Quark-quark Interactions Induced by Instantons

Multiquark t'Hooft interaction induced by instantons:

For $N_f = 3$, $q = u, d, s$, in the limit $m_u = m_d = m_s \to 0$

$$H_{\text{t'Hooft}} = \int d\rho n(\rho)(4\pi^2\rho^3)^3 \frac{1}{6N_c(N_C^2-1)} \epsilon^{f_1 f_2 f_3} \epsilon^{g_1 g_2 g_3} \times \left\{ \frac{2N_c+1}{2N_c+4} \overline{q}_R^{f_1} q_L^{g_1} \overline{q}_R^{f_2} q_L^{g_2} \overline{q}_R^{f_3} q_L^{g_3} + \frac{3}{8(N_c+2)} \overline{q}_R^{f_1} q_L^{g_1} \overline{q}_R^{f_2} \sigma_{\mu\nu} q_L^{g_2} \overline{q}_R^{f_3} \sigma_{\mu\nu} q_L^{g_3} + (R \leftrightarrow L) \right\}$$

t'Hooft quark-quark interaction has important effects in $K \to \pi\pi$ decays, $\Delta I = 1/2$ rule, CP violation, etc. (N.K. and V.Vento, "Instantons and the Delta(I) = 1/2 rule," Phys. Rev. Lett. **87** (2001) 111601)

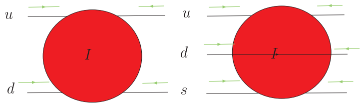


Figure: The quark-quark chirality-flip t'Hooft interaction induced by the instantons

Anomalous Quark-Gluon and Quark-Gluon-Pion Interaction

The general form of interaction vertex of massive quark with gluon can be written as

$$V_\mu(k_1^2, k_2^2, q^2) = -g_s t^a \left[\gamma_\mu F_1(k_1^2, k_2^2, q^2) + \frac{\sigma_{\mu\nu} q^\nu}{2M_q} F_2(k_1^2, k_2^2, q^2)\right],$$

where $k_{1,2}^2$ are the virtualities of incoming and outgoing quarks and q is the momentum transfered. It is similar to the photon-nucleon vertex

$$\Gamma_\mu^{QED} = \gamma_\mu F_1(q^2) + \frac{\sigma_{\mu\nu} q_\nu}{2M_N} F_2(q^2),$$

where $F_1(q^2), F_2(q^2)$ are Dirac and Pauli nucleon form factors respectively. The Anomalous Quark Chromomagnetic Moment (AQCM) (N.K. (1996)) is

$$\mu_a = F_2(0,0,0),$$

$$\Delta\mathcal{L} = -i\mu_a \frac{g_s}{4M_q} \bar{q} \sigma_{\mu\nu} t^a q G^a_{\mu\nu}.$$

Anomalous Quark-Gluon and Quark-Gluon-Pion Interaction

The form factor $F_2(k_1^2, k_2^2, q^2)$ given by the instanton model reads

$$F_2(k_1^2, k_2^2, q^2) = \mu_a F_q(\mid k_1 \mid \rho/2) F_q(\mid k_2 \mid \rho/2) F_g(\mid q \mid \rho),$$

where

$$F_q(z) = -z \frac{d}{dz}(I_0(z)K_0(z) - I_1(z)K_1(z)),$$
$$F_g(z) = \frac{4}{z^2} - 2K_2(z).$$

are the Fourier-transformed quark zero-mode and instanton fields respectively. $I_\nu(z)$ and $K_\nu(z)$ are the modified Bessel functions and ρ is the size of instanton. The value of AQCM is determined by the effective density of the instantons $n(\rho)$ in nonperturbative QCD vacuum

$$\mu_a = -\pi^3 \int \frac{d\rho n(\rho)\rho^4}{\alpha_s(\rho)}.$$

Anomalous Quark-Gluon and Quark-Gluon-Pion Interaction

In the mean field approximation, the value for AQCM is

$$\mu_a^{MF} \approx -0.4,$$

A similar number was recently obtained using a completely different approach based on the Dyson-Schwinger equations by Craig Roberts et el (arXiv:1203.5341)

The numerical results suggests that the value for AQCM is large, so should be it's contribution.

Within Shuryak's instanton liquid model, the relation between AQCM and the dynamical quark mass is given by

$$\mu_a = -\frac{3\pi(M_q\rho_c)^2}{4\alpha_s(\rho_c)},$$

where $\rho_c \approx 0.3$ fm and $\alpha_s(\rho_c) \approx 0.5.$

Anomalous Quark-Gluon and Quark-Gluon-Pion Interaction

The σ-model Lagrangian for the pion part from t'Hooft interaction reads (Diakonov,Petrov etc.)

$$\mathcal{L}_{eff} = \bar{q}[i\partial\!\!\!/ - M_q e^{i\gamma_5 \vec{\tau}\cdot\vec{\pi}/F_\pi}]q$$

Taking into account the AQCM effect, the effective quark-gluon-pion interaction is given as (Polyakov, Diakonov 2003):

$$\Delta\mathcal{L}_{eff} = -i\mu_a \frac{g_s}{4M_q} \bar{q}\sigma_{\mu\nu} e^{i\gamma_5 \vec{\tau}\cdot\vec{\pi}/F_\pi} t^a q G^a_{\mu\nu}$$

where F_π is the pion decay constant and G is the gluon field.

Constituent and Current Quarks

Current quark

Point-like particles with small mass, $m_{\mathsf{u}}, m_{\mathsf{d}} \approx 4 - 7$ MeV

Constituent quark

Quasi-particles with the size $\rho \approx 0.3\,\mathrm{fm}$ and heavy mass, $m_{\mathsf{u}}, m_{\mathsf{d}} \approx 100 - 400$ MeV.

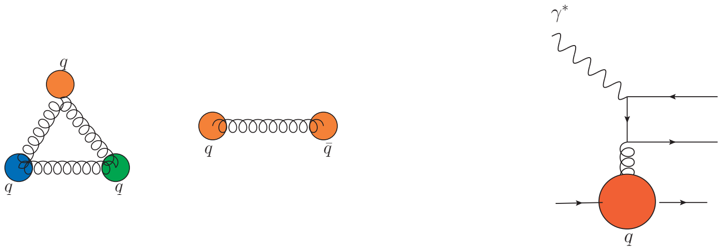


Figure: Constituent quark picture of hadron(left panel) and structure of constituent quark in Deeply-Inelastic Scattering (right panel).

The Gluon Distribution in the Constituent Quark

The unpolarized and polarized gluon distributions are of great importance for making theoretical predictions of high energy experiments with unpolarized or polarized beams.

In our study, we use DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) approach.

Splitting diagrams

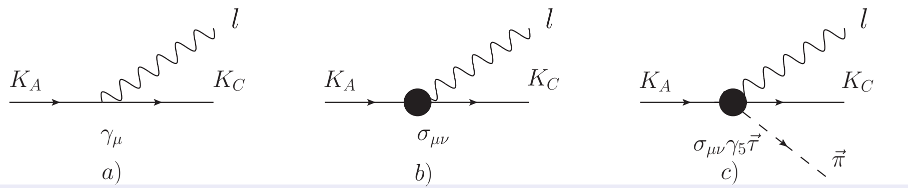


Figure: a) Perturbative QCD, b) and c) are the non-perturbative quark-gluon interaction and the quark-gluon-pion interaction respectively.

The Integrated Gluon Distributions in the Quarks

The integrated gluon distributions in the constituent quark are defined as

$$g(z,Q^2) = \int_{p^2_{\perp min}}^{p^2_{\perp max}} dp^2_\perp g(z,p^2_\perp),$$

$$\Delta g(z,Q^2) = \int_{p^2_{\perp min}}^{p^2_{\perp max}} dp^2_\perp \Delta g(z,p^2_\perp),$$

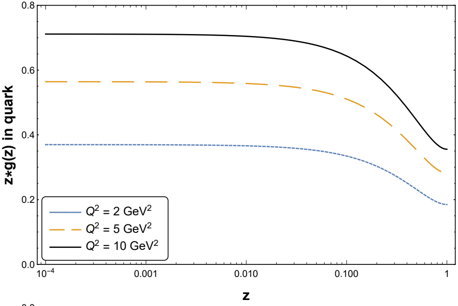
Integration limits for the non-perturbative case

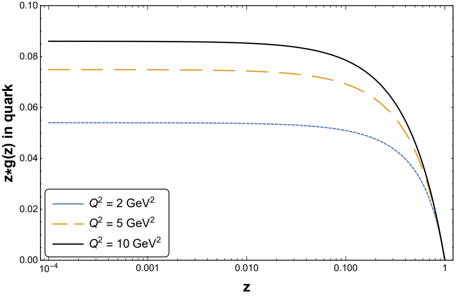
$p^2_{\perp min} = 0$, $p^2_{\perp max} = Q^2$

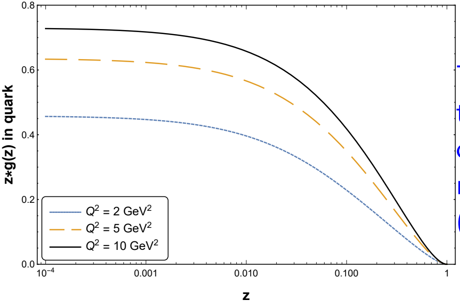
Integration limits for the perturbative case

$p^2_{\perp min} = 1/\rho_c^2$, $p^2_{\perp max} = Q^2$

The unpolarized Gluon Distributions in the Quarks







The perturbative QCD (first panel), the non-perturbative interaction without pion (second panel), and the non-perturbative interaction with pion (third panel).

The Polarized Gluon Distributions in the Quarks

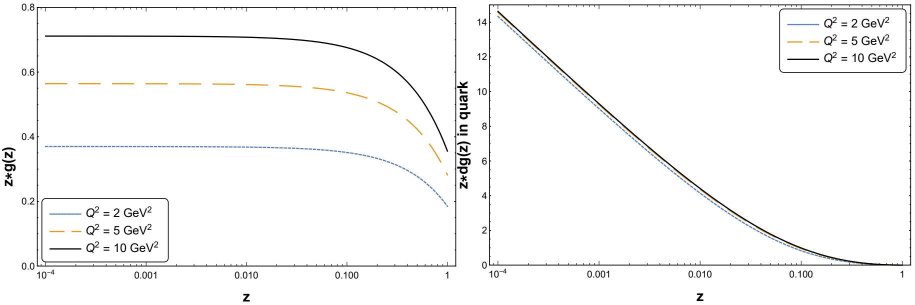


Figure: The z dependency of the contributions to the polarized gluon distribution in the constituent quark from the pQCD (left panel), and from the non-perturbative interaction with pion (right panel).

Scaling on Q^2 for non-perturbative interaction.

The Gluon Distributions in the Nucleon

We will apply the convolution model to obtain the gluon distributions in the nucleon from that in the constituent quark.

Definition of momentums in z direction

- **P** The momentum of the nucleon
- **xP** The momentum of the gluon
- **yP** The momentum of the constituent quark

The unpolarized and polarized gluon distributions in the nucleon are given by

$$g_N(x, Q^2) = \int_x^1 \frac{dy}{y} q_V(y) g_q(\frac{x}{y}, Q^2),$$

$$\Delta g_N(x, Q^2) = \int_x^1 \frac{dy}{y} \Delta q_V(y) \Delta g_q(\frac{x}{y}, Q^2),$$

Gluon Distributions in the Nucleon

Constituent quark distribution function

$$q_V(y) = 60y(1-y)^3.$$

- The normalization are fixed by the requirements $\int_0^1 dy q_V(y) = 3$ and $\int_0^1 dy y q_V(y) = 1$, since total momentum of nucleon at $Q^2 \to 0$ is divided into three constituent quarks.

Polarized constituent quark distribution function

$$\Delta q_V(y) = 2.4(1-y)^3.$$

- The normalization is fixed from the hyperon weak decay data as

$$\int_0^1 dy \Delta q_V(y) = \Delta u_V + \Delta d_V \approx 0.6$$

Gluon Distributions in the Nucleon

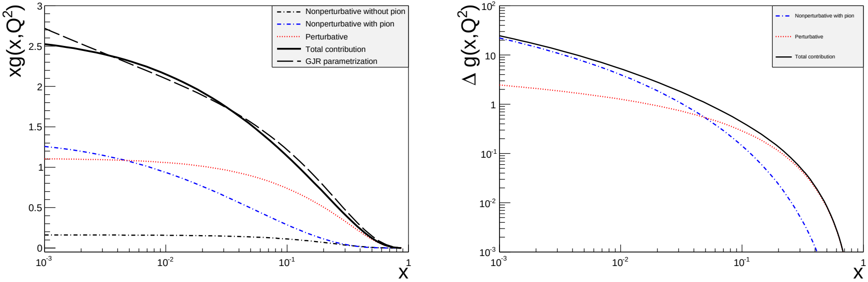


Figure: Unpolarized (left panel), and the polarized (right panel) gluon distributions in the nucleon at the scale $Q^2 = 2$ GeV^2.

The Decomposition of Proton Spin

The famous proton spin problem has been a longstanding puzzle in the QCD. The decomposition of the proton spin by Jaffe and Manohar is

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g,$$

where

Proton spin decomposition

- $\frac{1}{2}\Delta\Sigma$ The quark contribution,
- $\Delta G = \int_0^1 dx \Delta g(x)$ is the gluon polarization in nucleon,
- L_q The contribution from the orbital motion of the quarks,
- L_g The contribution from the orbital motion of the gluons.

At present, the typical value is $\Delta\Sigma(Q^2 = 2GeV^2) \approx 0.25$.

The Decomposition of Proton Spin

For three light quark flavors,

The axial anomaly effect in DIS

$$\Delta\Sigma_{DIS} = \Delta\Sigma - \frac{3\alpha_s}{2\pi}\Delta G,$$

Huge positive gluon polarization is needed

$$\Delta G \approx 3 \sim 4$$

in order to explain the small value of the $\Delta\Sigma_{DIS}$. The modern experimental data from the inclusive hadron productions and the jet productions excludes such large gluon polarization in the accessible intervals of x and Q^2. So does our model.

the axial anomaly effect, suggested by Efremov-Teryaev-Altarelli-Ross, cannot fully explain the proton spin problem.

The Decomposition of Proton Spin

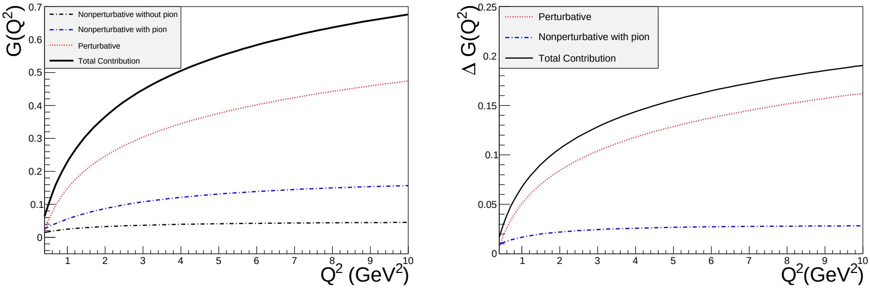


Figure: The part of the nucleon momentum carried by gluons (left panel), and the contribution of the gluons to nucleon spin (right panel) as the function of Q^2.

At $Q^2 = 10$ GeV^2, $\int_0^1 dx \Delta g(x) = 0.19$ in our model.

The Quark Electromagnetic Pauli Form Factor (EPFF)

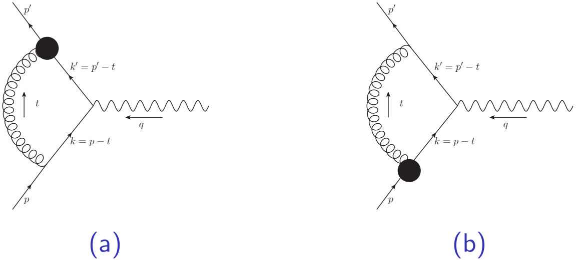


Figure: The diagrams with anomalous quark-gluon chromomagnetic vertex induced by instantons which contribute to EMFF of the quark. The vertex is denoted by a solid blob.

To perform analytical calculations the gaussian approximation for the form factors

$$F_g(k_E^2) \approx F_q(k_E^2) \approx e^{-k_E^2/\Lambda^2},$$

is used with $\Lambda = 2/\rho_c$.

The Amplitude from Feynman Diagrams

The total matrix element is

$$i\mathcal{M} \equiv -e_q C_F g_s^2 \frac{\mu_a}{M_q} \int \frac{d^4t}{(2\pi)^4} \frac{F_g(t^2)F_q(k'^2)N}{(k'^2 - M_q^2)(k^2 - M_q^2)t^2}$$
$$= -ie_q \overline{u}(p')\Gamma^\mu(p,p')u(p),$$

where $C_F = \text{tr}(T^a T^a) = \frac{4}{3}$ is the color factor and e_q is the electric charge of the quark and

$$N \equiv -i\overline{u}(p')\sigma^{\alpha\rho}(\not{k}' + M_q)\gamma^\mu(\not{k} + M_q)\gamma_\rho t_\alpha u(p).$$

One way to extract the Pauli form factor $F_2(Q^2)$ from $i\mathcal{M}$ is to rearrange the gamma matrices using Dirac equation, and find the term proportional to $i\sigma^{\mu\nu}/2M_q$. However, It is much easier to use the projector operator method. (S. J. Brodsky and J. D. Sullivan, Phys. Rev. **156**, 1644 (1967).)

The Projection Method

The projection method provides us a shortcut to calculate the Pauli form factor:

$$F_2(q^2) = \text{tr}\left\{(\not{p} + M_q)\Lambda_\rho^{(2)}(p', p)(\not{p}' + M_q)\Gamma^\rho(p', p)\right\},$$

where $q^2 = (p' - p)^2 \equiv -Q^2$ and

$$\Lambda_\rho^{(2)}(p', p) \equiv \frac{M_q^2}{k^2(4M_q^2 - k^2)}\left[\gamma_\rho + \frac{k^2 + 2M_q^2}{M_q(k^2 - 4M_q^2)}(p' + p)_\rho\right].$$

The general procedure is

1. Apply the projection method to the amplitude,
2. Perform Wick rotation in order to continue with the integral
3. Apply either Feynman parametrization or inverse Laplace transformation, in order to expand the integrand in the power of integrated momentum

The Numerical Results

In our model the form factor $F_2(Q^2)$ is proportional to the quark charge. Therefore, there is the relation between u- and d-quark form factors

$$F_2^d(Q^2) = -\frac{1}{2}F_2^u(Q^2).$$

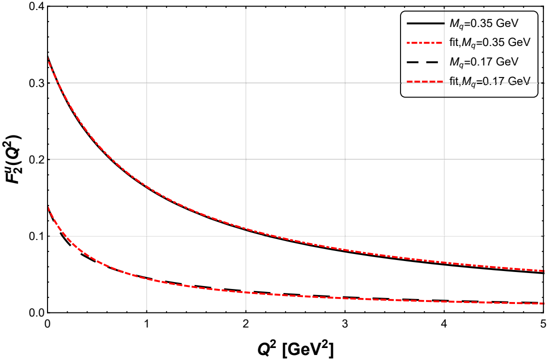


Figure: The F_2 form factor as a function of Q^2 for the different dynamical quark mass M_q in the comparison with the fit.

The Numerical Results

Our numerical result can be fitted very well by the formula

$$F_2(Q^2, M_q) = \frac{F_2(0, M_q)}{1 + \rho_c Q^2/(4.7 M_q)},$$

We would like to emphasize that positive sign of F_2 form factor for u-quark (see Fig.2), is fixed by the negative sign of the AQCM, $\mu_a = -\frac{3\pi(M_q\rho_c)^2}{4\alpha_s(\rho_c)}$.

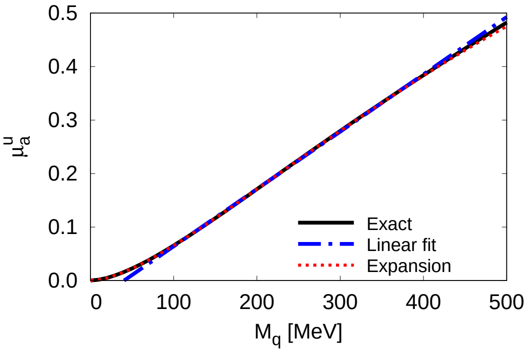


Fig: Behavior of μ_a^u versus quark mass. It can be fit very well with the linear function

$$\mu_a^u \approx \frac{2}{3}\left(-0.065 + 0.97(M_q\rho_c)\right)$$

The results for $\mu_a^{e,u} = F_2^u(Q^2 = 0)$ obtained in mean field approximation and Diakonov-Petrov model are

$$\mu_a^{e,u} = 0.33 \quad \text{for} \quad M_q = 350\,\text{MeV},$$
$$\mu_a^{e,u} = 0.14 \quad \text{for} \quad M_q = 170\,\text{MeV}.$$

The Behaviour of EPFF at High and Low Q^2

With the help of Tauber's Theorems, the Behaviour of Pauli form factor at high and low Q^2 can be obtained via inverse Laplace transformation. Moreover, additionally we can expansion over M_q^2/Λ^2 for the dynamic mass of quarks are small relative to Λ.
At high Q^2:

$$\begin{aligned} F_2(Q^2) &\approx 4e_q \frac{M_q^2}{Q^2}\Bigg(2\ln(2) - \frac{1}{2} \\ &+ \frac{(M_q\rho_c)^2}{4}\left[\ln\left(2(M_q\rho_c)^2\right) - 2 + \gamma_E\right]\Bigg). \end{aligned}$$

Therefore, at large Q^2 form factor behaves as $F_2(Q^2) \sim 1/Q^2$.
At low Q^2:

$$\begin{aligned} F_2(0) &\approx e_q (M_q\rho_c)^2\Bigg(y + \frac{(192y + 211)}{288}(M_q\rho_c)^2 \\ &+ \frac{(1536y + 3089)}{9216}(M_q\rho_c)^4 + ...\Bigg), \end{aligned}$$

where $y = \ln\left(\frac{2}{(M_q\rho_c)^2}\right) - \gamma_E - \frac{1}{4}$

- The interaction with pion gives the dominating contribution to the unpolarized and polarized gluon distributions in the constituent quark and in the nucleon,

- The interaction with pion gives the dominating contribution to the unpolarized and polarized gluon distributions in the constituent quark and in the nucleon,
- Instanton effect serves as a possible fundamental QCD mechanism to solve the "Spin Crisis" (calculation in progress).

- The interaction with pion gives the dominating contribution to the unpolarized and polarized gluon distributions in the constituent quark and in the nucleon,
- Instanton effect serves as a possible fundamental QCD mechanism to solve the "Spin Crisis" (calculation in progress).
- Anomalous quark-gluon chromomagnetic interaction induced by instantons leads to large magnetic moment of u- and d-quarks,

- The interaction with pion gives the dominating contribution to the unpolarized and polarized gluon distributions in the constituent quark and in the nucleon,
- Instanton effect serves as a possible fundamental QCD mechanism to solve the "Spin Crisis" (calculation in progress).
- Anomalous quark-gluon chromomagnetic interaction induced by instantons leads to large magnetic moment of u- and d-quarks,
- It would be interesting to study the electromagnetic properties of hadrons based on the constituent quark model with an effective quark-photon and quark-gluon vertices induced by instantons,

- The interaction with pion gives the dominating contribution to the unpolarized and polarized gluon distributions in the constituent quark and in the nucleon,
- Instanton effect serves as a possible fundamental QCD mechanism to solve the "Spin Crisis" (calculation in progress).
- Anomalous quark-gluon chromomagnetic interaction induced by instantons leads to large magnetic moment of u- and d-quarks,
- It would be interesting to study the electromagnetic properties of hadrons based on the constituent quark model with an effective quark-photon and quark-gluon vertices induced by instantons,
- Instanton effects leads to the flip of quark helicity , therefore can make contribution to various spin-dependent photon-hadron cross sections, including polarized semi-inclusive DIS (calculation in progress).

- The interaction with pion gives the dominating contribution to the unpolarized and polarized gluon distributions in the constituent quark and in the nucleon,
- Instanton effect serves as a possible fundamental QCD mechanism to solve the “Spin Crisis” (calculation in progress).
- Anomalous quark-gluon chromomagnetic interaction induced by instantons leads to large magnetic moment of u- and d-quarks,
- It would be interesting to study the electromagnetic properties of hadrons based on the constituent quark model with an effective quark-photon and quark-gluon vertices induced by instantons,
- Instanton effects leads to the flip of quark helicity , therefore can make contribution to various spin-dependent photon-hadron cross sections, including polarized semi-inclusive DIS (calculation in progress).

Thank you!