

# Measuring Gluon Orbital Angular Momentum at the Electron-Ion Collider

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# Outline

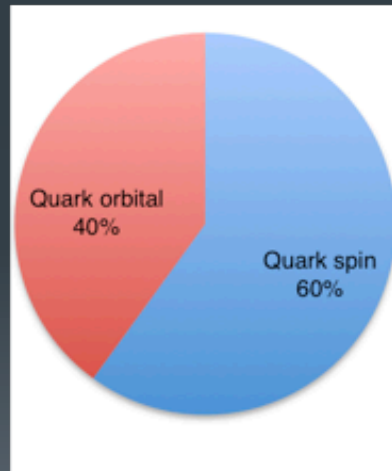
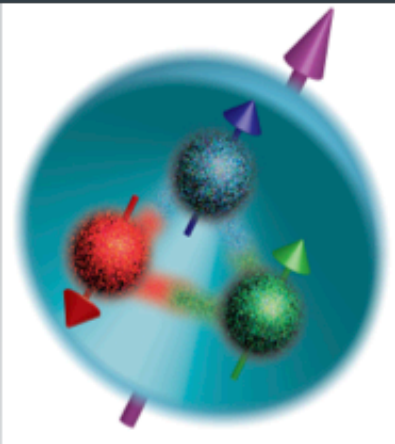
- Proton spin structure from DIS
- Gluon OAM and Wigner distribution
- Experimental observable

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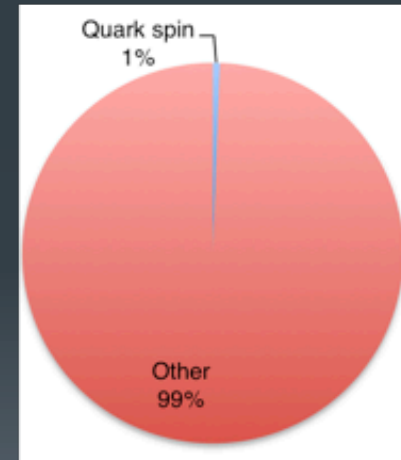
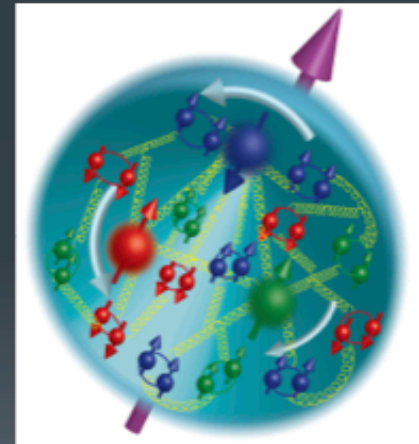
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# Proton, a platform where the gluon is indispensable

Quark Model Prediction:



EMC Measurement (1987):



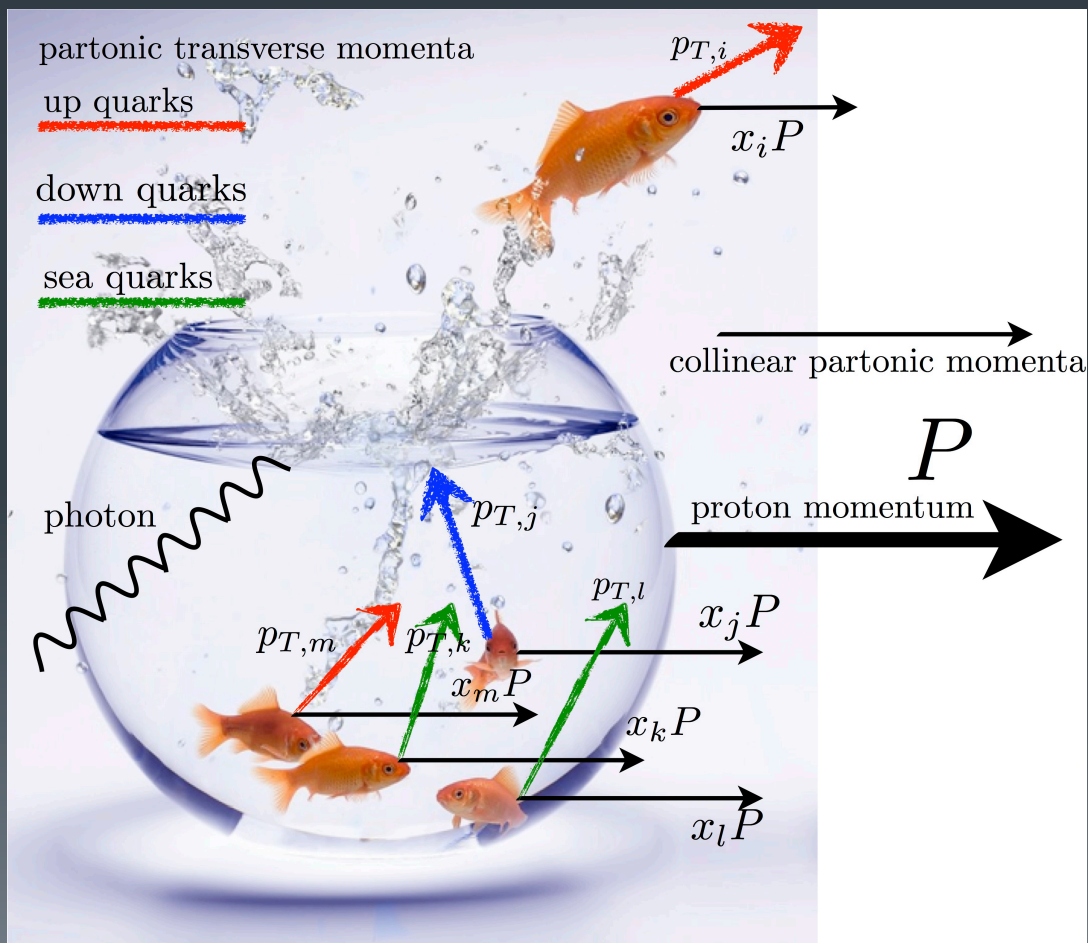
A proton spin crisis?

A naive spin sum rule:

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + l_q + l_g$$



# How to probe the nucleon spin structure?



To look into the proton?  
Smash it first!

**Deep inelastic scattering**

# State-of-the-art Polarized Deep Inelastic Scattering Experiments



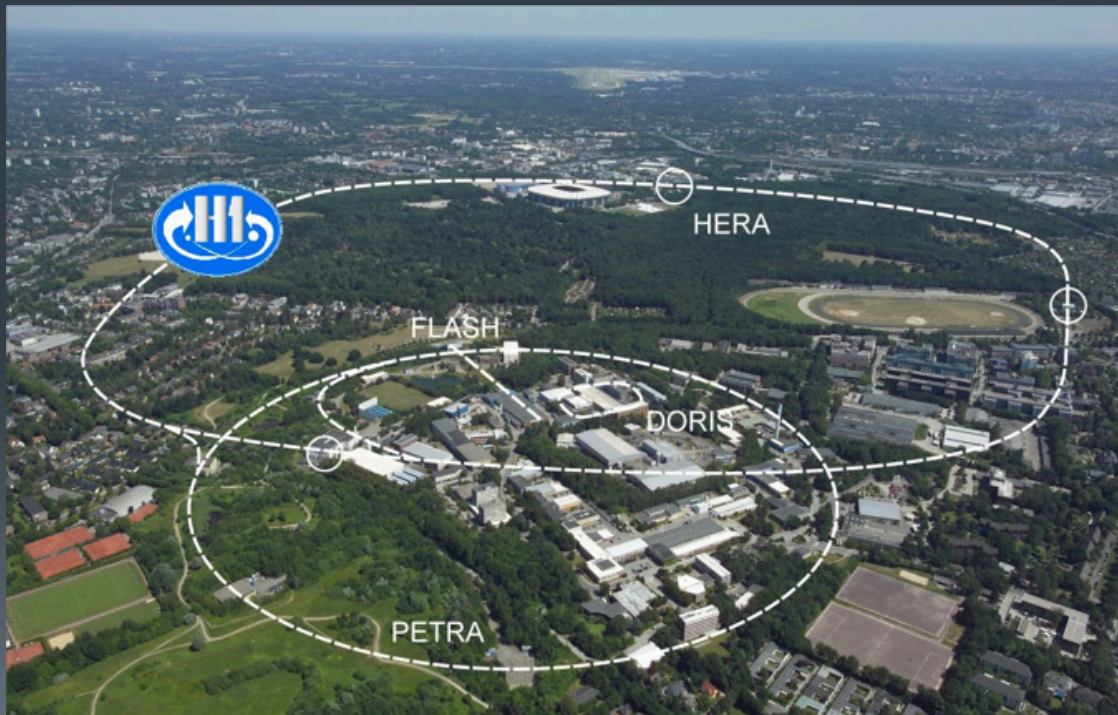
SLAC

HERMES @ HERA, DESY

COMPASS @ SPS, CERN

CLAS @ Jefferson Lab

# State-of-the-art Polarized Deep Inelastic Scattering Experiments



SLAC

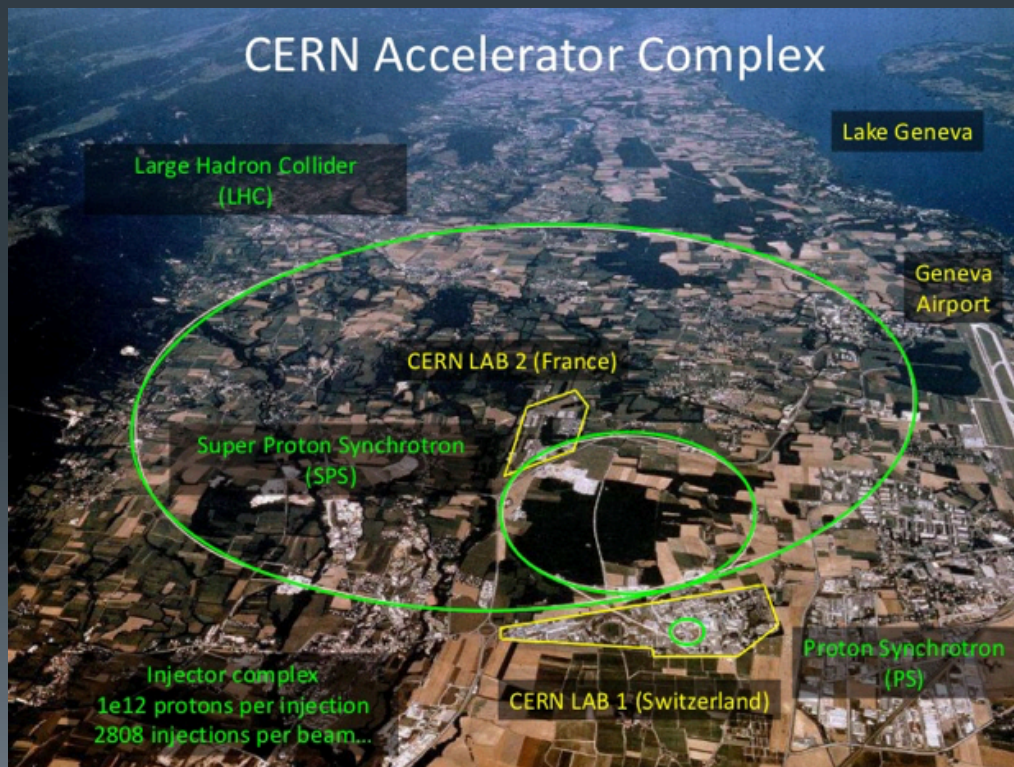
HERMES @ HERA, DESY

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# State-of-the-art Polarized Deep Inelastic Scattering Experiments



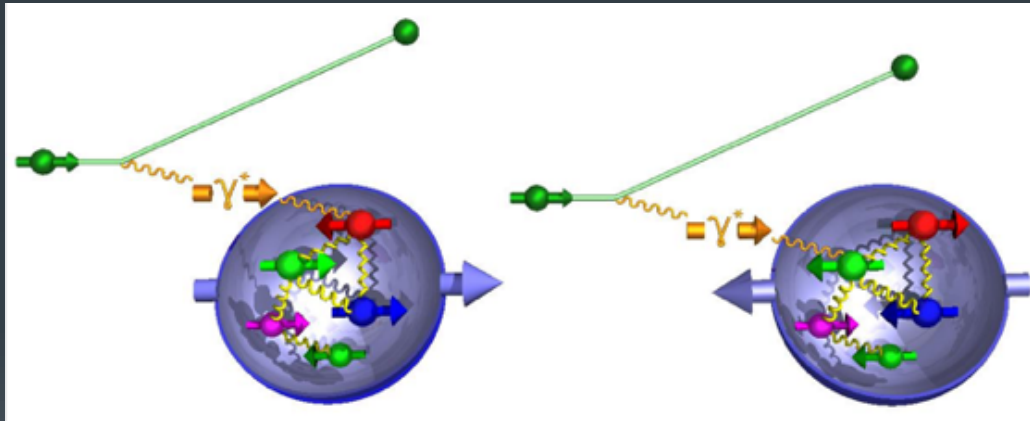
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# Polarized Lepton Proton Scattering:



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$$\frac{d\sigma_{\uparrow\uparrow} - d\sigma_{\uparrow\downarrow}}{d\sigma_{\uparrow\uparrow} + d\sigma_{\uparrow\downarrow}} \propto \sum_q e_q^2 C_q(x) \otimes \Delta q(x) + C_g(x) \otimes \Delta g(x)$$

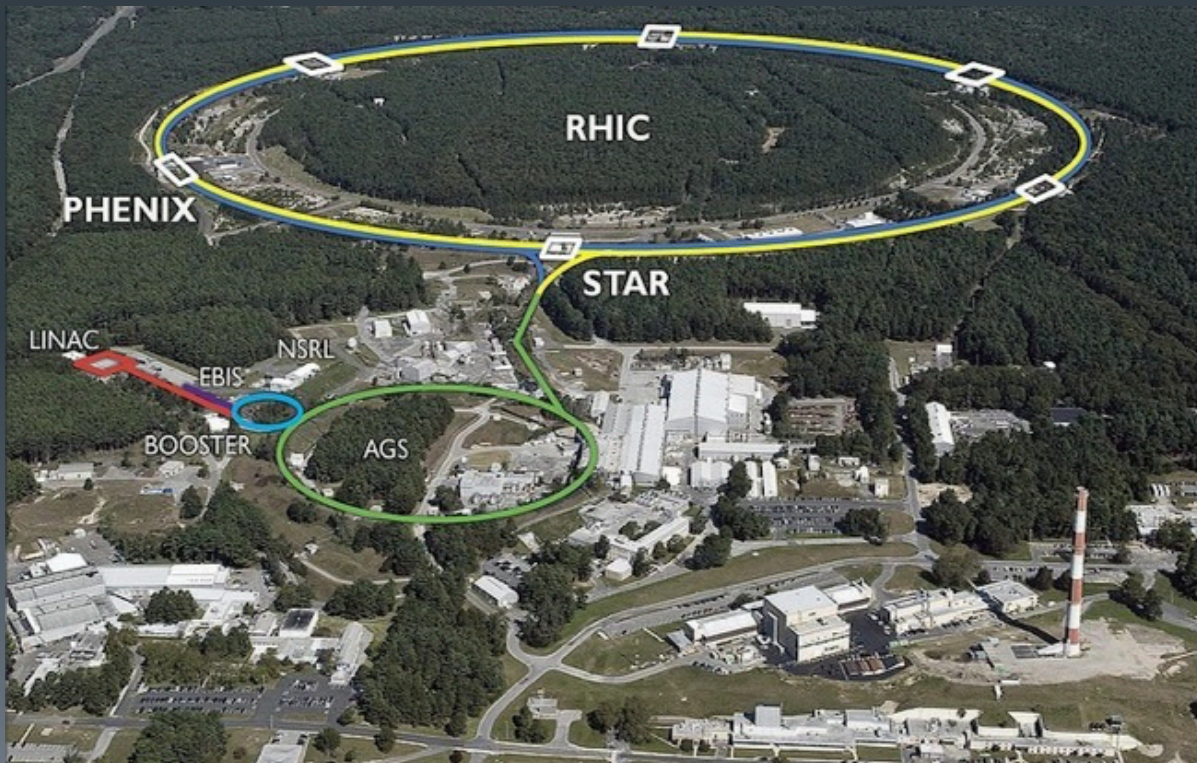
$$C_q(x) \sim 1 + O(\alpha_s)$$

$$C_g(x) \sim O(\alpha_s)$$

$$\Delta\Sigma = \int_0^1 dx \Delta q(x), \quad \Delta G = \int_0^1 dx \Delta g(x)$$



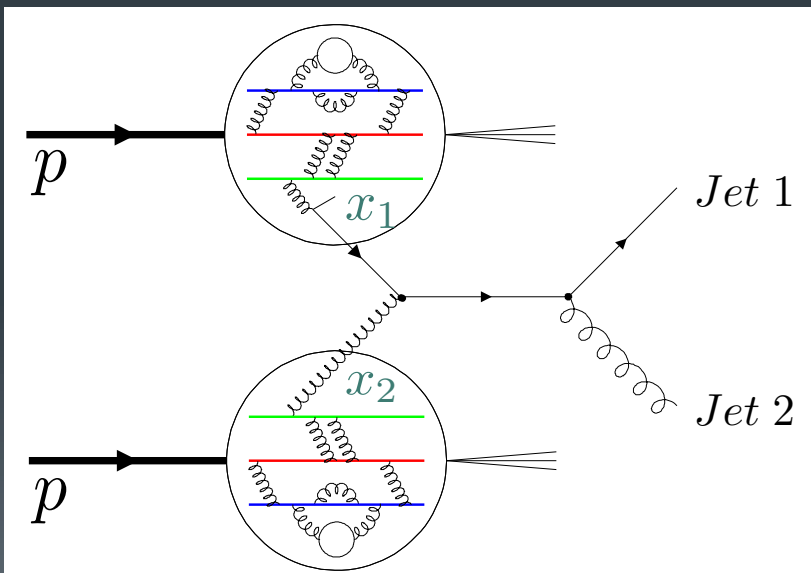
# Polarized Proton-Proton Collision Experiment



STAR, PHENIX,  
BRAHMS @ RHIC, BNL

RHIC has made the  
most precise  
measurement of the  
gluon polarization so far.

# Polarized Proton-Proton Collision Experiment



$$\frac{d\sigma_{\uparrow\uparrow} - d\sigma_{\uparrow\downarrow}}{d\sigma_{\uparrow\uparrow} + d\sigma_{\uparrow\downarrow}} \propto \frac{\Delta f_1(x_1) \otimes \Delta f_2(x_2) \otimes \Delta\sigma_{h/J} (\otimes D_f^h)}{f_1(x_1) \otimes f_2(x_2) \otimes \sigma_{h/J} (\otimes D_f^h)}$$

**L.O. Contribution!**

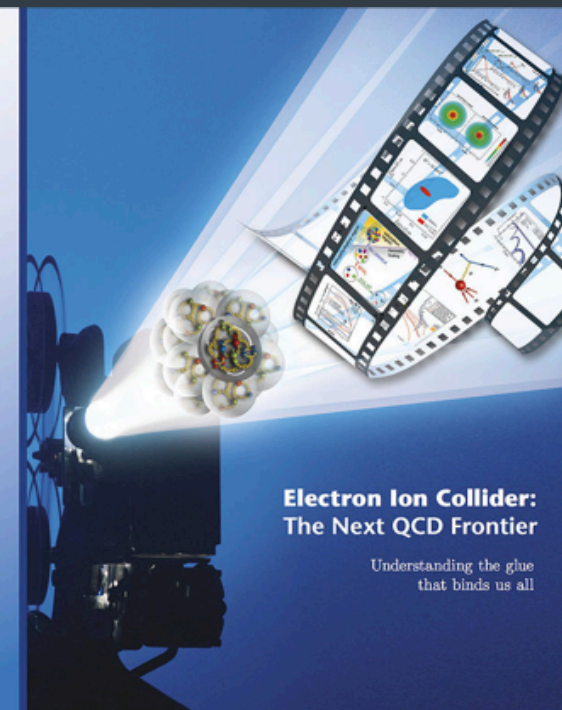
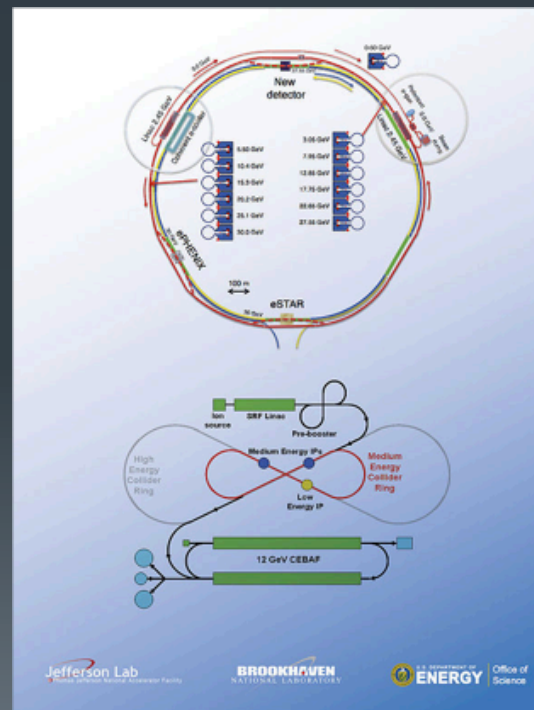
Probing sea quark polarization via  $W$  production: J. Zhang's talk



# Electron-Ion Collider

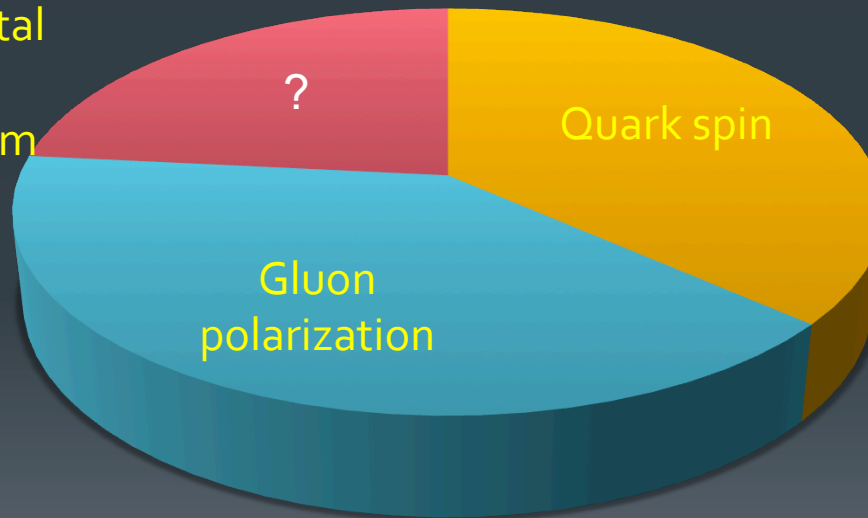
- Highly Polarized Beams
- Large Kinematic Range
- High Collision Luminosity

EIC will be capable of measuring the nucleon spin structure to a more precise level!



# The longitudinal nucleon spin structure

Quark and  
gluon orbital  
angular  
momentum



$$\Delta\Sigma(Q^2=10 \text{ GeV}^2) = 0.366, \text{ de Florian et al., 2009}$$

SLAC  
HERMES (DESY)  
COMPASS (CERN)  
JLab  
RHIC

**Naïve spin sum rule:**

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + l_q^z + l_g^z$$

$$\Delta G(Q^2=10 \text{ GeV}^2) = 0.2 \sim 0.3, \text{ de Florian et al., 2014;}$$

E. Nocera et al., 2014;  
Lattice QCD: Yang, Suffian, Y.Z., et al., 2016

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- Proton spin structure from DIS
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# Orbital angular momentum

- OAM in Ji sum rule (Ji, 1997):

- Measureable through twist-2 GPD in deeply virtual Compton scattering (DVCS);
- Parton density interpretation not clear.

$$\frac{1}{2} = J_q + J_g, \quad (L_g = J_g - \Delta G)$$

- OAM in Jaffe-Manohar sum rule (Jaffe and Manohar, 1989):

- Clear partonic interpretation;
- Related to a TMD (pretzelosity) in models (She, Zhu, and Ma, 2009; H. Avakian et al., 2009, 2010), accessible through SIDIS (Lefky and Prokudin, 2015; COMPASS, 2017);
- Model-independent observable not known until recently.

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + l_q^z + l_g^z$$

# The gluon orbital angular momentum (OAM) and Wigner distribution

- Moment of a phase space Wigner distribution

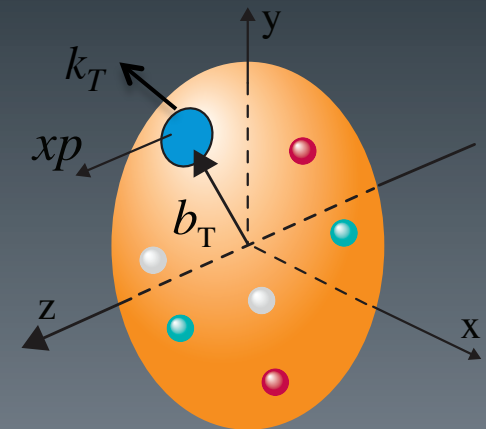
$$L_g(x) = \int db_{\perp}^2 d^2k_{\perp} (b_{\perp} \times k_{\perp}) W_{LC}^g(x, 0, k_{\perp}, b_{\perp})$$

- Wigner distribution or generalized transverse momentum distribution (GTMD)

$$W_{LC}^g(x, \xi, k_{\perp}, b_{\perp}) = \int d^2\Delta_{\perp} e^{-ib_{\perp} \cdot \Delta_{\perp}} f(x, \xi, k_{\perp}, \Delta_{\perp})$$

$$L_g(x) = \varepsilon_{\perp}^{\alpha\beta} \frac{\partial}{\partial i\Delta_{\perp}^{\alpha}} \bigg|_{\Delta=0} \int d^2k_{\perp} k_{\perp}^{\beta} f_g(x, \xi, k_{\perp}, \Delta_{\perp})$$

Belitsky, Ji, and Yuan, 2004;  
 Meissner, Metz and Schlegel, 2009;  
 Lorce and Pasquini, 2011;  
 Lorce et al., 2012;  
 Y. Hatta, 2012;  
 Ji, Xiong, and Yuan, 2012.



# The gluon orbital angular momentum (OAM) and Wigner distribution

- Parametrization of GTMD

$F_{l,4}^g$

$$f_g(x, \xi, k_\perp, \Delta_\perp) = F_g(x, \xi, |k_\perp|, |\Delta_\perp|) + i(\vec{k}_\perp \times \vec{\Delta}_\perp) S^+ \underline{F_g^{(l)}}(x, \xi, |k_\perp|, |\Delta_\perp|) + \dots$$

- Gluon OAM density as the moment of GTMD

$$L_g(x, \xi, |\Delta_\perp|) = - \int d^2 k_\perp k_\perp^2 \underline{F_g^{(l)}}(x, \xi, |k_\perp|, |\Delta_\perp|)$$

$$L_g(x) = L_g(x, 0, 0)$$

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# Experimental Process

$$f_g(x, \xi, k_{\perp}, \Delta_{\perp})$$

Two independent momenta

Momentum transfer of the proton, exclusive process

Consider  $\gamma^* p$  scattering:

2-2 process, final state momenta not independent;

2-3 process, one more independent momentum.

Intrinsic transverse momentum  $k_T$ ?

Answer: Exclusive dijet production in  $l+p$  scattering ✓

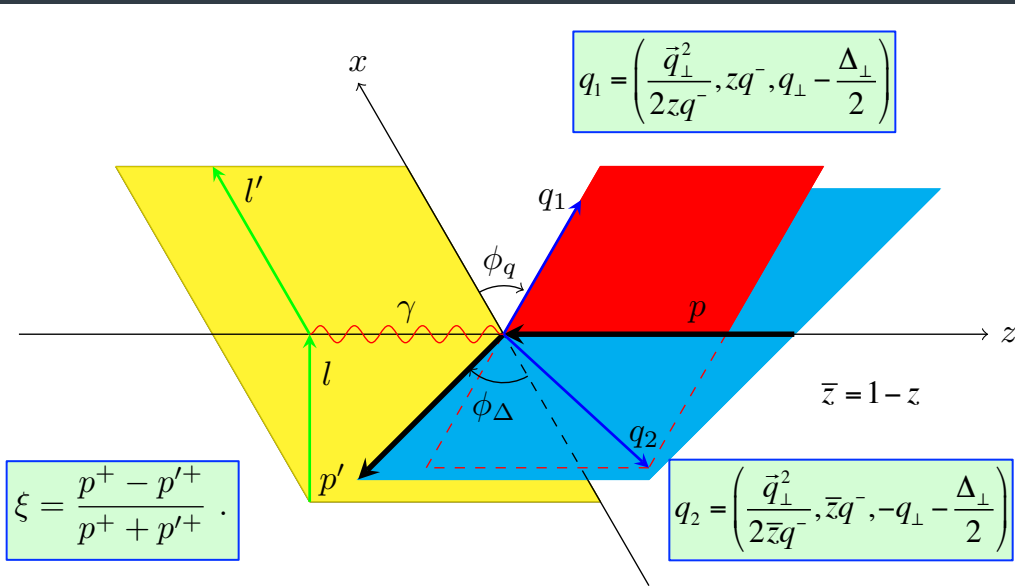
$$\ell + p \rightarrow \ell' + q_1 + q_2 + p'$$

Braun and Ivanov, 2005

Hatta, Xiao and Yuan, 2016



# Kinematics



$$q = l - l' , \quad q^2 = -Q^2$$

$$x_{Bj} = \frac{Q^2}{2q \cdot p} , \quad y = \frac{q \cdot p}{l \cdot p} ,$$

$$\Delta = p' - p , \quad P = \frac{p + p'}{2} ,$$

$$t = \Delta^2 , \quad (q + p)^2 = W^2 ,$$

$$(q - \Delta)^2 = (q_1 + q_2)^2 = M^2 .$$

$$Q^2 \sim W^2 \sim \vec{q}_\perp^2 \gg \Delta_\perp^2$$

$$\mu^2 = z\bar{z}Q^2 , \quad \beta = \frac{\mu^2}{\vec{q}_\perp^2 + \mu^2}$$

# Scattering Amplitude

Scattering amplitude:

Photon helicity  
decomposition:

$$g^{\mu\nu} = \sum_{\lambda=L,\perp} \epsilon_{\lambda}^{*\mu} \epsilon_{\lambda}^{\nu}$$

$$M = \frac{e_{em}}{Q^2} \sum_{\lambda=L,\perp} \bar{u}(l') \not{\epsilon}_{\lambda}(q) u(l) \epsilon_{\lambda,\nu} M_{\gamma^*}^{\nu} = \frac{e_{em}}{Q^2} \sum_{\lambda=L,\perp} \bar{u}(l') \not{\epsilon}_{\lambda}(q) u(l) \mathcal{A}_{\lambda}$$

Leptonic part, averaging  
initial spins and summing  
over final spins

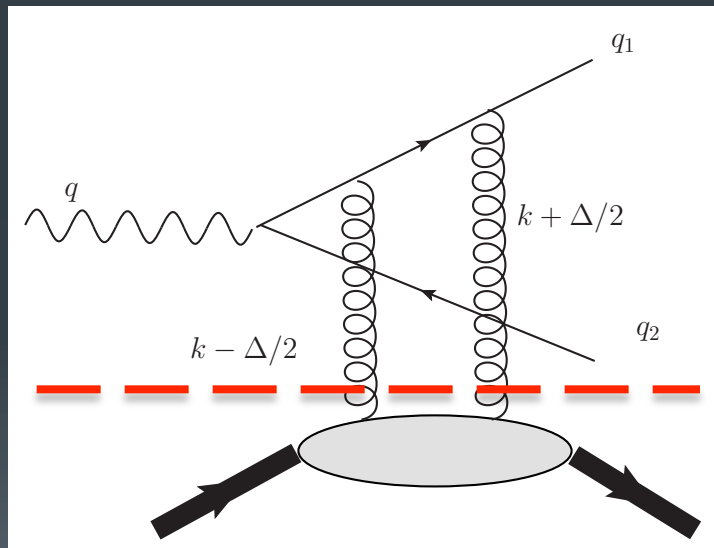
Hadronic part, summing  
over final state quark  
spins and color

$$\frac{d\sigma}{dy dQ^2 d\Omega} = \sigma_0 \left[ (1-y) |A_L|^2 + \frac{1+(1-y)^2}{2} |A_T|^2 \right]$$

$$\sigma_0 = \frac{\alpha_{em}^2 \alpha_s^2 e_q^2}{16\pi^2 Q^2 y N_c} \frac{4\xi^2 z \bar{z}}{(1-\xi^2)(\vec{q}_{\perp}^2 + \mu^2)^3}$$

# Collinear factorization of hadronic amplitude

- Leading order diagrams (6 in total)
- Collinear Factorization



Hard Part

$$\mathcal{H}(x, \xi, q_{\perp}, k_{\perp}, \Delta_{\perp})$$

Soft Part

$$x f^g(x, \xi, k_{\perp}, \Delta_{\perp})$$

$$i\mathcal{A}_f \propto \int dx d^2k_{\perp} \mathcal{H}(x, \xi, q_{\perp}, k_{\perp}, \Delta_{\perp}) x f^g(x, \xi, k_{\perp}, \Delta_{\perp}) ,$$

# Twist expansion

$$\mathcal{H}(x, \xi, q_{\perp}, k_{\perp}, \Delta_{\perp}) = \mathcal{H}^{(0)}(x, \xi, q_{\perp}, 0, \Delta_{\perp}) + k_{\perp}^{\alpha} \frac{\partial}{\partial k_{\perp}^{\alpha}} \mathcal{H}(x, \xi, q_{\perp}, 0, \Delta_{\perp}) + \cdots$$

Twist 2 (Target-spin independent):

Braun and Ivanov, 2005

$$i\mathcal{A}_f^{(0)} \propto \int dx \mathcal{H}^{(0)}(x, \xi, q_{\perp}, 0, 0) x F_g(x, \xi, \Delta_{\perp})$$

Gluon GPD

Twist 3 (Target-spin dependent):

$$\int d^2 k_{\perp} (\vec{q}_{\perp} \cdot \vec{k}_{\perp}) x f^g(x, \xi, k_{\perp}, \Delta_{\perp}) = -i S^+ (\vec{q}_{\perp} \times \vec{\Delta}_{\perp}) x L_g(x, \xi, \Delta_{\perp}) + \cdots,$$

# Differential cross section

■ Result:

$$\Delta\sigma = (\sigma(S^+) - \sigma(-S^+))/2$$

$$\frac{d\Delta\sigma}{dydQ^2d\Omega} = \sigma_0\lambda_p \frac{2(\bar{z} - z)(\vec{q}_\perp \times \vec{\Delta}_\perp)}{\vec{q}_\perp^2 + \mu^2} \left[ (1-y)A_{fL} + \frac{1 + (1-y)^2}{2} A_{fT} \right]$$

$\lambda_p$  Nucleon Polarization

$$|q_\perp| |\Delta_\perp| \sin(\phi_q - \phi_\Delta)$$

$$A_{fL} = 16\beta \operatorname{Im} \left( [\mathcal{F}_g^* + 4\xi^2 \bar{\beta} \mathcal{F}_g'^*] [\mathcal{L}_g + 8\xi^2 \bar{\beta} \mathcal{L}_g'] \right),$$

$$A_{fT} = 2 \operatorname{Im} \left( [\mathcal{F}_g^* + 2\xi^2(1-2\beta)\mathcal{F}_g'^*] \left[ \mathcal{L}_g + 2\bar{\beta} \left( \frac{1}{z\bar{z}} - 2 \right) (\mathcal{L}_g + 4\xi^2(1-2\beta)\mathcal{L}_g') \right] \right)$$

# Generalized Compton Form Factors

## ■ Definition:

$$\mathcal{F}_g(\xi, t) = \int dx \frac{1}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} F_g(x, \xi, t) ,$$

$$\mathcal{F}'_g(\xi, t) = \int dx \frac{1}{(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} F_g(x, \xi, t) .$$

$$\mathcal{L}_g(\xi, t) = \int dx \frac{x\xi}{(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} x L_g(x, \xi, t) ,$$

$$\mathcal{L}'_g(\xi, t) = \int dx \frac{x\xi}{(x + \xi - i\varepsilon)^3(x - \xi + i\varepsilon)^3} x L_g(x, \xi, t) .$$

## ■ $x$ -dependence cannot be measured:

Needs modelling of the GPD and GTMD;

Real part: principle value integration.

Imaginary part:  $F(\xi, \pm\xi, t)$ ,  $L(\xi, \pm\xi, t)$ .

$$\frac{1}{x + \xi - i\varepsilon} = \frac{1}{x + \xi} + i\pi\delta(x + \xi)$$

# Single longitudinal target-spin asymmetry

## ■ Definition:

$$A_{\sin(\phi_q - \phi_\Delta)} = \int d\phi_q d\phi_\Delta \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\phi_q d\phi_\Delta} \sin(\phi_q - \phi_\Delta) \bigg/ \int d\phi_q d\phi_\Delta \frac{d\sigma_\uparrow + d\sigma_\downarrow}{d\phi_q d\phi_\Delta}$$

$$A_{\sin(\phi_q - \phi_\Delta)} \propto \frac{(\bar{z} - z) |\vec{q}_\perp| |\vec{\Delta}_\perp|}{\vec{q}_\perp^2 + \mu^2}$$

## ■ Feature:

Asymmetric jets

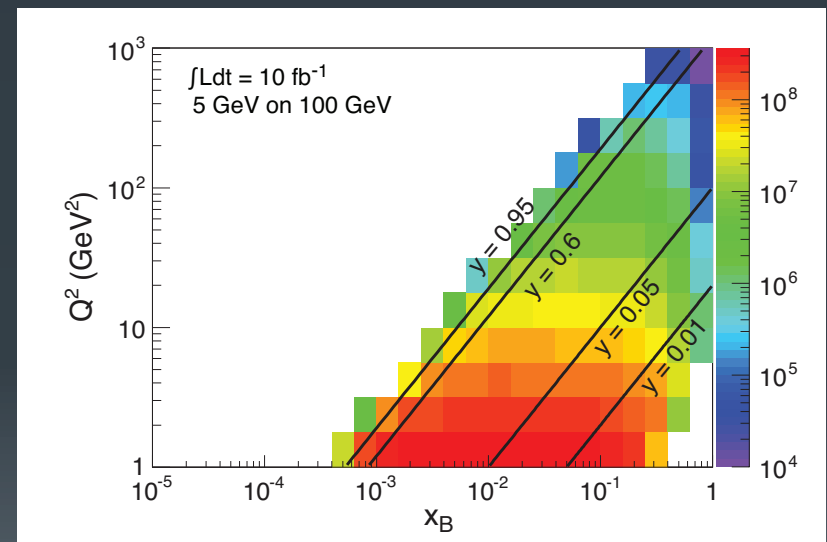
Suppressed effect  $\mathcal{O}(\Delta_T/Q)$

X. Ji, F. Yuan and Y.Z.,  
Phys. Rev. Lett. 118(2017) no.19, 192004

The same process at small  $x$ :  
Hatta, Nakagawa, Yuan and Y.Z., 2016  
Double exclusive Drell-Yan  $\pi p \rightarrow p \gamma^* \gamma^*$ :  
Bhattacharya, Metz, and Zhou, 2017

# Measurement at EIC

- Key measurements:
  - Dijet momenta
  - Final state nucleon momentum
- Kinematics:
  - Large Bjorken  $x$ , high  $Q^2$ ;
  - Nucleon deflection angle (determines  $t$  and  $\xi$ ).



A. Accardi et al., arXiv: 1212.1701

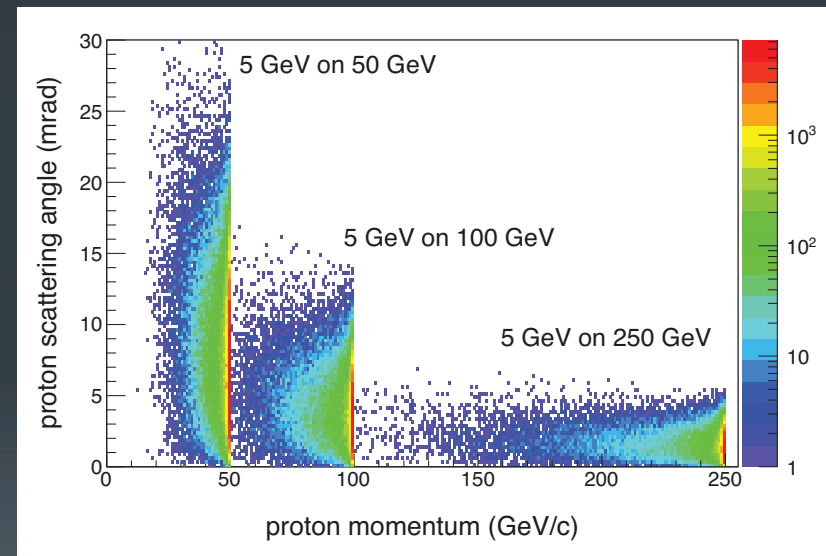


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$$E_p = 50 \text{ GeV}, \quad \theta = 10 \text{ mrad}$$

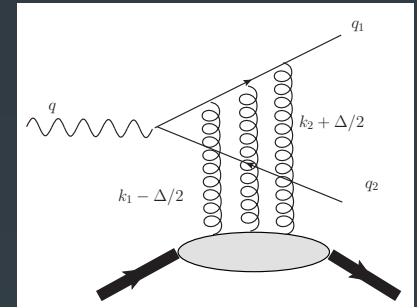
$$\xi \sim \frac{1 - \cos \theta}{3 + \cos \theta} \sim 0.0000125, \quad t \sim 0.25 \text{ GeV}^2.$$



A. Accardi et al., arXiv: 1212.1701

# Outlook

- Include genuinely twist-three diagrams (undergoing);
- One-loop radiative corrections, test validity of collinear factorization;
- Simulations.



After all, the leading contribution to the single target-spin asymmetry is strongly sensitive to the gluon OAM!

# Summary

- Gluon OAM in the Jaffe-Manohar sum rule can be measured through the Wigner distribution;
- The leading contribution to the single longitudinal target-spin asymmetry in exclusive dijet production from ep scattering is strongly sensitive to the gluon OAM.
- The differential cross section formula has been derived for the experimental observable at leading order.