Five-body calculation of heavy pentaquark system

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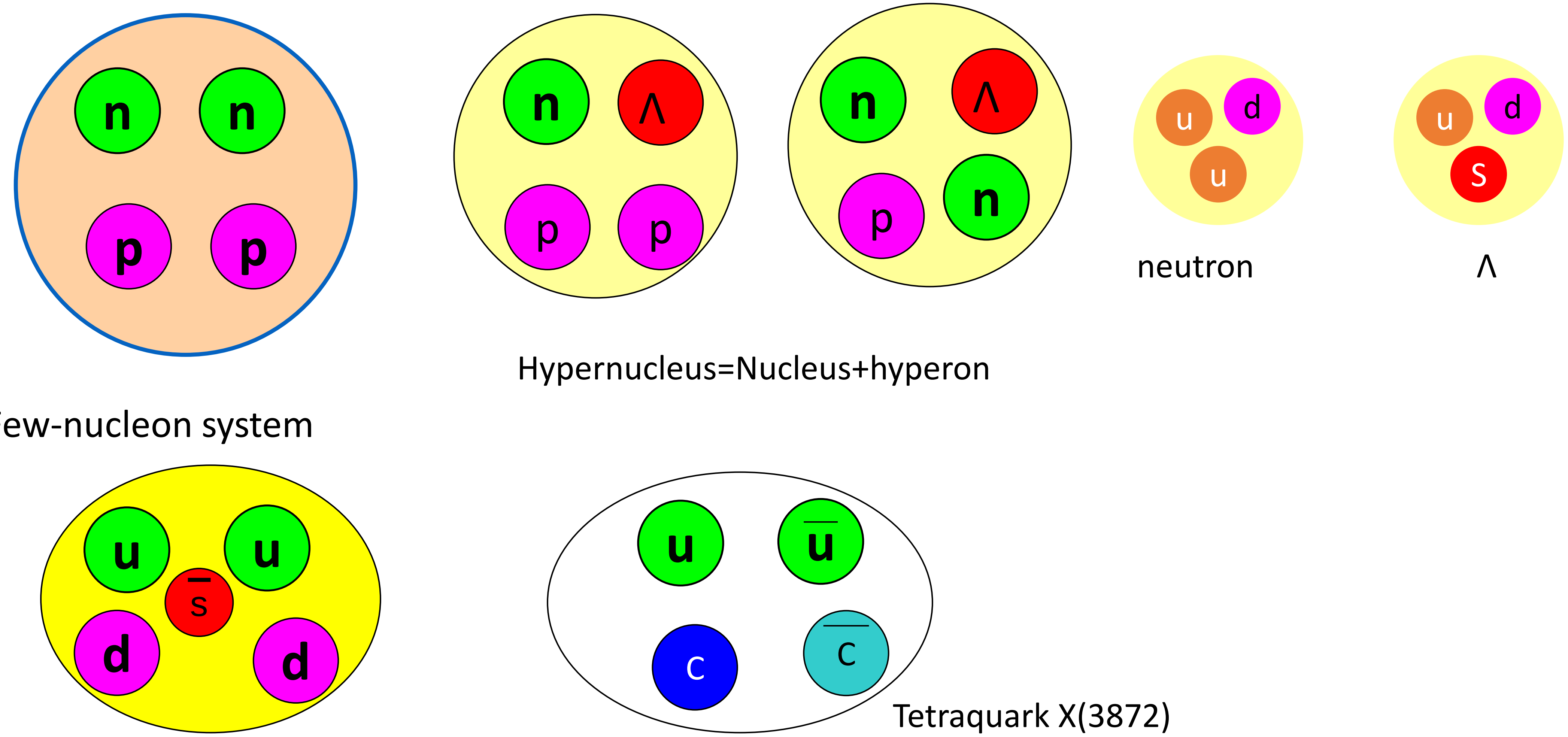
I have been studying the following subjects from view point of few-body problem using my own method called Gaussian Expansion Method.

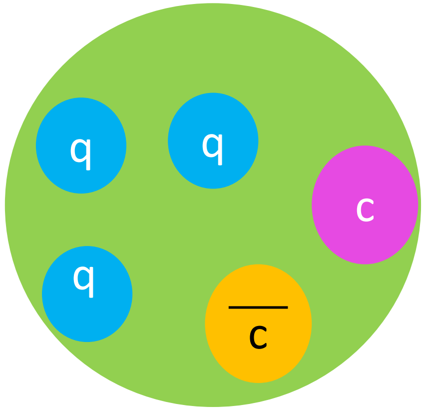


pentaquark

PRL 115, 072001 (2015) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 14 AUGUST 2015

Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays

R. Aaij *et al.**
(LHCb Collaboration)
(Received 13 July 2015; published 12 August 2015)



Observations of exotic structures in the $J/\psi p$ channel, which we refer to as charmonium-pentaquark states, in $\Lambda_b^0 \to J/\psi K^- p$ decays are presented. The data sample corresponds to an integrated luminosity of 3 fb^{-1} acquired with the LHCb detector from 7 and 8 TeV pp collisions. An amplitude analysis of the three-body final state reproduces the two-body mass and angular distributions. To obtain a satisfactory fit of the structures seen in the $J/\psi p$ mass spectrum, it is necessary to include two Breit-Wigner amplitudes that each describe a resonant state. The significance of each of these resonances is more than 9 standard deviations. One has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV, while the second is narrower, with a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. The preferred J^P assignments are of opposite parity, with one state having spin 3/2 and the other 5/2.

State	Mass (MeV)	Width (MeV)	Fit fraction (%)	Significance
$P_c(4380)^+$	4380±8±29	205±18±86	8.4±0.7±4.2	9σ
$P_c(4450)^+$	4449.8±1.7±2.5	39± 5±19	4.1±0.5±1.1	12σ

- Best fit has J^P=($3/2^-$, $5/2^+$), also ($3/2^+$, $5/2^-$) & ($5/2^+$, $3/2^-$) are preferred

To describe the data of Pc(4380)$^+$ and Pc(4459)$^+$ state, there are theoretical effort.

- Cusp?

  Phys. Rev. D92 071502 (2015), Phys. Lett. B751 59 (2015)

- Meson-Baryon state?

  Phys. Rev. Lett. 115 172001(2015), Phys. Rev. D92 094003 (2015)

  Phys. Rev. Lett. 132002 (2015), Phys. Rev. D92 114002 (2015)

  Phys. Lett. B753 547 (2016)

- Baryoncharmonnia

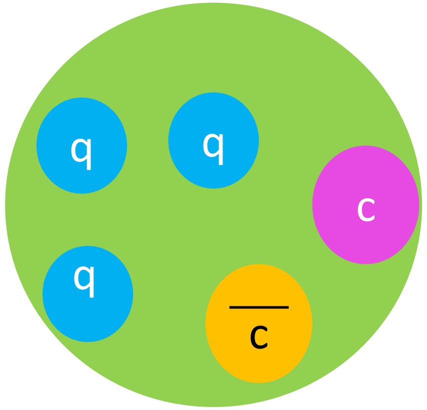
  Phys. Rev. D92 031502 (2015)

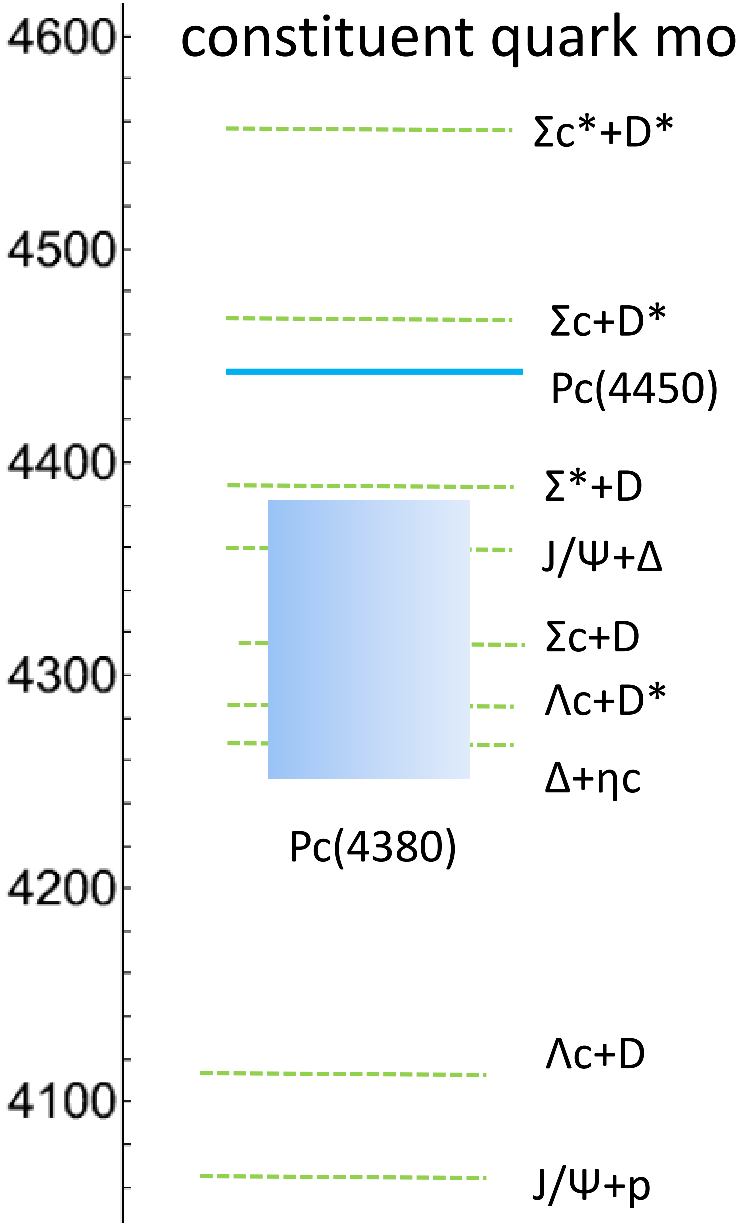
- Tightly bound pentaquark states

  Eur. Phys. J. A48 61 (2012), Phys. Lett. B 749 454 (2015),

  Phys. Lett. B749 289 (2015) , Phys. Lett. B764 254 (2017) etc.

Motivated by the experimental data of pentaquark system at LHCb, We calculate this system within the framework of non-relativistic constituent quark model.





To describe the experimental data,
It is necessary to reproduce the observed threshold.

The Hamiltonian is important
to reproduce the low-lying energy spectra of meson and baryon system.

Hamiltonian

$$H = \sum_i \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_G + V_{\text{Conf}} + V_{\text{CM}} - \Lambda/r$$

Λ=0.1653GeV^2

$$V_{\text{Conf}} = -\sum_{i<j} \sum_{\alpha=1}^{8} \frac{\lambda_i^\alpha}{2} \frac{\lambda_j^\alpha}{2} \left[\frac{k}{2} (\mathbf{x}_i - \mathbf{x}_j) \;\; + v_0 \right],$$

K=0.5069

$$V_{\text{CM}} = \sum_{i<j} \sum_{\alpha=1}^{8} \frac{\lambda_i^\alpha}{2} \frac{\lambda_j^\alpha}{2} \frac{\xi_\sigma}{m_i m_j} e^{-(\mathbf{x}_i - \mathbf{x}_j)^2/\beta^2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j .$$

$\xi_\alpha=(2\pi/3)kp$ $\beta=A((2m_i m_j)/(m_i+m_j))^{(-B)}$

Kp=1.8609 A=1.6553 B=0.2204

m_q=315 MeV, m_c=1836 MeV

B. Silvestre-Brac Few-body Systems 20, 1 (1996).

Cal.	Exp.
Baryon	
N: 953 MeV	939 MeV
Δ: 1265 MeV	1232
Λc: 2276 MeV	2286
Σc:2451 MeV	2465
Σc*:2531 MeV	2545
Meson	
D: 1862 MeV	1870
D*:2016 MeV	2010
J/Ψ:3102 MeV	3094
ηc :3007 MeV	2984
χ_c l=1,s=0: 3462.4 MeV	hc:3525 MeV
L=1,S=1 :3486.5 MeV	3530 MeV

Calculated energy spectra for meson and baryon systems are in good agreement with the observed data.

In order to solve few-body problem accurately,

Gaussian Expansion Method (GEM) , since 1987,

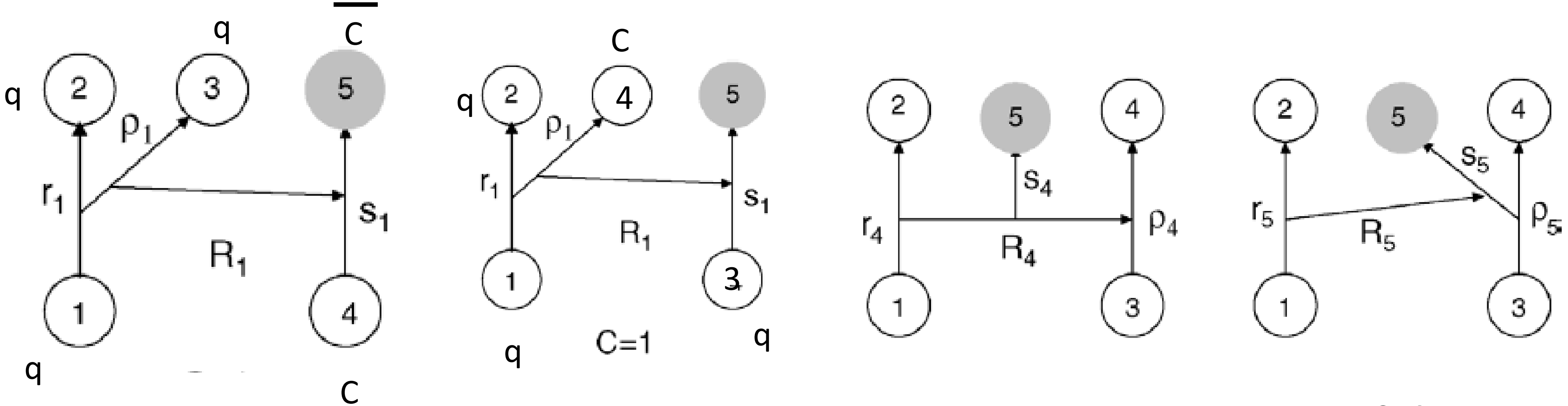
- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,
Kamimura and his collaborators.

Review article :
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

High-precision calculations of various 3- and 4-body systems:

Exotic atoms / molecules ,	Light hypernuclei,
3- and 4-nucleon systems,	3-quark systems,
multi-cluster structure of light nuclei,	^{4}He-atom tetramer

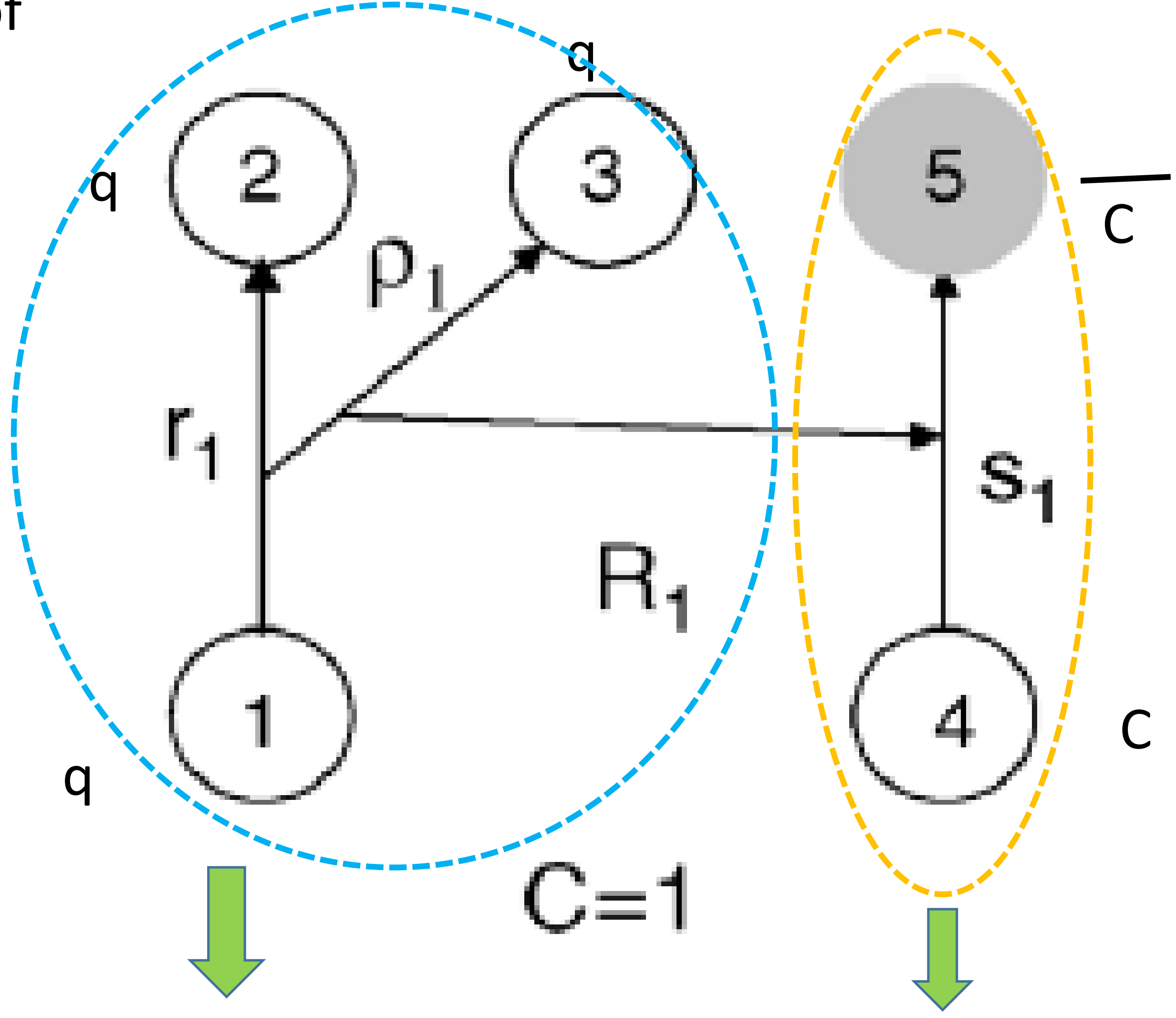


C=1(J/Ψ+p,ηc+p) C=2(Λc+D,Σc+D) C=3 C=4

$$\Psi_{JM}(qqqc\bar{c}) = \Phi_{JM}^{(C=1)} + \Phi_{JM}^{(C=2)} + \phi_{JM}^{(c=3)} + \Phi_{JM}^{(C=4)}$$

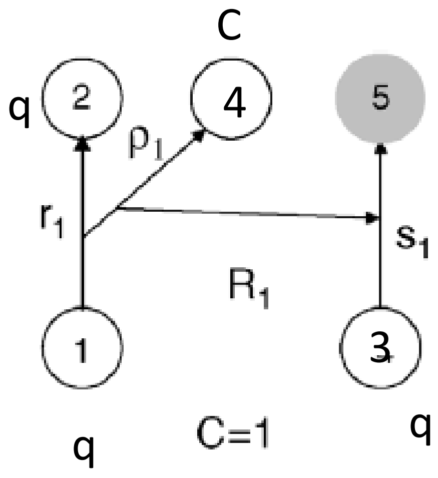
$$\Phi_{\alpha JM}(qqq\overline{cc}) = \mathcal{A}_{qqqq}\{[(\text{color})^{(c)}_{\alpha} \quad (\text{isospin})^{(c)}_{\alpha} \quad (\text{spin})^{(C)}_{\alpha} \quad (\text{spatial})^{(c)}_{\alpha}]_{JM}\}$$

Wavefunction of Color part



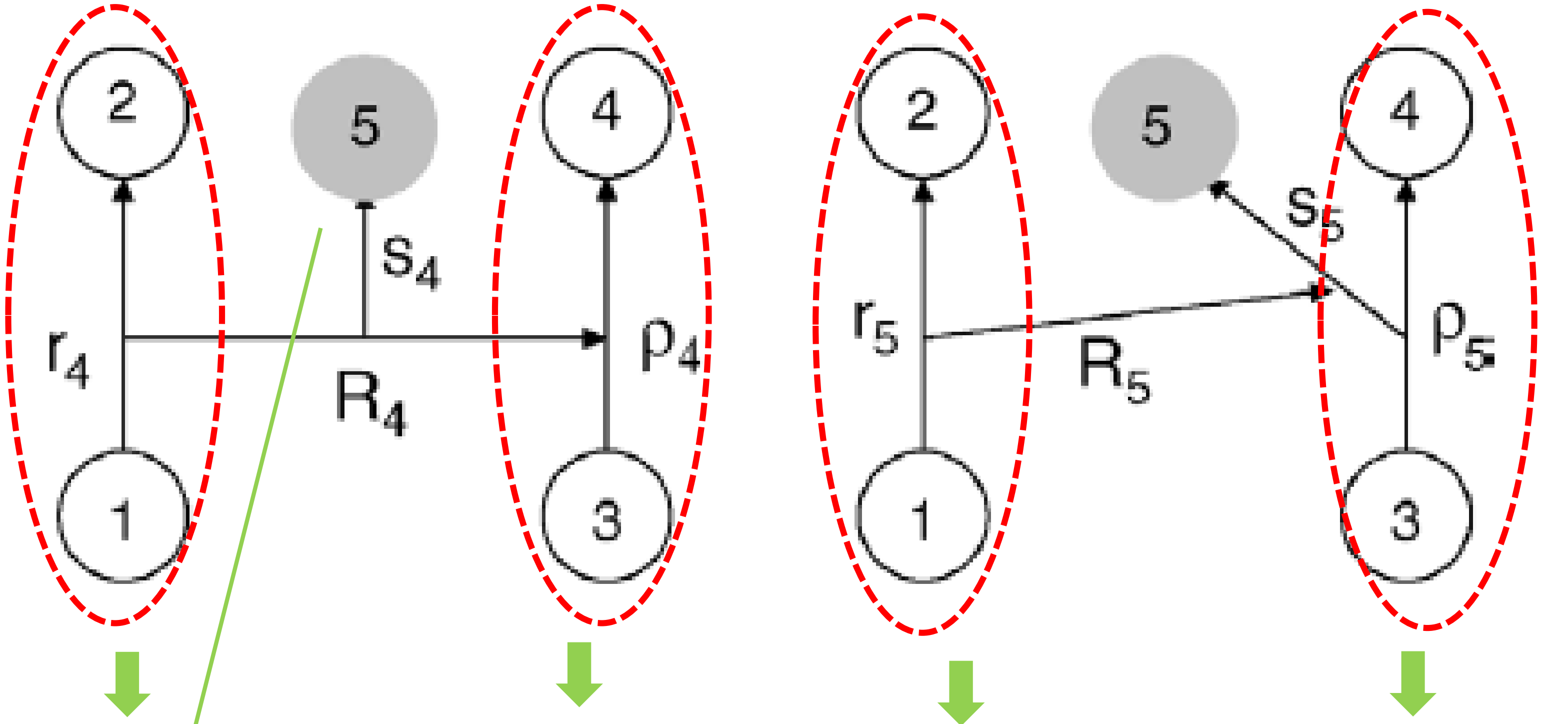
$1 \otimes 1 = 1$

Similar for C=2



C=2(Λc+D,Σc+D)

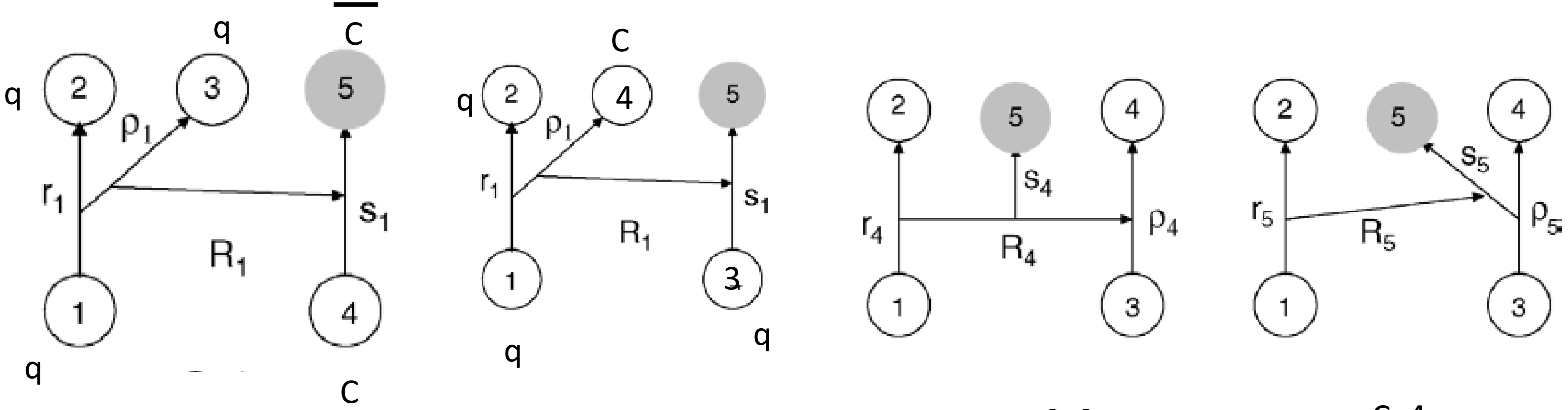
Confining channels



$\bar{3} \otimes \bar{3} = 3$ $\quad$ $\bar{3} \otimes \bar{3} = 3$

$\bar{3} \otimes 3 = 8 \oplus 1$ $\quad$ $\bar{3} \otimes 3 = 8 \oplus 1$

I take color singlet. $\quad$ I take color singlet.



C=1(J/Ψ+p,ηc+p)   C=2(Λc+D,Σc+D)   C=3   C=4

$$\Psi_{JM}(qqqc\bar{c}) = \Phi_{JM}^{(C=1)} + \Phi_{JM}^{(C=2)} + \phi_{JM}^{(c=3)} + \Phi_{JM}^{(C=4)}$$

$$\Phi_{\alpha JM}(qqqc\bar{c}) = \mathcal{A}_{qqqq}\{[(\mathrm{color})^{(c)}_{\alpha} \quad (\mathrm{isospin})^{(c)}_{\alpha} \quad (\mathrm{spin})^{(C)}_{\alpha} \quad (\mathrm{spatial})^{(c)}_{\alpha}]_{JM}\}$$

$$(\mathrm{spatial})^{(c)}_{\alpha} = \varphi_{nl}^{(c)}(r_c)\psi_{\nu\lambda}^{(c)}(\rho_c)\varphi^{(c)}_{kj}(s_c)\Phi_{n_R LM}^{(c)}(R_c)$$

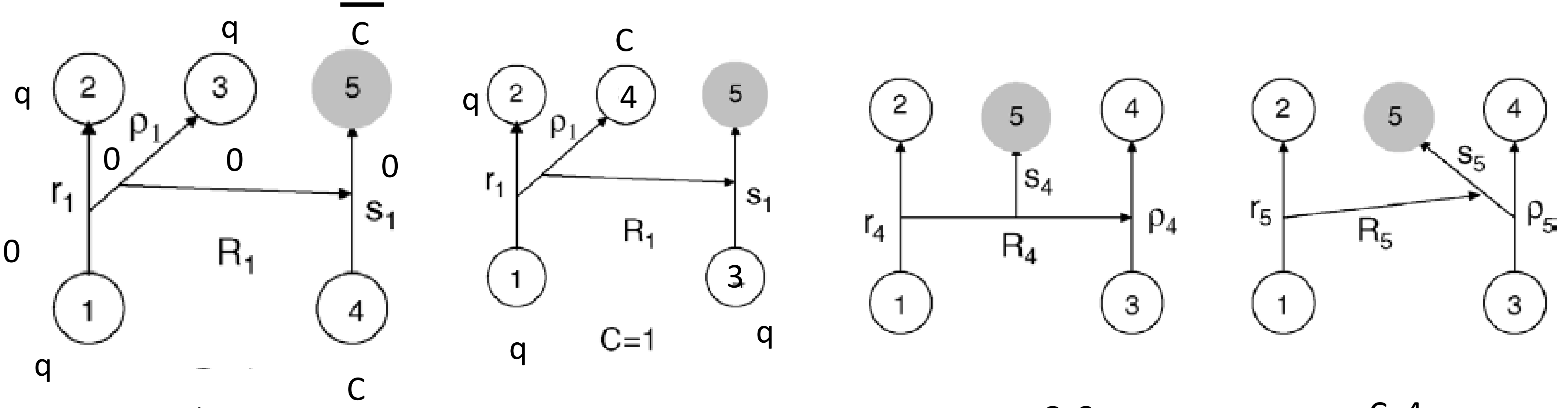
$$\phi_{n_R L_c M}(\mathbf{R}) = R^{L_c} e^{-(R/\bar{R}_{n_R})^2} Y_{L_c M}(\hat{\mathbf{R}}) \qquad \bar{R}_{n_R} = \bar{R}_1 a^{n_R - 1} \quad (n_R = 1 - n_R^{\mathrm{max}})$$

Same procedure is taken for r,ρ, and s.

For the Pc(4380) and (4450), we consider the following 9 candidates states,

Total orbital angular momentum: L=0, 1, 2
Total Spin : S=1/2, 3/2, 5/2

For example, in the case of total orbital angular momentum L=0, S=1/2, 3/2, 5/2, $J^{\pi}=1/2^{-}, 3/2^{-}, 5/2^{-}$
we take s-waves for all coordinates.

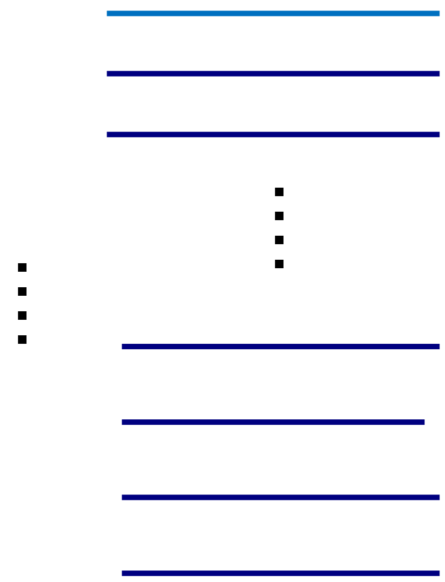


C=1(J/Ψ+p,ηc+p) C=2(Λc+D,Σc+D) C=3 C=4

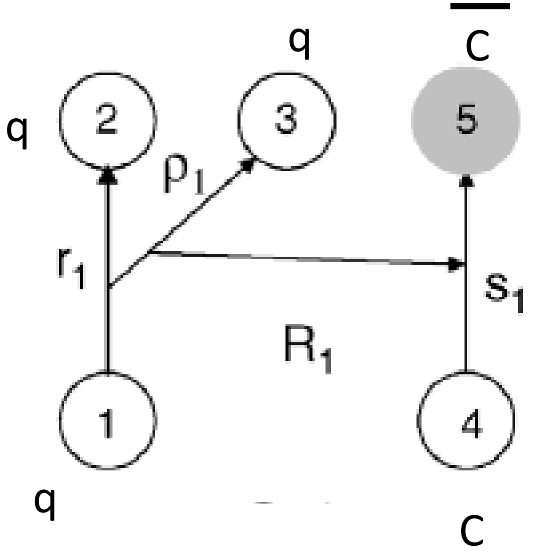
$$(H-E)\Psi=0$$

By the diagonalization of Hamiltonian, we obtain N eigenstates for each J^{π}.

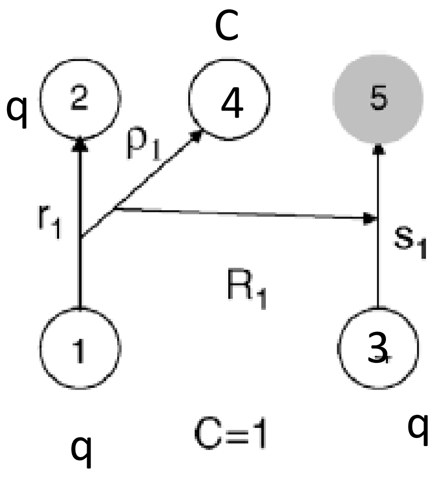
Here, we use about 40,000 basis functions.
Then, we obtained 40,000 eigenfunction for each J^{π}.
Here, we investigate $J=1/2^{-}$, namely, L(total angular momentum)=0, S(total spin)=1/2.



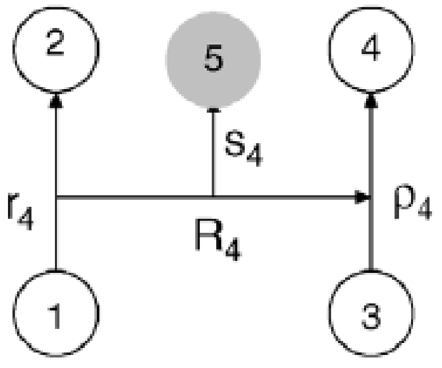
L=0,S=1/2 for example



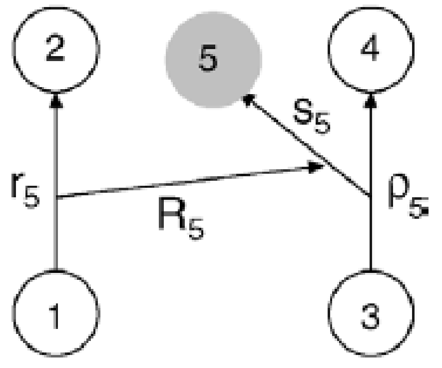
C=1(J/Ψ+p,ηc+p)



C=2(Λc+D,Σc+D)



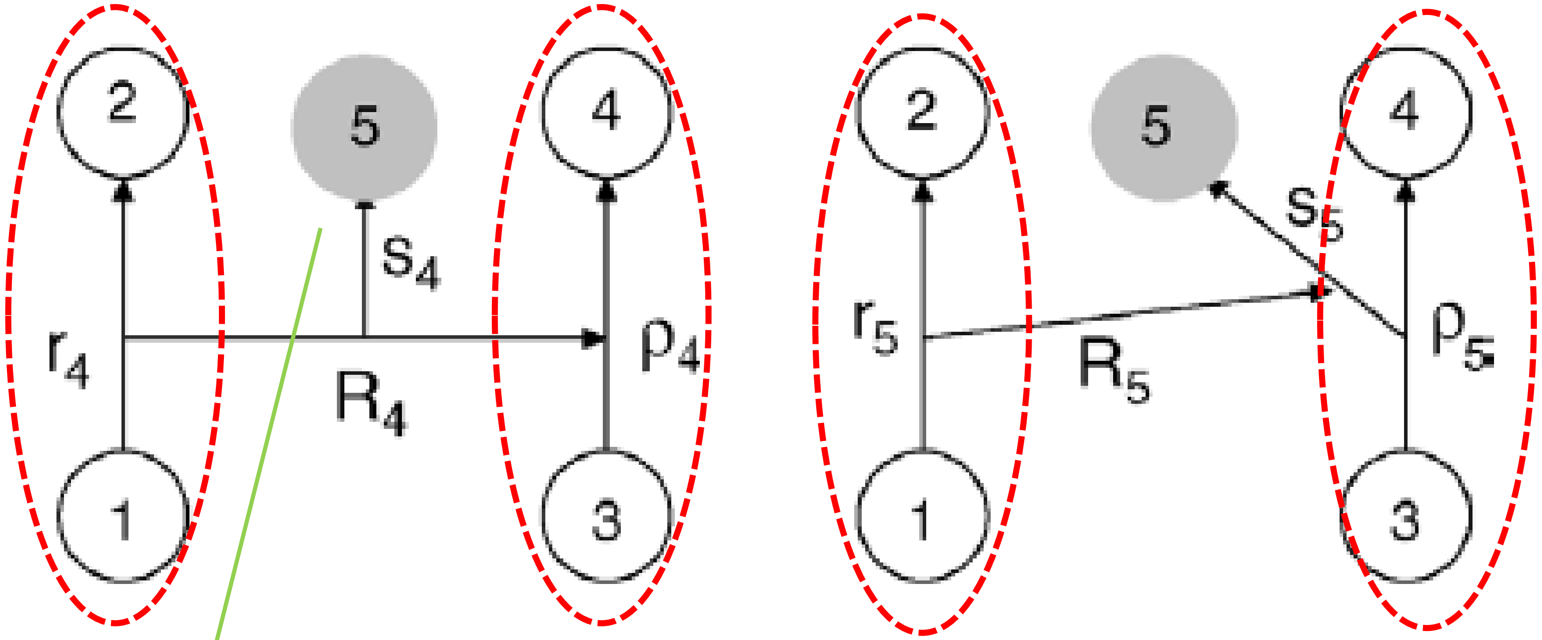
C=3



C=4

First, we take two channels.

Confining channels



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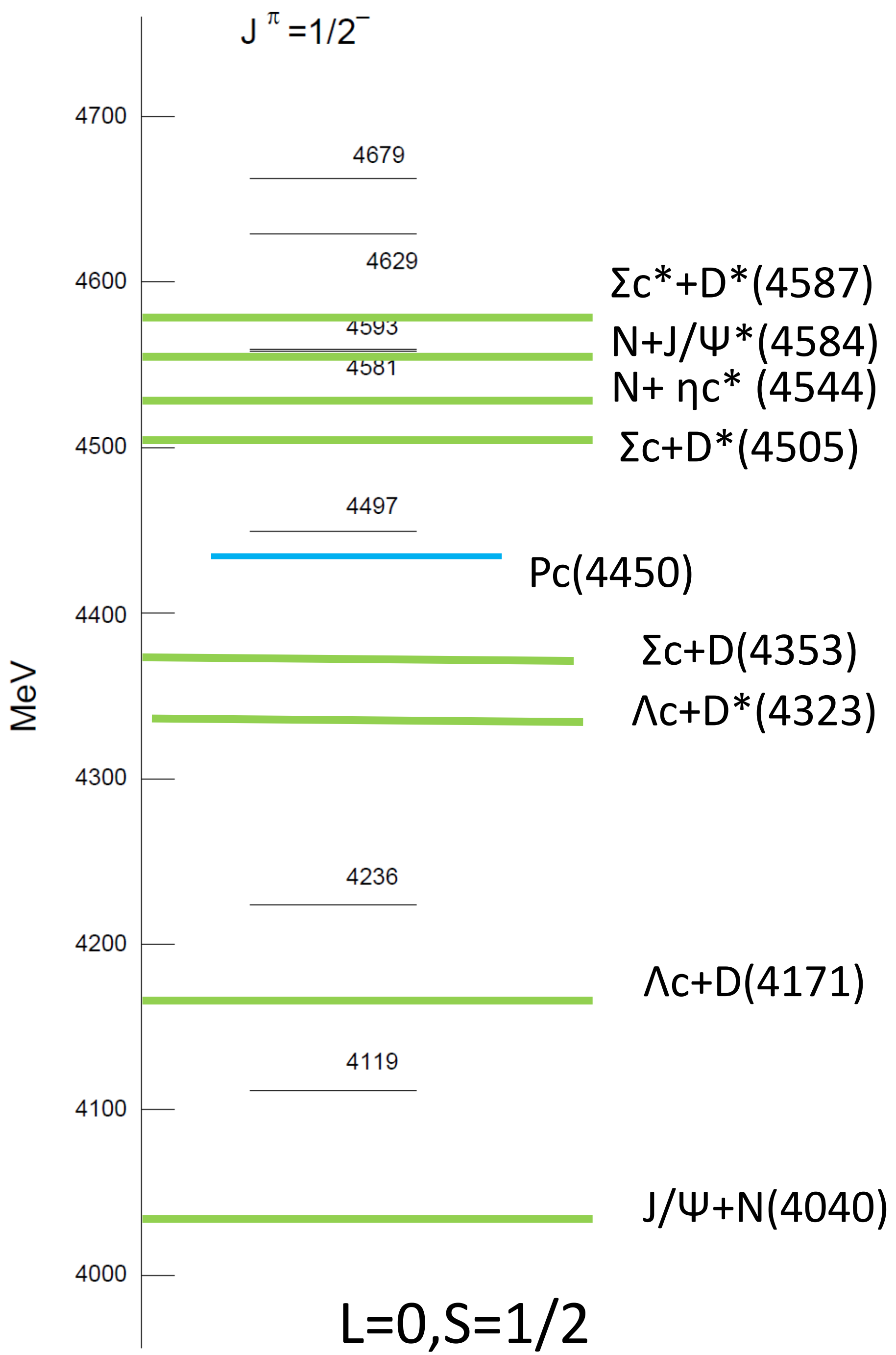
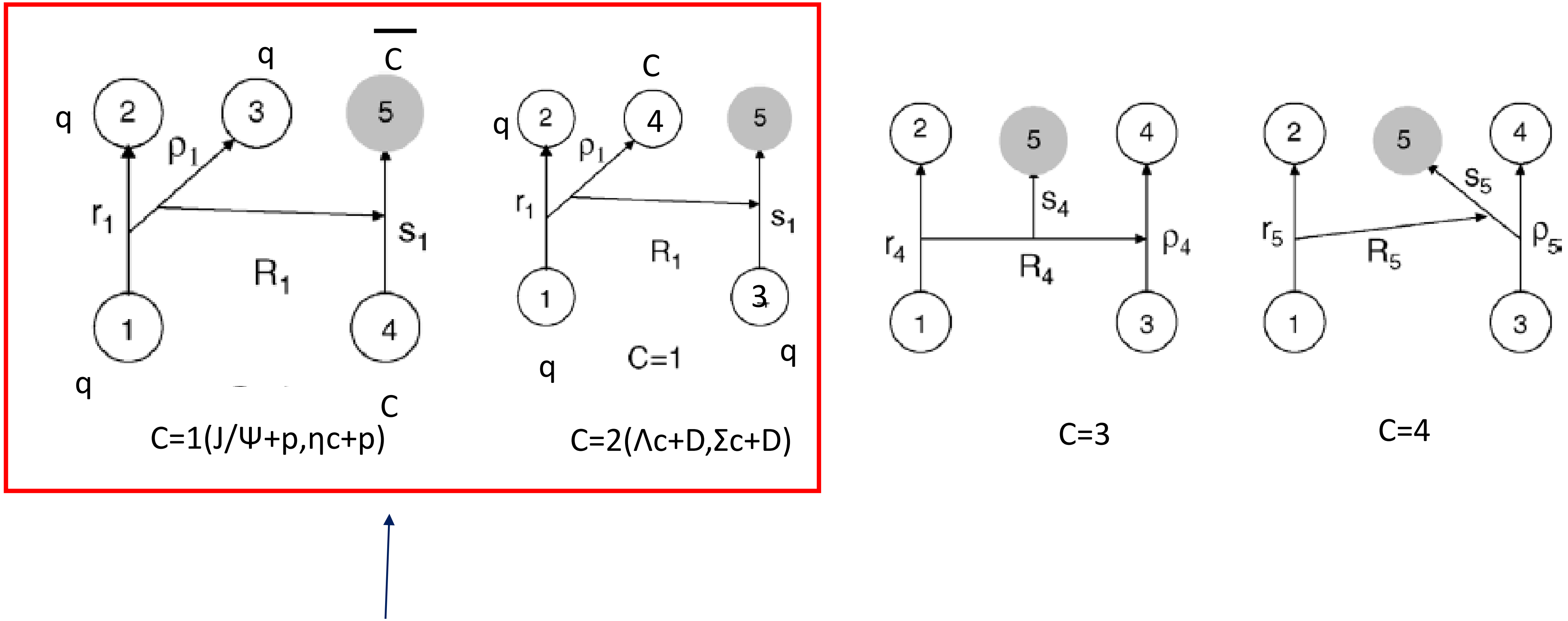


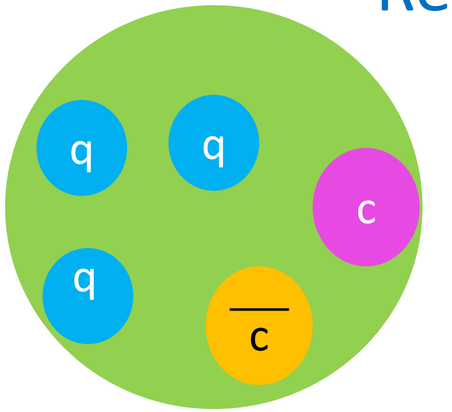
J π =1/2⁻
MeV
4700
4600
4500
4400
4300
4200
4100
4000
4679
4629
4593
4581
4497
4236
4119
Σc*+D*(4587)
N+J/Ψ*(4584)
N+ ηc* (4544)
Σc+D*(4505)
Pc(4450)
Σc+D(4353)
Λc+D*(4323)
Λc+D(4171)
J/Ψ+N(4040)
L=0,S=1/2

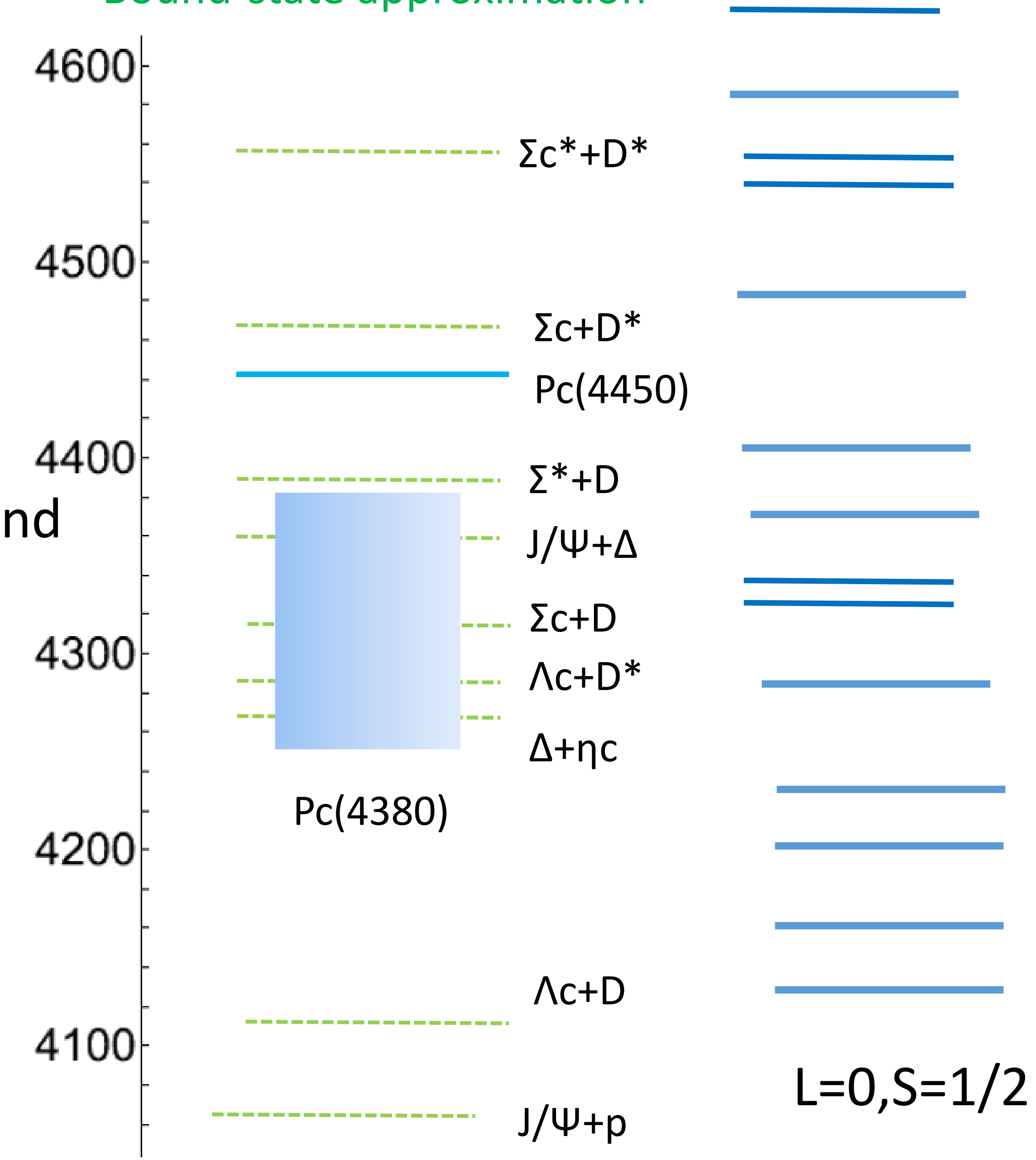


Next, we take two scattering channels.

Results before doing the scattering calculation

Bound-state approximation





Do these states correspond to resonance states or discrete non-resonance continuum states?

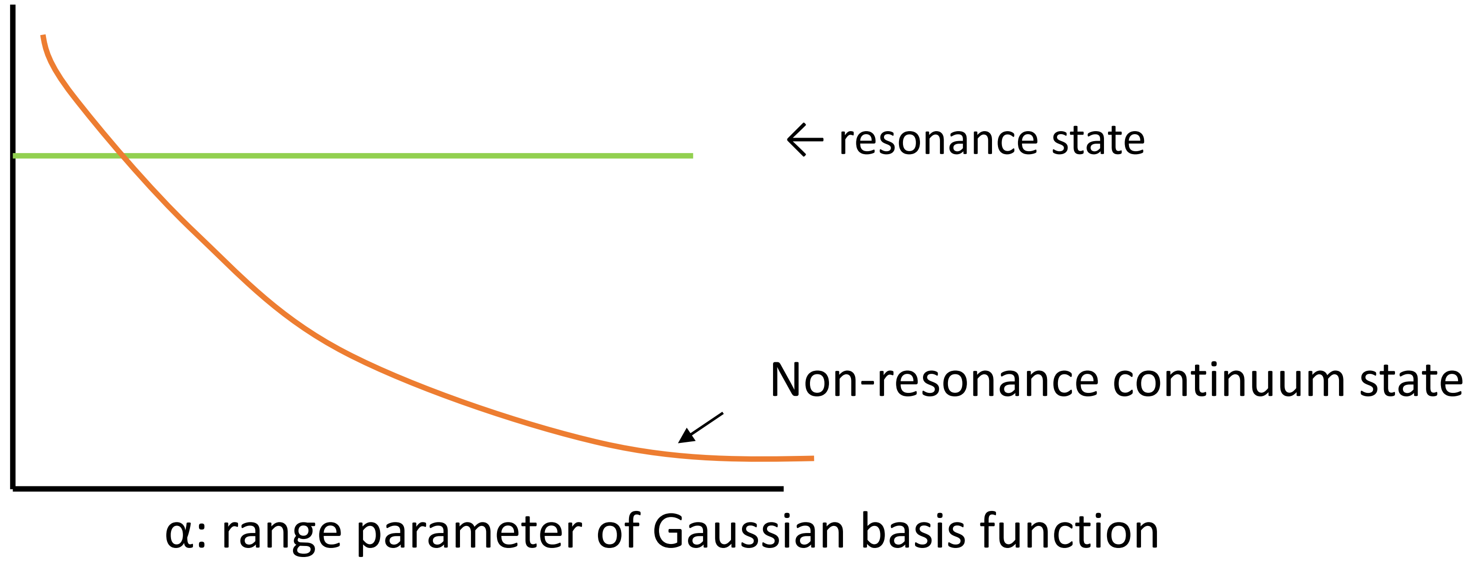
useful method: real scaling method

often used in atomic physics

In this method, we artificially scale the range parameters of our Gaussian basis functions by multiplying a factor α:

$r_n \rightarrow \alpha r_n$ in $r^l \exp(-r/r_n)^2$ for exmple $0.8 < \alpha < 1.5$

and repeat the diagonalization of Hamiltonian for many value of α.



[schematic illustration of the real scaling]

What is the result in our pentaquark calculation?

of nonstoichiometric samples, because they have an

[5]R. F. Brebrick, J. Phys. Chem. Solids 32, 551 (1971).

Resonance state lifetimes from stabilization graphs

Jack Simons[a)]

Chemistry Department, University of Utah, Salt Lake City, Utah 84112
(Received 20 January 1981; accepted 18 May 1981)

The stabilization method (SM) pioneered by Taylor and co-workers[1] has proven to be a valuable tool for estimating the energies of long-lived metastable states of electron–atom, electron–molecule, and atom–diatom complexes. In implementing the SM one searches for eigenvalues arising from a matrix representation of the relevant Hamiltonian H which are "stable" as the basis set used to construct H is varied.

To obtain lifetimes of metastable states, one can choose from among a variety of techniques[2–7] (e.g., phase shift analysis, Feshbach projection "golden rule" formulas, Siegert methods, and complex coordinate scaling methods), many of which use the stabilized *eigenvector* as starting information. Here we demonstrate that one can obtain an *estimate* of the desired lifetime directly from the stabilization graph in a manner which makes a close connection with the complex coordinate rotation method (CRM) for which a satisfactory mathematical basis exists.

The starting point of our development is the observation that both the stable eigenvalue (E_r) and the eigenvalue(s) (E_c) which come from above and cross E_r (see Fig. 1 and Refs. 9–11 and 13) vary in a nearly linear manner (with α) near their avoided crossing points. This observation leads us to propose that the two eigenvalues arising in each such avoided crossing can be thought of as arising from two "uncoupled" states having energies $\epsilon_r(\alpha)=\epsilon+S_r(\alpha-\alpha_c)$ and $\epsilon_c(\alpha)=\epsilon+S_c(\alpha-\alpha_c)$, where S_r and S_c are the *slopes* of the linear parts of the stable and "continuum" eigenvalues, respectively. α_c is the value of α at which these two straight lines would intersect, and ϵ is their common value at $\alpha=\alpha_c$. This modeling of ϵ_r and ϵ_c is simply based upon the *observa-*

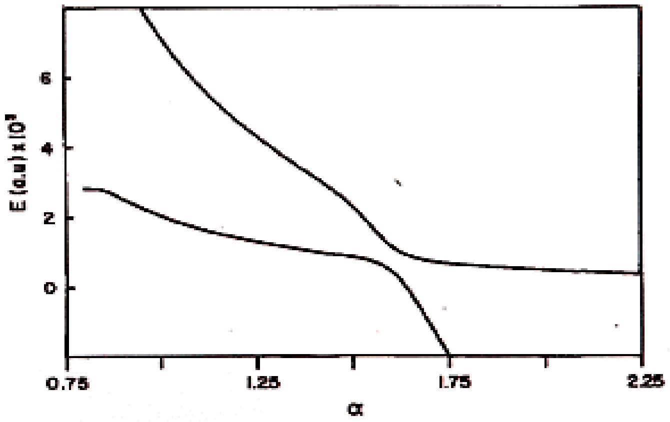


FIG. 1. Stabilization graph for the $^2\pi$ shape resonance state of LiH^- (Ref. 9).

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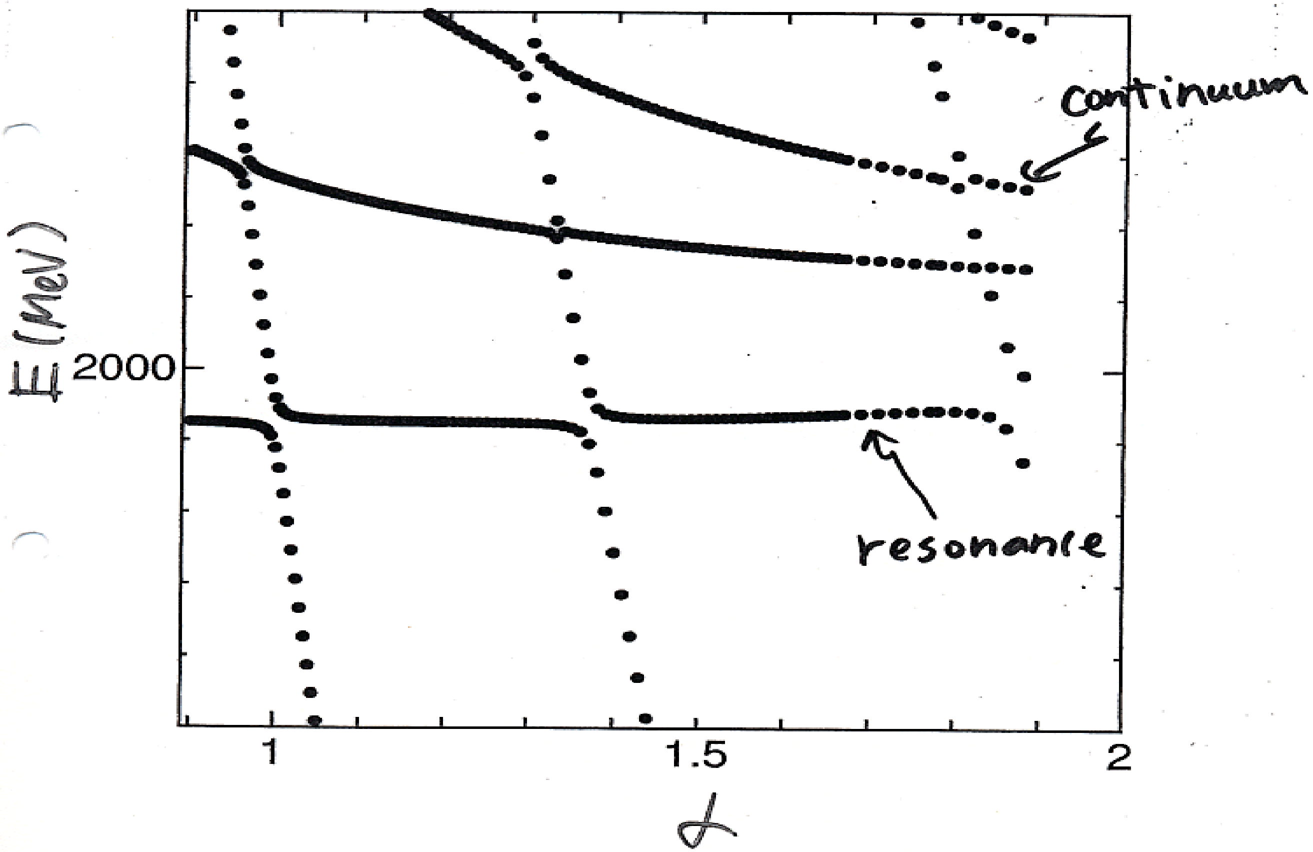
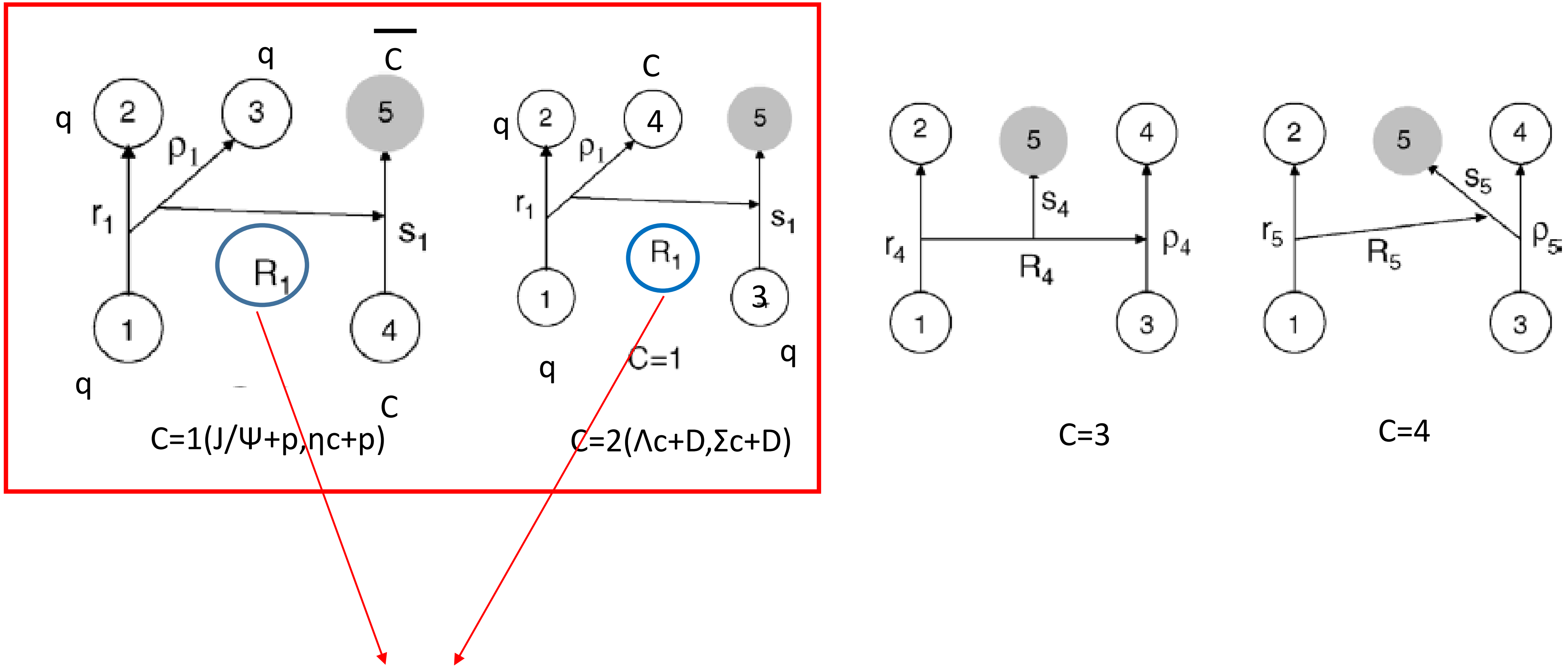
Example of real scaling

Not result of penta quark system



What is the result of our pentquark calculation?

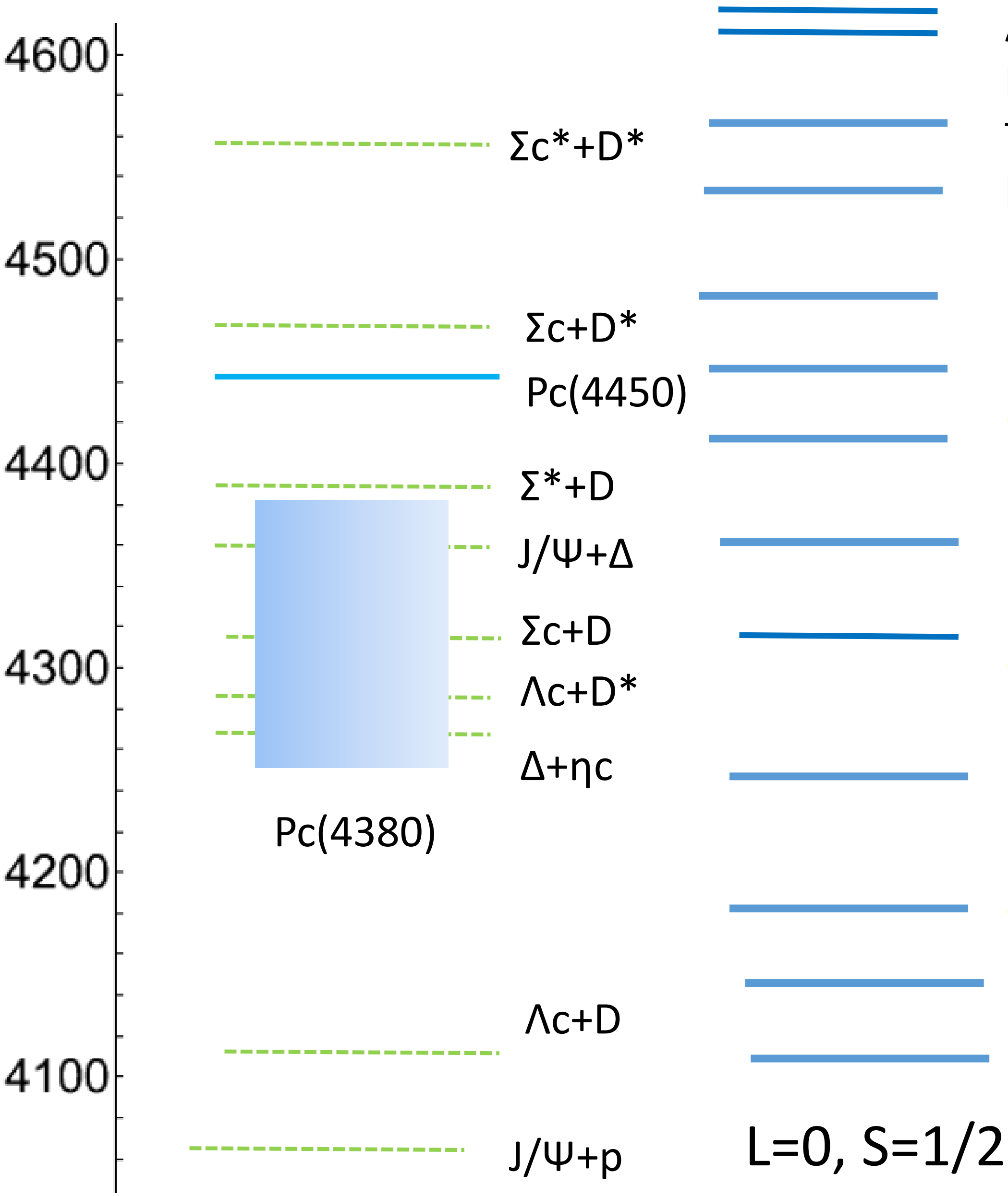


$$\phi_{n_R L_c M}(\mathbf{R}) = R^{L_c} e^{-(R/\bar{R}_{n_R})^2} Y_{L_c M}(\hat{\mathbf{R}}) \qquad \bar{R}_{n_R} = \bar{R}_1 a^{n_R - 1} \quad (n_R = 1 - n_R^{\max})$$

$$R_{nR} \to \alpha R_{nR}$$

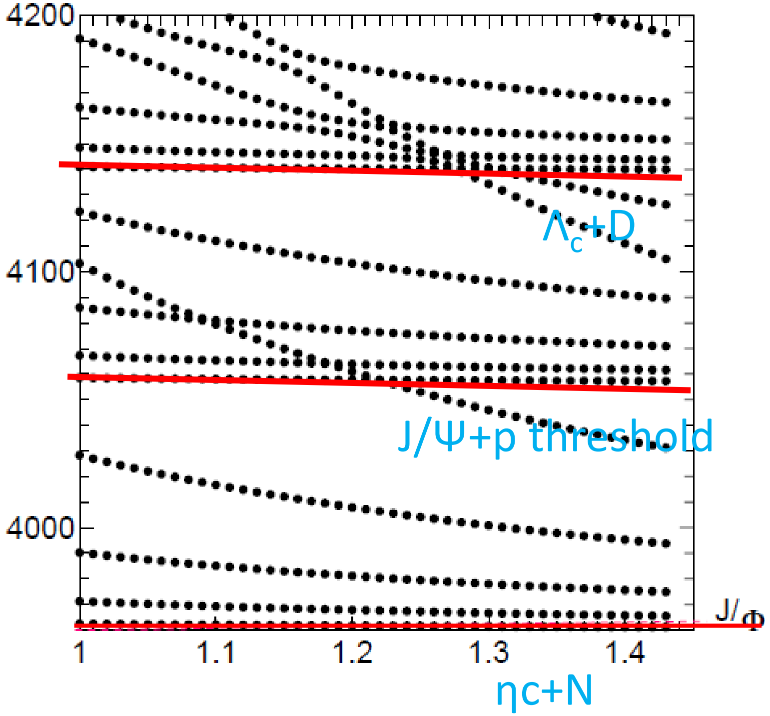
Results before doing the scattering calculation

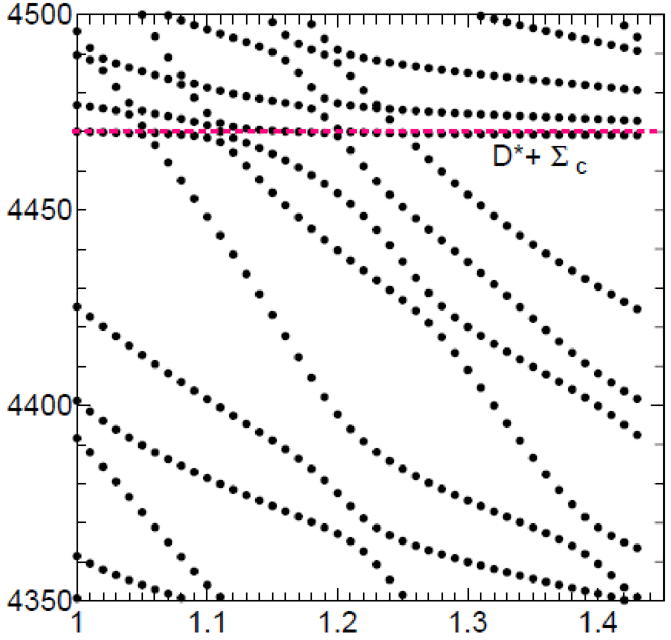
Bound-state approximation

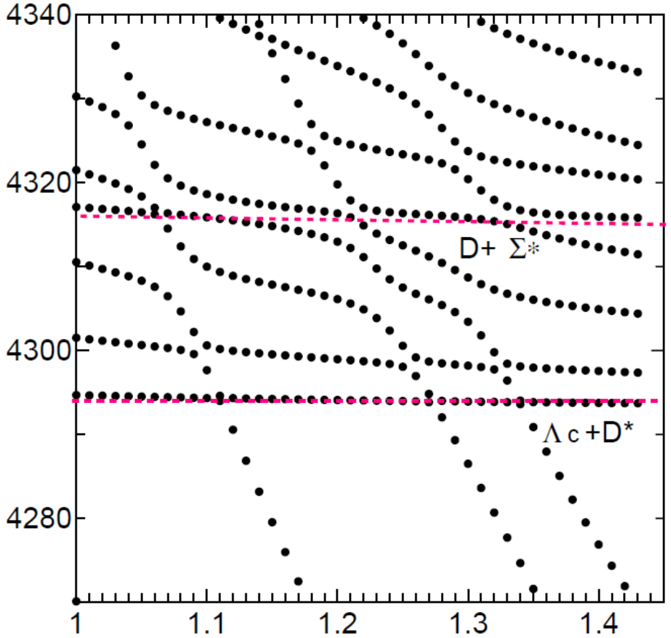


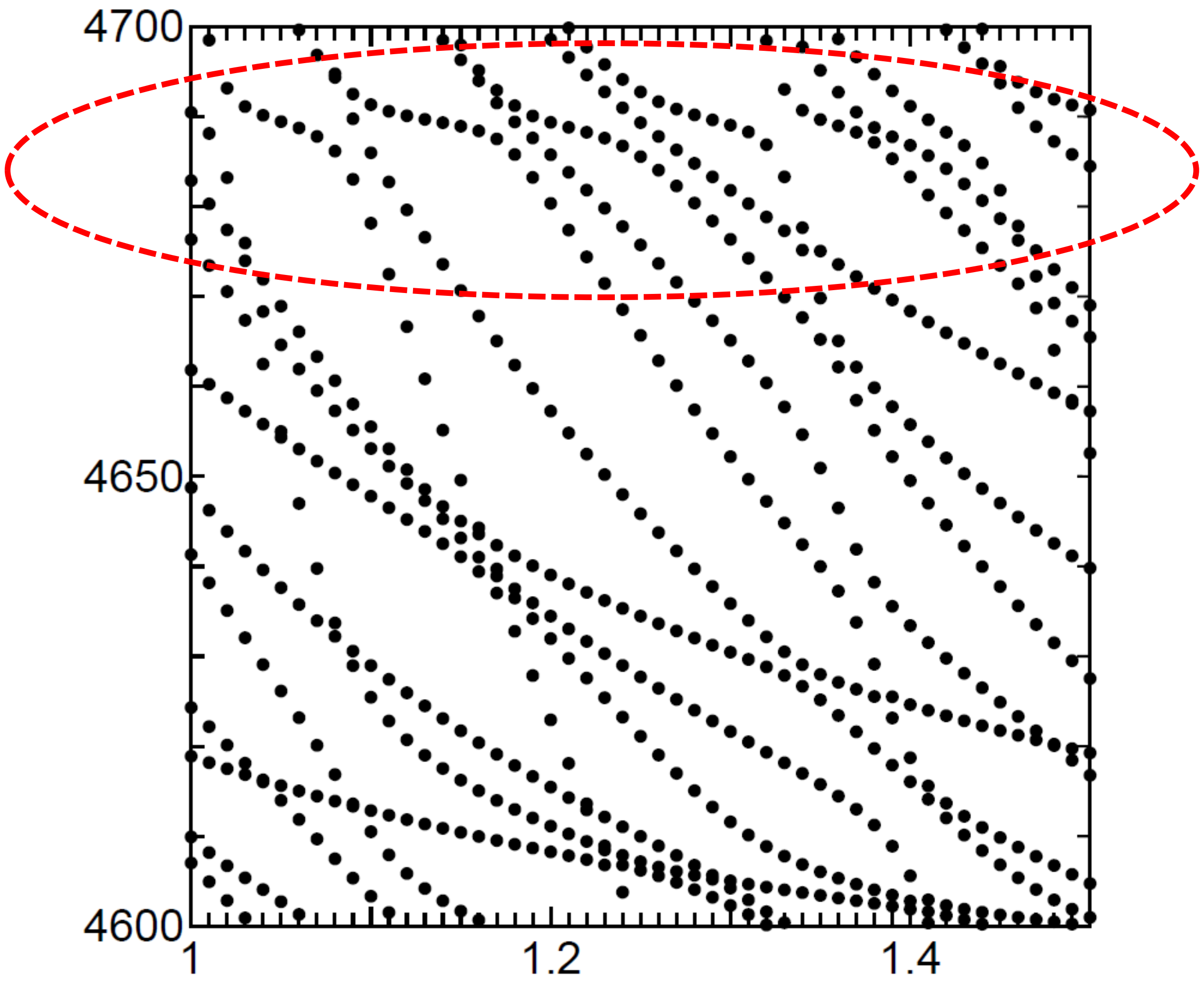
L=0, S=1/2

All states are melted into each meson-baryon continuum decaying state. Then, there is no resonant state between 4000 MeV to 4600 MeV.







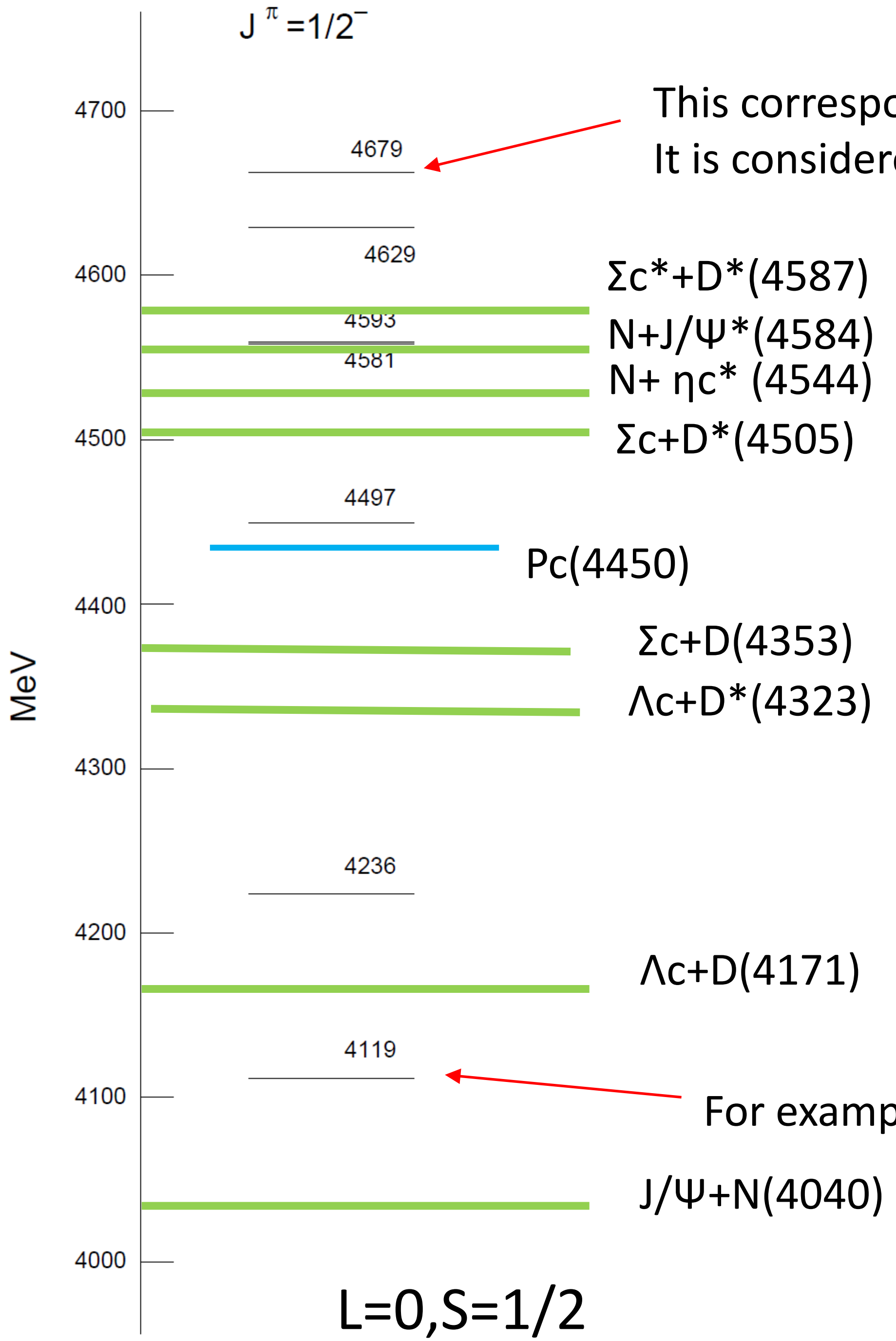


One resonance
at 4690 MeV

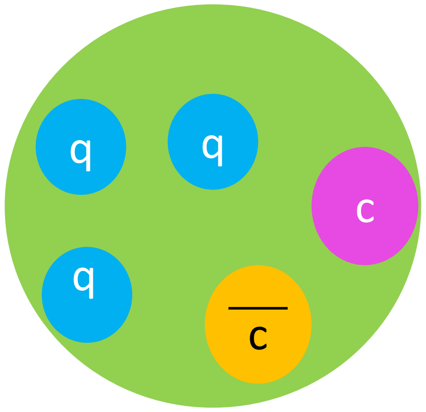
Much higher than the observed data

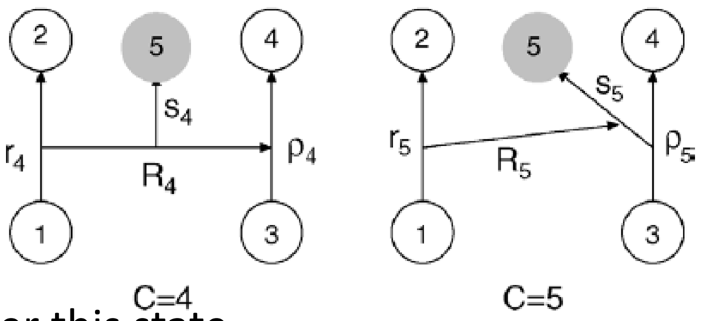
Why we have a resonance state at such higher energy?

This corresponds to resonant state, like a feshbach resonant state.
It is considered that other states are melted into various threshold.

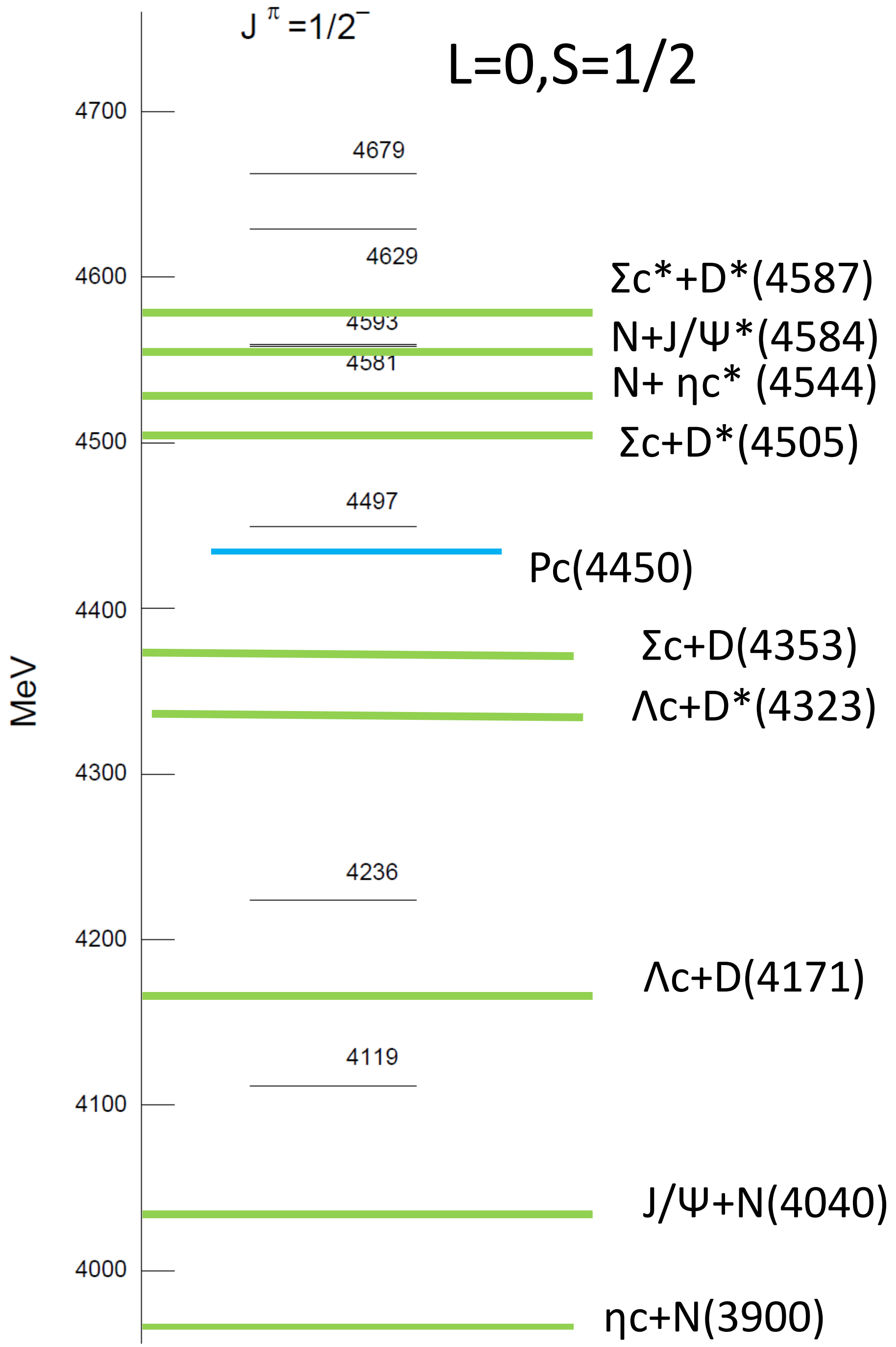


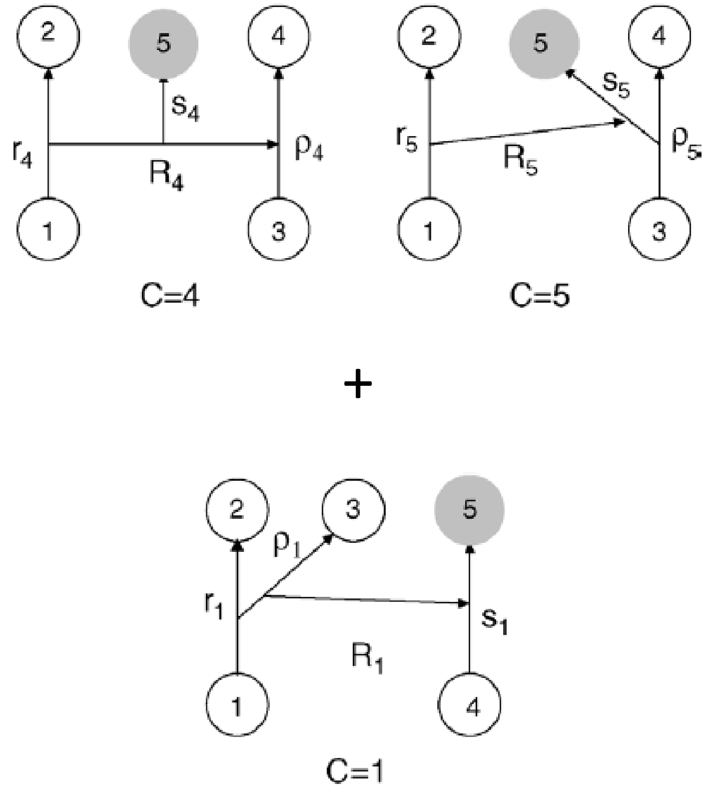
For example, let us consider this state.





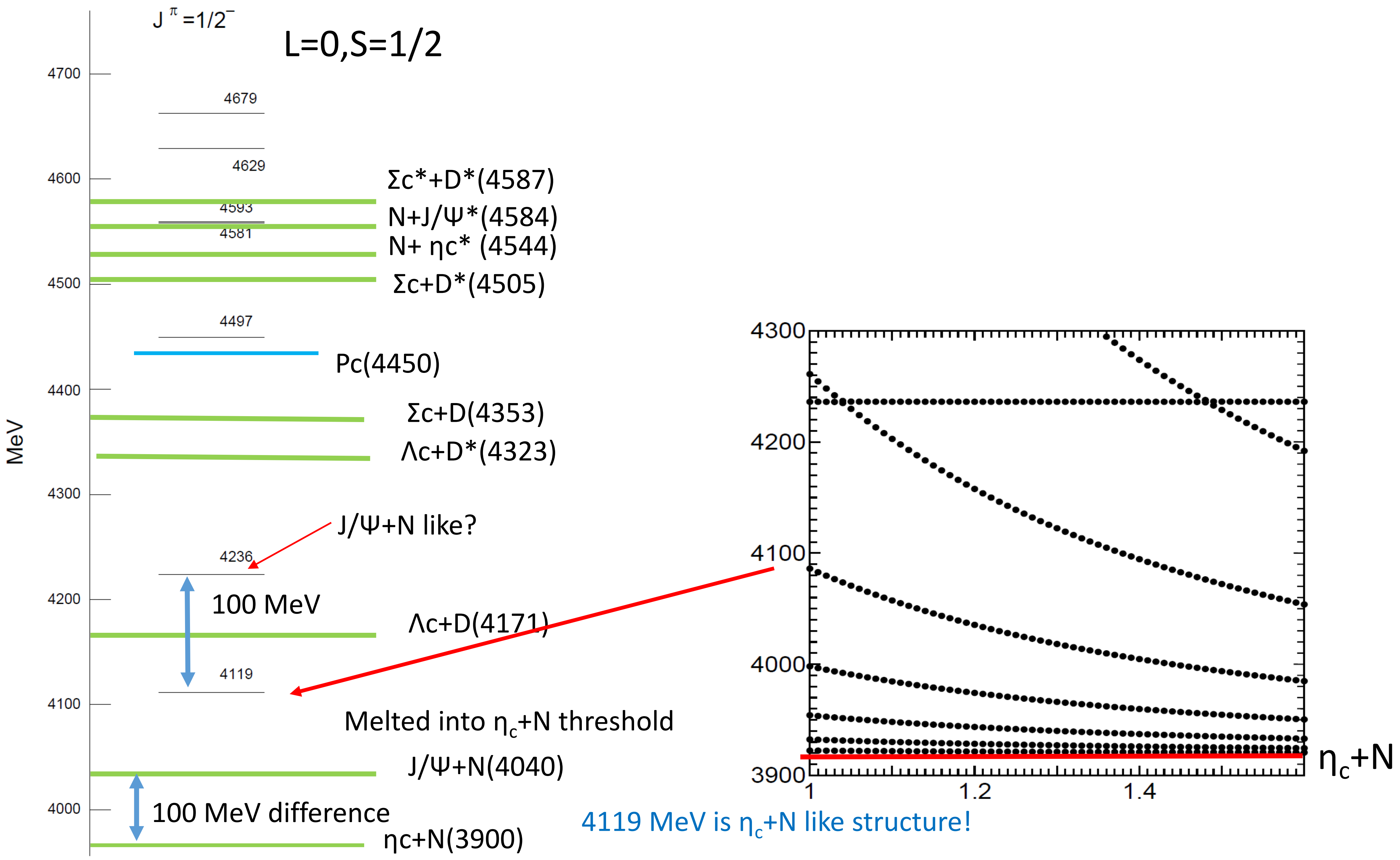
Confining channels





ηc+N channel

Conjecture: 4119 MeV can be describe as ηc+N like structure. However, due the restriction of the configurations, namely, by only C=4 and 5 channels, the mass energy is up than the ηc+N by about 200 MeV. In order to investigate this conjecture, we solve scattering states including ηc+N channel only with real scaling method. If 4119 MeV is ηc+N like structure, this state should be melted into ηc+N threshold.



J π =1/2−
L=0,S=1/2
MeV
4700
4600
4500
4400
4300
4200
4100
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4679
4629
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4119
Σc*+D*(4587)
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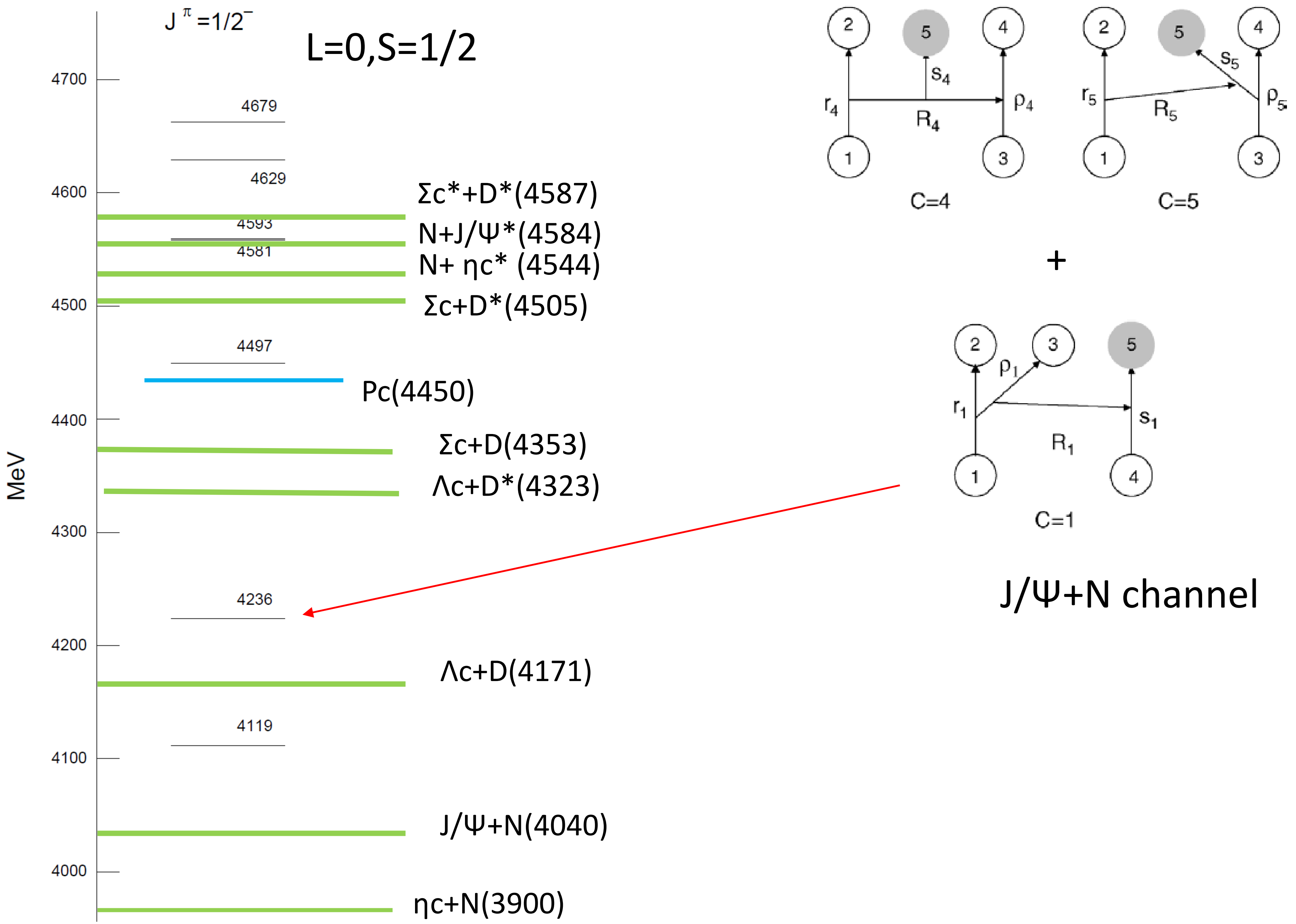
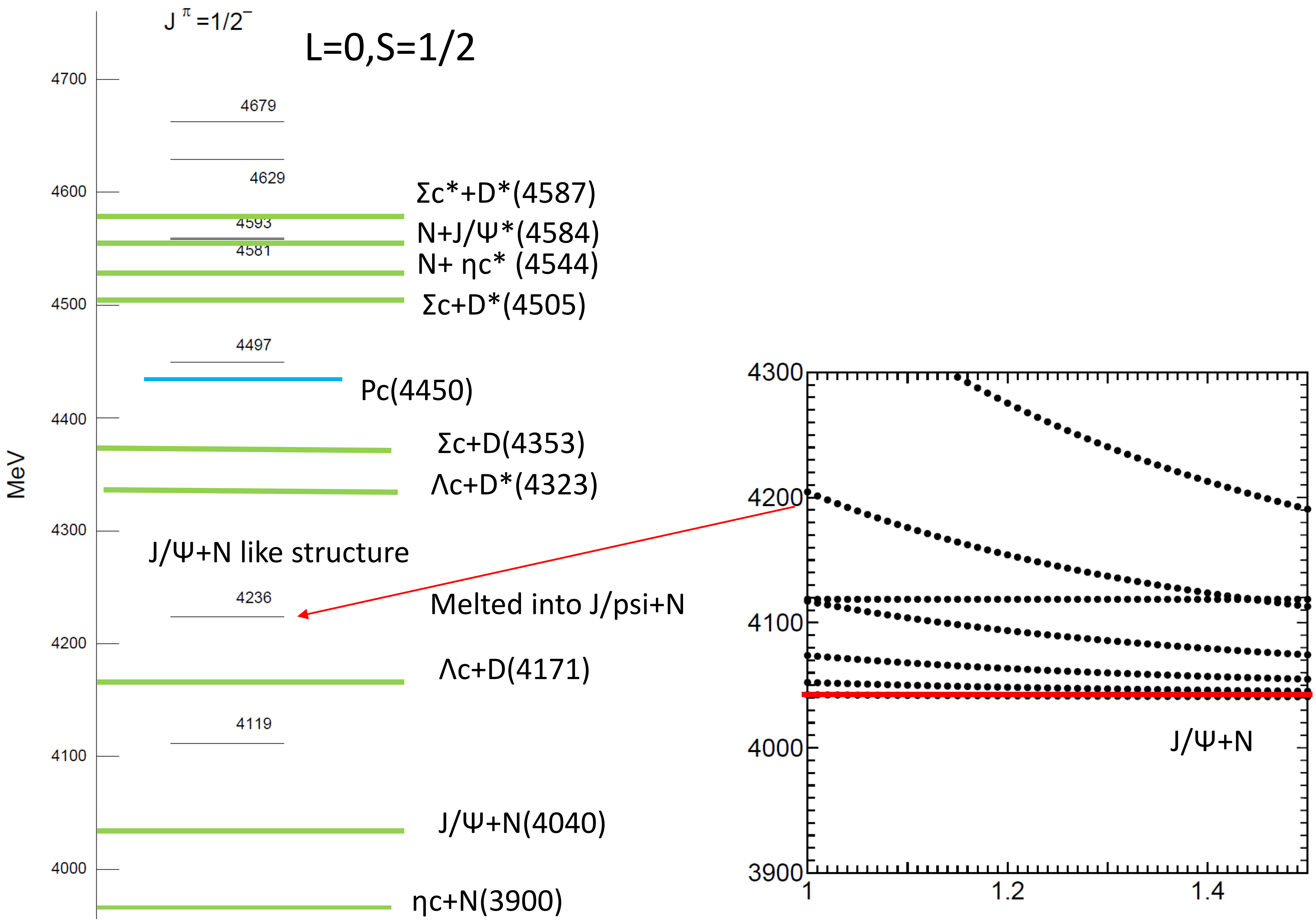
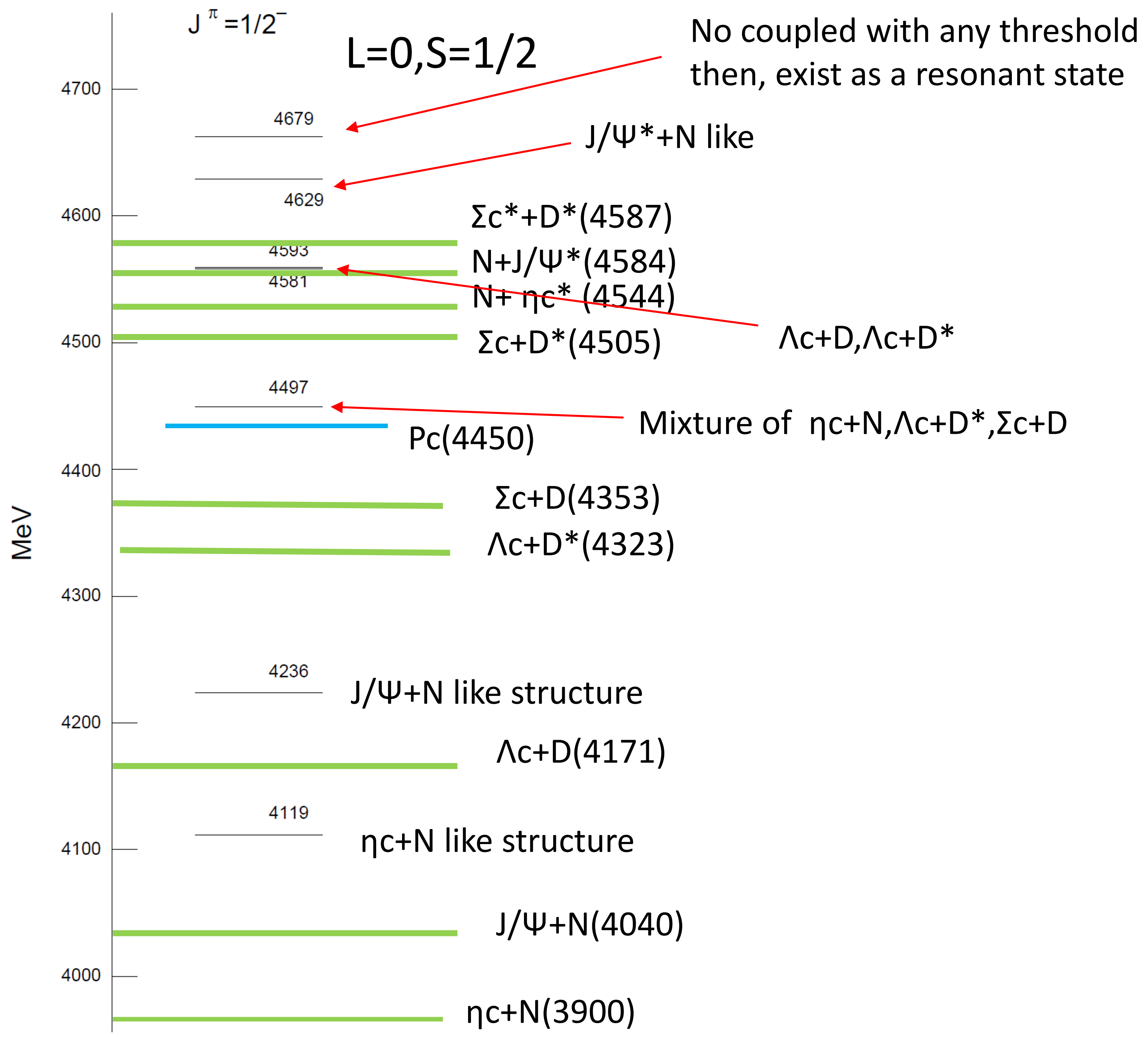
N+ ηc* (4544)
Σc+D*(4505)
Pc(4450)
Σc+D(4353)
Λc+D*(4323)
J/Ψ+N like?
100 MeV
Λc+D(4171)
Melted into η_c+N threshold
J/Ψ+N(4040)
100 MeV difference
ηc+N(3900)
4300
4200
4100
4000
3900
1
1.2
1.4
η_c+N
4119 MeV is η_c+N like structure!



L=0,S=1/2
$J^{\pi}=1/2^{-}$
MeV
4700
4600
4500
4400
4300
4200
4100
4000
4679
4629
4593
4581
4497
4236
4119
Σc*+D*(4587)
N+J/Ψ*(4584)
N+ ηc* (4544)
Σc+D*(4505)
Pc(4450)
Σc+D(4353)
Λc+D*(4323)
Λc+D(4171)
J/Ψ+N(4040)
ηc+N(3900)
C=4
C=5
+
C=1
J/Ψ+N channel



J π =1/2−
L=0,S=1/2
MeV
4700
4600
4500
4400
4300
4200
4100
4000
4679
4629
4593
4581
4497
4236
4119
Σc*+D*(4587)
N+J/Ψ*(4584)
N+ ηc* (4544)
Σc+D*(4505)
Pc(4450)
Σc+D(4353)
Λc+D*(4323)
J/Ψ+N like structure
Melted into J/psi+N
Λc+D(4171)
J/Ψ+N(4040)
ηc+N(3900)
4300
4200
4100
4000
3900
1
1.2
1.4
J/Ψ+N



J π =1/2−
L=0,S=1/2
No coupled with any threshold
then, exist as a resonant state
J/Ψ*+N like
Σc*+D*(4587)
N+J/Ψ*(4584)
N+ ηc* (4544)
Σc+D*(4505)
Λc+D,Λc+D*
Mixture of ηc+N,Λc+D*,Σc+D
Pc(4450)
Σc+D(4353)
Λc+D*(4323)
J/Ψ+N like structure
Λc+D(4171)
ηc+N like structure
J/Ψ+N(4040)
ηc+N(3900)
4679
4629
4593
4581
4497
4236
4119
4700
4600
4500
4400
4300
4200
4100
4000
MeV

Summary

・Motivated by the observed Pc(4380) and Pc(4450) systems at LHCb, we calculated energy spectra of $qqqc\bar{c}$ system using non-relativistic constituent quark model. To obtain resonant states, we also use real scaling method.

・Currently, we find no penta-quark like resonant states with L=0,S=1/2 at observed energy region.

However, we have one resonant state at 4690 MeV. This can be penta-quark state.

Future work

To investigate the structure, now I am calculating density distribution.

Also,

the calculation with L=0, S=3/2 is still on going.

Thank you!