Hadron Physics in China, 2017, Nanjing

The principle of maximum conformality and its application to high energy physics

Jian-Ming Shen (申建明)，Chongqing University
Sheng-Quan Wang (王声权), Guizhou Minzu University

In collaboration with Stanley J. Brodsky, Xing-Gang Wu

Contents

1 Scheme-and-Scale Ambiguities in pQCD

2 Renormalization Group Invariance

3 The Principle of Maximum Conformality

4 Summary

Scheme-and-Scale Ambiguities in pQCD

Consider an observable ρ(Q) in pQCD in some scheme with the following expansion,

$$\rho_n(Q,\mu) = \sum_{k=0}^{n} r_k(\mu/Q)\,\alpha_s^{k+p}(\mu)$$

$$\alpha_s = g_s^2 / 4\pi$$

Q ---- the kinematic scale

μ ---- the initial renormalization scale

RGE

$$\frac{\partial}{\partial \ln \mu^2}\left(\frac{\alpha_s(\mu)}{4\pi}\right) \equiv \beta(\alpha_s) = -\sum_{i=0}^{\infty} \beta_i \left(\frac{\alpha_s(\mu)}{4\pi}\right)^{i+2}$$

$$\frac{\partial \rho_\infty(\mu)}{\partial \mu} \equiv 0$$

$$\frac{\partial \rho_n(\mu)}{\partial \mu} \neq 0$$

Conventional Scale Setting

- infinite order, the prediction is free of scheme- and scale- dependence
- fixed-order, the prediction is scheme- and scale- dependent

- Guessing a renormalization scale, $\mu = Q$. only a guess work !
- Varying, e.g. $\mu \in$ [Q/2, 2 Q], to estimate its uncertainty

Scheme-and-Scale Ambiguities in pQCD

Problems with the conventional scale setting

- *Predictions are scheme-and-scale dependent ! At every fixed-order !*
- *Convergence is usually problematic !* renormalon $n!\beta_i^n\alpha_s^n$
- *Unrealistic estimate of higher-order terms: Only β-terms can be exposed by scale variation !*
- *Introduces an unnecessary systematic error !*

Scheme-and-Scale Ambiguities in pQCD

A brief introduction about scale-setting methods

- the renormalization group improved effective charge (FAC).

FAC $\Rightarrow$ $\rho = c_0 \alpha_s^p (\mu^{PAC})$ G. Grunberg, Phys. Lett. B **95**, 70 (1980)

- the principle of minimum sensitivity (PMS).

PMS $\Rightarrow$ $\frac{\partial \rho_n}{\partial \mu} = 0$ P.M. Stevenson, Phys. Lett. B **100**, 61 (1981);

- the Brodsky-Lepage-Mackenzie method (BLM) and its underlying principle of maximum conformality (PMC). S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys. Rev. D **28**, 228 (1983).

BLM $\Rightarrow$ n_f*-term* PMC $\Rightarrow$ *Systematic, all-orders, scale-setting*

Renormalization Group Invariance

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme or initial scale choice

At infinite order, the renormalization scheme/scale dependence from the running couplings and the pQCD calculable coefficients do exactly cancel

$$\rho(Q)=\sum_{i=0}^{\infty} C_i(\mu/Q)\,\alpha_s^{p+i}(\mu) \xrightarrow{\text{RGI}} \frac{\partial\rho}{\partial \mathrm{RS}}=0 \rightarrow \left.\frac{\partial\rho}{\partial \mathrm{RS}}\right|_{C_i}+\left.\frac{\partial\rho}{\partial \mathrm{RS}}\right|_{\alpha_s}=0 \rightarrow \left.\frac{\partial\rho}{\partial \mathrm{RS}}\right|_{C_i}=-\left.\frac{\partial\rho}{\partial \mathrm{RS}}\right|_{\alpha_s}$$

“RS” stands for either the scale or the scheme parameters

Renormalization Group Invariance

For a review, see

IOP Publishing

Reports on Progress in Physics

Rep. Prog. Phys. **78** (2015) 126201 (15pp)

doi:10.1088/0034-4885/78/12/126201

Review

Renormalization group invariance and optimal QCD renormalization scale-setting: a key issues review

Xing-Gang Wu[1], Yang Ma[1], Sheng-Quan Wang[1], Hai-Bing Fu[1], Hong-Hao Ma[1], Stanley J Brodsky[2] and Matin Mojaza[3]

[1] Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China
[2] SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA

The Principle of Maximum Conformality (PMC)

The concept of PMC

PHYSICAL REVIEW D **85,** 034038 (2012)

Scale setting using the extended renormalization group and the principle of maximum conformality: The QCD coupling constant at four loops

Stanley J. Brodsky[1,*] and Xing-Gang Wu[1,2,†]

[1]*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[2]*Department of Physics, Chongqing University, Chongqing 401331, China*
(Received 30 November 2011; published 22 February 2012)

A key problem in making precise perturbative QCD predictions is to set the proper renormalization scale of the running coupling. The extended renormalization group equations, which express the invariance of the physical observables under both the renormalization scale- and scheme-parameter transformations, provide a convenient way for estimating the scale- and scheme-dependence of the physical process. In this paper, we present a solution for the scale equation of the extended renormalization group equations at the four-loop level. Using the principle of maximum conformality (PMC)/Brodsky-Lepage-Mackenzie (BLM) scale-setting method, all nonconformal $\{\beta_i\}$ terms in the perturbative expansion series can be summed into the running coupling, and the resulting scale-fixed predictions are independent of the renormalization scheme. The PMC/BLM scales can be fixed order-by-order. As a

Main idea of PMC : *all nonconformal $\{\beta_i\}$ terms in the perturbative expansion series can be summed into the running coupling so that the remaining terms in the perturbative series are identical to that of a conformal theory, i.e., the corresponding theory with $\{\beta_i\}=\{0\}$.*

PMC provides the principle underlying BLM scale setting

The Principle of Maximum Conformality (PMC)

δ-Renormalization (R_δ) schemes

PRL **110,** 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza[*]
CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Stanley J. Brodsky[†]
SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Xing-Gang Wu[‡]
Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China
(Received 13 January 2013; published 10 May 2013)

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

The Principle of Maximum Conformality (PMC)

***R_δ* : illuminating renormalization scheme dependence of the QCD coupling**

***δ-Renormalization (Rδ) schemes*：** *during renormalization, subtract the finite part* $ln(4\pi) - \gamma_E - \delta$ *as well with the pole.*

$$\mathcal{R}_0 = \overline{\rm MS},$$

$$\mathcal{R}_{\ln 4\pi - \gamma_E} = {\rm MS},$$

$$\mathcal{R}_{-2} = {\rm G},$$

All R_δ's are connected by scale displacements

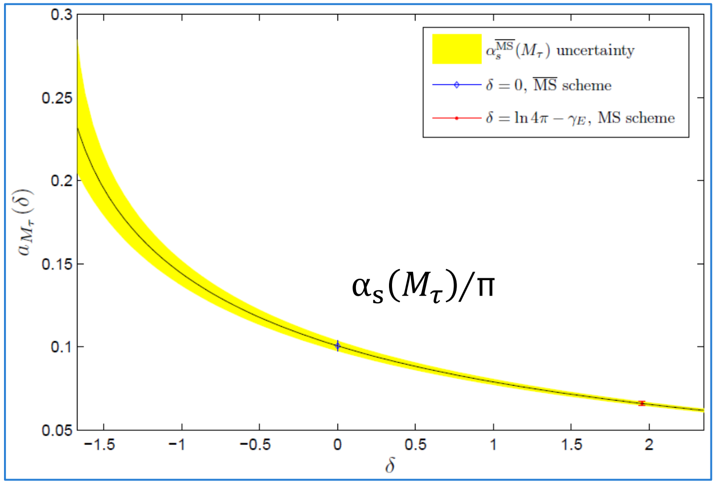
$$\alpha(\mu_0) = \alpha(\mu_\delta) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{{\rm d}^n \alpha(\mu)}{({\rm d} \ln \mu^2)^n} |_{\mu=\mu_\delta} \; (-\delta)^n$$

$$\alpha = \alpha_s / 4\pi$$

$$\ln \mu_0^2/\mu_\delta^2 = -\delta$$

Both the scheme dependence and scale-evolution of R_δ couplings are governed by the same RGE

$$\frac{\partial \alpha(\mu_\delta)}{\partial \delta} = \mu_\delta^2 \frac{\partial \alpha(\mu_\delta)}{\partial \mu_\delta^2} = \beta(\alpha(\mu_\delta))$$



The δ subtraction defines an infinite set of renormalization schemes

The Principle of Maximum Conformality (PMC)

R_δ : exposing the renormalization scheme dependence of the coefficients

$$\rho_0(Q) = \alpha(\mu_0)^p \sum_{k=0}^{\infty} r_{k+1}(Q^2/\mu_0^2)\alpha(\mu_0)^k$$

$$\alpha(\mu_0) = \alpha(\mu_\delta) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\mathrm{d}^n \alpha(\mu)}{(\mathrm{d}\ln\mu^2)^n}|_{\mu=\mu_\delta} (-\delta)^n$$

$$\alpha = \alpha_s / 4\pi$$

$$\begin{aligned}\rho_\delta(Q) =& r_1\alpha_1(\mu_\delta)^p + [r_2 + p\beta_0 r_1\delta_1]\alpha_2(\mu_\delta)^{p+1} + \left[r_3 + p\beta_1 r_1\delta_1 + (p+1)\beta_0 r_2\delta_2 + \frac{p(p+1)}{2}\beta_0^2 r_1\delta_1^2\right]\alpha_3(\mu_\delta)^{p+2} \\ &+ \left[r_4 + p\beta_2 r_1\delta_1 + (p+1)\beta_1 r_2\delta_2 + (p+2)\beta_0 r_3\delta_3 + \frac{p(3+2p)}{2}\beta_0\beta_1 r_1\delta_1^2 + \frac{(p+1)(p+2)}{2}\beta_0^2 r_2\delta_2^2 \right. \\ &\left. + \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_1\delta_1^3\right]\alpha_4(\mu_\delta)^{p+3} + \mathcal{O}(\alpha^{p+4}) ,\end{aligned}$$

shows the general way that nonconformal $\{\beta_i\}$ terms enter an observable

- exposing the pattern of $\{\beta_i\}$-terms in the coefficients at each order;
- some of the coefficients of the $\{\beta_i\}$-terms are degenerate, i.e., the coefficient of $\beta_0\alpha^{p+1}$ and $\beta_1\alpha^{p+2}$ can be set equal.

$$\begin{aligned}\rho(Q) =& r_{1,0}\alpha(\mu)^p + [r_{2,0} + p\beta_0 r_{2,1}]\alpha(\mu)^{p+1} + \left[r_{3,0} + p\beta_1 r_{2,1} + (p+1)\beta_0 r_{3,1} + \frac{p(p+1)}{2}\beta_0^2 r_{3,2}\right]\alpha(\mu)^{p+2} \\ &+ \left[r_{4,0} + p\beta_2 r_{2,1} + (p+1)\beta_1 r_{3,1} + (p+2)\beta_0 r_{4,1} + \frac{p(3+2p)}{2}\beta_0\beta_1 r_{3,2} + \frac{(p+1)(p+2)}{2}\beta_0^2 r_{4,2} \right. \\ &\left. + \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_{4,3}\right]\alpha(\mu)^{p+3} + \mathcal{O}(\alpha^{p+4}) ,\end{aligned}$$

The Principle of Maximum Conformality (PMC)

$$\frac{\partial \rho_\delta}{\partial \delta} = -\frac{\partial \alpha(\mu_\delta)}{\partial \delta}\frac{\partial \rho_\delta}{\partial \alpha(\mu_\delta)} = -\mu_\delta^2\frac{\partial \alpha(\mu_\delta)}{\partial \mu_\delta^2}\frac{\partial \rho_\delta}{\partial \alpha(\mu_\delta)} = -\beta(\alpha(\mu_\delta))\frac{\partial \rho_\delta}{\partial \alpha(\mu_\delta)}$$

- *At infinite order, the δ-dependence from the running couplings and the pQCD calculable coefficients do exactly cancel.*
- *Both the scheme variation and scale-evolution of the R_δ coupling are governed by the same RGE.*
- *The δ-dependence of the couplings and the coefficients are both associated with the scale-dependence of the running coupling at each order. All these dependence are associated with β-function.*
- *If one can sum all $\{\beta_i\}$-terms to shifting and setting the renormalization scales in the running couplings, then one eliminates the scheme dependence of the coefficients and couplings simultaneously, and obtain a scale-fixed, sheme-independent conformal series.*

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme or initial scale choice

The Principle of Maximum Conformality (PMC)

PMC multi-scale approach

systematically determine the argument of the coupling order by order in pQCD

$$\rho(Q) = r_{1,0}\alpha(\mu)^p + [r_{2,0} + p\beta_0 r_{2,1}]\,\alpha(\mu)^{p+1} + \left[r_{3,0} + p\beta_1 r_{2,1} + (p+1)\beta_0 r_{3,1} + \frac{p(p+1)}{2}\beta_0^2 r_{3,2}\right]\alpha(\mu)^{p+2}$$
$$+ \left[r_{4,0} + p\beta_2 r_{2,1} + (p+1)\beta_1 r_{3,1} + \frac{p(3+2p)}{2}\beta_1\beta_0 r_{3,2} + (p+2)\beta_0 r_{4,1} + \frac{(p+1)(p+2)}{2}\beta_0^2 r_{4,2}\right.$$
$$\left. + \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_{4,3}\right]\alpha(\mu)^{p+3} + \cdots,$$

$$r_{1,0}\alpha^p(Q_1) \xleftarrow{\text{PMC}} r_{1,0}\alpha^p(\mu) + p\beta_0 r_{2,1}\alpha^{p+1}(\mu) + \left[p\beta_1 r_{2,1} + \frac{p(p+1)}{2}\beta_0^2 r_{3,2}\right]\alpha^{p+2}(\mu) + \left[p\beta_2 r_{2,1} + \frac{p(3+2p)}{2}\beta_1\beta_0 r_{3,2} + \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_{4,3}\right]\alpha^{p+3}(\mu) + \cdots$$

......

$N^{n-1}LO$ PMC scale Q_n satisfies

$$\ln\frac{Q_n^2}{\mu^2} + \Delta_n^{(1)}\ln^2\frac{Q_n^2}{\mu^2} + \Delta_n^{(2)}\ln^3\frac{Q_n^2}{\mu^2} + \cdots = R_{n,1} + \Delta_n^{(1)} R_{n,2} + \Delta_n^{(2)} R_{n,3} + \cdots$$

$$R_{n,j} = (-1)^j\frac{r_{n+j,j}}{r_{n,0}},\quad \Delta_n^{(1)} = \frac{1}{2}\left[\frac{\partial\beta}{\partial\alpha} + (p+n-2)\frac{\beta}{\alpha}\right],\quad \Delta_n^{(2)} = \frac{1}{3!}\left[\left(\frac{\partial\beta}{\partial\alpha}\right)^2 + \beta\frac{\partial^2\beta}{(\partial\alpha)^2} + 3(p+n-2)\frac{\beta}{\alpha}\frac{\partial\beta}{\partial\alpha} + (p+n-2)(p+n-3)\frac{\beta^2}{\alpha^2}\right]$$

$$\rho(Q) = r_{1,0}\alpha^p(Q_1) + r_{2,0}\alpha^{p+1}(Q_2) + r_{3,0}\alpha^{p+2}(Q_3) + r_{4,0}\alpha^{p+3}(Q_4) + \cdots$$

Scale-fixed, scheme-independent, conformal series

The Principle of Maximum Conformality (PMC)

PMC single-scale approach (PMC-s)

PHYSICAL REVIEW D **95,** 094006 (2017)

Novel all-orders single-scale approach to QCD renormalization scale-setting

Jian-Ming Shen,[1,*] Xing-Gang Wu,[1,†] Bo-Lun Du,[1,‡] and Stanley J. Brodsky[2,§]

[1]*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China*

[2]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

(Received 28 January 2017; published 11 May 2017)

The principle of maximal conformality (PMC) provides a rigorous method for eliminating renormalization scheme-and-scale ambiguities for perturbative QCD (pQCD) predictions. The PMC uses the renormalization group equation to fix the β pattern of each order in an arbitrary pQCD approximant, and it then determines the optimal renormalization scale by absorbing all $\{\beta_i\}$ terms into the running coupling at each order. The resulting coefficients of the pQCD series match the scheme-independent conformal series with $\beta = 0$. As in QED, different renormalization scales appear at each order; we call this the multiscale approach. In this paper, we present a novel single-scale approach for the PMC, in which a single effective scale is constructed to eliminate all nonconformal β terms up to a given order simultaneously. The PMC single-scale approach inherits the main features of the multiscale approach; for example, its predictions are

The Principle of Maximum Conformality (PMC)

PMC single-scale approach (PMC-s)

The pQCD expansion for $\rho(Q)$ can be reorganized into the following compact form,

$$\rho(Q) = \sum_{n\geq 1} r_{n,0}\alpha(\mu)^{n+p-1} + \sum_{n\geq 1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq 1}(-1)^j \Delta_n^{(j-1)} r_{n+j,j}$$

$$r_{i,j} = \sum_{k=0}^{j} C_j^k \hat{r}_{i-k,j-k} L^k$$ where, $\hat{r}_{i,j} = r_{i,j}|_{\mu=Q}$ $\quad L = \ln \mu^2/Q^2$

$$\rho(Q) = \sum_{n\geq 1} \hat{r}_{n,0}\alpha^{p+n-1}(\mu) + \sum_{n\geq 1}\left[(p+n-1)\alpha^{p+n-2}\beta\right]\sum_{j\geq 1}(-1)^j \Delta_n^{(j-1)} \hat{r}_{n+j,j}$$

$$+\sum_{k\geq 1}\ln^k\frac{\mu^2}{Q^2}\sum_{n\geq 1}\left[(p+n-1)\alpha^{p+n-2}\beta\right]\sum_{j\geq k}(-1)^j C_j^k \Delta_n^{(j-1)} \hat{r}_{n+j-k,j-k},$$

$\mu^2 \to \mu_\delta^2 = \mu^2 e^\delta$

$\rho_\delta(Q)$

fixes $\mu = Q_\star$ ***all nonconformal terms vanish !***

$$\rho(Q) = \sum_{n\geq 1} \hat{r}_{n,0}\alpha^{p+n-1}(Q_\star)$$

The PMC-s fixes the renormalization scale by directly requiring all the RG-dependent nonconformal terms up to a given order to vanish.

The Principle of Maximum Conformality (PMC)

PMC single-scale approach (PMC-s)

Equation

$$\sum_{n\geq1}\left[(p+n-1)\alpha^{p+n-2}(Q^*)\beta\left(\alpha(Q^*)\right)\right]\sum_{j\geq1}(-1)^{j-1}\Delta_n^{(j-1)}\left(\alpha(Q^*)\right)\hat{r}_{n+j,j}$$
$$=\sum_{k\geq1}\ln^k\frac{Q_*^2}{Q^2}\sum_{n\geq1}\left[(p+n-1)\alpha^{p+n-2}(Q^*)\beta\left(\alpha(Q^*)\right)\right]\sum_{j\geq k}(-1)^j C_j^k\Delta_n^{(j-1)}\left(\alpha(Q^*)\right)\hat{r}_{n+j-k,j-k},$$

Solution

$$\ln\frac{Q_*^2}{Q^2}=S_0+S_1\alpha(Q_*)+S_2\alpha^2(Q_*)+\cdots$$

$$S_0=-\frac{\hat{r}_{2,1}}{\hat{r}_{1,0}},\quad S_1=\frac{(p+1)(\hat{r}_{2,0}\hat{r}_{2,1}-\hat{r}_{1,0}\hat{r}_{3,1})}{p\hat{r}_{1,0}^2}+\frac{(p+1)(\hat{r}_{2,1}^2-\hat{r}_{1,0}\hat{r}_{3,2})}{2\hat{r}_{1,0}^2}\beta_0,$$

$$S_2=\frac{(p+1)^2\left(\hat{r}_{1,0}\hat{r}_{2,0}\hat{r}_{3,1}-\hat{r}_{2,0}^2\hat{r}_{2,1}\right)+p(p+2)\left(\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,0}-\hat{r}_{1,0}^2\hat{r}_{4,1}\right)}{p^2\tilde{r}_{1,0}^3}$$
$$+\frac{(p+1)^2(\hat{r}_{1,0}\hat{r}_{2,0}\hat{r}_{3,2}-\hat{r}_{2,0}\hat{r}_{2,1}^2)+(p+1)(p+2)\left(2\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,1}-\hat{r}_{2,0}\hat{r}_{2,1}^2-\hat{r}_{1,0}^2\hat{r}_{4,2}\right)}{2p\tilde{r}_{1,0}^3}\beta_0$$
$$+\frac{(p+1)(p+2)\left(3\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,2}-2\hat{r}_{2,1}^3-\hat{r}_{1,0}^2\hat{r}_{4,3}\right)}{6\hat{r}_{1,0}^3}\beta_0^2+\frac{(p+2)\left(\hat{r}_{2,1}^2-\hat{r}_{1,0}\hat{r}_{3,2}\right)}{2\hat{r}_{1,0}^2}\beta_1,$$

- the effective scale Q_* is explicitly independent of the choice of initial choice of the renormalization scale μ at any fixed order. It thus has universal properties.
- it also converges rapidly as shall be shown below, thus any residual scale dependence due to uncalculated higher-order terms is greatly suppressed.

The Principle of Maximum Conformality (PMC)

Front. Phys. 11(1), 111201 (2016)
DOI 10.1007/s11467-015-0518-5
REVIEW ARTICLE

Progress in Particle and Nuclear Physics 72 (2013) 44–

Contents lists available at SciVerse ScienceDi

Progress in Particle and Nuclear

journal homepage: www.elsevier.com/locate/p

ELSEVIER

Review

The renormalization scale-setting problem in QC

Xing-Gang Wu [a,*], Stanley J. Brodsky [b], Matin Mojaza [b,c]

[a] Department of Physics, Chongqing University, Chongqing 401331, PR China
[b] SLAC National Accelerator Laboratory, Stanford University, CA 94039, USA
[c] CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230, Denmark

A R T I C L E   I N F O

Keywords:

A B S T R A C T

IOP Publishing   Reports on Progress in Physics

Rep. Prog. Phys. 78 (2015) 126201 (15pp)   doi:10.1088/0034-4885/78/12/126201

Review

Renormalization group invariance and optimal QCD renormalization scale-setting: a key issues review

Xing-Gang Wu[1], Yang Ma[1], Sheng-Quan Wang[1], Hai-Bing Fu[1], Hong-Hao Ma[1], Stanley J Brodsky[2] and Matin Mojaza[3]

[1] Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China
[2] SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA
[3] CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230,

- S.J. Brodsky and X.G. Wu, Phys.Rev.Lett.109,042002(2012).
- M. Mojaza, S.J. Brodsky and X.G. Wu, Phys.Rev.Lett.110,192001(2013).
- S.J. Brodsky and X.G. Wu, Phys.Rev.D85,034038(2012).
- S. J. Brodsky and L. Di Giustino, Phys. Rev. D 86, 085026 (2012).
- S.J. Brodsky, M. Mojaza and X.G. Wu, Phys.Rev.D89, 014027 (2014).

The Principle of Maximum Conformality (PMC)

Application of PMC : R-ratio

$$R_{e^+e^-}(Q) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2[1 + R(Q)],$$

$$\ln\frac{Q_\star^2}{Q^2} = 0.2249 + 1.6369\alpha_s(Q) + 1.5559\alpha_s^2(Q)$$

The difference between the two nearby values becomes smaller and smaller when more loop terms are included.

Eliminate scale dependence

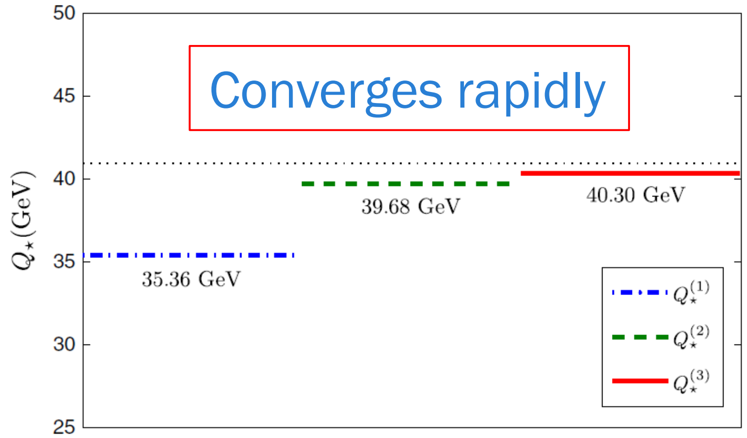


FIG. 1. The determined PMC scale $Q_\star$ for $R(Q)$ up to N^{2}LL accuracy. $Q_\star^{(1)}$ is at the LL accuracy, $Q_\star^{(2)}$ is at the NLL accuracy, and $Q_\star^{(3)}$ is at the N^{2}LL accuracy. $Q = 31.6$ GeV.

TABLE II. The value of each loop term (LO, NLO, N^{2}LO, or N^{3}LO) for the four-loop prediction $R_3(Q)$ under three scale-setting approaches. $Q = 31.6$ GeV and $\mu \in [Q/2, 2Q]$. The central values for the conventional scale-setting (Conv.) are for $\mu = Q$. We observe that each loop term for the PMC or PMC-s is almost unchanged for $\mu \in [Q/2, 2Q]$.

	LO	NLO	N^{2}LO	N^{3}LO	Total
Conv.	$0.04482^{+0.00652}_{-0.00501}$	$0.00283^{-0.00612}_{+0.00361}$	$-0.00115^{-0.00109}_{+0.00147}$	$-0.00033^{+0.00061}_{+0.00008}$	$0.04617^{-0.00008}_{+0.00015}$
PMC	0.04275	0.00350	−0.00004	−0.00002	0.04619
PMC-s	0.04292	0.00339	−0.00008	−0.00004	0.04619

The predictions are close to each other for both the total and the separate loop terms

Summary

$$\frac{\partial \rho_\delta}{\partial \delta} = -\frac{\partial \alpha(\mu_\delta)}{\partial \delta}\frac{\partial \rho_\delta}{\partial \alpha(\mu_\delta)} = -\mu_\delta^2\frac{\partial \alpha(\mu_\delta)}{\partial \mu_\delta^2}\frac{\partial \rho_\delta}{\partial \alpha(\mu_\delta)} = -\beta(\alpha(\mu_\delta))\frac{\partial \rho_\delta}{\partial \alpha(\mu_\delta)}$$

- *The scheme-invariance of the full pQCD series is realized by cancellation between the pQCD calculable coefficients and the QCD couplings. (RGI)*
- *Both the scheme dependence and the scale-evolution of the R_δ coupling are governed by the same RGE.*
- *The PMC uses R_δ to expose the renormalization scheme dependence of the pQCD calculable coefficients and the QCD coupling, and uses RGE fix the {β_i} terms of each order.*
- *The PMC eliminates the scheme/scale dependence of the coefficients and couplings simultaneously by shifting and setting the optimal renormalization scales, i.e., absorbing all {β_i} terms into the running coupling at each order.*

Summary

The PMC provides a rigorous method for eliminating renormalization scheme-and-scale ambiguities for pQCD predictions.

- ***Determine renormalization scales without ambiguity***
- ***Eliminate scheme-and-scale dependence***
- ***Test QCD to maximum precision at colliders***
- ***Maximize sensitivity to new physics beyond SM***
- ***Obtain high precision determination of fundamental parameters***

Summary

Regularization

Redefining integrals in a way to control the divergences

Renormalization

Redefining parameters to remove the well-defined divergences

Renormalization scale setting

Removing the arbitrariness of the previous two steps, and connecting the theory to experiments

The PMC eliminates renormalization scheme-and-scale ambiguities for pQCD predictions.

Thus, in a sense, the PMC optimizes the QCD renormalization.

Thank you !