重慶大學
CHONGQING UNIVERSITY

QCD Bound-State Problem
via
Dyson-Schwinger Equation

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(秦思学)

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(ANL, Frankfurt U, PKU)

1 Background: *What* are bound-states?

$$E_{\text{sys}} < \sum E_i$$

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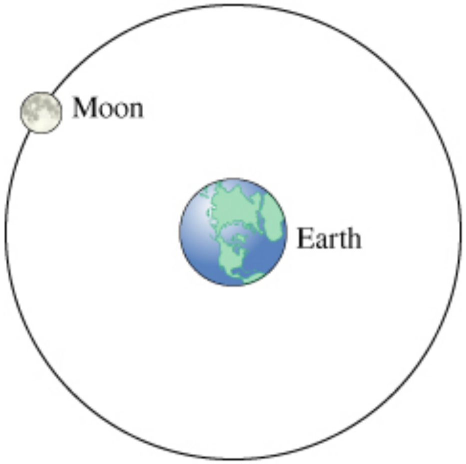

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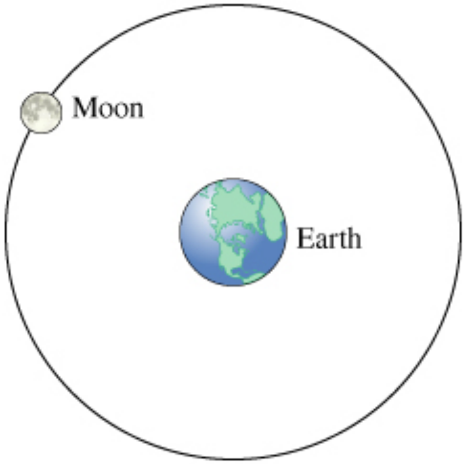

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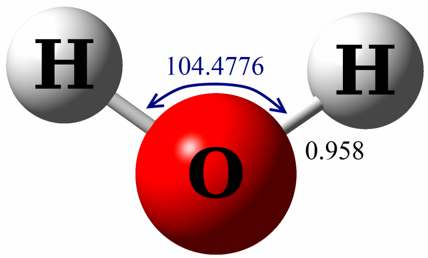

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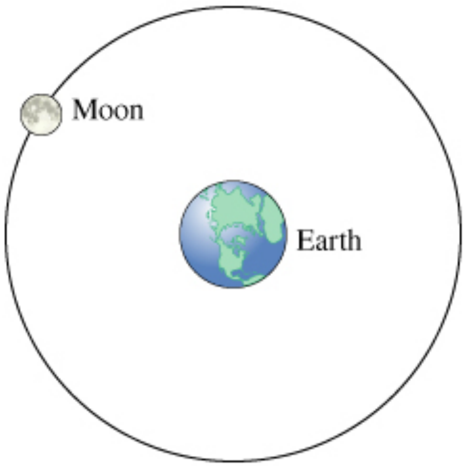

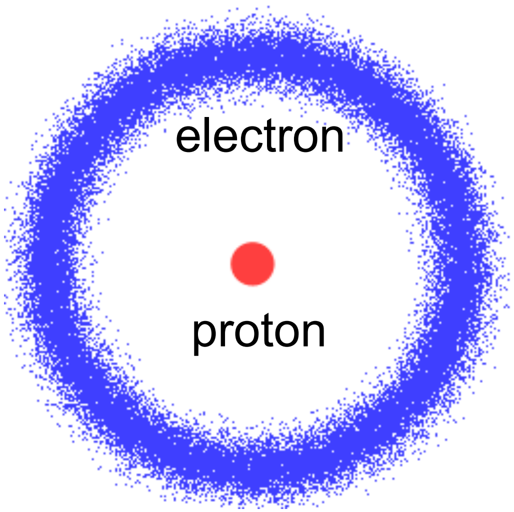

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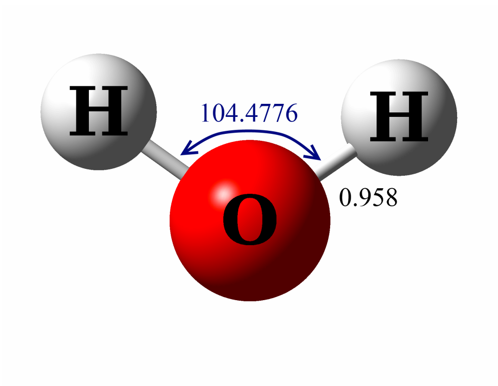

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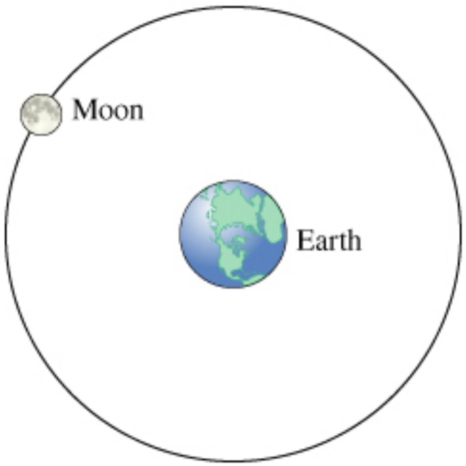

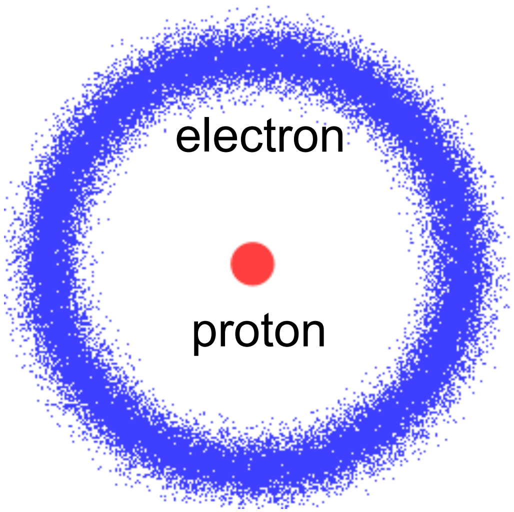

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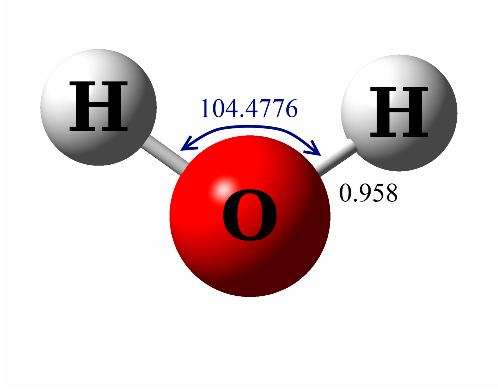

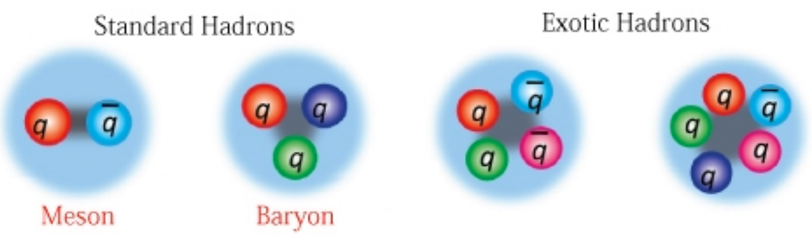

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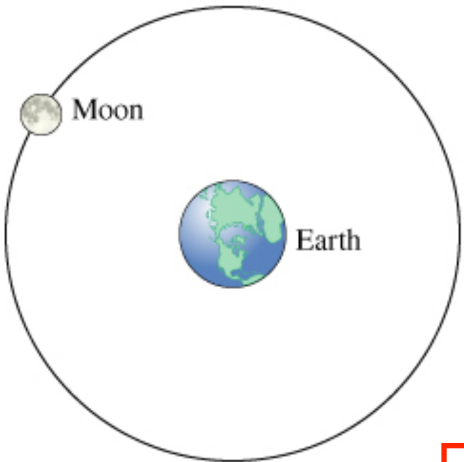
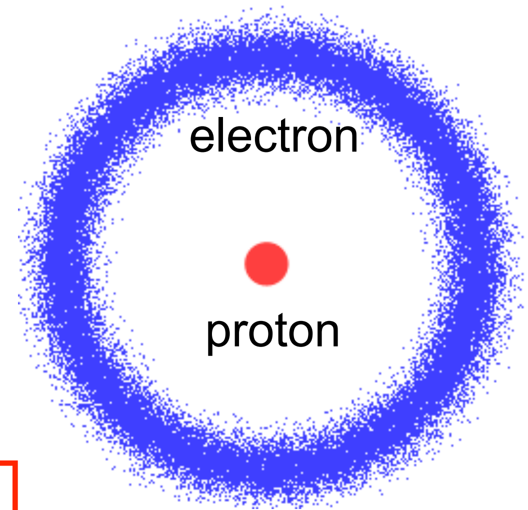
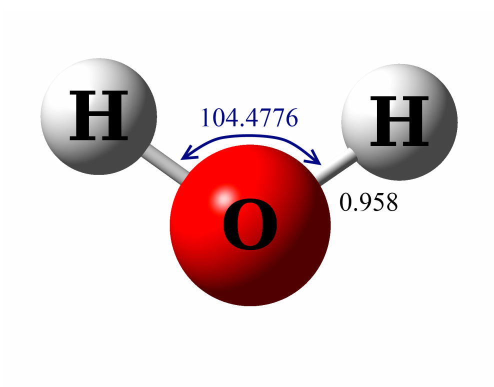
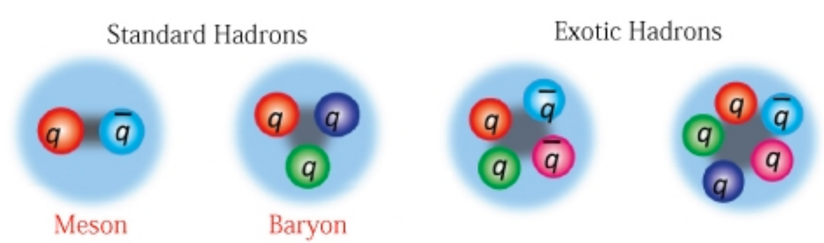




What matter is possible
&
How is it constituted?

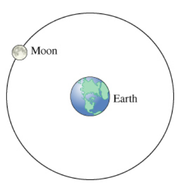


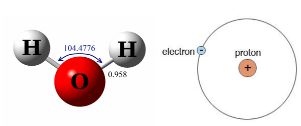


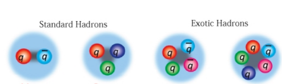
1 Background: *Why* do we study bound-states?

Experiment

"Easy" objects involving interactions







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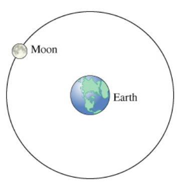

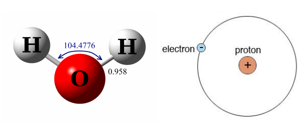

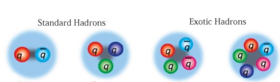

+ etc.

Theory

"Simple" objects involving dynamics

- Newtonian Mechanics
- Quantum Mechanics
- Quantum Field Theory

+ etc.

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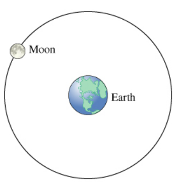

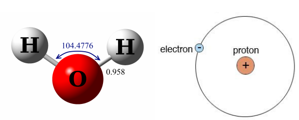

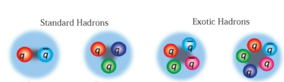

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Newtonian Mechanics

Quantum Mechanics

Quantum Field Theory

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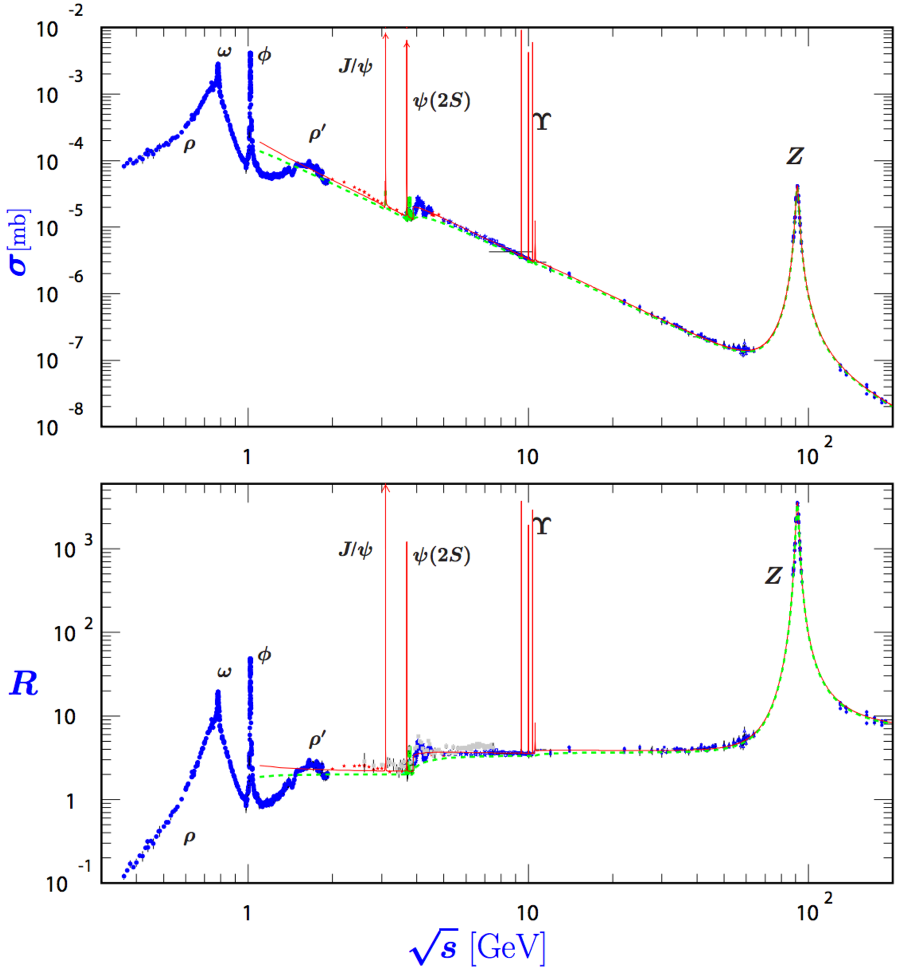

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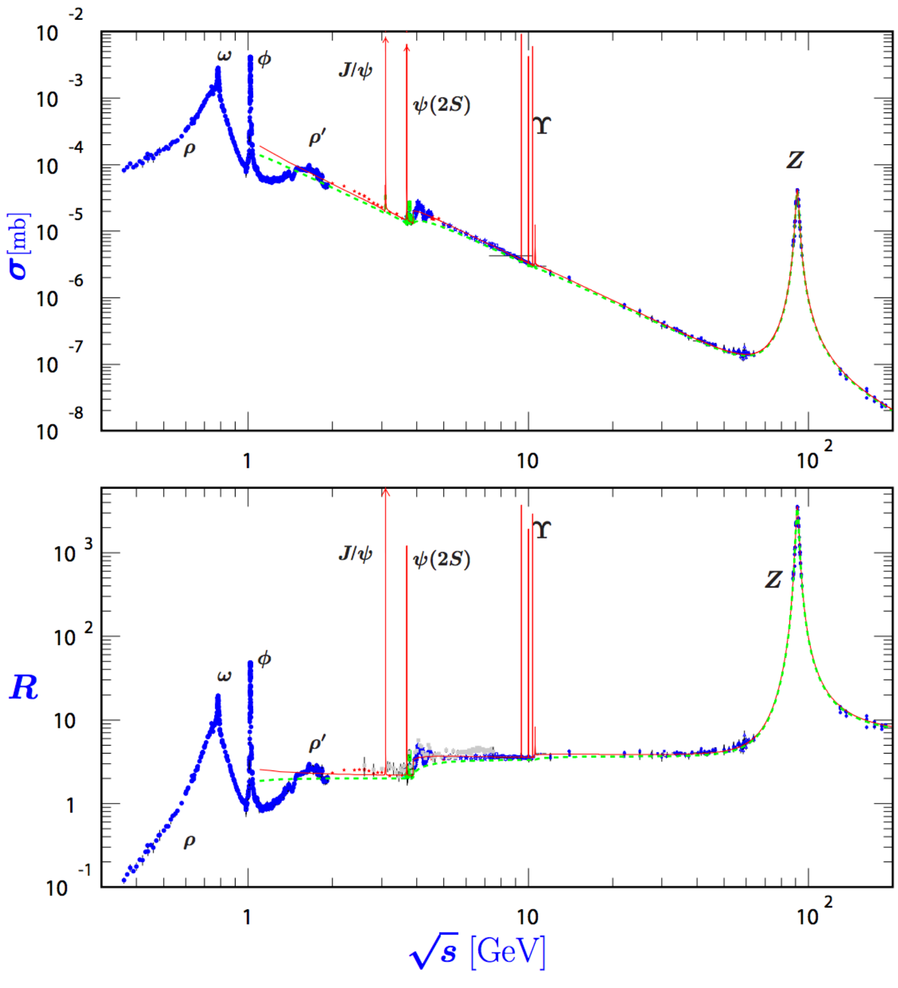

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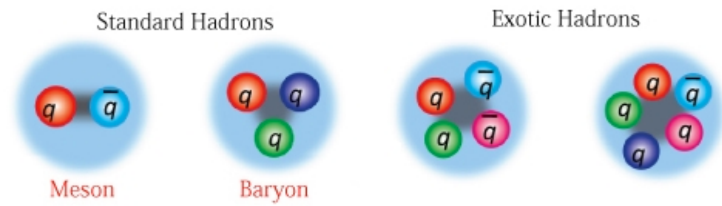
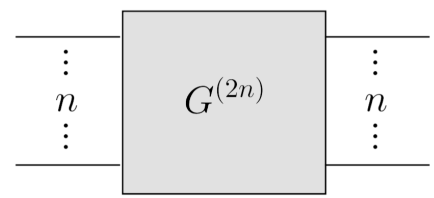
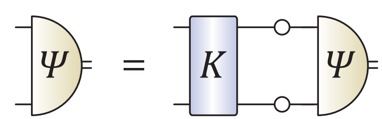


Quantum Field Theory

- Green functions



- Bethe-Salpeter equation



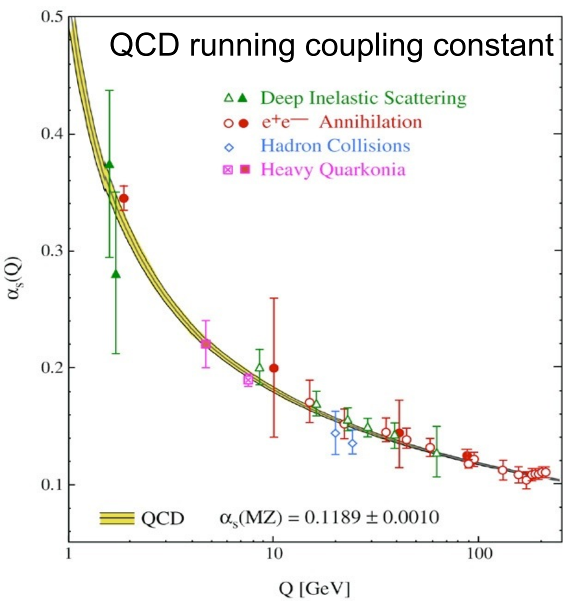
1 Background: *Why* is QCD bound-state problem difficult?

• Relativistic bound states

"These problems are those involving bound states […] such problems necessarily involve a breakdown of ordinary perturbation theory. […] The pole therefore can only arise from a divergence of the sum of all diagrams […]"

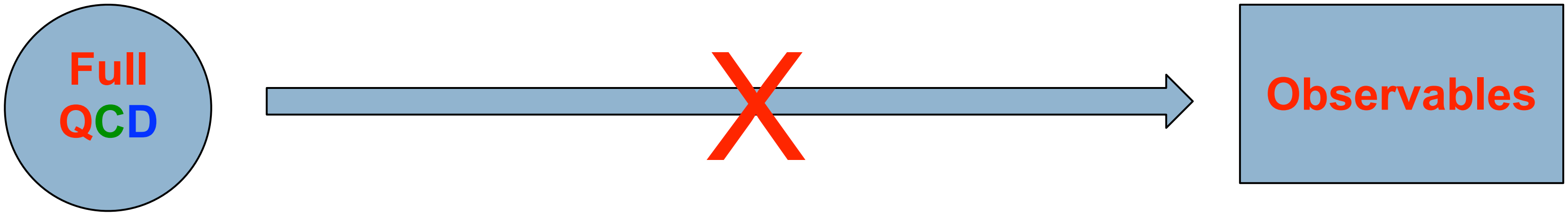
The QFT book vol1 p564 Weinberg

• Strongly coupled systems



- **Asymptotic freedom**: Bonds between particles become asymptotically weaker as energy increases and distance decreases (Nobel Prize).
- **Quark and Gluon Confinement**: No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
- **Dynamical Chiral Symmetry Breaking**: Mystery of bound state masses, e.g., current quark mass (Higgs) is small, and no degeneracy between *parity partners.*

1 Background: Non-perturbative approaches of QCD



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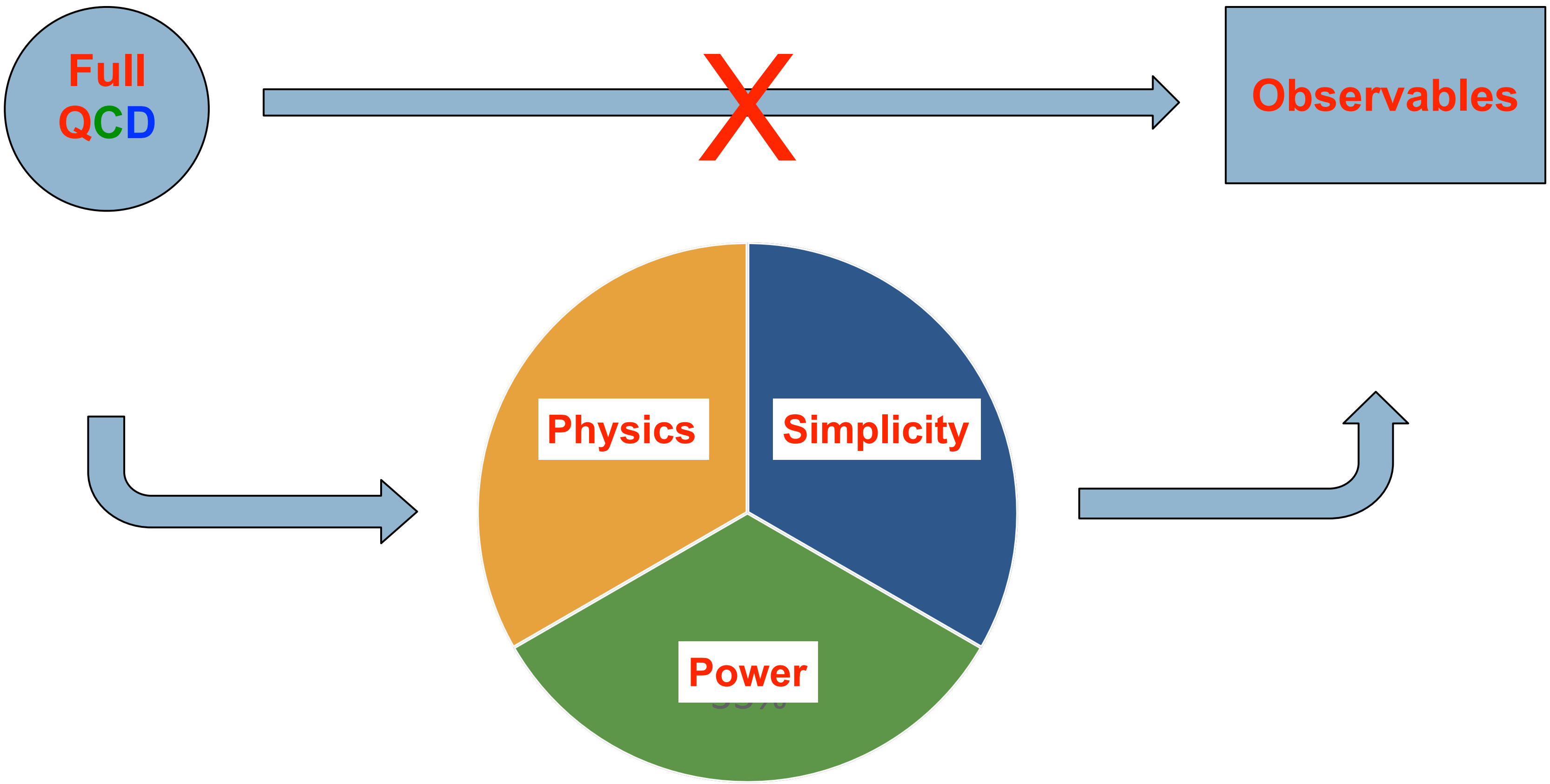

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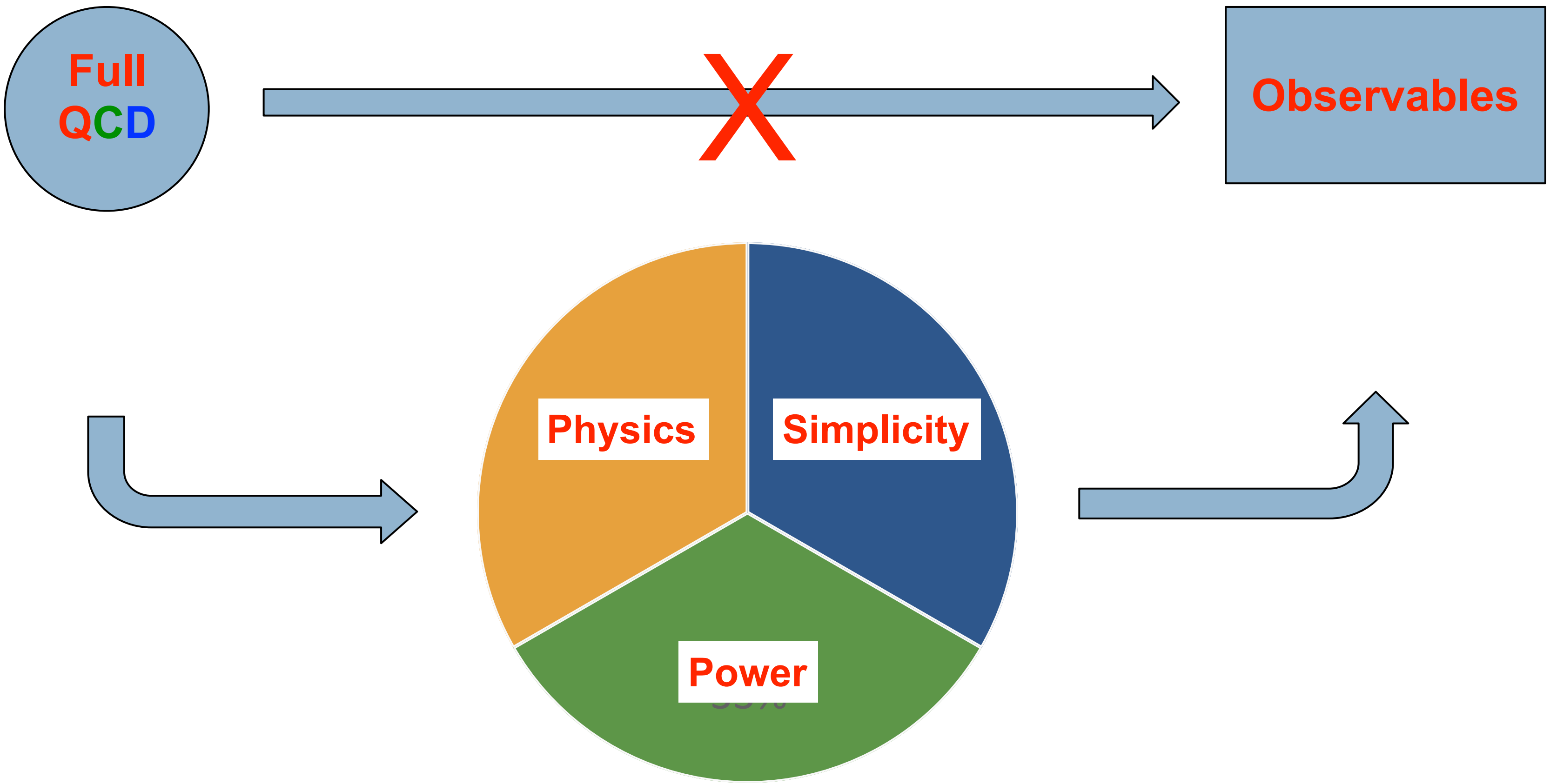

Lattice QCD, Dyson-Schwinger equations, chiral perturbation, AdS/QCD, NJL model, …

2 DSE: EoM of QCD's Green functions

Classical Mechanics | Quantum Field Theory

Degrees of freedom

Generalized coord. | Fields on spacetime

Principle of Least Action

$$\frac{\delta S[q]}{\delta q} = 0$$

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Equations of Motion (EoM)

Euler-Lagrange Equation | Dyson-Schwinger Equations

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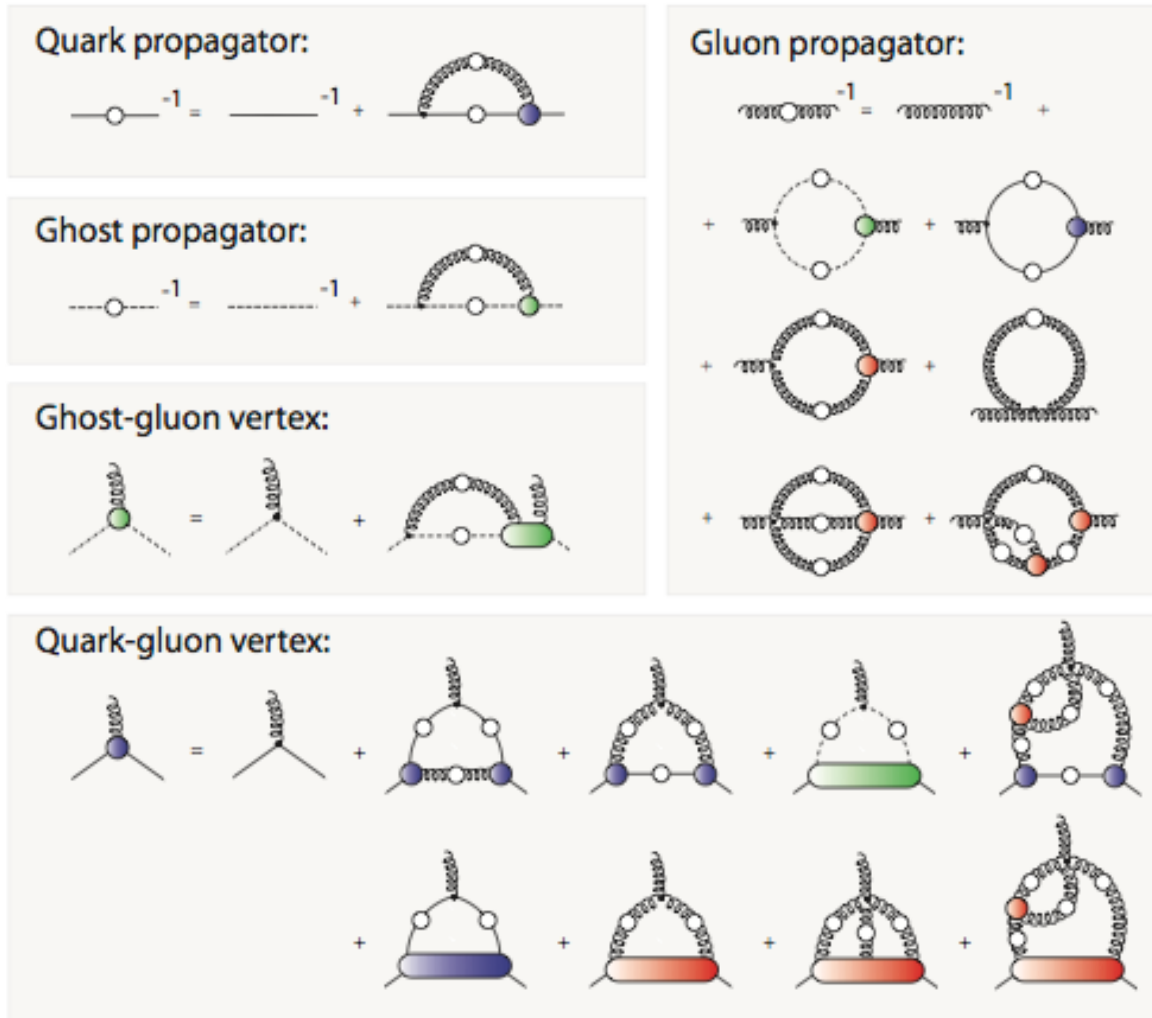

G. Eichmann, arXiv:0909.0703

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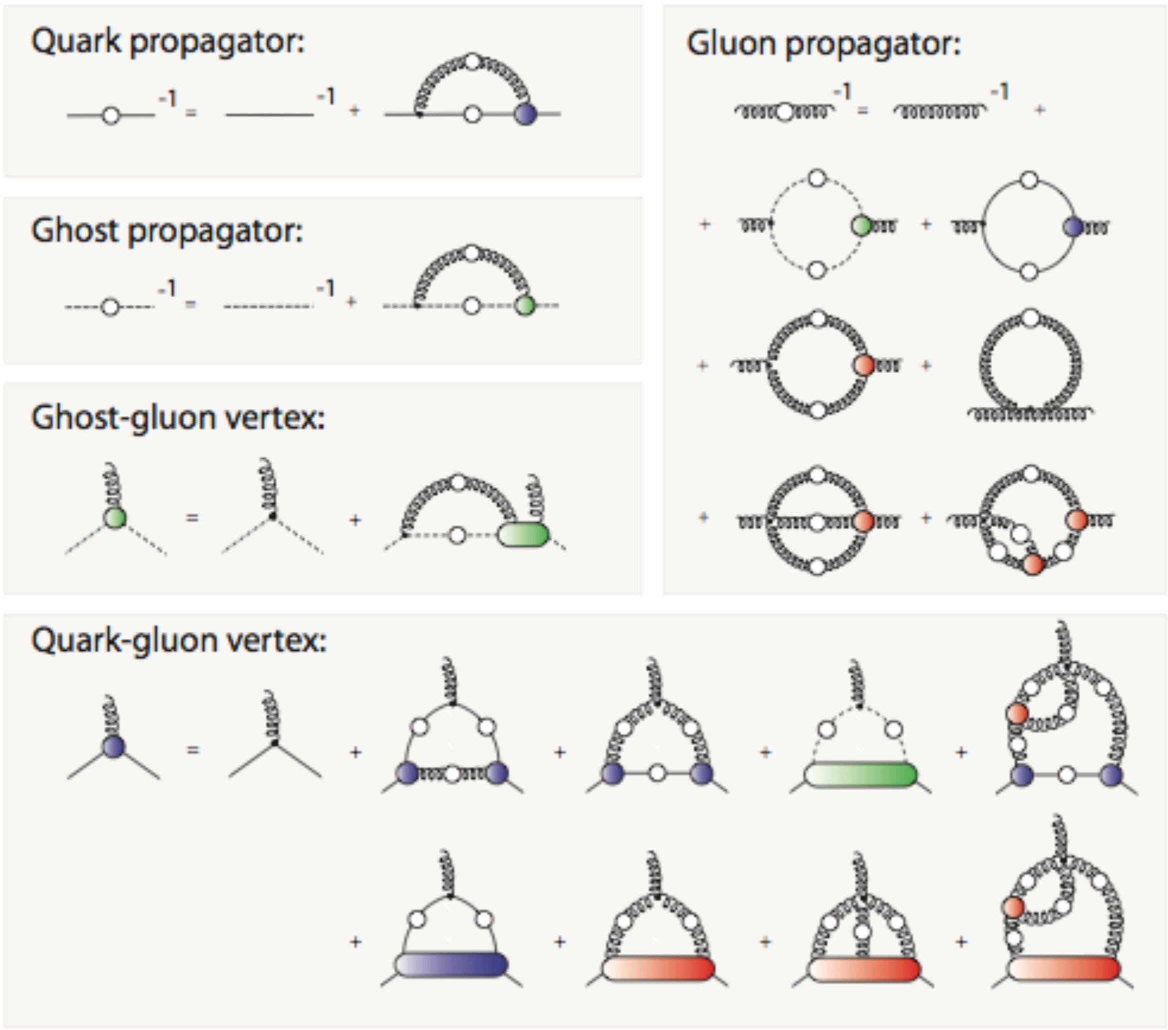
- ✦ Most equations are very complicated.

- ✦ Green functions of different orders couple together.



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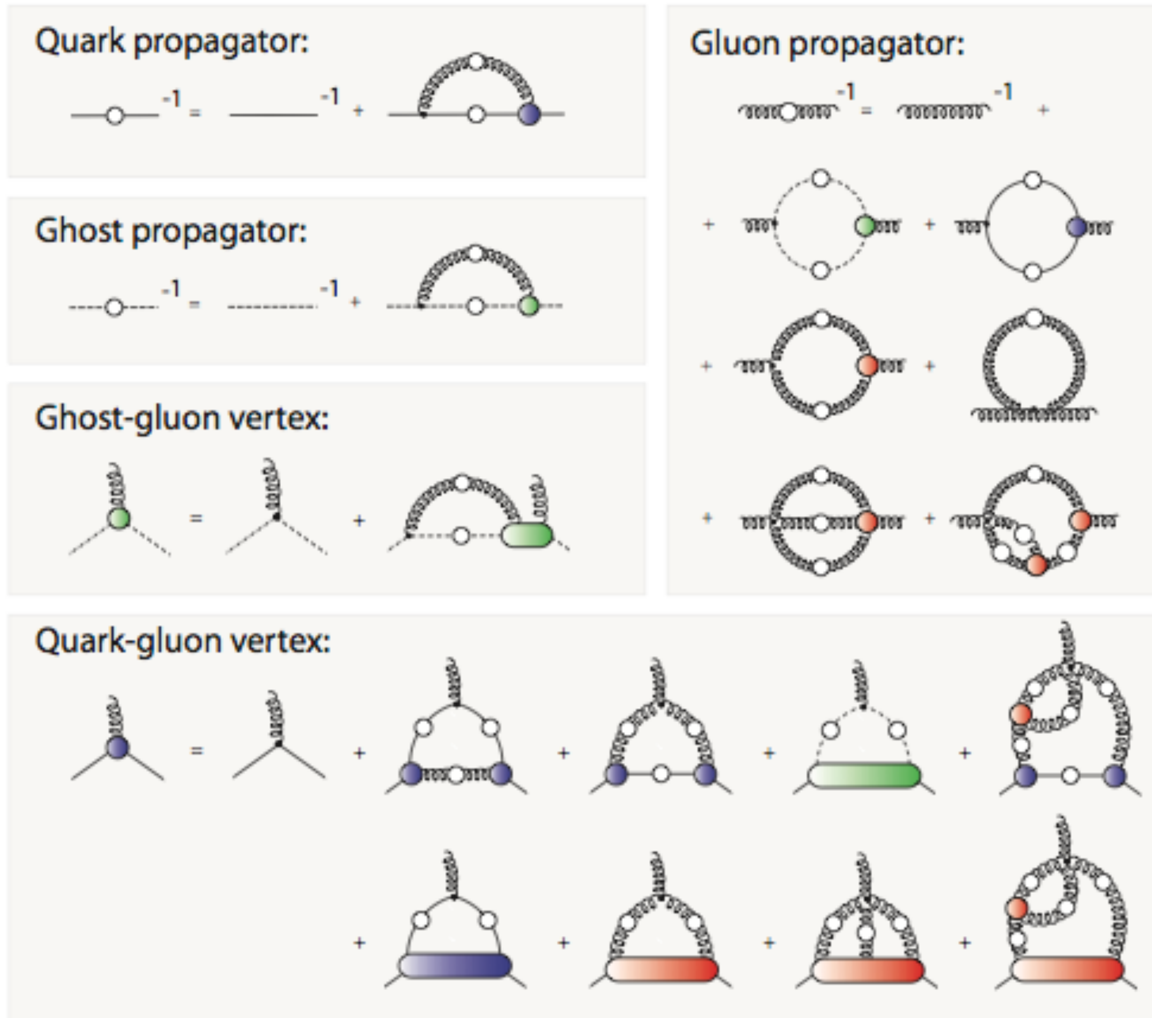
- ❑ *Modeling*



- ✦ Green functions of different orders couple together.

- ❑ **Truncation**

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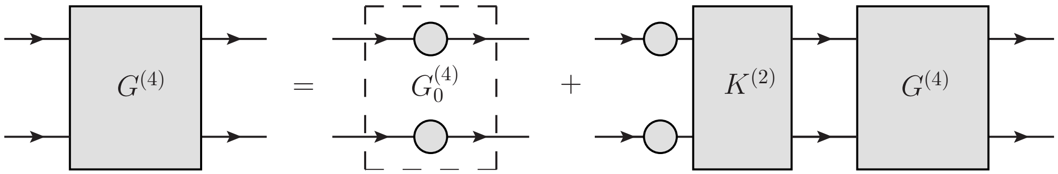
2 DSE: Bound-states in terms of Green functions

- In QFT, bound-states are encoded in Green functions.



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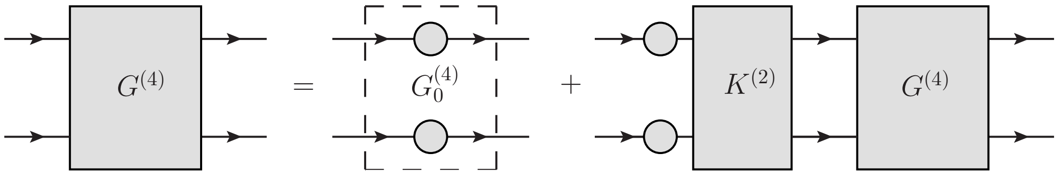

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- The **kernel** can be decomposed by its orthogonal eigenbasis, which are classified by J^P quantum number and radial quantum number n_r,

$$K^{(2)} = \sum_i \lambda_i^{-1} \, |\Gamma_i\rangle\langle\Gamma_i| \qquad |\Gamma_i\rangle = \lambda_i \, K^{(2)} \cdot G_0^{(4)} \cdot |\Gamma_i\rangle \qquad \langle\Gamma_i|G_0^{(4)}|\Gamma_j\rangle = \delta_{ij}$$

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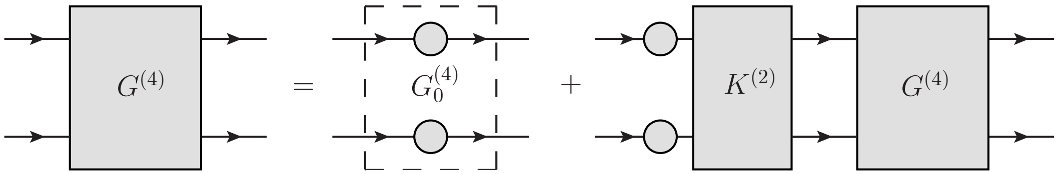

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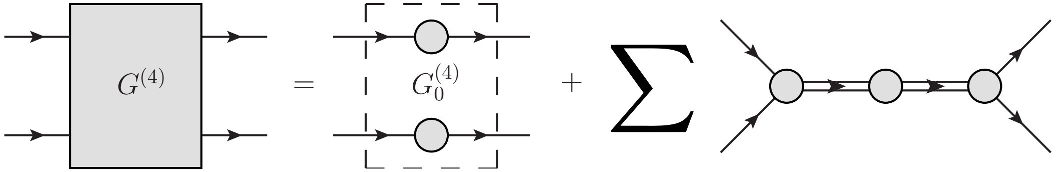
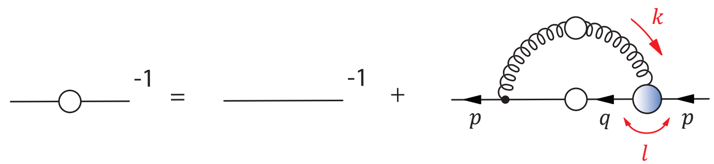
- Accordingly, the four-point Green function can be decomposed:



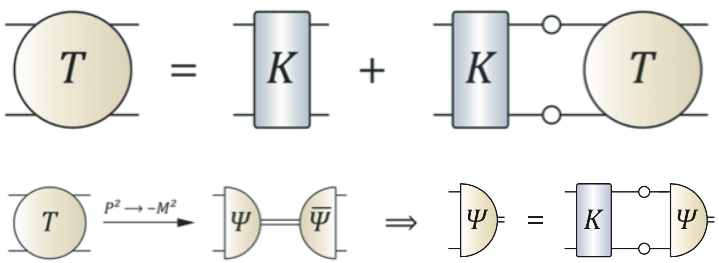
$$\boxed{G^{(4)} = G_0^{(4)} + \sum_i |\chi_i\rangle \frac{1}{\lambda_i(P^2) - 1} \langle\chi_i|}$$

2 DSE: Most frequently used equations

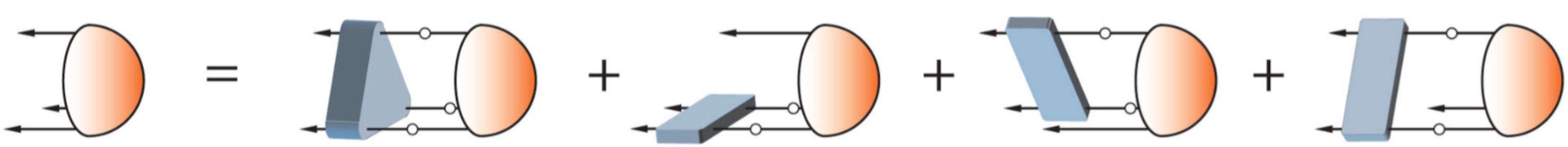
- **One-body gap equation**



- **Two-body Bethe-Salpeter equation**



- **Three-body Faddeev equation**



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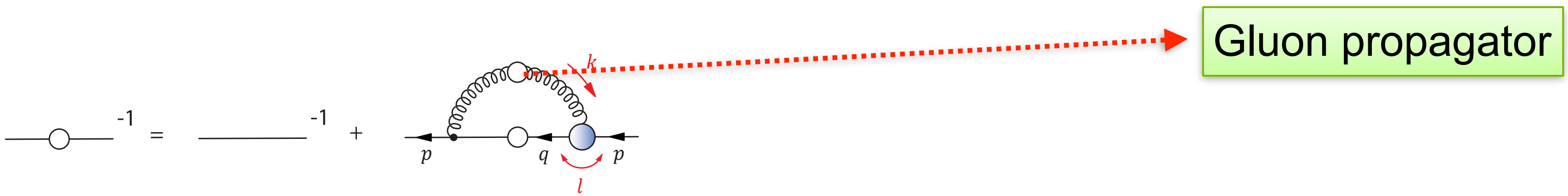

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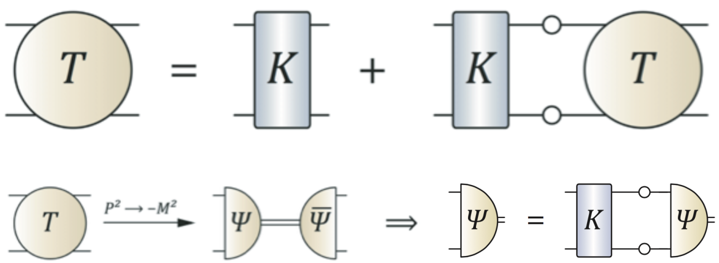

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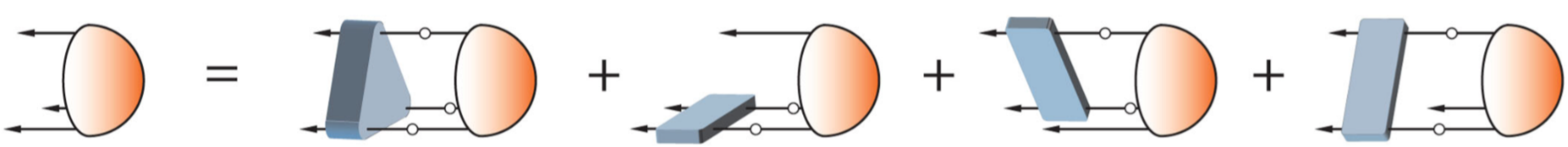

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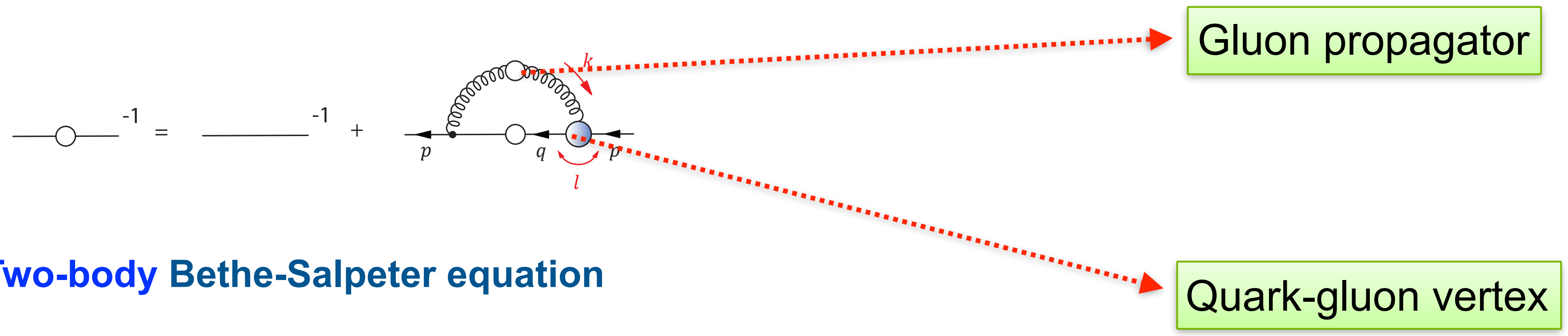

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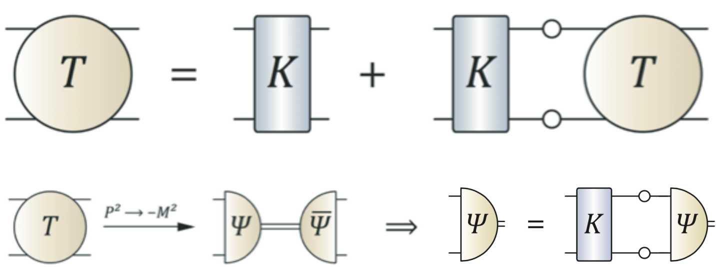

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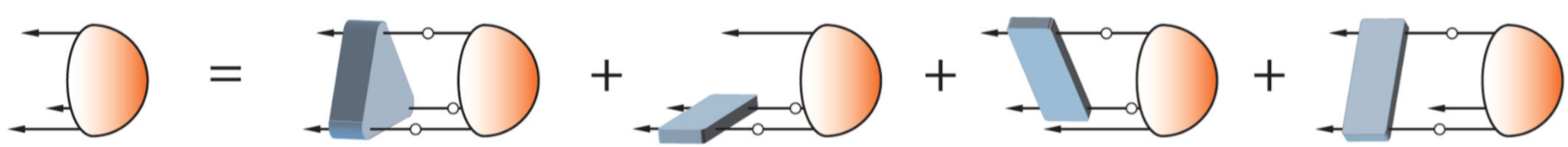

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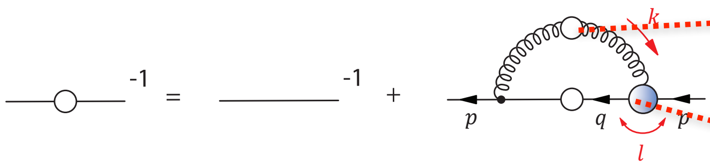

Gluon propagator

Quark-gluon vertex

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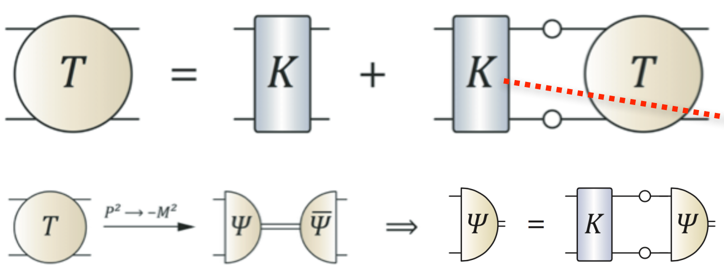
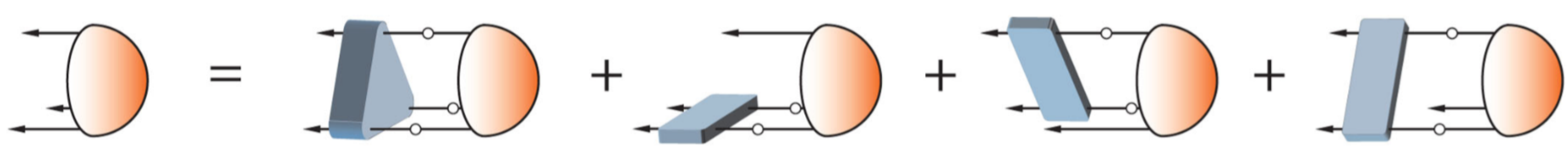


Scattering Kernel

- **Three-body Faddeev equation**



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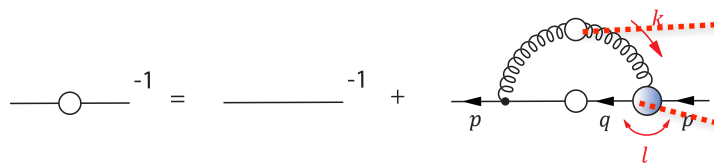

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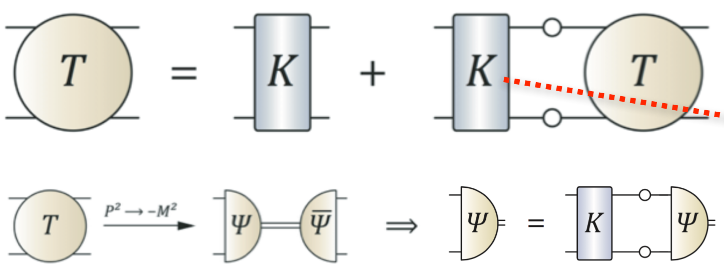

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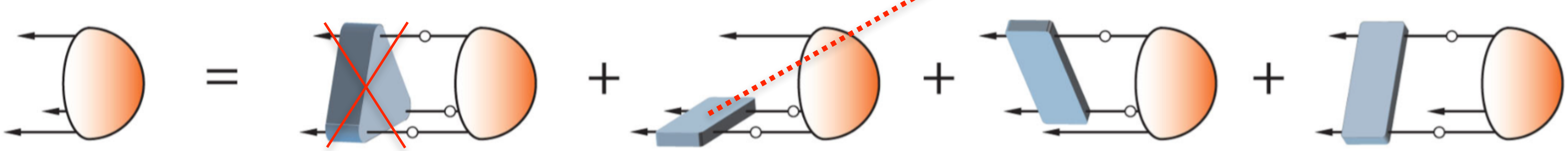


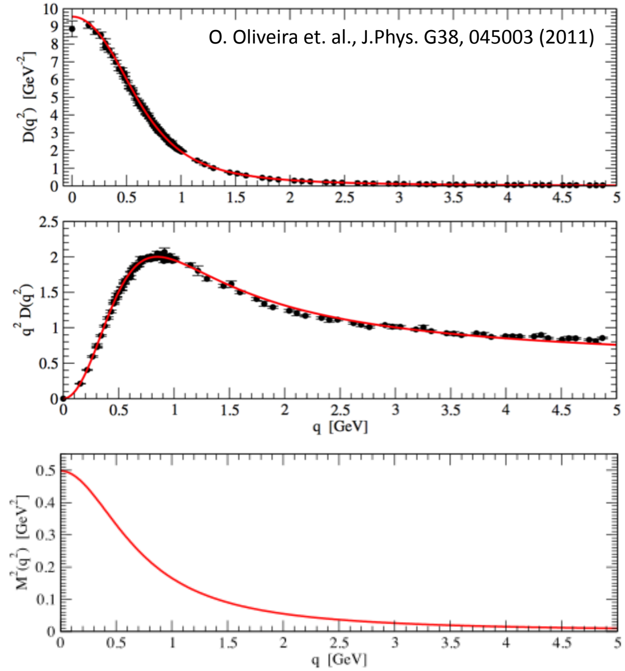
2.1 DSE: Dynamically massive gluon

- In Landau gauge (Lorentz covariant and LQCD favored):

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

- Modeling the dress function: ***gluon mass scale*** + ***effective running coupling constant***

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}, \qquad m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$



The gluon propagator is modeled as two parts: Infrared + Ultraviolet. The former is an expansion of delta function; The latter is a form of one-loop perturbative calculation.

$$\delta^4(k) \overset{\omega\sim 0}{\approx} \frac{1}{\pi^2}\frac{1}{\omega^4}e^{-k^2/\omega^2} \qquad \mathcal{G}(s) = \frac{8\pi^2}{\omega^4}De^{-s/\omega^2} + \frac{8\pi^2\gamma_m\mathcal{F}(s)}{\ln\left[\tau + (1+s/\Lambda_{\rm QCD}^2)^2\right]}$$

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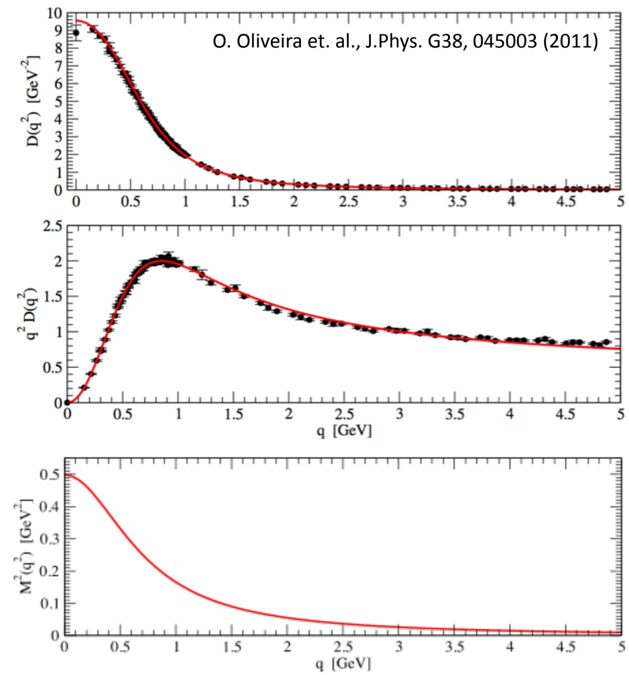

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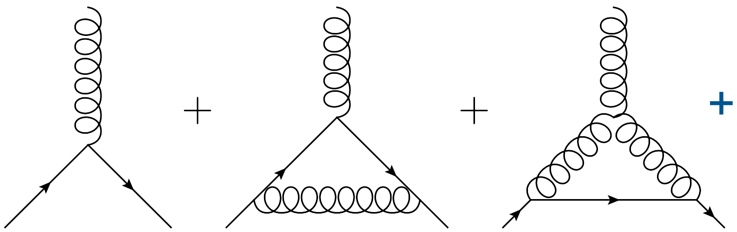
$$\delta^4(k) \overset{\omega \sim 0}{\approx} \frac{1}{\pi^2}\frac{1}{\omega^4} e^{-k^2/\omega^2} \qquad \mathcal{G}(s) = \frac{8\pi^2}{\omega^4} D e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m \mathcal{F}(s)}{\ln\left[\tau + (1 + s/\Lambda_{\rm QCD}^2)^2\right]}$$

- The gluon mass scale is *typical values of lattice QCD*: *Mg* in [0.6, 0.8] GeV.
- The gluon mass scale is inversely proportional to the confinement length.

2.2 DSE: DCSB in quark-gluon vertex (Abelian)



+ etc.

$$[\Gamma_\mu(p,q)]_{\alpha\beta} = \{\gamma_\mu, p_\mu, q_\mu\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$$

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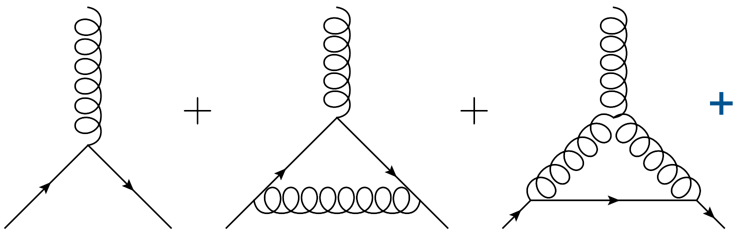

$$[\Gamma_\mu(p,q)]_{\alpha\beta} = \{\gamma_\mu, p_\mu, q_\mu\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$$

- **Gauge symmetry:** vector WGTI

$$iq_\mu\Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)$$

- **Chiral symmetry:** axial-vector WGTI

$$q_\mu\Gamma^A_\mu(k,q) = S^{-1}(k)i\gamma_5 + i\gamma_5 S^{-1}(p) - 2im\Gamma_5(k,p)$$

- **Lorentz symmetry + :** transverse WGTIs

$$\begin{aligned} q_\mu\Gamma_\nu(k,p) - q_\nu\Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu}S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda\epsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p)\,, \end{aligned}$$

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He, PRD, 80, 016004 (2009)

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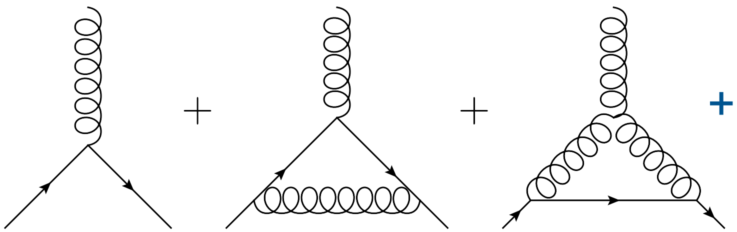

$$[\Gamma_\mu(p,q)]_{\alpha\beta} = \{\gamma_\mu, p_\mu, q_\mu\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$$

- The WGTIs express the curls and divergences of the vertices.
- The WGTIs of the vertices in different channels couple together.
- The WGTIs involve contributions from high-order Green functions.

❑ **Gauge symmetry:** vector WGTI

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$\nabla \cdot \Phi$

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2.2 DSE: DCSB in quark-gluon vertex (Abelian)

- Defining proper projection tensors and contract them with the transverse WGTIs, one can decouple the WGTIs and obtain a group of equations for the vector vertex:

$$T^1_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_{\rm D}\,, \qquad T^2_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta\,.$$

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p),$$

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- It is a group of full-determinant linear equations and a unique solution:

$$\Gamma^{\mathrm{Full}}_\mu(k,p) = \Gamma^{\mathrm{BC}}_\mu(k,p) + \Gamma^{\mathrm{T}}_\mu(k,p) + \Gamma^{\mathrm{FP}}_\mu(k,p)$$

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❖ The unknown high-order terms contribute to the transverse part, i.e., the longitudinal part has been completely determined by the quark propagator.

❖ The quark propagator contributes to the longitudinal and transverse parts. The **DCSB** terms are highlighted.

$$\Gamma^{\rm BC}_\mu(k,p) = \gamma_\mu\Sigma_A + t_\mu \not{t}\frac{\Delta_A}{2} - it_\mu\Delta_B$$

$$\Gamma^{\rm T}_\mu(k,p) = -\sigma_{\mu\nu}\Delta_B + \gamma^T_\mu q^2\frac{\Delta_A}{2} - \left(\gamma^T_\mu[\not{q},\not{t}] - 2t^T_\mu\not{q}\right)\frac{\Delta_A}{4}$$

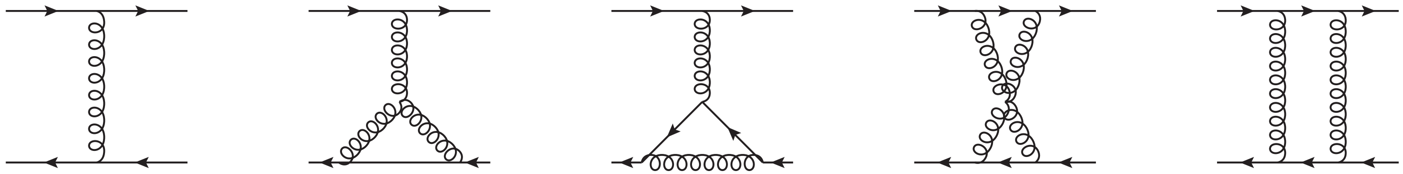
$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

$$\Sigma_\phi(x,y) = \frac{1}{2}[\phi(x) + \phi(y)],$$

$$\Delta_\phi(x,y) = \frac{\phi(x) - \phi(y)}{x - y}.$$

$$X^T_\mu = X_\mu - \frac{q\cdot X q_\mu}{q^2}$$

2.3 DSE: Symmetries of kernel (discrete)



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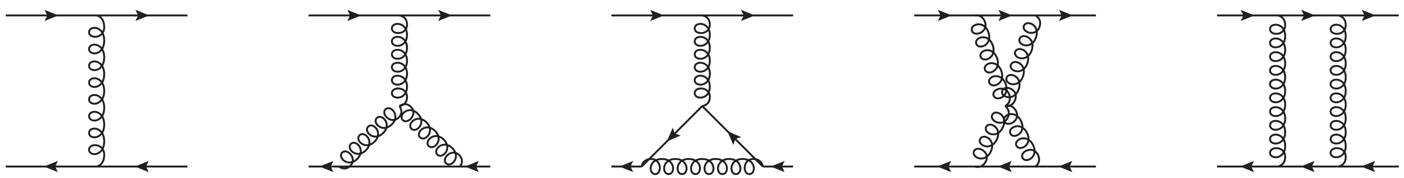

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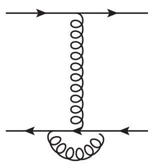
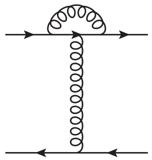
✦ Permutation: $\mathscr{P}\mathcal{K}(q_\pm, k_\pm) = \mathcal{K}^*(q_\pm, k_\pm) = C\, K_R^\mu(-q_\mp, -k_\mp)\, C^{-1} \otimes C\, K_L^\mu(-q_\mp, -k_\mp)\, C^{-1}$

$\frac{1}{2!}$  +  + etc.

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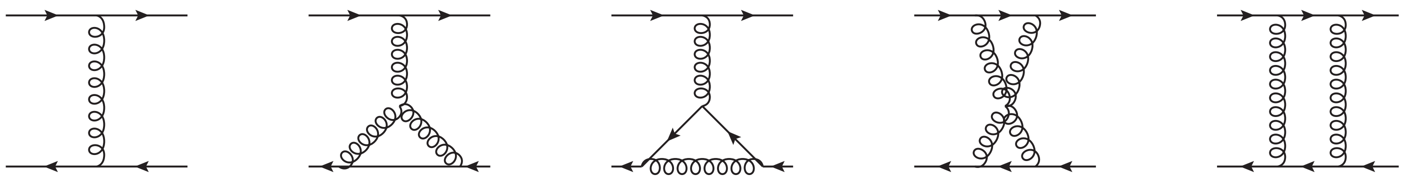

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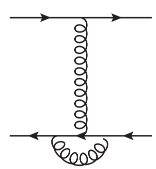
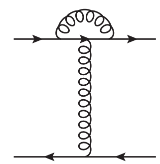
$\frac{1}{2!}$  $+$  $+$ etc.

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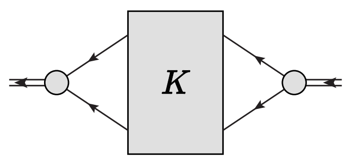
$\langle \chi_i | K | \chi_j \rangle =$ $\bar{\chi}_i$ 

χ_j $= \delta_{ij}$

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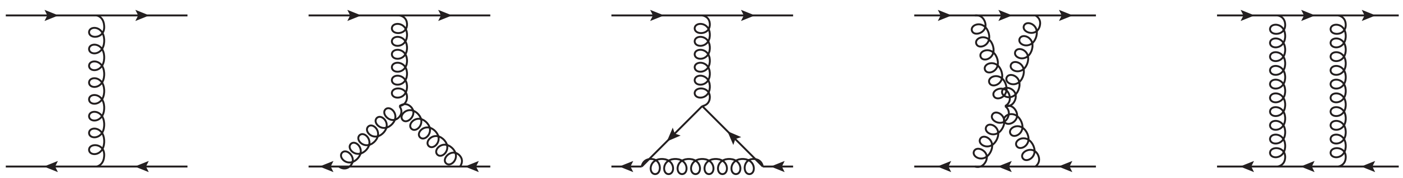

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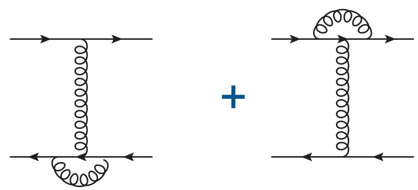
$\frac{1}{2!}$  + etc.

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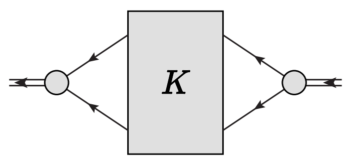
$\langle \chi_i | K | \chi_j \rangle =$ $\bar{\chi}_i$  χ_j $= \delta_{ij}$

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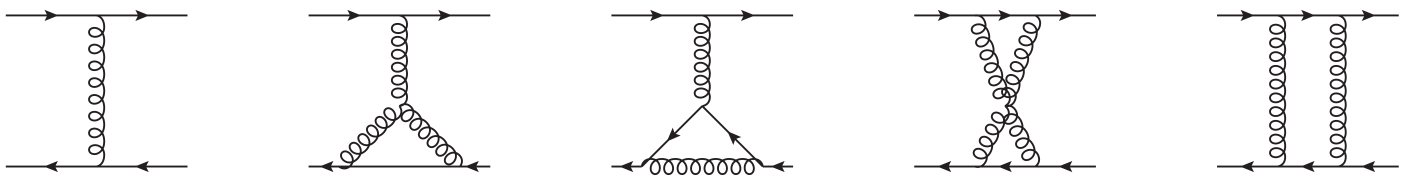

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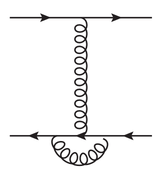
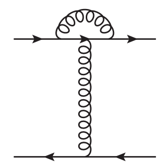
$\frac{1}{2!}$  $+$  $+$ etc.

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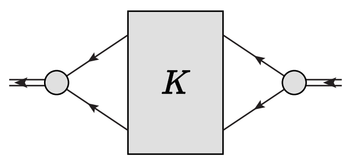
$\langle\chi_i|K|\chi_j\rangle = \quad \bar{\chi}_i$ 

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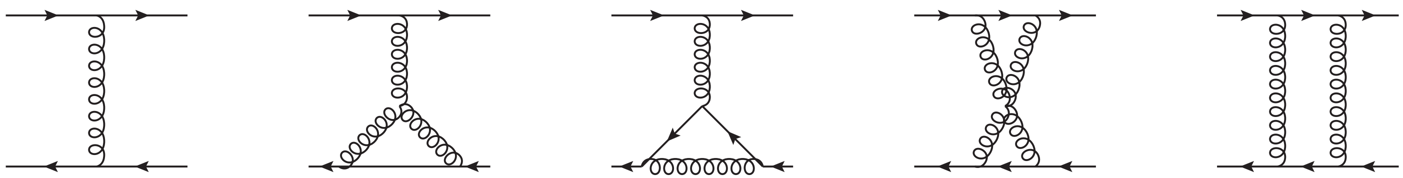

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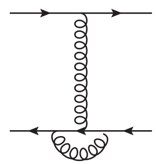
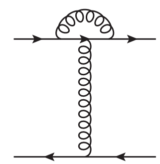
$\frac{1}{2!}$  +  + etc.

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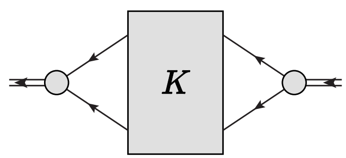
$\langle\chi_i|K|\chi_j\rangle =$ $\bar{\chi}_i$ 

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$$K = \mathbf{1}\otimes\mathbf{1} + \gamma_5\otimes\gamma_5 + \cancel{\mathbf{1}\otimes\gamma_5 + \gamma_5\otimes\mathbf{1}}$$

Lorentz covariance guarantees CPT-symmetry; T-symmetry is obtained for free.

2.3 DSE: Symmetries of kernel (continuous)

✦ In the chiral limit, the color-singlet av-WGTI (**chiral symmetry**) is written as

$$P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

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✦ Assuming there is a radially excited pion, its leptonic decay constant vanishes

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DCSB means **much more** than massless pseudo-scalar meson.

2.3 DSE: Symmetries of kernel (continuous)

The Bethe-Salpeter equation and the quark gap equation are written as

$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'},$$

$$S^{-1}(k) = S_0^{-1}(k) + \int_q D_{\mu\nu}(k-q)\gamma_\mu S(q)\Gamma_\nu(q,k),$$

The color-singlet axial-vector and vector WGTIs are written as

$$P_\mu\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-),$$

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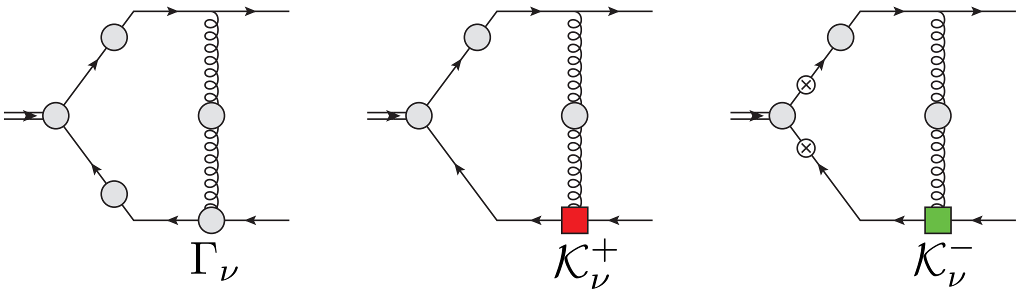
The **kernel** satisfies the following WGTIs: quark propagator + quark-gluon vertex

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)],$$

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)].$$

Assuming the scattering **kernel** has the following structure:



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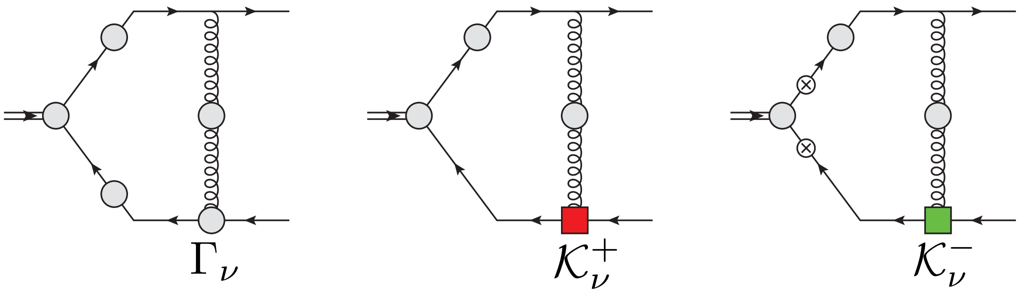

Ladder-like term

Symmetry-rescuing term

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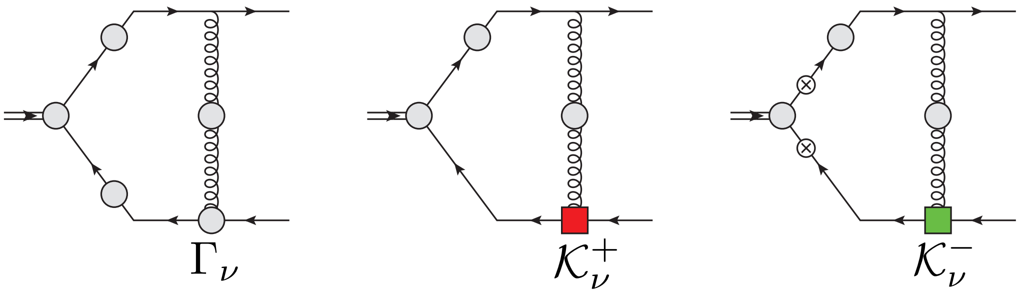

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Inserting the ansatz for the kernel into its WGTIs, we have

$$\int_q D_{\mu\nu}\gamma_\mu S_+(\Gamma_\nu^+ - \Gamma_\nu^-) = \int_q D_{\mu\nu}\gamma_\mu S_+(S_+^{-1} - S_-^{-1})\mathcal{K}_\nu^+ + \int_q D_{\mu\nu}\gamma_\mu S_+\gamma_5(S_+^{-1} - S_-^{-1})\gamma_5\mathcal{K}_\nu^-,$$

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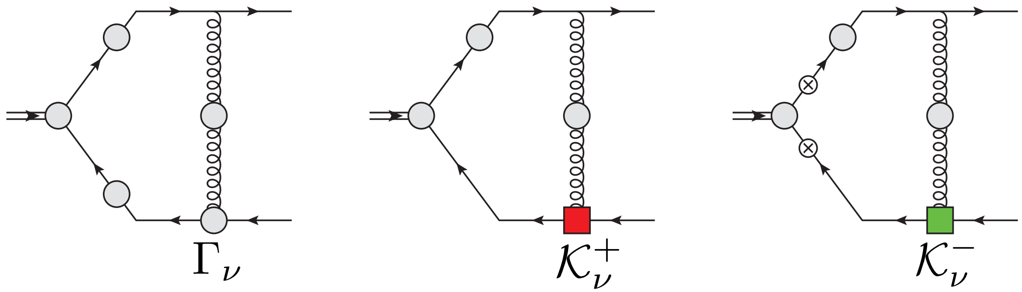

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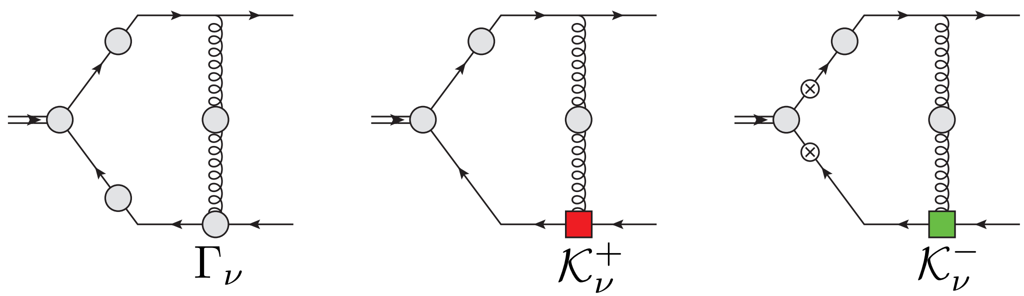

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

$$\Gamma^\Sigma_\nu = \Gamma^+_\nu + \gamma_5\Gamma^+_\nu\gamma_5 \qquad \Gamma^\Delta_\nu = \Gamma^+_\nu - \Gamma^-_\nu$$

$$B_\Sigma = 2B_+ \qquad B_\Delta = B_+ - B_-$$

$$A_\Delta = i(\gamma\cdot q_+)A_+ - i(\gamma\cdot q_-)A_-$$

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$$\int_q D_{\mu\nu}\gamma_\mu S_+ \boxed{(\Gamma^+_\nu - \Gamma^-_\nu)} = \int_q D_{\mu\nu}\gamma_\mu S_+ \boxed{(S^{-1}_+ - S^{-1}_-)\mathcal{K}^+_\nu} + \int_q D_{\mu\nu}\gamma_\mu S_+ \boxed{\gamma_5(S^{-1}_+ - S^{-1}_-)\gamma_5\mathcal{K}^-_\nu},$$

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Eventually, the solution is straightforward:

$$\boxed{\mathcal{K}^\pm_\nu = (2B_\Sigma A_\Delta)^{-1}[(A_\Delta \mp B_\Delta)\Gamma^\Sigma_\nu \pm B_\Sigma\Gamma^\Delta_\nu].}$$

- ✦ The form of scattering kernel is simple.
- ✦ The kernel has no kinetic singularities.
- ✦ All channels share the same kernel.

2 DSE: Summary

- **Gluon propagator**: Solve the gluon DSE or extract information from lattice QCD. The dressing function of gluon has a mass scale as that of quark.

- **Quark-gluon vertex**: Solve the WGTIs resulting from the fundamental symmetries (gauge, chiral, and Lorentz symmetries). The vertex is significantly modified by DCSB.

- **Scattering kernel**: Analyze continuous (color-singlet WGTIs) and discrete symmetries. The kernel preserves the chiral symmetry which makes pion to play a twofold role: Bound-state and Goldstone boson.

3 Application: Simplest approximation of DSEs

I. Gluon propagator

II. Quark-gluon vertex

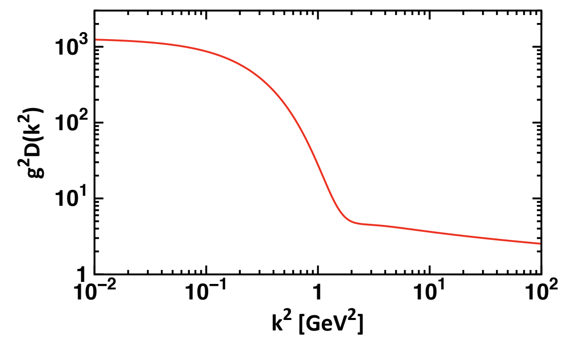
III. Scattering kernel

3 Application: Simplest approximation of DSEs

I. Gluon propagator

Massive gluon model

$$g^2 D_{\mu\nu}^{ab}(k) = \delta_{ab} D_{\mu\nu}^{\text{free}}(k) \mathcal{G}(k^2)$$



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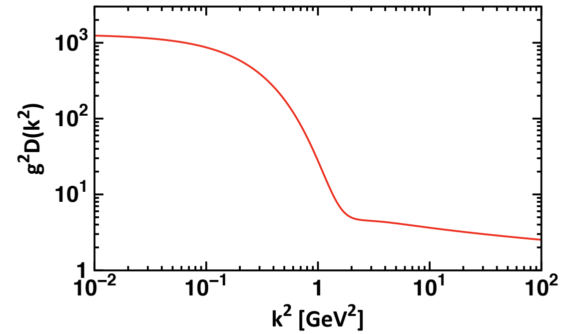
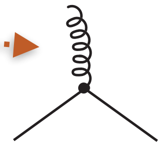


II. Quark-gluon vertex — Rainbow approximation



III. Scattering kernel

I. Gluon propagator

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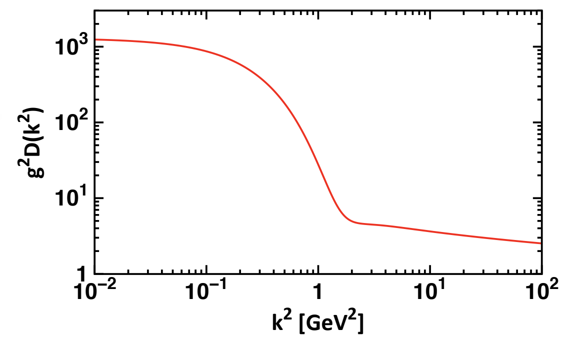

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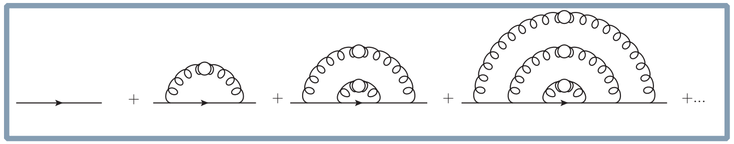

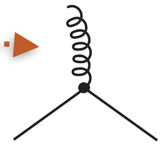

III. Scattering kernel

3 Application: Simplest approximation of DSEs

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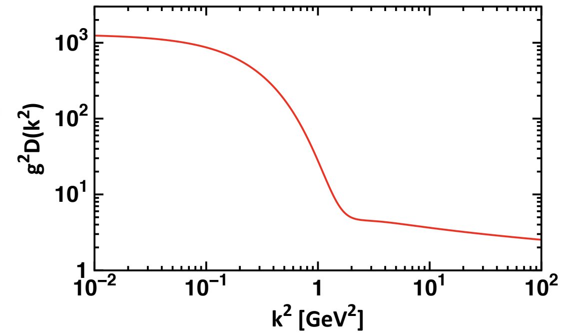

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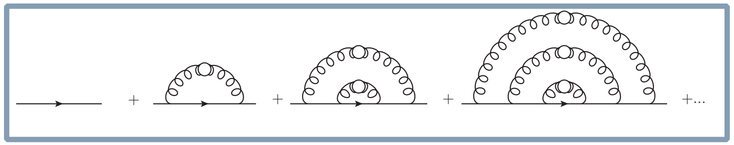
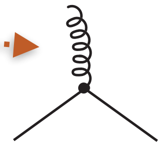
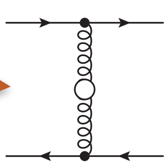




III. Scattering kernel

Ladder approximation



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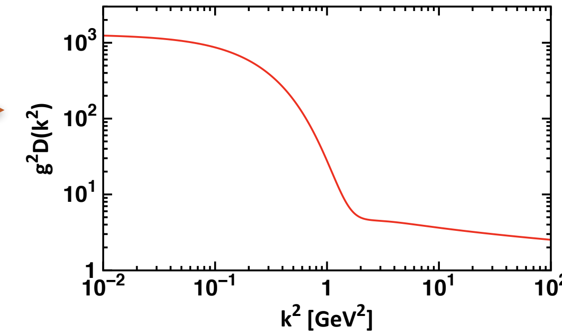
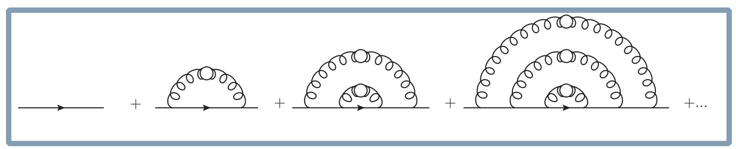
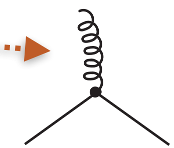
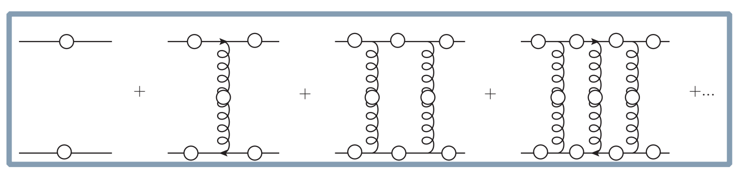


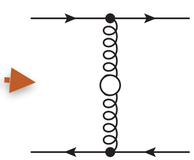
II. Quark-gluon vertex Rainbow approximation





III. Scattering kernel Ladder approximation



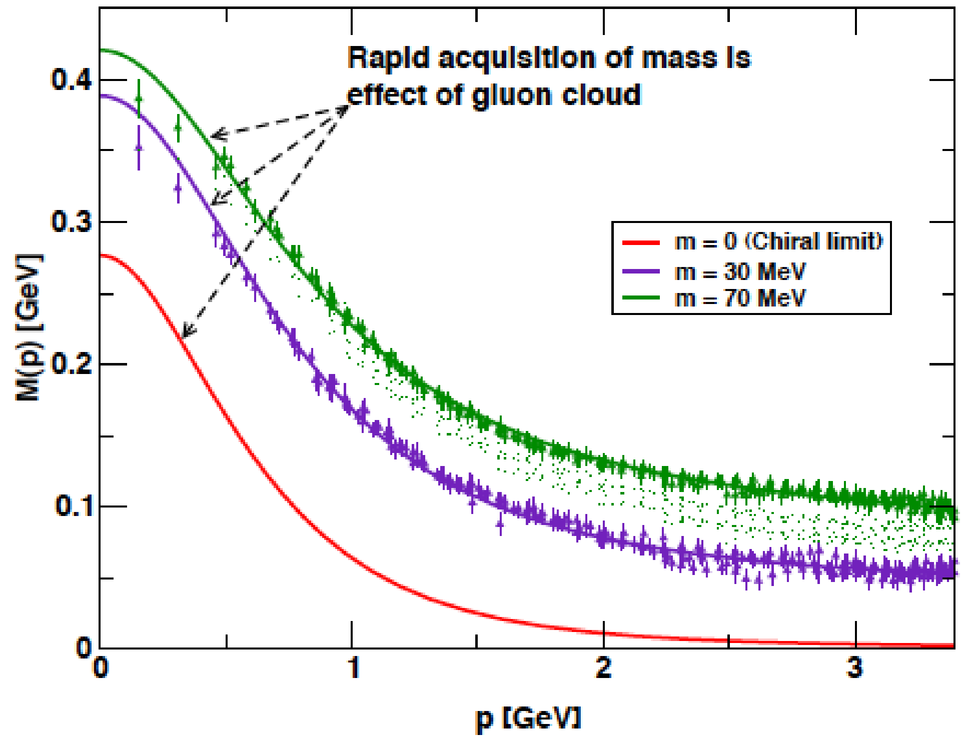


3 Application: Realization of DCSB & Confinement

✦ **DCSB:**

1. The quark's **effective mass** runs with its momentum.
2. The most of **constituent quark mass** comes from a cloud of gluons.

$$S(p) = \frac{1}{i\gamma \cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



✦ **Confinement:**

Although we exactly know few knowledge about confinement, the positivity violation of quark spectral density supports a fact that a asymptotically free quark is unphysical. In this sense, we say that quarks are confined.

3 Application: Rainbow-Ladder spectrum

✦ Light ground mesons

Summary of light meson results
$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle_\mu^0$	$(0.236\text{ GeV})^3$	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r_π^2	0.44 fm^2	0.45
$r_{K^+}^2$	0.34 fm^2	0.38
$r_{K^0}^2$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
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Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ \text{s}^{-1}$	7.38
$\Gamma(K_{\mu3})$	$5.2 \cdot 10^6\ \text{s}^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^*K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^*K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^*K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a_0^0	0.220	0.170
a_0^2	0.044	0.045
a_1^1	0.038	0.036

Tandy @ Beijing Lectures 2010

3 Application: Rainbow-Ladder spectrum

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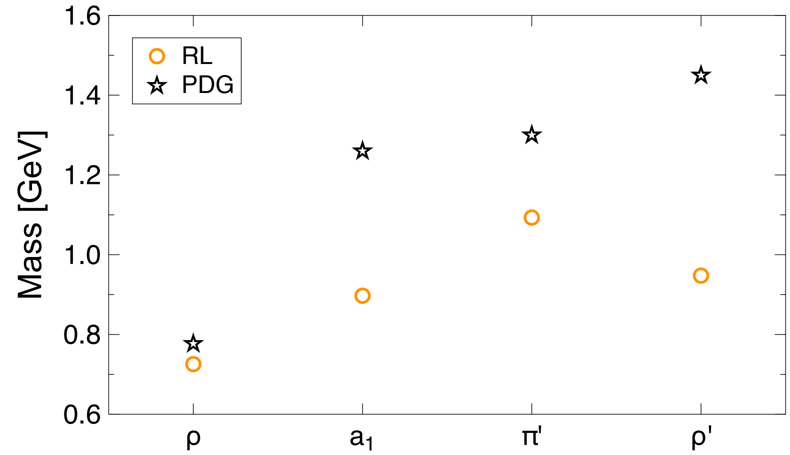
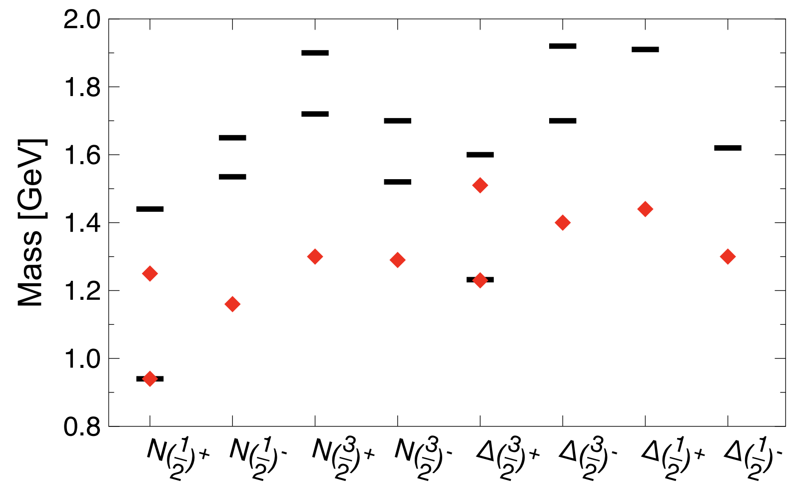
✦ Heavy ground and radially excited states





3 Application: Rainbow-Ladder spectrum

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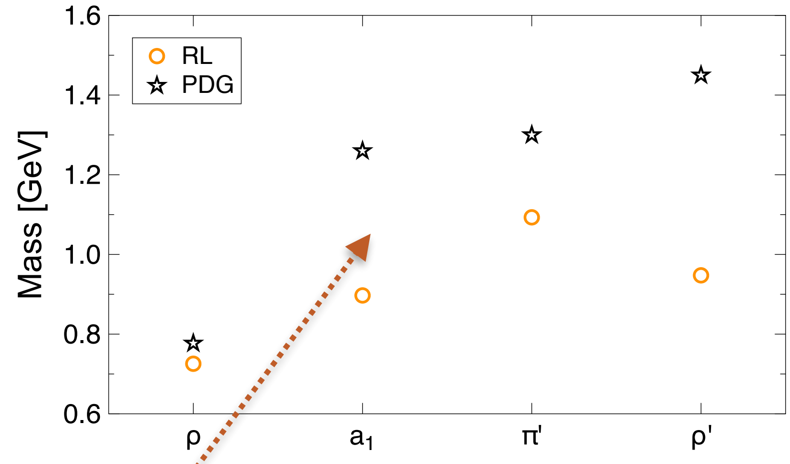
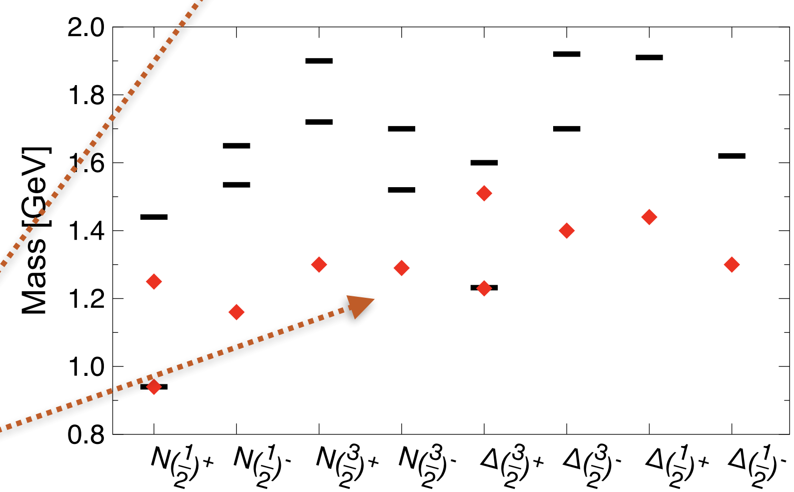
a_1^1	0.038	0.036

Tandy @ Beijing Lectures 2010

✦ Heavy ground and radially excited states





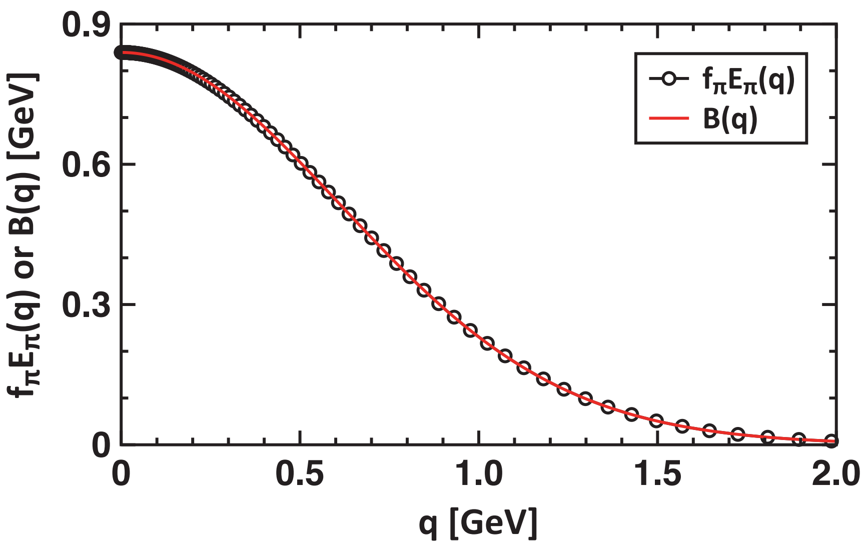
Hadron spectrum: systematically wrong ordering and magnitudes.

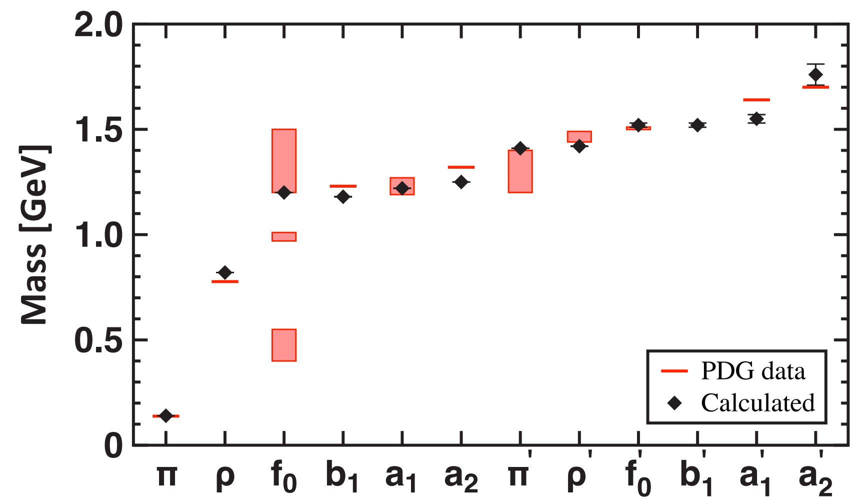
3 Application: Sophisticated spectrum

Let the quark-gluon vertex includes both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma_\mu^{\rm BC}(p,q) + \eta\,\Gamma_\mu^{\rm T}(p,q) \qquad \Gamma_\mu^{\rm T}(p,q) = \Delta_B \tau_\mu^8 + \Delta_A \tau_\mu^4$$

$$\boxed{\begin{aligned}\tau_\mu^4 &= l_\mu^{\rm T}\gamma\cdot k + i\gamma_\mu^{\rm T}\sigma_{\nu\rho}l_\nu k_\rho,\\ \tau_\mu^8 &= 3\,l_\mu^{\rm T}\sigma_{\nu\rho}l_\nu k_\rho/(l^{\rm T}\cdot l^{\rm T}).\end{aligned}}$$





	$-\langle\bar{q}q\rangle_0^{1/3}$	$\rho_\pi^{1/2}$	f_π	m_π	m_ρ	m_σ	m_{b_1}	m_{a_1}	m_{a_2}	$m_{\pi'}$	$m_{\rho'}$	$m_{\sigma'}$	$m_{b_1'}$	$m_{a_1'}$	$m_{a_2'}$
this work	0.283	0.493	0.093	0.14	0.82	1.20	1.18	1.22	1.25	1.41	1.42	1.52 ± 0.01	1.52 ± 0.01	1.55 ± 0.02	1.76 ± 0.05
PDG	-	-	0.092	0.14	0.78	0.50	1.24	1.26	1.32	1.30	1.45	-	-	1.64	1.70

TABLE I: The meson spectrum (Full vertex, $(D\omega)^{1/3} = 0.637$ GeV, $\omega = 0.60$ GeV, $\eta = 1.00$ and $m_q = 3.0$ MeV).

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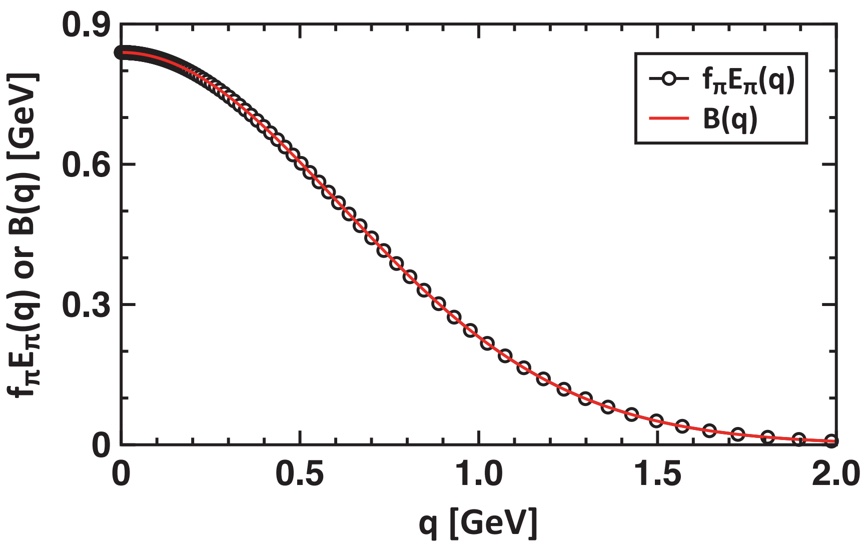

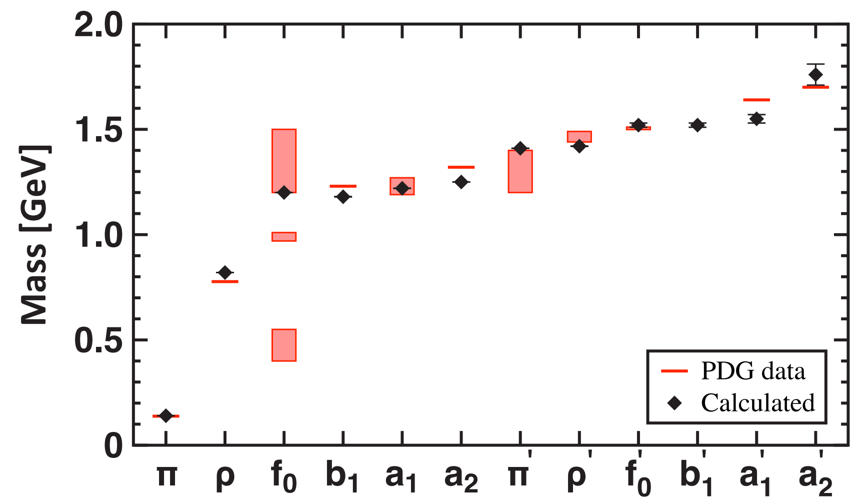

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✦ Based on LQCD and QCD's symmetries, a **systematic method** to construct the **gluon propagator, quark-gluon vertex,** and **scattering kernel**, is proposed.

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Outlook

◆ With the **sophisticated method** to solve the DSEs, we can push the approach to a wide range of applications in **QCD bound-state** problems, e.g., baryons and structures.

◆ Hopefully, after more and more **successful applications** are presented, the DSEs may provide a **faithful path** to understand **QCD** and a powerful tool for general physics.