Extracting parton physics from "lattice cross sections"

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Based on works done with Tomomi Ishikawa, Jian-Wei Qiu and Shinsuke Yoshida

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Outline

I. Introduction to PDFs

II. New ideas to calculate PDFs on lattice

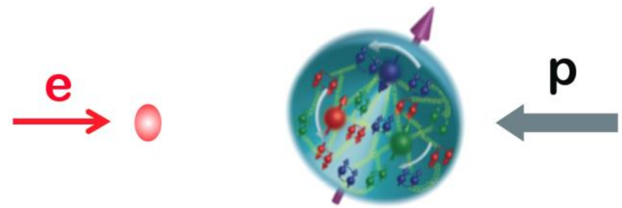
III. A case study: Quasi PDFs

IV. Renormalizability of quasi PDFs

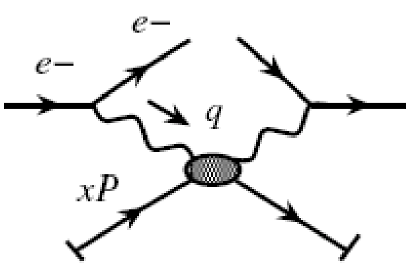
V. Summary

- The key and a first principle method to relate experimental data to QCD theory

- Electron-hadron:



$\sigma_{\text{tot}}^{\text{DIS}}$ =



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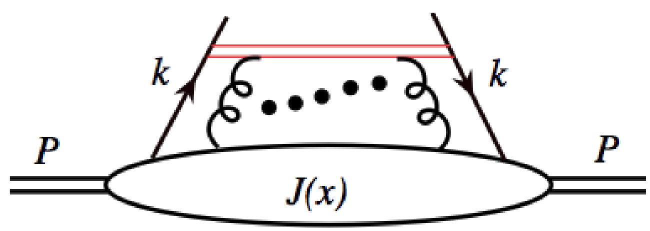


$+ O(\frac{1}{QR})$

Hard-part Probe	Parton-distribution Structure	Correction Approximation

Operator definition of PDFs

- Spin-averaged quark distribution

$$f_{q/p}(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\,\gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\}\psi(0)|P\rangle$$

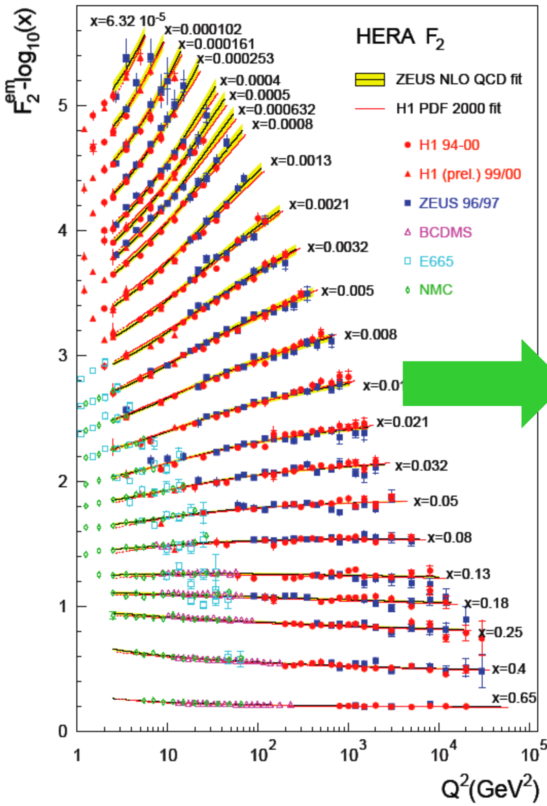
  - Simplest of all parton correlation functions
  - Unlike cross section, not direct physical observable; but well defined in QCD

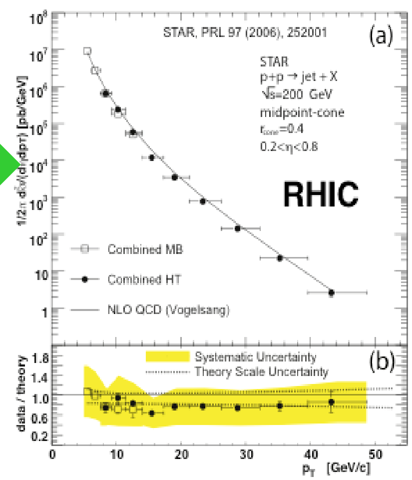
- Boost invariant along "+" direction
- Parton interpretation emerges in $A_+ = 0$ gauge
- Logarithmic UV divergent, renormalizable
- Time dependent!

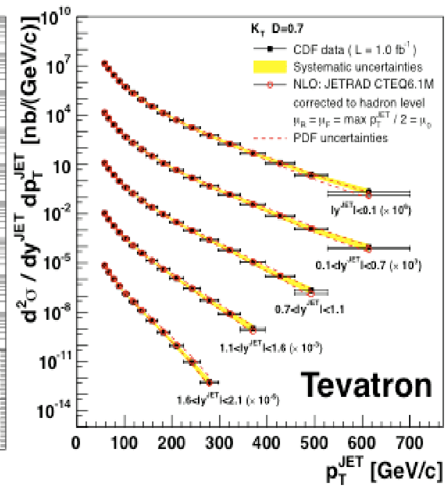
Extract PDFs by fitting data

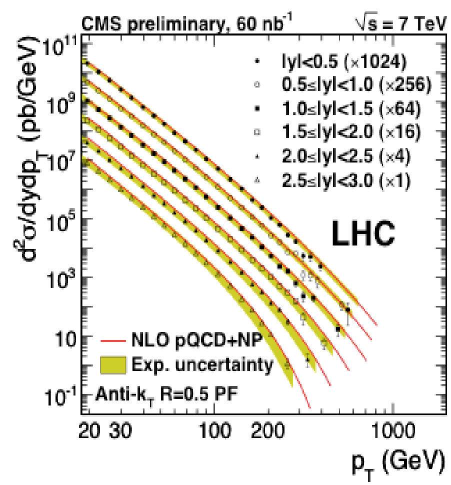
➢ **Successful**

Measure e-p at 0.3 TeV (HERA)
Predict p-p at 0.2, 1.96, and 7 TeV

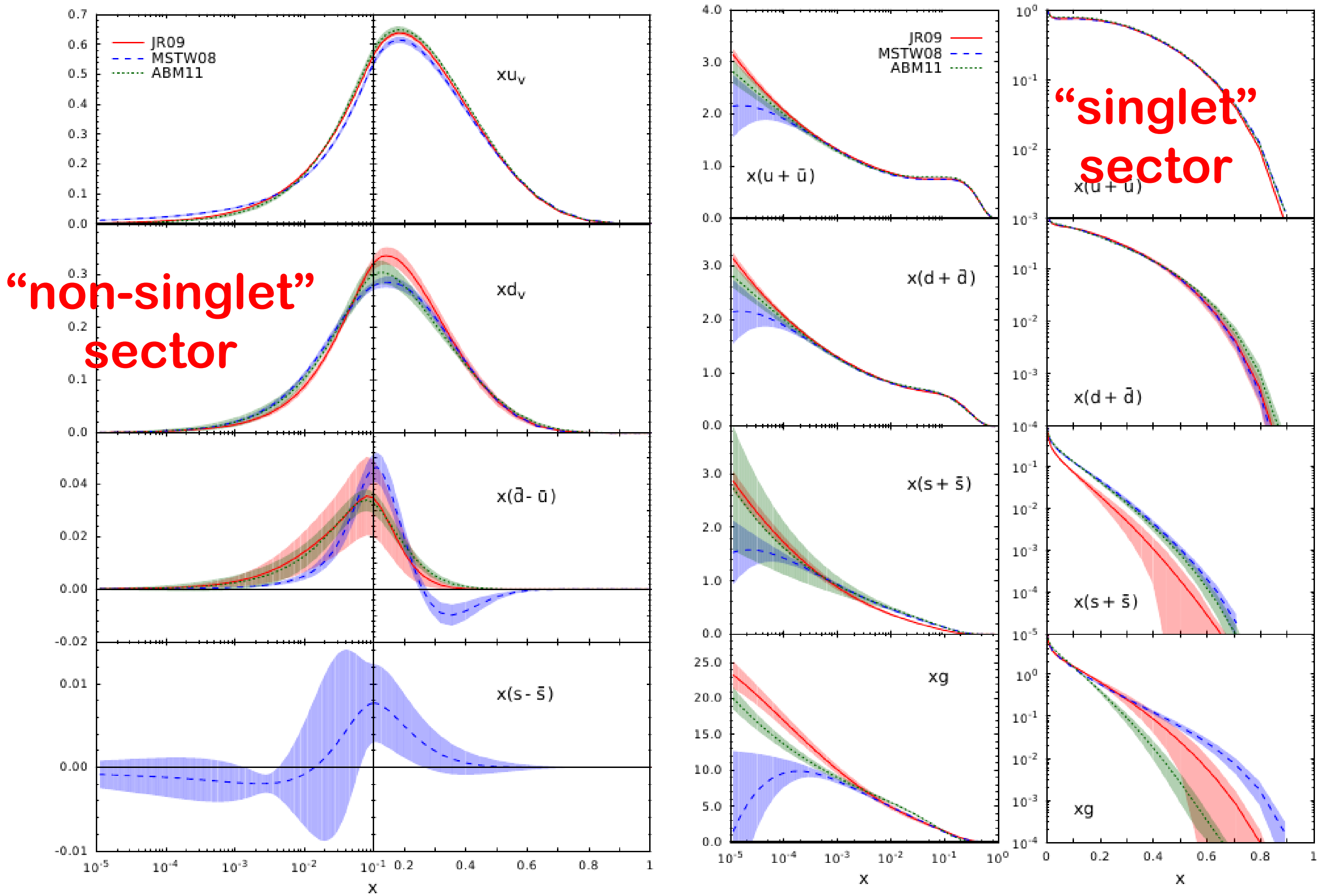








Uncertainty of PDFs

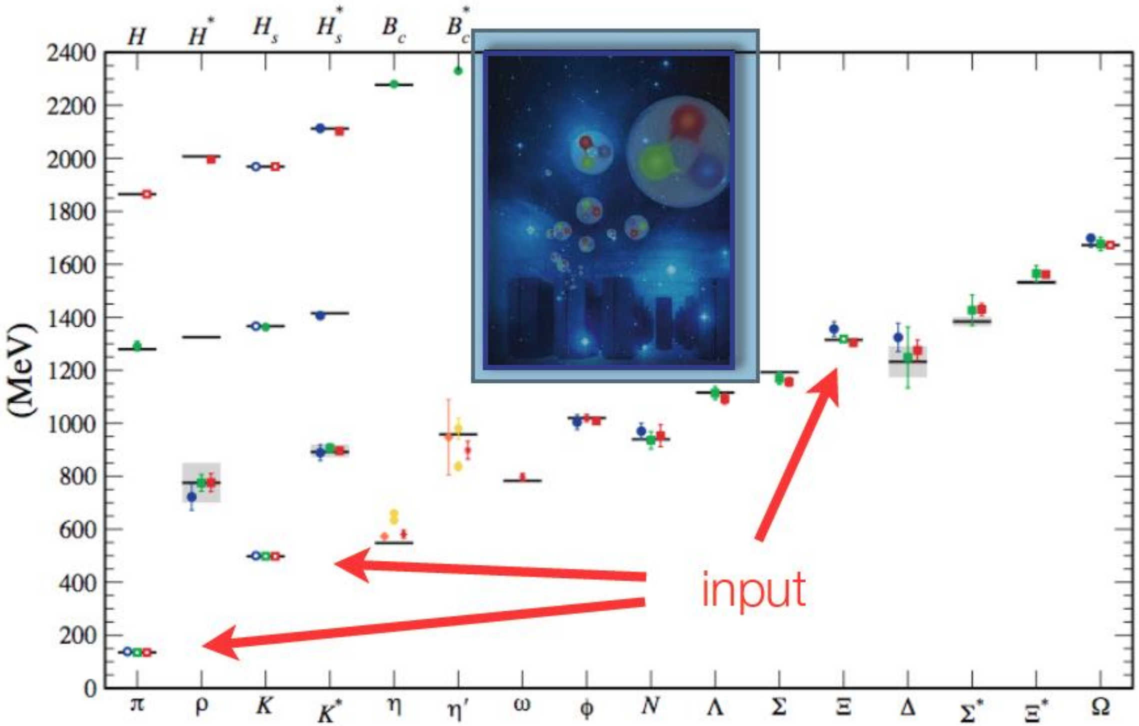


Question

Is it possible to determine PDFs nonperturbatively from first principle?

Lattice QCD

- The main nonperturbative approach to solve QCD
- Predict the hadron mass



(Kronfeld)

- An intrinsically Euclidean time: $\tau = i\,t$

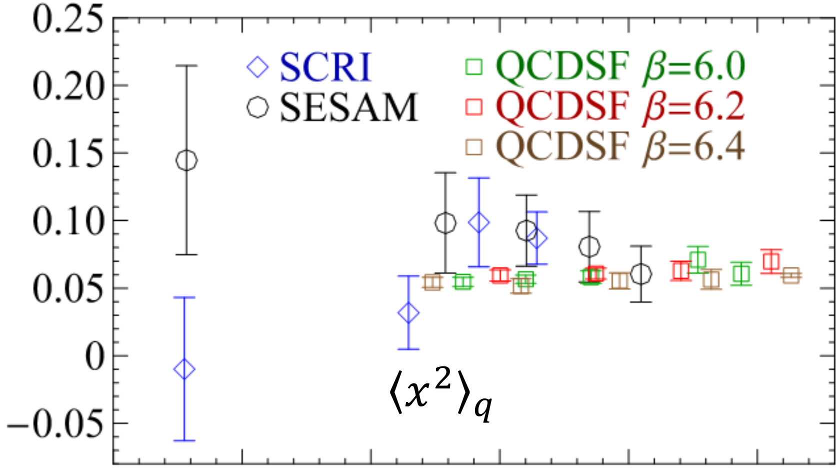
Cannot calculate PDFs directly

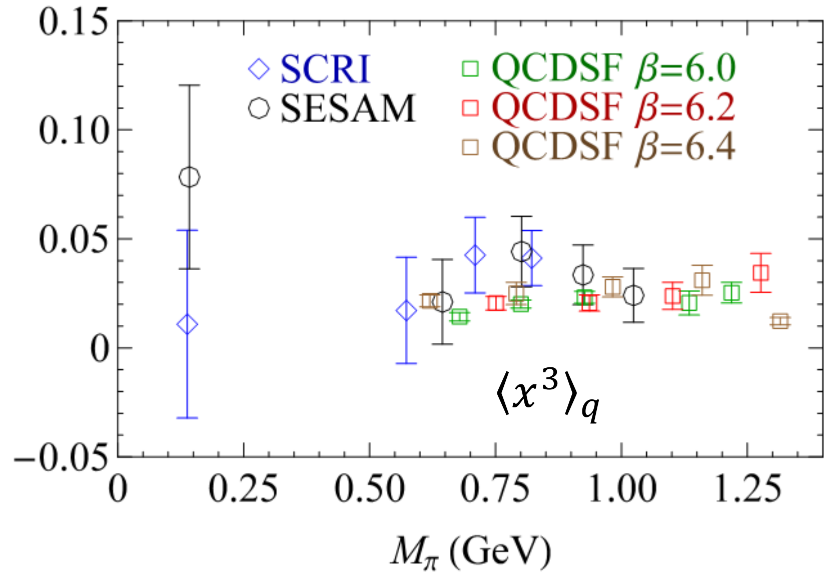
PDFs from lattice QCD

- **Moments: matrix elements of local operators**

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n f_{q/p}(x,\mu^2)$$

- **Works, but only for limited moments**





Dolgov et al., hep-lat/0201021

Gockeler et al., hep-ph/0410187

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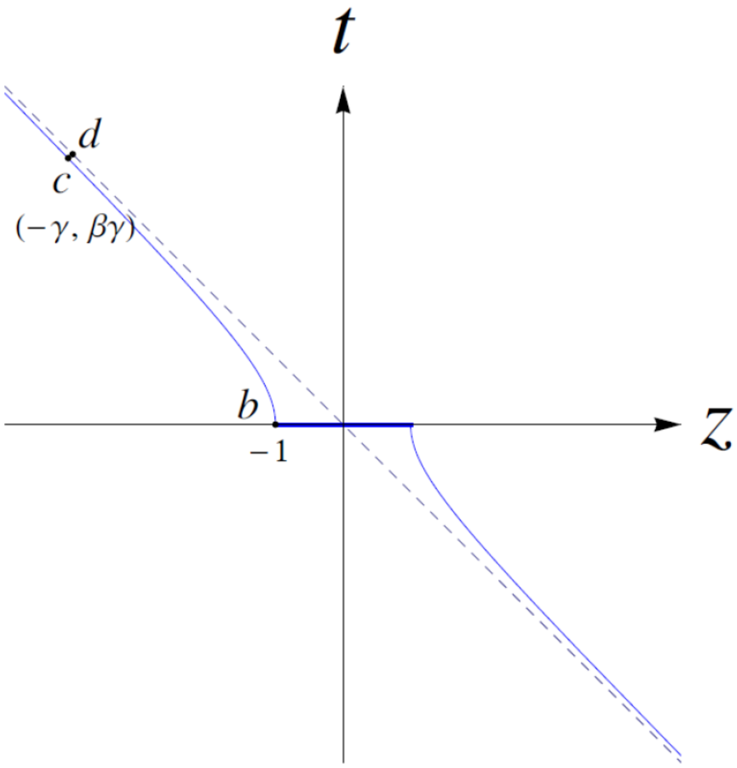
Ji's idea

- What if quark bilinear is slightly off light cone?



- Exist a frame where quark bilinear is equal time, but proton is moving

Ji's approach

Ji, 1305.1539

➢ Quasi distribution:

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int \frac{d\xi_z}{4\pi} e^{-ix\xi_z P_z} \langle P|\overline{\psi}(\xi_z)\,\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z\, A_z(\eta_z)\right\}\psi(0)|P\rangle$$

➢ Features of quasi PDFs

- Fields separated along the z-direction
- No time dependence: calculable using standard lattice method
- Quasi PDFs goes to PDFs as $P_z \to \infty$

LaMET: relation quasi to normal

Ji, 1404. 6680
Ji, Zhang, Zhao, 1706.07416

- Relation between "Quasi" PDFs and normal PDFs at finite P_z :

See Xiangdong Ji's talk

$$F(P^z/\Lambda) = Z(P^z/\Lambda, \Lambda/\mu) f(\mu) + \mathcal{O}(1/(P^z)^2) + \ldots$$

Quasi PDFs

Normal PDFs

Higher dimensional operators in LaMET

Normal PDFs: an effective field theory of "quasi" PDFs

- Finite P_z effects can be improved by solving RG equation :

$$\gamma(\alpha_s) = \frac{1}{Z}\frac{\partial Z}{\partial \ln P^z} \qquad \frac{\partial F(P^z)}{\partial \ln P^z} = \gamma(\alpha_s) F(P^z) + \mathcal{O}(1/(P^z)^2)$$

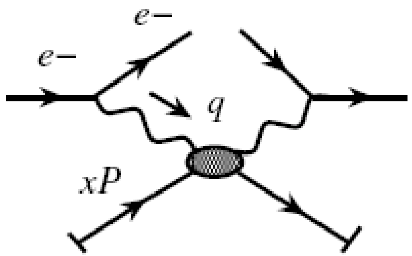
How to go beyond quasi PDFs

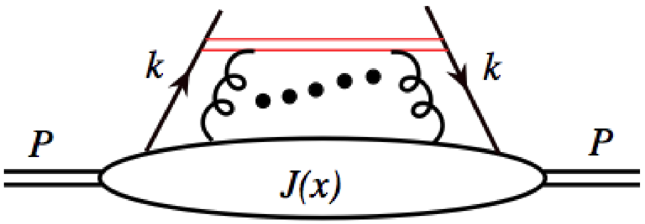
- **Key for Ji's idea**
  - Relate lattice calculable quantities (quasi PDFs) to physically interesting quantities (normal PDFs) which cannot be calculated on lattice easily
  - Using Large Momentum Effective Theory: Quasi PDFs goes to PDFs as $P_z \to \infty$
- **Can we generalize Ji's idea?**

Observation (1)

➢ What is factorization (e.g. DIS)?

- Traditional understanding: because of space-time scale separation, quantum interference between two successive processes are negligible

$\sigma_{\text{tot}}^{\text{DIS}}$ = 

$\otimes$ 

$+ O(\frac{1}{QR})$

An equivalent statement: PDFs form a complete set to expand the nonperturbative structure of DIS process

➢ PDFs are infrared finite, collinear divergent

- Form a complete set of CO structure

Observation (2)

➢ CO divergence is time-independent

- Leading power perturbative CO divergences of single hadron matrix elements: logarithmic $\propto \int dk_T^2/k_T^2$
- Divergence from the region when $k_T \to 0$
- Divergence: the same for both Minkowski and Euclidean time

➢ Explain why can extract PDFs from lattice

- The information that we need from lattice is CO structure of PDFs, which is time-independent

QCD factorization can help to relate different quantities that have the same CO structure

Ma-Qiu's approach

YQM, Qiu, 1404.6860, 1412.2688

- "Lattice cross section": $\tilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, 1/a, P_z)$
  - Hadronic matrix element
  - $P_z \leftrightarrow \sqrt{s}$: "collision energy"
  - $1/a \leftrightarrow Q$: hard scale, resolution
  - $\tilde{x} \leftrightarrow x$: parameter

- Condition for a good "lattice cross section"
  ① Calculable on Euclidean lattice QCD
  ② UV and IR safe perturbatively (renormalizable)
  ③ CO divergence: factorizable (similar to DIS cross section)

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x}, \tilde{\mu}^2, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2) \mathcal{C}_i\left(\frac{\tilde{x}}{x}, \tilde{\mu}^2, \mu^2, P_z\right) + O(1/\tilde{\mu}^2)$$

- The last condition relates "Lattice cross sections" to PDFs

"Lattice cross sections"

- **Relaxed condition relative to quasi-PDFs**
  - They not demanded to go to PDFs in any limit
  - Any quantity calculable on lattice can be used to constrain PDFs, as far as its CO structure can be factorized by PDFs

Factorization is the essential question!

- **Candidates of good "lattice cross section"**
  - Quasi-PDFs Ji, 2013
  - Pseudo-PDFs Radyushkin, 2017
  - Conserved currents Liu et. al.
  - …
- **Ma-Qiu's approach is a generalization of Ji's**

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Quasi PDFs

- "Quasi quark" PDF as an example

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int \frac{d\xi_z}{4\pi} e^{-ix\xi_z P_z} \langle P|\overline{\psi}(\xi_z)\,\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

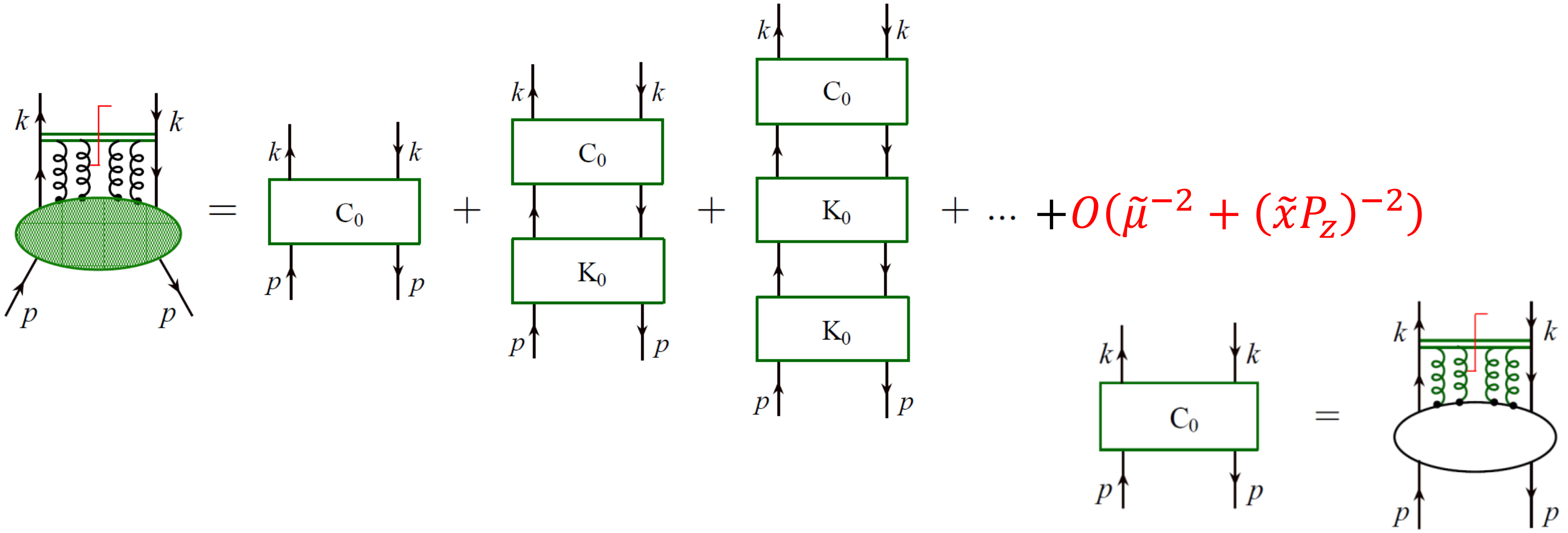
- Useful only if it is a good "lattice cross section"

- A good "lattice cross section"?
  - ✓ No time dependence: calculable on Euclidean lattice
  - ✓ IR divergence: cancelled by unitarity YQM, Qiu, 1404.6860, 1412.2688
  - ? CO divergence: factorizable?
  - ? UV safe perturbatively: renormalizable?

Ladder decomposition

YQM, Qiu, 1404.6860, 1412.2688

➢ Generalized ladder diagrams decomposition



- C_0, K_0: 2PI kernels
- Ordering in virtuality $p^2 \ll k^2 \sim \tilde{\mu}^{-2} + (\tilde{x}P_z)^{-2}$

➢ Using physical gauge, 2PI diagrams are finite

Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

Factorization

YQM, Qiu, 1404.6860, 1412.2688

- Factorize the last kernel, and then recursively:

$\hat{\mathcal{P}}$: pick up the singular part of integration

$$\tilde{f}_{q/p} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K_0^i + \text{UVCT} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K_0^i$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=0}^{m-1} K^i (1-\hat{\mathcal{P}})K\right]_{\text{ren}} + \tilde{f}_{q/p}\hat{\mathcal{P}}\,K$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=1}^{m} \left[(1-\hat{\mathcal{P}})K\right]^i\right]_{\text{ren}} + \tilde{f}_{q/p}\hat{\mathcal{P}}\,K\,,$$

$$\Rightarrow \quad \tilde{f}_{q/p} = \left[C_0 \frac{1}{1-(1-\hat{\mathcal{P}})K}\right]_{\text{ren}} \left[\frac{1}{1-\hat{\mathcal{P}}K}\right]$$

Normal PDFs

All CO divergences of quasi PDF

Finite

$$\Rightarrow \quad \tilde{\sigma}_{\text{M}}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x,\mu^2)\, \mathcal{C}_i\!\left(\frac{\tilde{x}}{x},\tilde{\mu}^2,\mu^2,P_z\right) + O\!\left(\tilde{\mu}^{-2} + (\tilde{x}P_z)^{-2}\right)$$

- Factorizable as far as quasi-PDFs are multiplicatively renormalized

How about renormalization?

➢ Coordinate space definition

$$\tilde{F}_{q/p}(\xi_z, \tilde{\mu}^2, p_z) = \frac{e^{ip_z\xi_z}}{p_z} \langle h(p)|\overline{\psi}_q(\xi_z)\,\frac{\gamma_z}{2}\,\Phi_{n_z}^{(f)}(\{\xi_z, 0\})\,\psi_q(0)|h(p)\rangle$$

➢ Conjecture of all-orders renormalization

$$\tilde{F}^R_{i/p}(\xi_z, \tilde{\mu}^2, p_z) = e^{-C_i|\xi_z|} Z_{wi}^{-1} Z_{vi}^{-1} \tilde{F}^b_{i/p}(\xi_z, \tilde{\mu}^2, p_z)$$

Ishikawa, YQM, Qiu, Yoshida, 1609.02018
Chen, Ji, Zhang, 1609.08102
Constantinou, H. Panagopoulos, 1705.11193
...

- Rigorous proof is needed!

Proof: Importance and difficulty

➢ Why proof is important?

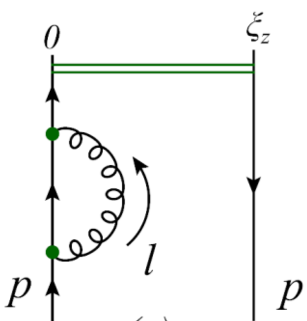
- All-order proof of factorization needs multiplicative renormalization YQM, Qiu, 1404.6860, 1412.2688
- Whether mixing with other operators under renormalization? A close set of operators are needed

➢ Why proof is difficult

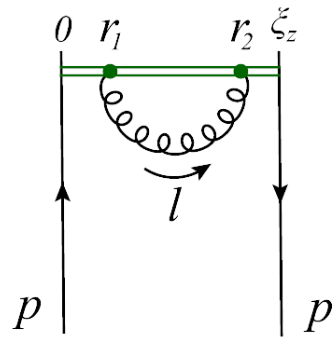
- Because of z-direction dependence, Lorentz symmetry is broken, hard to exhaust all possible UV divergences
- Renormalization of composite operator is needed

➢ Identifying UV divergences

- Renormalization of QCD in covariant gauge: only from 4-dimensional loop integration, all components become large
- Quasi-PDFs: 3-dimensional integration as while as 4-dimensional integration can generate UV divergences



(c)



(a)

$$l^\mu = \bar{l}^\mu + l_z n_z^\mu$$

UV: 4-D integration

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2 (p-l)^2}$$

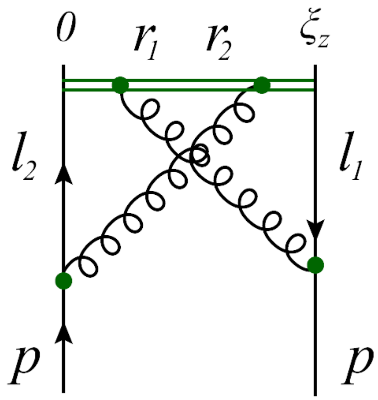
UV: 3-D integration

$$\int \frac{d^3 \bar{l}}{l^2} = \int \frac{d^3 \bar{l}}{\bar{l}^2 - l_z^2}$$

Broken of Lorentz symmetry con.

➢ Hard to identify all UV regions

- **Need to consider 3-D and 4-D integrations for each loop**



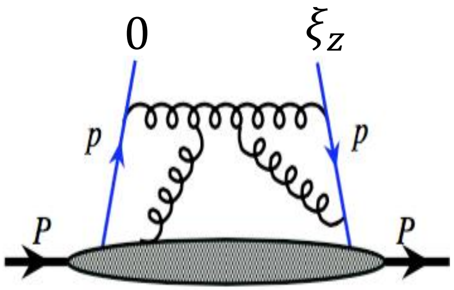
① l_1: 3-D, l_2: 3-D
② l_1: 3-D, l_2: 4-D
③ l_1: 4-D, l_2: 3-D
④ l_1: 4-D, l_2: 4-D

- **A n-loop diagram, to identify all possible UV divergences, needs consider 2^n different cases!**

Composition operator renormalization

➢ Quasi-quark PDF in $\mathrm{A_z} = 0$ gauge: no gauge link

$$\tilde{F}_{q/p}(\xi_z, \tilde{\mu}^2, p_z) = \frac{e^{ip_z\xi_z}}{p_z} \langle h(p)|\overline{\psi}_q(\xi_z)\, \frac{\gamma_z}{2}\, \psi_q(0)|h(p)\rangle$$



- Renormalization of quark field $\bar{\psi}_q$ and ψ_q: taking care by renormalized QCD Lagrangian
- Renormalization of the bi-local operator as a whole: still needs to study

➢ Comparison: Quark PDF in $\mathrm{A_+} = 0$ gauge

- Similar for quark field renormalization
- Renormalization of the bi-local operator as a whole: needed!
- It is this renormalization that mixes quark PDF with gluon PDF

Keys for a rigorous proof

Ishikawa YQM, Qiu, Yoshida, 1707.03107

- **Working in Feynman gauge**
  - Because renormalization of QCD Lagrangian in Feynman gauge is well known

- **Key to prove the renormalization: show that UV divergences are local in space-time**
  - Nontrivial conclusion! E.g. UV divergences for normal PDFs are non-local in "−" direction
  - The most difficult part in our proof
  - One can guess this, but a rigorous proof is badly needed

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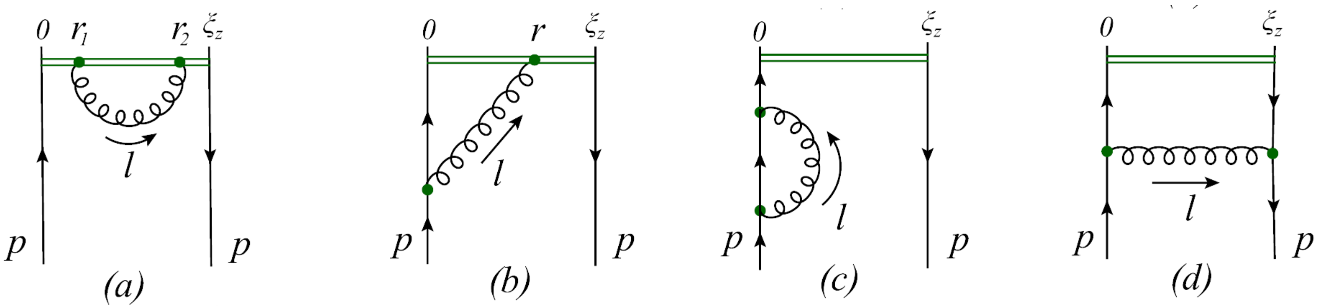
One-loop calculation

One-loop diagrams: quark in a quark

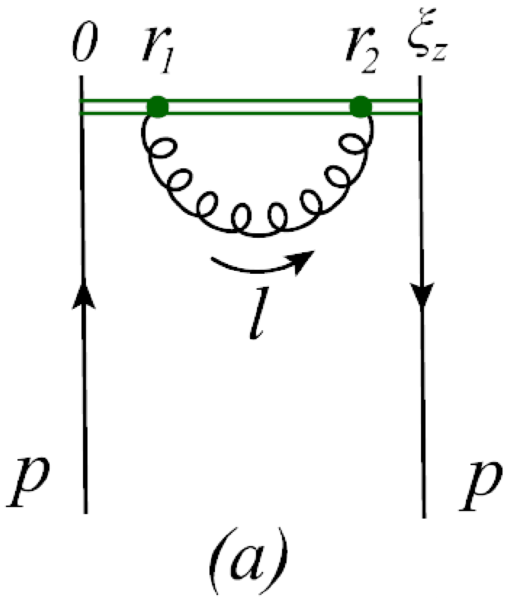
➢ Quasi quark PDFs at one loop level



- **Will demonstrate that UV divergences are local in space-time, which is significantly different from normal PDFs**

- **Note: normal PDFs, UV divergences from the regio** $(l_+, l_-, l_\perp) \sim (1, \lambda^2, \lambda)$ **with** $\lambda \to \infty$**, nonlocal in '-' direction in coordinate space.**
- **Thus, renormalization of normal PDFs is a convolution, while renormalization of quasi-PDFs is multiplicative factor**

Fig.1 (a)



$$M_{1a} = \frac{e^{ip_z\xi_z}}{p_z}\frac{1}{N_c}\mathrm{Tr}_c[T^aT^a]\int_a^{\xi_z-2a}dr_1\int_{r_1+a}^{\xi_z-a}dr_2 \times \int\frac{d^4l}{(2\pi)^4}e^{-ip_z\xi_z}e^{il_z(r_2-r_1)}\left(\frac{-ig_{\mu\nu}}{l^2}\right) \times (-ig_sn_z^\mu)(-ig_sn_z^\nu)\mathrm{Tr}\left[\frac{1}{2}\not{p}\,\frac{1}{2}\gamma_z\right] = \frac{\alpha_sC_F}{4i\pi^3}\int_a^{\xi_z-2a}dr_1\int_{r_1+a}^{\xi_z-a}dr_2\int d^4l\frac{e^{il_z(r_2-r_1)}}{l^2}$$

- **Cutoff “a” between fields along gaugelink**
- **Conclusion independent of regulators**

$$\int\frac{d^3\bar{l}}{l^2} = \int\frac{d^3\bar{l}}{\bar{l}^2-l_z^2} = \int d^3\bar{l}\left(\frac{1}{\bar{l}^2}+\frac{l_z^2}{(\bar{l}^2-l_z^2)\bar{l}^2}\right)$$

$$d^4l = d^3\bar{l}\,dl_z \qquad l^2=\bar{l}^2-l_z^2$$

$$\int dl_z e^{il_z(r_2-r_1)} = 2\pi\delta(r_2-r_1)$$

- **First term vanishes because $r_1 \neq r_2$, thus 3D integration is finite**

Fig. 1(a) cont.

- Fix 3D, l_z integration is finite
- UV divergent only if all 4 components of l^μ go to infinity

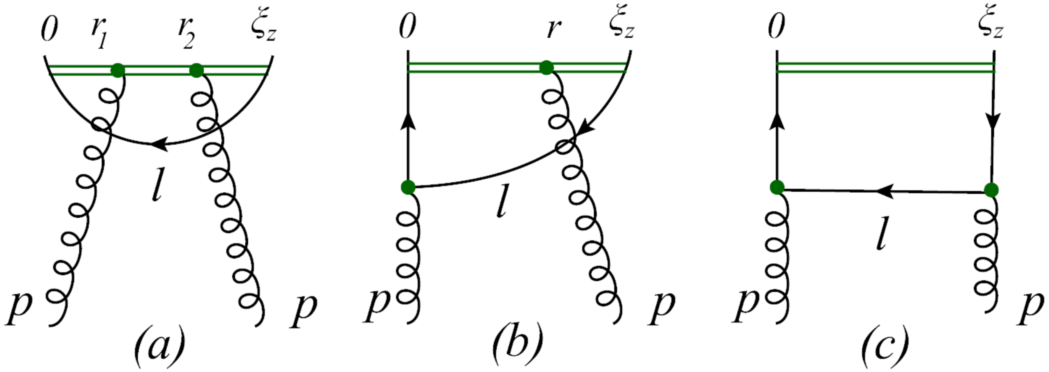
$$M_{1a} \overset{\text{div}}{=} -\frac{\alpha_s C_F}{\pi}\frac{|\xi_z|}{a} + \frac{\alpha_s C_F}{\pi}\ln\frac{|\xi_z|}{a}$$

- At this order, UV divergences only come from the region where all loop momenta go to infinity, thus localized in coordinate space.
- Will show next: this behavior remains true up to all order in perturbation theory.

$$\begin{aligned} M^{(1)} &\overset{\text{div}}{=} M_{1a} + 2\times M_{1b} + 2\times\frac{1}{2}M_{1c} + M_{1d} \\ &= \frac{\alpha_s C_F}{\pi}\left(-\frac{|\xi_z|}{a} + 2\ln\frac{|\xi_z|}{a} - \frac{1}{4\epsilon}\right). \end{aligned}$$

One-loop diagrams: quark in a gluon

➢ Gluon to quark



$$M_{2a} \propto \int_0^{\xi_z} dr_1 \int_{r_1}^{\xi_z} dr_2 \int d^4 l\, e^{-il_z\xi_z} \frac{l_z}{l^2}$$

$$= \frac{\xi_z^2}{2} \int dl_z\, e^{-il_z\xi_z}\, l_z \int d^3\bar{l} \left(\frac{1}{\bar{l}^2} + \frac{l_z^2}{(\bar{l}^2 - l_z^2)\bar{l}^2}\right)$$

- **UV divergence from 3-D $\propto \delta'(\xi_z)$, vanishes for finite ξ_z**

One-loop diagrams: quark in a gluon con.

➢ Finite term

$$
\begin{aligned}
&\frac{\xi_z^2}{2}\int dl_z\, e^{-il_z\xi_z}\, l_z \int d^3\bar{l}\frac{l_z^2}{(\bar{l}^2-l_z^2)\bar{l}^2}\\
\propto&\frac{\xi_z^2}{2}\int dl_z\, e^{-il_z\xi_z}\, \frac{l_z^3}{|l_z|}\\
=&\frac{2i}{\xi_z},
\end{aligned}
$$

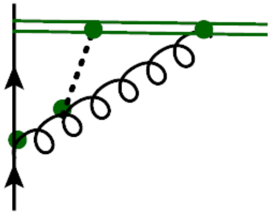
- Divergent as $\xi_z \to 0$
- Result in bad large $\tilde{x}$ behavior in momentum space

Outline

Power counting

Divergence index

➢ UV divergence at higher loops

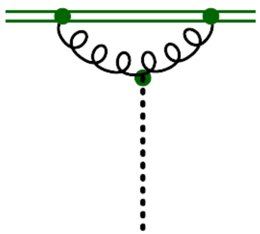


- Construct higher-loop diagrams from lower-loop diagrams by adding gluons to it
- Define divergence index ω_3 (ω_4) for 3D (4D) integration
- Using $\Delta\omega_3$ $(\Delta\omega_4)$ to denote divergence index changes for 3D (4D) integration

➢ Condition for renormalizability

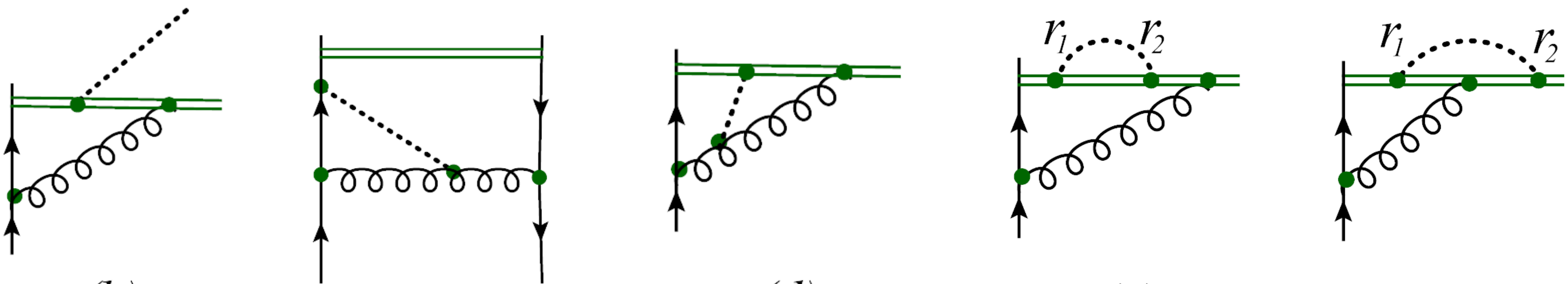
- Finite number of divergent topologies
- Sufficient condition: $\Delta\omega_3 \leq 0$ and $\Delta\omega_4 \leq 0$ for all cases, but not a necessary condition

Cases I-V



(a)

Case I: Only one end of the gluon is attached to parton lines of a loop, like Fig. 3(a). In this case we have one more propagator and one more QCD vertex in the loop, which results in $\Delta\omega_3 = -1$ and $\Delta\omega_4 = -1$. Thus a linear divergence can be changed to a logarithmic divergence, and a logarithmic divergent diagram is changed to be finite.



(b) *(c)* *(d)* *(e)* *(f)*

II. $\Delta\omega_3 = 0$, $\Delta\omega_4 = -1$

III. $\Delta\omega_3 = -1$, $\Delta\omega_4 = 0$

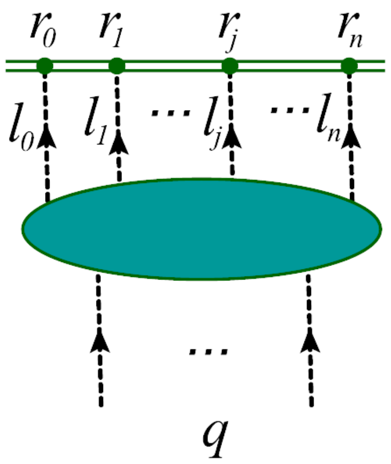
IV. $\Delta\omega_3 = 0$, $\Delta\omega_4 = 0$

V. $\Delta\omega_3 = 1$, $\Delta\omega_4 = 0$

- $\Delta\omega_3 > 0$ **for case V, may result in infinite topologies of UV div.**
- **Dangerous for the renormalizability**

GLI diagram

➢ Gauge-link-irreducible (GLI) diagram



- Diagram is connected no matter how many cuts are applied on the gauge link, or remove it
- Similar as the terminology 1PI

$$l_0 = q - l_1 - \cdots - l_n$$

- Can be generated from one-loop diagrams combined with insertions in Cases I, III, IV, all of which has $\Delta\omega_3 \leq 0$ and $\Delta\omega_4 \leq 0$
- Thus superficial UV divergence index $\omega \leq 1$

Finiteness of 3-D integration for GLI

➢ Dependence on l_j

$$e^{iq_z r_0}\prod_{j=1}^{n}\int_{r_{j-1}+a}^{r_{max}-a} dr_j \int \frac{d^4 l_j}{(2\pi)^4} e^{il_{jz}(r_j-r_0)}\mathcal{M}(q,l_1,\cdots,l_n)$$

- Numerator in M: decompose to $\bar{l}_j$ and l_{jz}
- Denominator in M:

$$\frac{1}{(l_j+k)^2} = \frac{1}{\Delta - 2k_z l_{jz} - l_{jz}^2}$$
$$= \frac{1}{\Delta} + \frac{2k_z l_{jz}}{\Delta^2} + \frac{(\Delta + 4k_z^2 + 2k_z l_{jz}) l_{jz}^2}{(\Delta - 2k_z l_{jz} - l_{jz}^2)\Delta^2}$$

$$\Delta = (\bar{l}_j + \bar{k})^2 - k_z^2$$

- Last term: finite for integration of $\bar{l}_j$

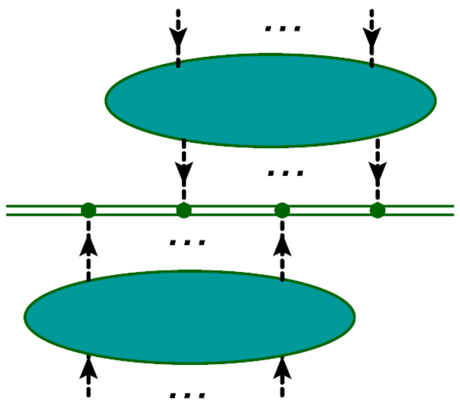
➢ UV divergence from integration of $\bar{l}_j$

- l_{jz} dependence is factorized out, vanish for finite $r_j - r_0$

$$\int dl_{jz} e^{il_{jz}(r_j-r_0)} l_z^m \propto \delta^{(m)}(r_j - r_0)$$

Quasi-PDFs: UV divergences local

➢ A non-GLI diagram made up by 2 GLI dia.



- Superficial UV divergence index $\omega \leq 2$
- For each GLI sub-diagram, similar argue for GLI diagram. UV finite if any 3-D integration is applied

➢ Easily generate to any non-GLI diagram:

- Overall UV divergence, obtained by fixing "z" component of any loop momentum, eventually vanishes after the integration of this "z" component
- UV divergences of quasi-PDFs: from the region whether all loop momenta become large → local in space-time
- As $\Delta\omega_4 \leq 0$ for all cases: finite div. topology, renormalizable

PDFs: UV divergences non-local

➢ 3-D' (l_- and $l_\perp$) integration of PDFs

$$\frac{1}{(l+k)^2} = \frac{1}{\hat{\Delta} + 2l_+(l_- + k_-)}$$
$$= \frac{1}{\hat{\Delta}} - \frac{2(l_- + k_-)l_+}{\hat{\Delta}^2} + \frac{4(l_- + k_-)^2 l_+^2}{(\hat{\Delta} + 2l_+(l_- + k_-))\hat{\Delta}^2}$$

$$\hat{\Delta} = 2k_+(l_- + k_-) - (\vec{l}_\perp + \vec{k}_\perp)^2$$

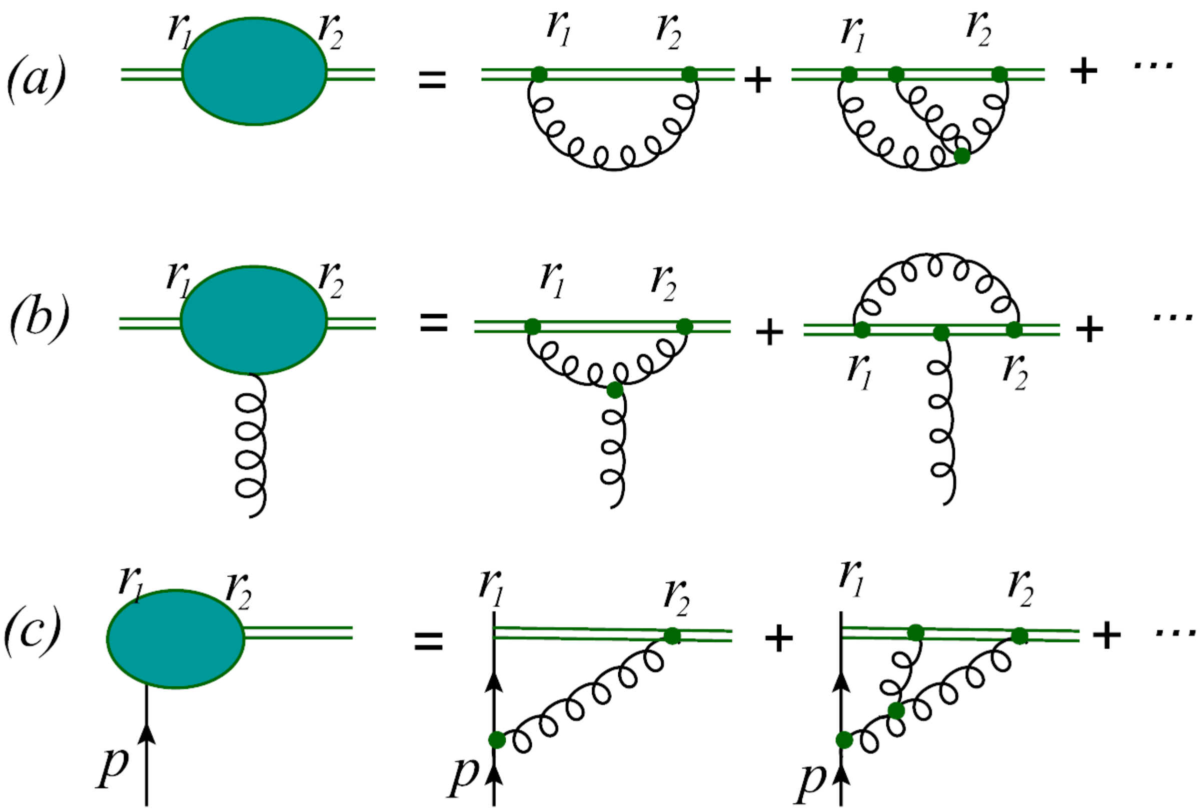
- Similar argue as quasi-PDFs: l_+ is factorized in the first two terms ,vanish under 3-D' integration
- But the last term is still UV divergent under 3-D' integration

➢ UV divergent region and non-locality

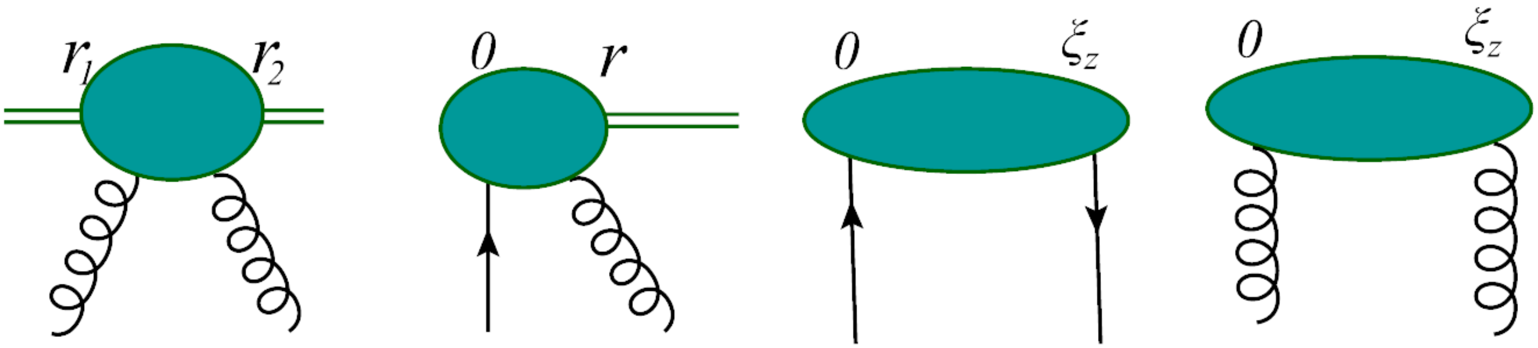
$$(l_+, l_-, \vec{l}_\perp) \sim (1, \lambda^2, \lambda) \text{ as } \lambda \to \infty$$
$$l_- l_+ \sim l_\perp^2 \sim \lambda^2$$

- Non-local in "-" direction in space-time

UV divergent topologies



UV finite topologies

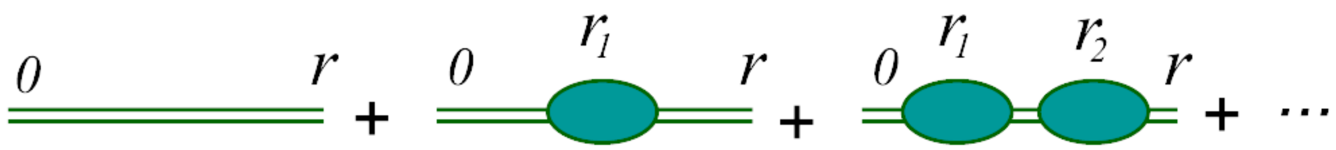


- **The last diagram: no mixing between quasi-quark PDF and quasi-gluon PDF**

Outline

Renormalization

➢ Renormalization



$$1 + c\int_0^r dr_1 + c^2 \int_0^r dr_1 \int_{r_1}^r dr_2 + \cdots$$
$$= \mathcal{P} e^{\,c\int_0^r dr'} = e^{\,c\,r}\,,$$

- It is allowed to introduce an overall factor $e^{-c|\xi_z|}$ to remove all power UV divergences

➢ Interpretation

- Mass renormalization of test particle

Dotsenko, Vergeles, NPB (1980)

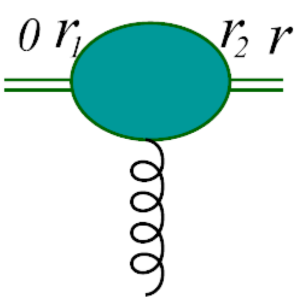
Log divergence related to gaugelink

Dotsenko, Vergeles, NPB (1980)

➢ Log div. from gaugelink self energy

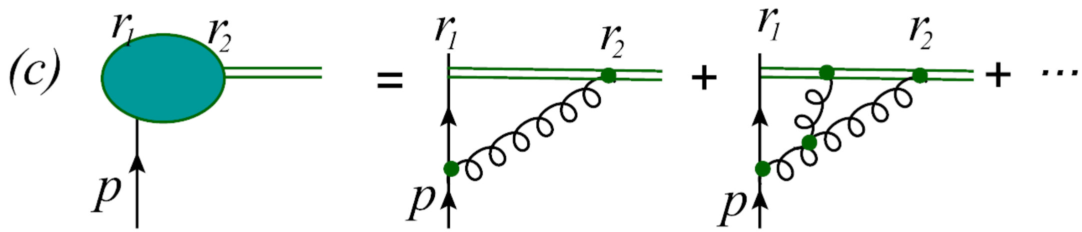
- Besides power divergence, there are also logarithmic UV divergences
- It is known that these divergences can be removed by a "wave function" renormalization of the test particle, Z_{wq}^{-1}.

➢ Log div. from gluon-gaugelink vertex



- Logarithmic UV: can be absorbed by the coupling constant renormalization of QCD.

UV from vertex correction



➢ Remove UV div. at fixed order

- The most dangerous UV diagram, may mix with other operators
- Locality of UV divergence: no dependence on $r_2 - r_1$ or p
- UV divergence is proportional to quark-gaugelink vertex at lowest order, with a constant coefficient
- A constant counter term is able to remove this UV divergence.

➢ Renormalization to all-orders

- Using bookkeeping forests subtraction method, the net effect is to introduce a constant multiplicative renormalizaton factor Z_{vq}^{-1} for the quark-gaugelink vertex.

Renormalization

Ishikawa YQM, Qiu, Yoshida, 1707.03107

- **Using renormalized QCD Lagrangian:**
  - All UV divergences (too all orders) can be removed by the following renormalization

$$\tilde{F}^{R}_{i/p}(\xi_z, \tilde{\mu}^2, p_z) = e^{-C_i|\xi_z|} Z_{wi}^{-1} Z_{vi}^{-1} \tilde{F}^{b}_{i/p}(\xi_z, \tilde{\mu}^2, p_z)$$

- **Renormalization: multiplicative factor, not mix with other operators**
  - Significantly different from normal PDFs
- **Quasi quark PDF is indeed a "good lattice cross section"**

Proof using auxiliary field

- Replace gauge link by auxiliary field

$$O(x,y) = \overline{\psi}(x)\Gamma L(x,y)\psi(y)$$

Ji, Zhang, Zhao, 1706.08962

$$O(x,y) = \overline{\psi}(x)\Gamma Q(x)\overline{Q}(y)\psi(y)$$

  - Nonlocal operator becomes a bi-local operator

- Same conclusion, although with different methods

See Ji's and Zhao's talks

Summary

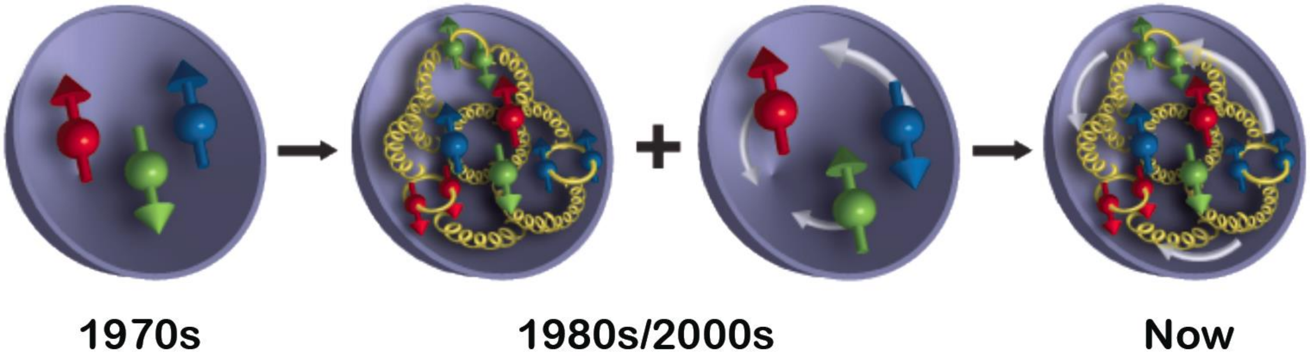
- "Lattice cross section" = hadronic matrix elements that are **calculable** + **renormalizable** + **factorizable**
- Candidates of good "Lattice cross section": Quasi-PDFs, Pseudo-PDFs, Conserved currents, ...
- Quasi-PDFs are now proven to be good "Lattice cross section"
- Find good "lattice cross section" for other nonperturbative quantities: GPDs, TMDs, ...

Thank you!

Back up

Hadron's internal structure

➢ **Hadron: bound state of quarks and gluons under strong interaction**



- Understanding hadron internal structure completely is still beyond the capability of the best minds in the world

➢ **"Structure" functions**

- PDFs, TMDs, GPDs, Wigner functions …

PDFs by fitting data

- **QCD factorization**

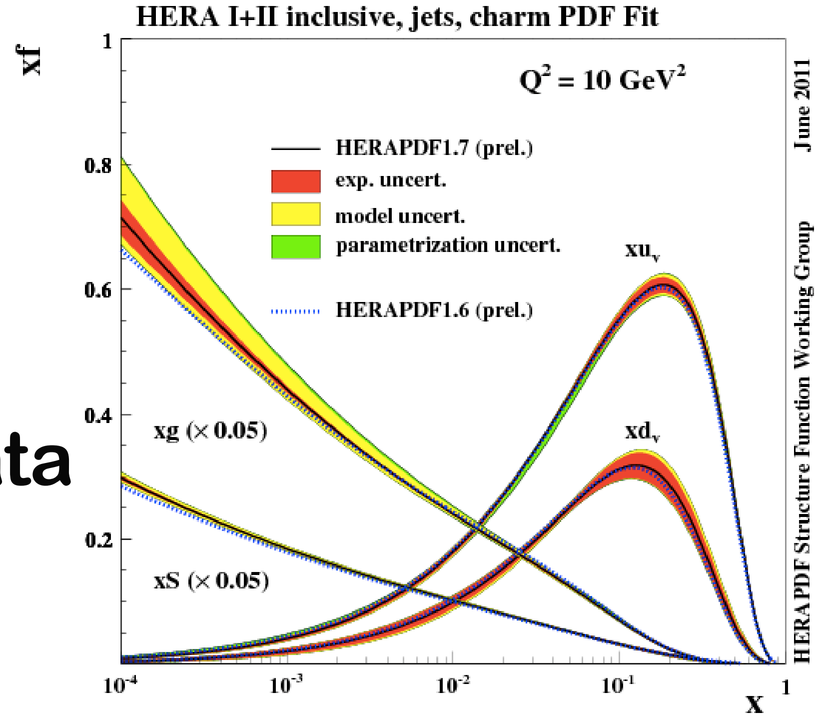
DIS: $\mathrm{F}_2(\mathrm{x_B},\mathrm{Q}^2)=\sum_i C_i(x_B/x,\mu^2/Q^2)\otimes f_{\mathrm{i}/p}(x,\mu^2)$

HH: $\frac{d\sigma}{dydp_T^2}=\sum_{i,j}\frac{d\hat{\sigma}_{ij}}{dydp_T^2}\otimes f_{\mathrm{i}/p}(x,\mu^2)\otimes f_{\mathrm{j}/p}(\mathrm{x}',\mu^2)$

- **DGLAP evolution:**

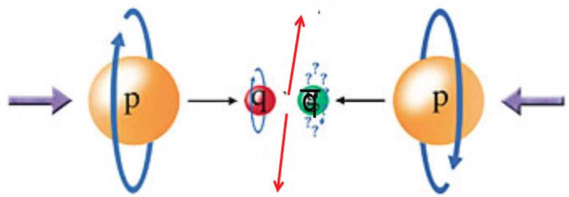
$$\frac{\partial f_{\mathrm{i}/p}(x,\mu^2)}{\partial\ \ln\mu^2}=\sum_j P_{ij}(x/x')\otimes f_{\mathrm{j}/p}(\mathrm{x}',\mu^2)$$

- **Extract PDFs by fitting data**

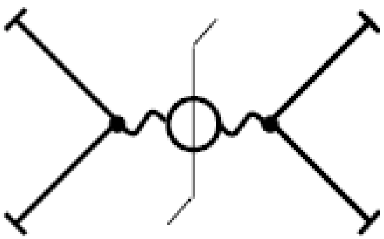


Factorization for more than one hadron

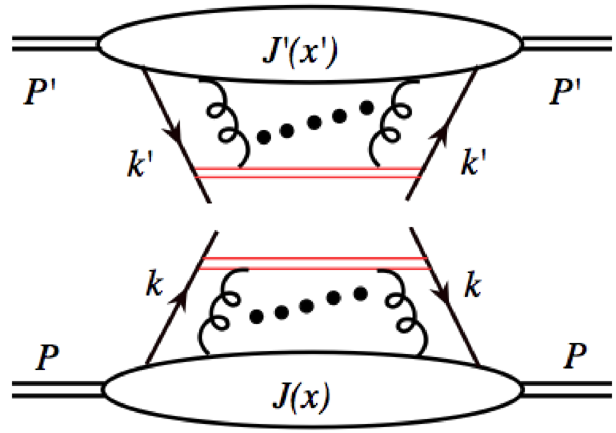
- **Two hadrons:**



$\sigma_{\text{tot}}^{\text{DY}} =$



$\otimes$



$+ O(\frac{1}{QR})$

Hard-part Probe	Parton-distribution Structure	Correction Approximation

- **Predictive power: universality of PDFs**
  - Cancellation of soft interaction between two PDFs

Relation to PDFs

Ji, 1305.1539, 1404.6680

➢ Naïve OPE

$$\overline{\psi}\lambda\cdot\gamma\Gamma\left(\lambda\cdot D\right)^{n}\psi=\lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$$

$$\left\langle P\left|O^{(\mu_1\cdots\mu_n)}\right|P\right\rangle=2a_nP^{(\mu_1}\cdots P^{\mu_n)}$$

- Matching between quasi-PDFs and normal PDFs

$$\tilde{q}(x,\mu^2,P_z)=\int_x^1\frac{dy}{y}Z\left(\frac{x}{y},\frac{\mu}{P_z}\right)q(y,\mu^2)+\mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

- quasi PDFs → normal PDFs, as $P_z\to\infty$.

➢ Question:

- Suffering from UV divergence, is naïve OPE reliable?
- Is the relation between quasi and normal correct?

Ji's approach V.S. Ma-Qiu's approach

➢ **For quasi PDFs:**

- Ji's approach: naïve OPE

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

- Ma-Qiu's approach: QCD factorization (if it is possible)

$$\tilde{\sigma}_{\rm M}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x,\mu^2)\,\mathcal{C}_i\!\left(\frac{\tilde{x}}{x},\tilde{\mu}^2,\mu^2,P_z\right) + O(\tilde{\mu}^{-2} + (\tilde{x}P_z)^{-2})$$

➢ **"Lattice cross sections"- complementary to colliders**

- High energy scattering experiments: sensitive to small x physics
- "Lattice cross sections": sensitive to large x physics

One loop matching: quark→quark

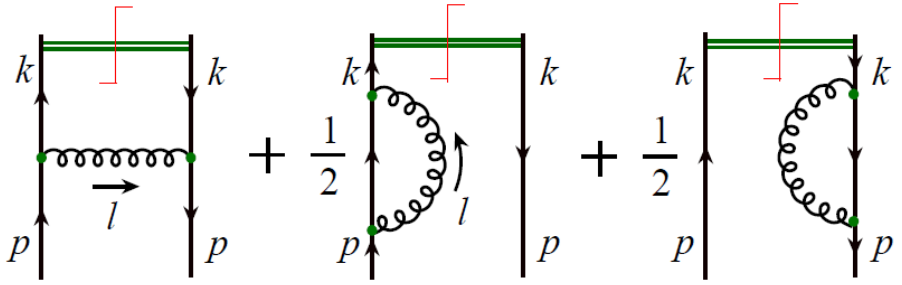
- **Expand the factorization formula**

$$\tilde{f}^{(1)}_{q/q}(\tilde{x}) = f^{(0)}_{q/q}(x) \otimes \mathcal{C}^{(1)}_{q/q}(\tilde{x}/x) + f^{(1)}_{q/q}(x) \otimes \mathcal{C}^{(0)}_{q/q}(\tilde{x}/x)$$

$$\Rightarrow \mathcal{C}^{(1)}_{q/q}(t, \tilde{\mu}^2, \mu^2, P_z) = \tilde{f}^{(1)}_{q/q}(t, \tilde{\mu}^2, P_z) - f^{(1)}_{q/q}(t, \mu^2)$$

- **Feynman diagrams**

Same diagrams for both, but with different gauge



- **Gauge choice**

$A_z = 0$ for $\tilde{f}_{q/q}$

Gluon propagator:

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2 l^\alpha l^\beta}{l_z^2}$$

$A_+ = 0$ for $f_{q/q}$

$$d^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n^\beta + n^\alpha l^\beta}{l_+}$$

One-loop expression

- After the integration of energy component by using residue theory

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta\left(1-\tilde{x}-y\right)-\delta\left(1-\tilde{x}\right)\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$

$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

where $y = l_z/P_z,\ \lambda^2 = l_\perp^2/P_z^2,\ C_F = (N_c^2-1)/(2N_c)$

- Cancellation of CO divergence

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

Only the first term is CO divergent for, which is the same as normal PDF - necessary!

One-loop coefficient functions

- $\overline{MS}$ scheme for normal PDF

$$\mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z) = \tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z) - f^{(1)}_{q/q}(t,\mu^2)$$

$t = \tilde{x}/x$

$$\frac{\mathcal{C}^{(1)}_{q/q}(t)}{C_F\frac{\alpha_s}{2\pi}} = \left[\frac{1+t^2}{1-t}\ln\frac{\tilde{\mu}^2}{\mu^2}+1-t\right]_+ + \left[\frac{t\Lambda_{1-t}}{(1-t)^2}+\frac{\Lambda_t}{1-t}\right.$$
$$\left. + \frac{\mathrm{Sgn}(t)\Lambda_t}{\Lambda_t+|t|} - \frac{1+t^2}{1-t}\left[\mathrm{Sgn}(t)\ln\left(1+\frac{\Lambda_t}{2|t|}\right)+\mathrm{Sgn}(1-t)\ln\left(1+\frac{\Lambda_{1-t}}{2|1-t|}\right)\right]\right]_N$$

where $\Lambda_t = \sqrt{\tilde{\mu}^2/P_z^2+t^2}-|t|$, $\mathrm{Sgn}(t) = 1$ if $t\geq 0$, and -1 otherwise

Coefficient functions for all partonic channels are free of CO div.

Exploratory studies

Lin et al. 1402.1462
Alexandrou et al. 1504.07455

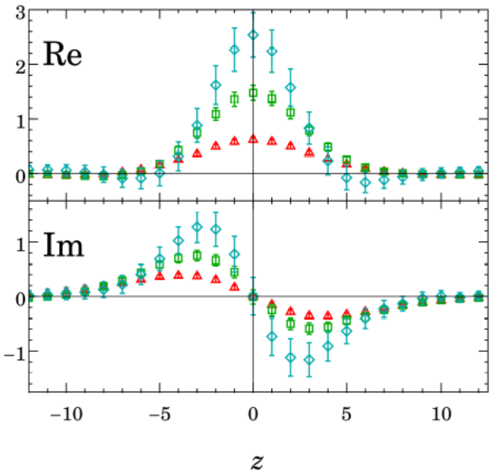


FIG. 1. The real (top) and imaginary (bottom) parts of the nonlocal isovector matrix element h of Eq. 3 computed on a lattice with the nucleon momentum P_z (in units of $2\pi/L$) = 1 (red triangles), 2 (green squares), 3 (cyan diamonds).

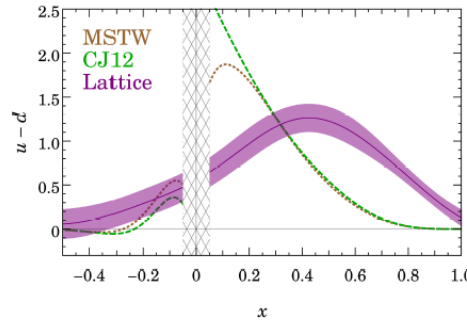


FIG. 2. The unpolarized isovector quark distribution $u(x) - d(x)$ computed on the lattice (purple band), compared with the global analyses by MSTW [13] (brown dotted line), and CTEQ-JLab (CJ12, green dashed line) [14] with medium nuclear correction near $(1.3\text{GeV})^2$. The negative x region is the sea quark distribution with $\bar{q}(x) = -q(-x)$.

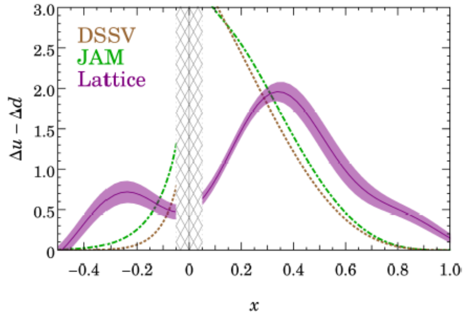


FIG. 3. (top) The isovector helicity distribution $\Delta u(x) - \Delta d(x)$ (purple band) computed on the lattice, along with selected global polarized analyses by JAM [19] (green dot-dashed) and DSSV09 [3] (brown dotted line). The corresponding sea-quark distributions are $\Delta\bar{q}(x) = \Delta q(-x)$.

- Works, good convergence
- Not consistent with experimental data
- Renormalization and further matching is needed

Matching

➢ Additional matching:

$$\tilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, 1/a, P_z) \overset{\mathcal{Z}}{\longleftrightarrow} \tilde{\sigma}_{\mathrm{E}}(\tilde{x}, \tilde{\mu}^2, P_z)$$

$$\Updownarrow$$

$$\tilde{\sigma}_{\mathrm{M}}(\tilde{x}, \tilde{\mu}^2, P_z) \overset{\mathcal{C}}{\longleftrightarrow} f_{i/h}(x, \mu^2)$$

- **Lattice perturbation theory**
- **Nonperturbative matching**

In progress