

# Longitudinal beam dynamics for the accelerator chain

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- Survey of LHC longitudinal dynamics
- Parameter design of SPPC longitudinal dynamics
- Preliminary consideration of longitudinal dynamics of SPPC injector chain
- Summary

Survey of LHC longitudinal dynamics

# LHC beam production and requirements

LHC proton beam production:

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- Source  $\rightarrow$  Linac2 (Linac4)  $\rightarrow$  PSB  $\rightarrow$  PS  $\rightarrow$  SPS  $\rightarrow$  LHC
- Extraction kinetic energies or momenta: 50 MeV (160 MeV) → 1.4 GeV (2 GeV) → 25 (26) GeV/c → 450 GeV/c → 7 TeV
- The requirements from luminosity:
- the beam emittance (small aperture of SC magnet)
- the total beam intensity k<sub>b</sub>N<sub>b</sub> (synchrotron radiation)
- the beam-beam effect ( $\propto N_b / \varepsilon_n \rightarrow$  tune spread, footprint)
- the space-charge limits in the injectors
- Longitudinal emittance requirements:
- ✓ small at injection (small ∆p/p to ease beam transport from the SPS through the two ~2.5 km long lines) (to avoid injection loss)
- Iarger in collision (avoid transverse emittance blow-up by Intra-beam scattering)



 $L = \frac{k_b N_b^2 f_{rev} \gamma}{4\pi\varepsilon_{-}\beta^*}$ 

transverse beam brightness ~Ν<sub>b</sub>/ε<sub>n</sub>

**学園科学院為能物現研究所** Institute of High Energy Physics Chinese Academy of Sciences Survey of LHC longitudinal dynamics

## LHC RF considerations

- 400MHz (main) + 200MHz(capture)
- At injection into the LHC :
  - 200 MHz RF system only for capture.
  - Four cavities, 0.75 MV each, which can be pushed up to 1 MV.
  - For capture the operational total voltage at 200 MHz is 3 MV;
- After capture:
  - 400 MHz RF system adiabatically is increased up to 8 MV
  - 200 MHz RF system is decreased to zero;
  - Acceleration only with 400 MHz.
- On the flat top:
  - the emittance is 2.5 eVs.
  - $f_{rf}$ =400MHz,  $V_{rf(max)}$ =16MV to produce ~1 ns long bunches.



#### LHC RF considerations

- The main RF system provides some damping of injection errors and natural Landau damping. (No longitudinal damper)
- Emittance increase during acceleration to reduce IBS growth rate but this will be in the capabilities of the RF system.
- Controlled emittance increase by excitation with band-limited noise. (  $_{\mathcal{E}_s} \propto \sqrt{E}$  to optimise instability threshold)
- The longitudinal emittance at top energy defined by intra-beam scattering lifetime (considering synchrotron radiation damping), RF lifetime and instability threshold considerations.
- A transverse feedback system damps transverse injection errors and stabilizes the beam against the resistive wall instability.

#### 

#### LHC main beam and RF parameters

Table 6.1: The Main Beam and RF Parameters

	Unit	Injection	Collision	
		450 GeV	7 TeV	
Bunch area $(2\sigma)^*$	eVs	1.0	2.5	
Bunch length $(4\sigma)^*$	ns	1.71	1.06	
Energy spread $(2\sigma)^*$	10-3	0.88	0.22	
Intensity per bunch	$10^{11}$ p	1.15	1.15	
Number of bunches		2808	2808	
Transverse emittance V/H	μm	3.75	3.75	
Intensity per beam	A	0.582	0.582	
Synchrotron radiation loss/turn	keV	-	7	
Longitudinal damping time	h	( <b>L</b> )	13	
Intrabeam scattering growth time - H	h	38	80	
- L	h	30	61	
Frequency	MHz	400.789	400.790	
Harmonic number		35640	35640	
RF voltage/beam	MV	8	16	
Energy gain/turn (20 min. ramp)	keV	485		
RF power supplied during acceleration/ beam	kW	~275		
Synchrotron frequency	Hz	63.7	23.0	
Bucket area	eVs	1.43	7.91	
RF (400 MHz) component of beam current	A	0.87	1.05	

controlled emittance blow-up

\* The bunch values at 450 GeV are an upper value for the situation after filamentation, ~ 100 ms after each batch injection. The bunch parameters at injection are described in the text.



# Some formulas

• Momentum compaction factor:

 $\alpha_p = \frac{1}{\delta} \frac{\Delta C}{C} = \frac{1}{C} \oint \frac{D(s)}{\rho(s)} ds \approx \frac{\langle D \rangle}{R} \approx \frac{1}{v_x^2}$ 

- Phase slip factor:  $\eta = \alpha_p \frac{1}{\gamma^2}$
- Synchrotron frequency:

 $f_s = \frac{\omega_0}{2\pi} \sqrt{-\frac{heV_{RF}\eta\cos\phi_s}{2\pi\beta_s^2 E}}$ 

• Bucket half height:

$$\left(\frac{\Delta E}{E}\right)_{\max} = \pm \beta \sqrt{\frac{eVG(\varphi_s)}{\pi h \eta E_s}}$$
$$G(\varphi_s) = (\pi - 2\varphi_s)\sin\varphi_s - 2\cos\varphi_s$$

Bucket area:

$$A_{\rm B} = \frac{16}{\omega_{\rm RF}} \sqrt{\frac{\beta^2 E e V}{2\pi h |\eta|}} \alpha_b(\phi_s)$$

- Bunch area:  $\varepsilon_s = 4\pi\sigma_t\sigma_{\Delta E/E}E$
- Bunch length:

$$\sigma_{s} = RA_{\rm rms}^{1/2} \left(\frac{\omega_{0}}{\pi\beta^{2}E}\right)^{1/2} \left(\frac{2\pi\beta^{2}E|\eta|}{heV|\cos\phi_{s}|}\right)^{1/4}$$

• Momentum spread:

$$\sigma_{\delta} = A_{rms}^{1/2} \left(\frac{\omega_0}{\pi\beta^2 E}\right)^{1/2} \left(\frac{heV|\cos\phi_s|}{2\pi\beta^2 E|\eta|}\right)^{1/4}$$

- The relation between bunch length and momentum spread:  $\sigma_z = \frac{R|\eta|}{\upsilon_s} \sigma_{\delta}$
- Energy loss per turn:

$$U_0(\text{KeV}) = 7.783 \times 10^{-12} \times \frac{E^4(\text{GeV})}{o(\text{m})}$$

• Synchrotron radiation damping time:  $\tau_E = \frac{1}{2} \frac{T_0 E_0}{U_0}$   $\tau_x = 2\tau_E$ 



#### The basic starting point

- According to the basic requirements for luminosity, and meanwhile taking luminosity upgrade into account.
- Besides, leaving a margin of certain parameter space for RF system to upgrade.

# Luminosity





#### Luminosity



- Only a little effect on luminosity by shortening bunch length.
- Luminosity upgrade measures:
  - decreasing β<sup>\*</sup>, meanwhile shortening bunch length;
  - increasing beam-beam parameter;
  - increasing bunch numeber in order to increase single beam current

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#### The requirements for SPPC longitudinal dynamics

Input parameters	Value		
Circumference (km)	100		
Energy (TeV)	37.5		
Transition gamma $\gamma_{tr}$	99.21		
Bunch intensity	1.5 x10 <sup>11</sup>		
Number of bunches	10080		
Bunch spacing (ns)	25		
Bunch length during physics (m)	0.0755		
RF frequency (MHz)	400		

#### Two constraints:

- Intra-beam Scattering
- Beam instability limits

#### > The goals:

- How to achieve this goal of bunch length 7.55cm.
- The variable range of bunch length for luminosity upgrade.

#### The design of longitudinal dynamics

- Determination of longitudinal emittance (IBS and beam instabilities need to be considered, pivotal issue);
- Given bunch length, we can get momentum spread;
- Determination of RF cavity voltage (f<sub>rf</sub> = 400MHz)



## Intra-beam Scattering (IBS)

#### Intrabeam Scattering (IBS):

Multiple small-angle Coulomb scattering of charged particle beams
An increase in both transverse and longitudinal emittance.

#### The effects of IBS:

- Redistribution of beam momenta
- Beam diffusion with impact on the beam quality (Brightness, luminosity, etc)

IBS growth rate:

$$\frac{1}{T_i} \propto \frac{N_b}{\gamma^4 \varepsilon_{xn} \varepsilon_{yn} \varepsilon_{sn}} f(optics, \gamma, \varepsilon_{xn}, \varepsilon_{yn}, \varepsilon_{sn})$$

- Theoretical models and their approximations
  - Classical models of Piwinski (P) and Bjorken Mtingwa (BM)
    - Complicated integrals averaged around the rings.
    - Depend on optics and beam properties.
  - High energy approximations Bane, CIMP, J.Wei, etc
    - Integrals with analytic solutions
- ➤ Tools: MADX, BETACOOL, etc



#### Intra-beam Scattering (IBS)



- At injection, longitudinal emittance between 1.5 eVs and 2.5 eVs may be a good choice; (injection time 840s)
- In collision, longitudinal emittance between 6 eVs and 9 eVs may be a good choice; (time during physics 14.2h)



#### **Beam instability limits**

- Two instability bottlenecks:
  - Loss of Landau damping
  - Transverse mode coupling instability (TMCI)
- Loss of Landau damping
- Landau damping of the instability can come from the spread in the synchrotron frequency due to the nonlinearity of the synchrotron force in the bunch.
- The coherent tune shift induced by the broad-band impedance for a longitudinal mode of order n can be approximate to

$$\Delta Q_n^L \approx \frac{|n|}{|n|+1} \frac{Q_s I_b}{3h V_{rf}} \left(\frac{2\pi R}{L}\right)^3 \operatorname{Im}\left(\frac{Z_L}{n}\right)_{eff}$$

• The synchrotron tune spread  $\Delta Q_s = (\pi^2 / 16)(hL / 2\pi R)^2 Q_s$ 

**Requiring**  $\Delta Q_L \leq \frac{1}{4} \Delta Q_s$ 

$$\frac{\left|\operatorname{Im} Z_{\prime\prime}\right|}{n} \leq \frac{3\pi^2}{64} \frac{h^3 V_{rf}}{I_b} \left(\frac{L}{2\pi R}\right)^2$$

 $L = 4\sigma_{c}$ 



#### **Beam instability limits**



#### Longitudinal impedance of less than 0.1 $\Omega$ needs to be studied !

- In order to Landau damp longitudinal coupled-bunch instability, a large spread in synchrotron frequency inside the bunch is required.
- One way: install a higher harmonic cavity (800MHz RF cavity needs to be studied)



#### Beam instability limits

- Transverse mode coupling instability (TMCI)
- This instability arises when the relative tune shift of two adjacent head-tail modes equals the synchrotron tune.
- The corresponding impedance threshold :  $\operatorname{Im}(Z_T)_{eff} \leq 2 \frac{E}{e} \frac{Q_s}{I_b \beta_{av}} \frac{L}{R}$



 more stringent requirement for transverse impedance.

Tansverse impedance of less than  $60M\Omega/m$  needs to be studied !



The relation of longitudinal emittance, momentum spread and bunch length

Momentum spread and bunch area vary with RF Voltage:



- bunch length: σz=7.55cm
- RF frequency: 400MH



 $\sigma_s \propto 1$ 

#### Bunch length varies with RF voltage:





 Momentum spread less than 1x10-4 will be a good choice !



The relation of longitudinal emittance, momentum spread and bunch length

Under different RF voltages, the relation between longitudinal emittance and bunch length is



 $\varepsilon_{s} = 4\pi\sigma_{\Delta E/E}\sigma_{z}\frac{E}{v} = 4\pi\beta\sigma_{\delta}\sigma_{z}\frac{E}{c}$  $= 4\pi\beta\frac{\upsilon_{s}}{R|\eta|}\sigma_{z}^{2}\frac{E}{c} \qquad \propto \sigma_{z}^{2}$ 

Filling factor in momentum



#### **SPPC longitudinal dynamics parameters**

1			LHC		HE-LHC	SPPC		
2			Injection	Collision	Collision	injection	collision1	
3	Proton energy	[GeV]	450	7000	16500	2100	37500	
4	Relativistic gamma		480.61	7461.52	17586.52	2239.16	39968.09	
5	Relativistic beta		1.00	1.00	1.00	1.00	1.00	
6	Bending radius	[m]	2803.95	2803.95	2803.95	10415.4	10415.4	
7	Number of particles per bunch	10 <sup>11</sup>	1.15	1.15	1.30	1.50	1.50	
8	Number of bunches		2808	2808	1404	10080	10080	
9	Longitudinal emittance(4o)	[eVs]	1	2.5	4	2	8.4	
10	Transverse normalized emittance	[µm rad]	3.5	3.75	2.59	2.4	2.4	
11	Circulating beam current	[A]	0.582	0.582	0.329	0.727	0.727	
12	Stored energy per beam	[MJ]	23.3	362	482.5	508.7	9083.3	
13	Momentum compaction	10-4	3.225	3.225	3.225	1.016	1.016	
14	Slip factor η	10-4	3.182	3.225	3.225	1.014	1.016	
15	Gamma transition $\gamma_{tr}$		55.68	55.68	55.68	9 <mark>9.21</mark>	99.21	
16	Ring circumference	[m]	26658.883	26658.883	26658.883	1.00E+05	1.00E+05	
17	Revoluntion frequency	[kHz]	11.245	11.245	11.245	3.00	3.00	
18	RF frequency	[MHz]	400.8	400.8	400.8	400.0	400.0	
19	Bunch filling factor		0.788	0.788	0.788	0.756	0.756	
20	Bunch seperation	ns	25	25	25	25	25	
21	Harmonic number		35640	35640	35640	133333	133333	
22	Synchrotron phase	rad	Π	3.1412	3.1352	Π	3.1046	
23	Total RF voltage	[MV]	8	16	32	20	40	
24	Synchrotron frequency	[Hz]	63.7	23.0	21.2	13.6	4.55	
25	Bucket area	[eVs]	1.43	7.91	17.18	4.48	26.73	
26	Bucket half height(ΔE/E)	[10 <sup>-3</sup> ]	1.00	0.36	0.33	0.67	0.22	
27	RMS bunch length	[cm]	11.24	7.55	6.48	9.00	7.55	
28	Full bunch length(40)	[ns]	1.50	1.01	0.86	1.20	1.01	
29	Momentum spread $\sigma_{\delta}$	10-4	4.72	1.13	0.89	2.53	0.71	
30	Energy loss per turn U <sub>o</sub>	[keV]	1.15E-04	6.67	205.78	0.01	1477.88	
31	Longitudinal damping time τ <sub>ε</sub>	[h]	48524.46	12.97	0.99	6680.90	1.17	
32	Intrabeam scattering growth time-H	[h]	38	80	(*************************************	46.7	166	
33	Intrabeam scattering growth time-L	[h]	30	61		39.9	171	

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- longitudinal emittance:
- 2 eVs(inj.) 8.4eVs (col.)
- Bunch length:

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- 9.0 cm(inj.) 7.55 cm(col.)
- Momentum spread(x10<sup>-4</sup>):
- 2.53(inj.) 0.71(col.)
- RF voltage:
- 20MV(inj.) 40MV(col.)



# Possible problems faced in injectors

- Matching between injector chain;
- Space charge effect at low energy;
- Beam instability (Which instability dominate)
- IBS (may not important, but still to calculate);
- Beam loading in the RF cavity.....
- Need special study for p-RCS;



### Matching between injector chain

- How to transfer beam from accelerator A to B?
- Accelerator A

Accelerator B

Matched bunch-to-bucket transfer



Synchrotron phase is  $\pi$  at injection and extraction of each injectors; i.e. extraction will happen at flat top of each injector;

- More stable;
- Time structure preserved;

#### > Other considerations:

- Longitudinal emittance is conserved during transfer to the next;
- Longitudinal emittance may blow up during acceleration at each ring, but keep constant as far as possible;



Iongitudinal dynamics of SPPC injector chain

#### Parameter design of SS

1			5	S
2			Injection	Extraction
3	Proton energy	[GeV]	180	2100
4	Relativistic gamma		192.84	2239.16
5	Relativistic beta		1.00	1.00
6	Ring circumference	[m]	7200	7200
7	Beam rigidity	[Tm]	603.5	7008.0
8	Dipole fied	[T]	0.71	8.26
9	Bending radius	[m]	848.42	848.42
10	Dipole filling factor		0.	74
11	Repetition period	[s]	3	80
12	Fillings per cycle			2
13	Number of particles per bunch	10 <sup>11</sup>	1.5	1.5
14	Number of bunches		672	672
15	Bunch spacing	[ns]	25	25
16	Accummulated particles	10 <sup>14</sup>	1.01	1.01
17	Revolution frequency	[kHz]	41.7	41.7
18	Revolution period	[µs]	24.0	24.0
19	Circulating beam current	[A]	0.67	0.67
20	Stored energy	[MJ]	2.9	33.9
21	Longitudinal emittance(4o)	[eVs]	1.0	2.0
22	Gamma transtion		25.65	25.65
23	Momentum compaction factor	10-3	1.52	1.52
24	Slipping factor	10 <sup>-3</sup>	1.49	1.52
25	RF frequency	[MHz]	200	200
26	Harmonic number		4800	4800
27	Ramping up time	[s]	1	2
28	Total RF voltage	[MV]	6	8
29	Synchrotron phase	rad	π	π
30	Synchrotron frequency	[Hz]	256.92	87.63
31	Bucket area	[eVs]	6.24	24.38
32	Bucket half height( $\Delta E/E$ )	[10 <sup>-3</sup> ]	1.72	0.58
33	RMS bunch length	[cm]	19.18	13.72
34	Full bunch length(40)	[ns]	2.56	1.83
35	Momentum spread $\sigma_{\delta}$	10 <sup>-3</sup>	0.69	0.17
36	Intrabeam scattering growth time-H	[h]	0.86	253
37	Intrabeam scattering growth time-L	[h]	4.97	87

Controlled emittance blow-up

- Note: the bunch length is 1.83ns at extraction of SS, but 1.2ns(SPPC inj. @400MHz), so 200MHz RF cavity is needed for SPPC capture.
- IBS growth time is enough.



# longitudinal dynamics of SPPC injector chain

#### **Parameter design of MSS**

1	injection into SPPC		MSS		1	1 beam application		MSS		
2	injection into SPPC		Injection	Extraction		2	beam application		Injection	Extraction
3	Proton energy	[GeV]	10	180	80MH7	3	Proton energy	[GeV]	10	180
4	Relativistic gamma		11.66	192.84		4	Relativistic gamma		11.66	192.84
5	Relativistic beta		1.00	1.00	RFcavity	5	Relativistic beta		1.00	1.00
6	Ring circumference	[m]	3478.24	3478.24	is needed	6	Ring circumference	[m]	3478.24	3478.24
7	Beam rigidity	[Tm]	36.4	603.5		7	Beam rigidity	[Tm]	36.4	603.5
8	Dipole fied	[T]	0.10	1.699		8	Dipole fied	[T]	0.10	1.699
9	Bending radius	[m]	355.23	355.23		9	Bending radius	[m]	355.23	355.23
10	Dipole filling factor		0.	75		10	Dipole filling factor		0.75	
11	Repetition rate	[Hz]	0	.5		11	Repetition rate	[Hz]	C	.5
12	Fillings per cycle			3		12 Fillings per cycle			3	
13	Number of particles per bunch	1011	1.5	1.5		13	Number of particles per bunch	1011	7.59	7.59
14	Number of bunches		336	336		14	Number of bunches		336	336
15	Bunch spacing	[ns]	25	25		15	Bunch spacing	[ns]	25	25
16	Accummulated particles	10 <sup>14</sup>	0.50	0.50		16	Accummulated particles	10 <sup>14</sup>	2.55	2.55
17	Revolution frequency	[kHz]	86.3	86.3		17	Revolution frequency	[kHz]	86.3	86.3
18	Revolution period	[µs]	11.6	11.6		18	Revolution period	[µs]	11.6	11.6
19	Average beam current	[µA]	4.04	4.04		19	Average beam current	[µA]	20.43	20.43
20	Beam power	[MW]	0.04	0.73		20	Beam power	[MW]	0.20	3.68
21	Transverse normalized emittance	[µm rad]	2.4	2.4		21	Transverse normalized emittance	[µm rad]	20.4	20.4
22	Longitudinal emittance(4o)	[eVs]	0.5	1.0		22	Longitudinal emittance(4o)	[eVs]	0.5	1.0
23	Gamma transtion		87.36i	87.36i		23	Gamma transtion		87.36i	87.36i
24	Momentum compaction factor	10-4	-1.31	-1.31		24	Momentum compaction factor	10-4	-1.31	-1.31
25	Slipping factor	10-4	-74.89	-1.58		25	Slipping factor	10-4	-74.89	-1.58
26	RF frequency	[MHz]	40.02	40.02		26	RF frequency	[MHz]	40.02	40.02
27	Harmonic number	1991 H 1991 H	464	464		27	Harmonic number		464	464
28	Ramping up time	[s]	0	.8		28	Ramping up time	[s]	C	).8
29	Total RF voltage	[MV]	3	5		29	Total RF voltage	[MV]	3	5
30	Synchrotron phase	rad	0	0		30	Synchrotron phase	rad	0	0
31	Synchrotron frequency	[Hz]	3513.20	155.22		31	Synchrotron frequency	[Hz]	3513.20	155.22
32	Bucket area	[eVs]	2.36	88.97		32	Bucket area	[eVs]	2.36	88.97
33	Bucket half height(ΔE/E)	[10 <sup>-3</sup> ]	2.34	4.91		33	Bucket half height(ΔE/E)	[10 <sup>-3</sup> ]	2.34	4.91
34	RMS bunch length	[cm]	110.22	25.38		34	RMS bunch length	[cm]	110.22	25.38
35	Full bunch length(40)	[ns]	14.70	3.38		35	Full bunch length(4σ)	[ns]	14.70	3.38
36	Momentum spread $\sigma_{\delta}$	10-3	1.08	0.52		36	Momentum spread $\sigma_{\delta}$	10 <sup>-3</sup>	1.08	0.52
37	Intrabeam scattering growth time-H	[s]	167	146		37	Intrabeam scattering growth time-H	[s]	4366	2746
20	Intrahoam coattoring growth time I	[c]	2001	1003		00	Introheam coattoring growth time I	[c]	1250	2612



#### Summary

- The longitudinal dynamics parameters of SPPC have been given to achieve the required bunch length and stability.
- Based on the beam instability, a shorter bunch length will set a more stringent requirement for impedance. Longitudinal impedance of less than 0.1Ω and transverse impedance of less than 60MΩ/m need to be studied.
- Preliminary parameters design of SS and MSS have been given taking matching between injectors and IBS into considerasion.

# Next to do

- Longitudinal dynamics of injector chain considering space charge and beam instability;
- Special study for p-RCS is needed;
- Longitudinal dynamics during acceleration;



# Thanks for your attention !

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# Back up

# SPS and PS requirements

#### > SPS

- the SPS is an "old" machine and is not optimised as an LHC injector.
- The intensity the SPS can be accelerated (at most 4 PS pulses) (Why does SPS ring have this limitation???)
- The momentum spread acceptance of the PS-SPS line (TT2, TT10) is about  $\pm 0.2\%$  in  $\Delta p/p$ , while the total bunch length has to be below 4 ns to fit into the buckets of the SPS 200 MHz accelerating system, implying a longitudinal emittance of 0.35 eVs per PS bunch.
- the longitudinal emittance will be increased from 0.35 to 1 eVs during SPS acceleration, there is little margin for transverse emittance blow-up in this machine.
- The main challenges for PS complex:
- the unprecedented transverse beam brightness ( $\sim N_b / \epsilon_n$ )
- the production of a bunch train with the LHC spacing of 25 ns before extraction from PS (25GeV)  $\Delta Q \propto -\frac{N}{\sqrt{1-2}}$
- Space charge issues in PS and PSB  $(\beta \gamma^2)_{rel} \varepsilon_n$ International workshop on CEPC 2017 ---- SPPC parallel session



#### Beam instability limits

- Longitudinal microwave instability
- The microwave instability is induced by the resistive part of the coupling impedance. It produce a fast increase of the mαnentum spread when the bunch intensity goes beyond a threshold value and splits a bunch into many mini-bunches.
- Keil-schnell-Boussard criterion:

$$\left|\frac{Z_{II}}{n}\right| \leq \frac{\sqrt{2\pi\eta} \left(\frac{E}{e}\right) \left(\frac{\sigma_z}{R}\right) \left(\frac{\sigma_E}{E}\right)^2}{I_b} \qquad \qquad \left|\frac{Z_{II}}{n}\right| \leq \frac{3}{2} \frac{hV_{rf}}{I_b} \left(\frac{L}{2\pi R}\right)^3$$



$$\begin{cases} \sigma_{z} = \frac{R|\eta|}{\upsilon_{s}} \sigma_{\delta} \\ \upsilon_{s} = \sqrt{-\frac{heV_{RF}\eta\cos\phi_{s}}{2\pi\beta_{s}^{2}E}} \Rightarrow \sigma_{z} = \sigma_{\delta}\sqrt{\frac{C_{ring}\cdot|\eta|\cdot E_{s}c\beta_{s}^{3}}{2\pi e\cdot V_{rf}f_{rf}\cdot|\cos\phi_{s}|}} \\ R = \frac{C_{ring}}{2\pi} \\ h = \frac{f_{rf}}{f_{0}} = \frac{f_{rf}\cdot C_{ring}}{\beta_{s}c} \end{cases}$$