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Georgi-Machacek Model Beyond Tree Level

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CWC, AL Kuo, K Yagyu, PLB774 (2017) 119 [1707.04176]
CWC, AL Kuo, K Yagyu, [1712,xxxxx]

Overview

- Higgs measurements
- Custodial Higgs models
- Georgi-Machacek (GM) model
- Renormalization of GM model
- Numerical results
- Summary

Higgs Physics Program

- Higgs mechanism in SM offers an elegant and minimal way to give mass to weak bosons and charged fermions:

$m_{W,Z}$ through gauge interactions

m_f through Yukawa interactions

$$\mathcal{L}_\Phi \supset |\mathcal{D}_\mu \Phi|^2 + \mu^2 |\Phi|^2 - \lambda |\Phi|^4 - Y \overline{\psi_L} \Phi \psi_R + \text{H.c.}$$

self interaction

- It also features in a self interaction that plays a role in the phase transition in the early Universe.
- After the discovery of 125-GeV Higgs boson, it has become an important program in particle physics to determine its interactions with SM particles (including itself).

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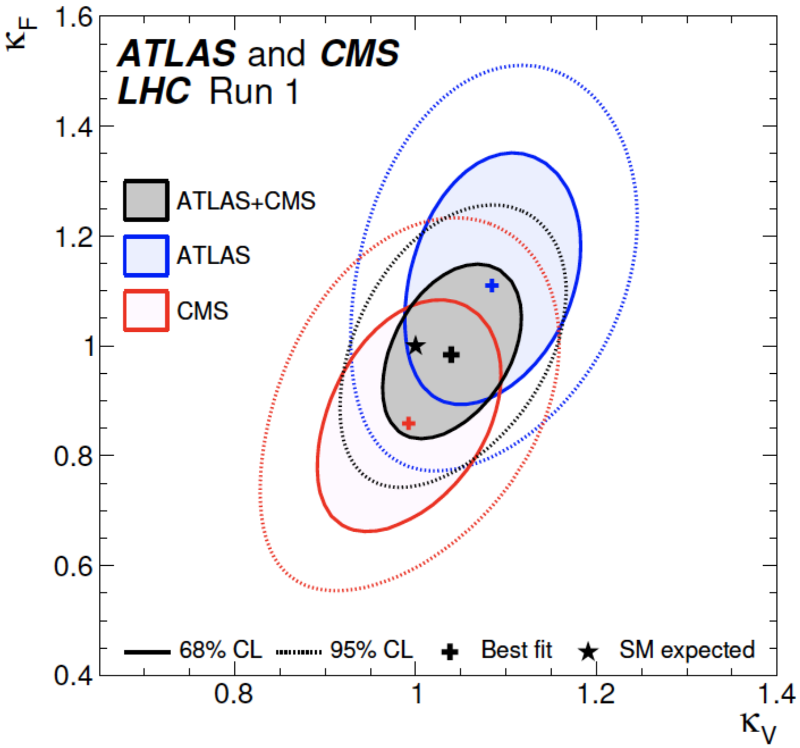
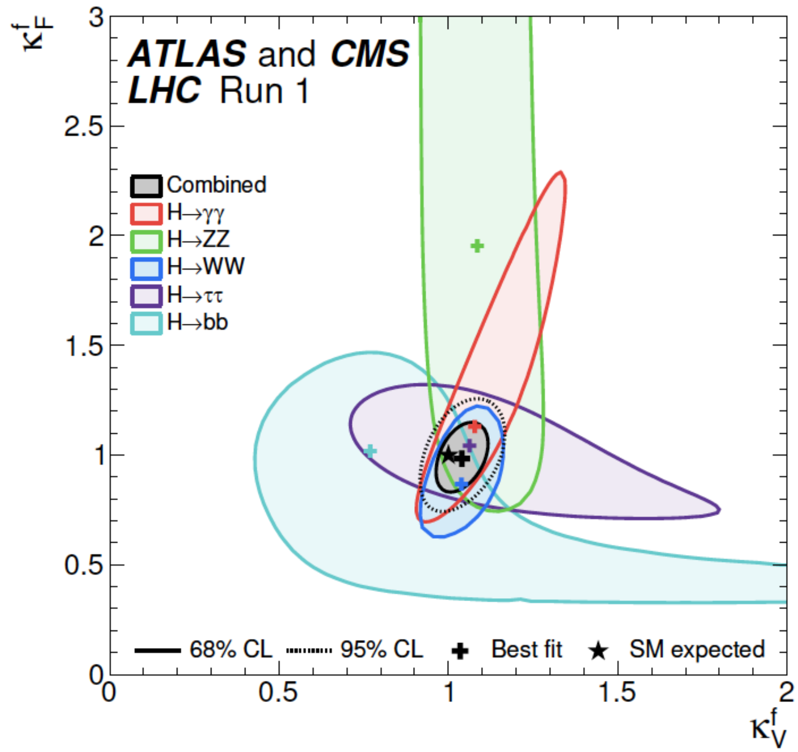
Higgs Physics Program

- Global fits of Higgs couplings (assuming universal scaling factors $\kappa_{F,V}$) from LHC Run-I
  ➠ quite consistent with SM

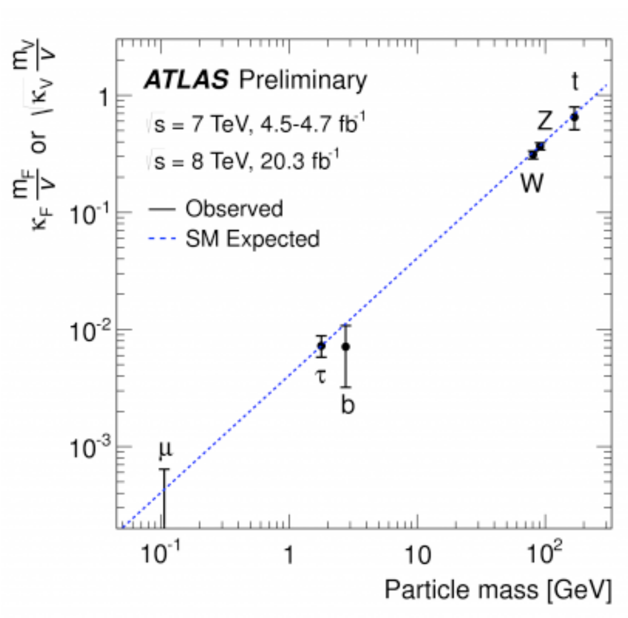


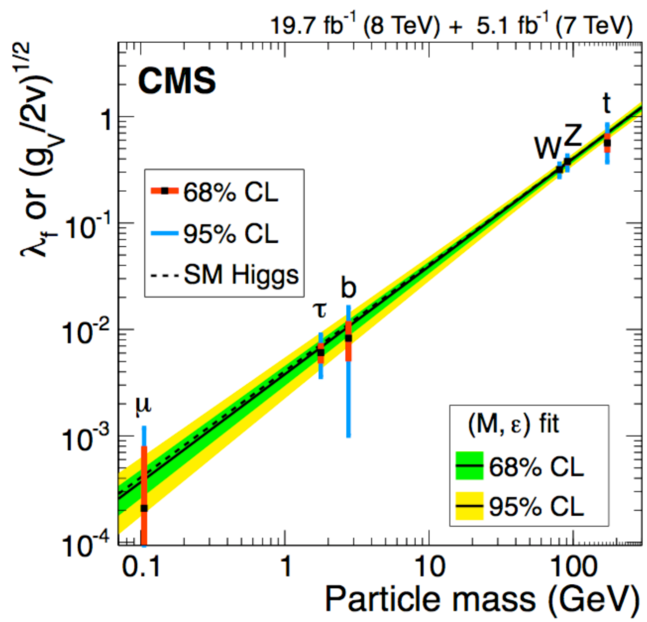


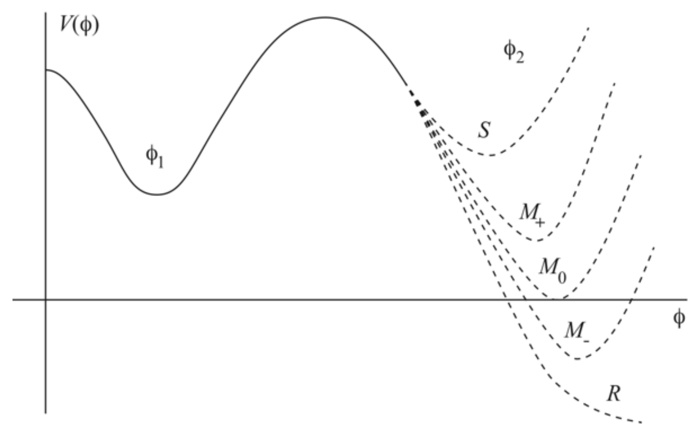
circa 2016 summer

Higgs Physics Program

- Precision coupling measurements are required!
  - ➠ tiny deviations from SM expectations?
  - ➠ hint a non-standard structure in the Higgs sector?
  - ➠ an extended Higgs sector?
  - ➠ how EWSB exactly happens?







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An Extended Higgs Sector

- Compared to fermion and gauge sectors, the scalar sector is less explored experimentally.

- Other than usual symmetries, no guiding principles in constructing the scalar sector:
  - ➠ representations of scalar bosons
  - ➠ numbers of scalar bosons
  - ➠ extra symmetries (continuous/discrete)
  - ➠ required by new physics
    (neutrino mass, DM, EWBG, SUSY, etc)

cf. 3 generations of fermions and 3 gauge interactions

Study of predictions and constraints of models with an extended Higgs sector

Higgs Extensions

- Higgs extensions are subject to a stringent constraint

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1.00040 \pm 0.00024$$ PDG 2014

- In models with an extended Higgs sector, at tree level

$$\rho_{\rm tree} = \frac{\sum_i v_i^2 \left[T_i(T_i+1) - Y_i^2\right]}{\sum_i 2Y_i^2 v_i^2}$$

- If only one new $SU(2)_L$ rep is added to the SM, $\rho_{tree} = 1$ gives the following possibilities:
  - (0,0) – real singlet, ➠ interacting mainly with h_{SM}
  - (1/2,1/2) – doublet, ➠ a popular choice (e.g., 2HDM)
  - (3,2) – septet, (25/2, 15/2), (48,28), (361/2,209/2), etc.

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- One can also choose to add a custodial symmetric rep (n,n) ($n \in \mathbb{N}$) under ($SU(2)_L$,$SU(2)_R$) with vacuum alignment.
  ➠ generalized GM model   Logan, Rentala 2015

Higgs Extensions

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- Simplest CP-conserving custodial Higgs models:
  - real Higgs singlet model (rHSM): Φ_{SM} + S
  - two Higgs doublet model (2HDM): Φ_{SM} + Φ'
  - GM model: Φ_{SM} + Δ

Georgi-Machacek Model

- The Higgs sector includes SM doublet field $\phi(2,1/2)$ and triplet fields $\chi(3,1)$ and $\xi(3,0)$

Georgi, Machacek 1985
Chanowitz, Golden 1985

$$\Phi = \begin{pmatrix} v_\phi & \phi^+ \\ \phi^- & v_\phi \end{pmatrix}, \qquad \Delta = \begin{pmatrix} v_\Delta & \xi^+ & \chi^{++} \\ \chi^- & v_\Delta & \chi^+ \\ \chi^{--} & \xi^- & v_\Delta \end{pmatrix}$$

transformed under $SU(2)_L \times SU(2)_R$ as

$$\Phi \to U_L \, \Phi \, U_R^\dagger \quad \text{and} \quad \Delta \to U_L \, \Delta \, U_R^\dagger$$

with $U_{L,R} = \exp(-i\, \theta_{L,R}^a T^a)$ and T^a being corresponding SU(2) generators.

- Take $v_\chi = v_\xi \equiv v_\Delta$ (aligned VEV).

➠ $SU(2)_L \times SU(2)_R \to$ custodial $SU(2)_V$

➠ $\rho = 1$ at tree level

Georgi-Machacek Model

- The most general Higgs potential allowed by gauge and Lorentz symmetries and built in with custodial symmetry is

assume $m_1^2 < 0$, but $m_2^2 > 0$

$$
\begin{aligned}
V(\Phi,\Delta) =& \frac{1}{2} m_1^2 \mathrm{tr}[\Phi^\dagger \Phi] + \frac{1}{2} m_2^2 \mathrm{tr}[\Delta^\dagger \Delta] + \lambda_1 (\mathrm{tr}[\Phi^\dagger \Phi])^2 + \lambda_2 (\mathrm{tr}[\Delta^\dagger \Delta])^2 \\
&+ \lambda_3 \mathrm{tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 \mathrm{tr}[\Phi^\dagger \Phi]\mathrm{tr}[\Delta^\dagger \Delta] + \lambda_5 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2}\right] \mathrm{tr}\left[\Delta^\dagger T^a \Delta T^b\right] \\
&+ \mu_1 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2}\right] (P^\dagger \Delta P)_{ab} + \mu_2 \mathrm{tr}\left[\Delta^\dagger T^a \Delta T^b\right] (P^\dagger \Delta P)_{ab}
\end{aligned}
$$

$$
\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -(\phi^+)^* & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} (\chi^0)^* & \xi^+ & \chi^{++} \\ -(\chi^+)^* & \xi^0 & \chi^+ \\ (\chi^{++})^* & -(\xi^+)^* & \chi^0 \end{pmatrix}, \quad P = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}
$$

- Decoupling limit: $m_2 \to \infty$

- v_Δ induced by v_Φ through μ_1

Higgs Spectrum

SU(2)L ⊗ SU(2)R

SU(2)V

1 complex triplet + 1 real triplet

Δ: **3 ⊗ 3**

5 ⊕ 3 ⊕ 1

1 complex doublet

Φ: **2 ⊗ 2**

3 ⊕ 1

5 physical parameters to scan

mixing angle β

mixing angle α

$$H_5 \equiv \begin{bmatrix} H_5^{++} \\ H_5^{+} \\ H_5^{0} \\ H_5^{-} \\ H_5^{--} \end{bmatrix}$$

CP-even

mH5

fermiophobic

$$H_3 \equiv \begin{bmatrix} H_3^{+} \\ H_3^{0} \\ H_3^{-} \end{bmatrix}$$

CP-odd

mH3

$$\Phi_3 \equiv \begin{bmatrix} w^{+} \\ z^{0} \\ w^{-} \end{bmatrix}$$

gaugephobic

$$H_1 \equiv [H_1^0]$$

mH1

h (125 GeV)

mh

CP-even

$$\tan\beta = \frac{v_\phi}{2\sqrt{2}v_\Delta}$$

$$v^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} = (246\text{ GeV})^2$$

Custodial Higgs Models

- Couplings of h modified by exotic Higgs fields due to their EW charges, and mixing with Φ_{SM}.
- At tree level, their hVV couplings satisfy

$$\kappa_W = \kappa_Z$$

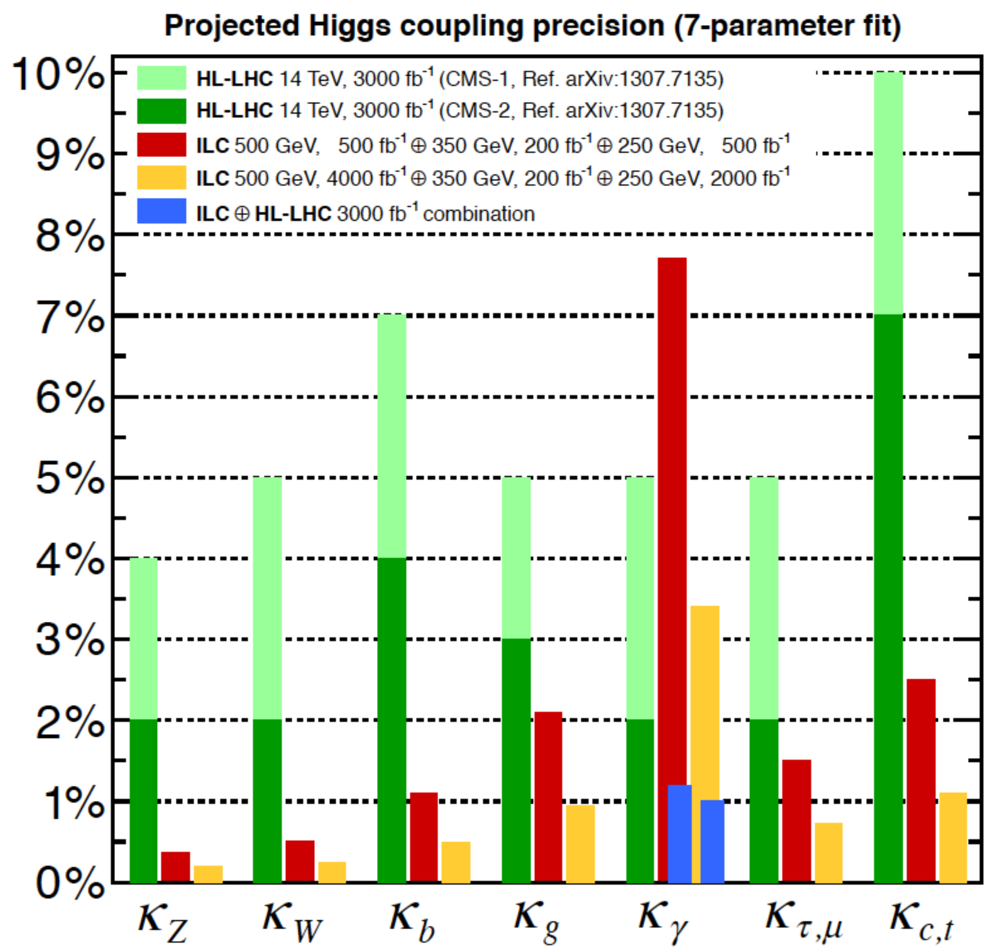
	rHSM	2HDM	GM model
$\Phi^{\rm new}$	S	$H, A, H^\pm$	$H_1, (H_3^0, H_3^\pm), (H_5^0, H_5^\pm, H_5^{\pm\pm})$
parameters $(m_h, v) = (125, 246)$ GeV	$m_H, \alpha, m_S,$ $\mu_S, \lambda_{\Phi S}, \lambda_S$	$m_H, m_A, m_{H^\pm},$ $\mu, \alpha, \tan\beta = v_2/v_1$	$m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha,$ $\tan\beta = v_\Phi/(2\sqrt{2}v_\Delta)$
$\kappa_{W,Z} = g_{hVV}/g_{hVV}^{\rm SM}$	$\cos\alpha$	$\sin(\beta-\alpha)$	$\sin\beta\cos\alpha - \sqrt{\frac{8}{3}}\cos\beta\sin\alpha$

always ≤ 1 through mixing

group factor that makes it possible for the entire factor to be > 1 (mixing required)

Expected Coupling Precision

- All Higgs couplings will be determined by HL-LHC + ILC to O(1) or sub percent level.
  ➠ need to know radiative corrections



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Radiative Corrections

- Radiative corrections can lead to at least two effects:
  - changes in the magnitudes of various couplings
  - deviations from tree-level relations among couplings

  due to various custodial symmetry breaking parameters (couplings, masses).

	rHSM	2HDM	GM model
$\Phi^{\rm new}$	S	$H, A, H^\pm$	$H_1, (H_3^0, H_3^\pm), (H_5^0, H_5^\pm, H_5^{\pm\pm})$
parameters $(m_h, v) = (125, 246)$ GeV	$m_H, \alpha, m_S,$ $\mu_S, \lambda_{\Phi S}, \lambda_S$	$m_H, m_A, m_{H^\pm},$ $\mu, \alpha, \tan\beta = v_2/v_1$	$m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha,$ $\tan\beta = v_\Phi/(2\sqrt{2}v_\Delta)$
$\kappa_{W,Z} = g_{hVV}/g_{hVV}^{\rm SM}$	$\cos\alpha$	$\sin(\beta-\alpha)$	$\sin\beta\cos\alpha - \sqrt{\frac{8}{3}}\cos\beta\sin\alpha$
$\delta\kappa_V$	$-\sin\alpha\,\delta\alpha$	$\cos(\beta-\alpha)(\delta\beta-\delta\alpha)$	$\frac{\partial\kappa_V}{\partial\alpha}\delta\alpha + \frac{\partial\kappa_V}{\partial\beta}\delta\beta + \frac{\partial\kappa_V}{\partial\rho}\delta\rho$

Renormalization of GM Model

- Independent counter terms in the model:

gauge sector

$$m_W^2 \to m_W^2 + \delta m_W^2 \ ,$$
$$m_Z^2 \to m_Z^2 + \delta m_Z^2 \ ,$$
$$\alpha_{\rm em} \to \alpha_{\rm em} + \delta\alpha_{\rm em} \ ,$$
$$B_\mu \to \left(1 + \frac{1}{2}\delta Z_B\right) B_\mu \ ,$$
$$W_\mu^a \to \left(1 + \frac{1}{2}\delta Z_W\right) W_\mu^a \ .$$

scalar sector

$$m_X^2 \to m_X^2 + \delta m_X^2 \ ,$$
$$(X = H_5, H_3, H_1, h)$$
$$\mu_i \to \mu_i + \delta\mu_i \ , (i = 1, 2)$$
$$v_\Delta \to v_\Delta + \delta v_\Delta \ ,$$
$$\nu \to 0 + \delta\nu \ , \ (\nu = v_\xi - v_\chi)$$
$$\alpha \to \alpha + \delta\alpha \ .$$

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Renormalization Conditions

- In addition to the renormalization conditions in the SM to get physical G_F, m_Z, and α_{EM}, the GM model allows one additional condition, which we take to make

$$\alpha_{\rm em} T = \frac{\Pi^{\rm 1PI}_{ZZ}(0) - \Pi^{\rm 1PI}_{ZZ}(0)\big|_{\rm SM}}{m_Z^2} - \frac{\Pi^{\rm 1PI}_{WW}(0) - \Pi^{\rm 1PI}_{WW}(0)\big|_{\rm SM}}{m_W^2} + \delta\rho$$

  equal to 0 or its experimental value.

$$\delta\rho = \frac{8 v_\Delta \delta v}{v^2}$$

- Use on-shell conditions to fix other counter terms.
- Observe gauge dependence in mixed 2-point functions.
  ➠ fixed by use of pinch technique in the physical f f → f f processes due to pinch terms

  Cornwall 1982, 1989; Papavassiliou 1990; Degrassi, Sirlin 1992; Papavassiliou, Pilaftsis 1998

  ➠ unlike 2HDM, one needs to sum up H_3-G, H_3-H_3, and G-G diagrams in GM model to see the gauge dependence cancellation

hVV Couplings

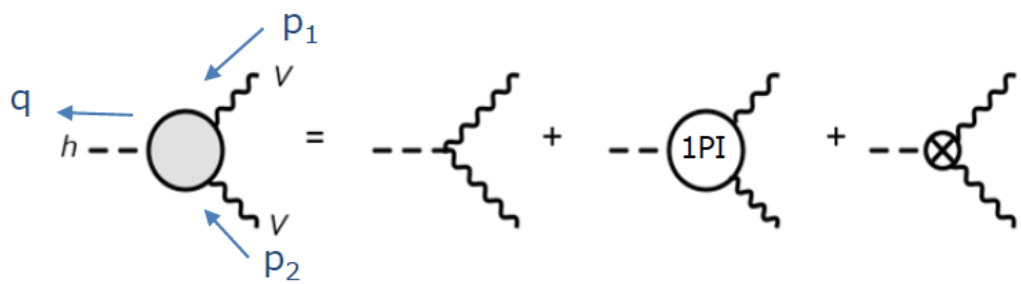
- In general, the renormalized $hV_\mu V_\nu$ vertices can be decomposed as

$$\hat{\Gamma}^{\mu\nu} = \hat{\Gamma}_1 g^{\mu\nu} + \hat{\Gamma}_2 p_1^\mu p_2^\nu + \hat{\Gamma}_3 \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}$$

where the last two form factors start to appear at 1-loop level from 1-particle irreducible (1PI) diagram contributions, while the first form factor

$$\hat{\Gamma}_1 = \frac{2m_V^2}{v}\kappa_V + \Gamma^{1\text{PI}} + \delta\Gamma$$

has contributions from the tree-level coupling, 1PI diagrams, and counter terms.



κ_Z and κ_W

- hVV scaling factors at 1-loop with momentum dependence are defined as:

$$\hat{\kappa}_V(p^2) \equiv \frac{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{\rm NP}}{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{\rm SM}}$$

- At 1σ, $\kappa_{W,Z}$ are (will be) determined to be

LHC Run-I	$\kappa_Z = [0.94, 1.13]$	$\kappa_W = [0.78, 1.00]$	
HL-LHC	$\Delta\kappa_Z = 2-4\%$	$\Delta\kappa_W = 2-5\%$	14 TeV, 3000/fb
ILC	$\Delta\kappa_Z = 0.58\%$	$\Delta\kappa_W = 0.81\%$	500 GeV, 500/fb ⊕ 350 GeV, 200/fb ⊕ 250 GeV, 500/fb

1606.02266 [hep-ex]
1310.8361 [hep-ex]
1506.05992 [hep-ex]

- Radiative corrections in SM:

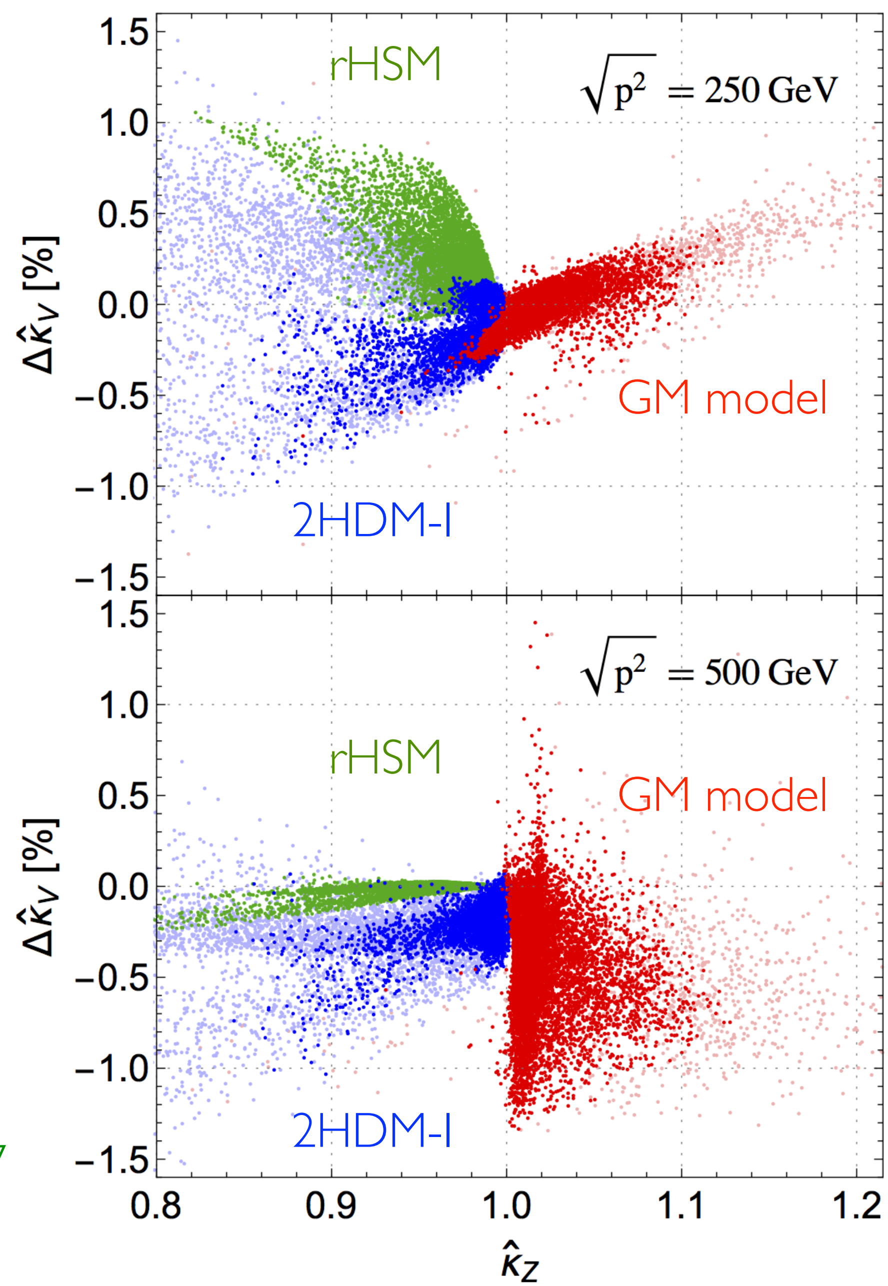
$$\frac{g_{hVV}^{1-\rm loop}}{g_{hVV}^{\rm tree}} \simeq \begin{cases} -1.2 \ \ (+1.0)\ \% \ \ (hZZ)\ , \\ +0.4 \ \ (+1.3)\ \% \ \ (hWW)\ , \end{cases} \quad \text{for } \sqrt{p^2} = 250\ (500)\ \text{GeV}$$

1-Loop Results

- Lighter dots satisfy theoretical constraints (unitarity, stability, perturbativity, and oblique parameters [S and T]).
- Darker dots further satisfy Higgs data from LHC Run-I.
- Other types of 2HDM are expected to have a similar result as 2HDM-I.
- It is possible to discriminate among the SM, rHSM, 2HDMs and GM model.

CWC, Kuo, and Yagyu 2017

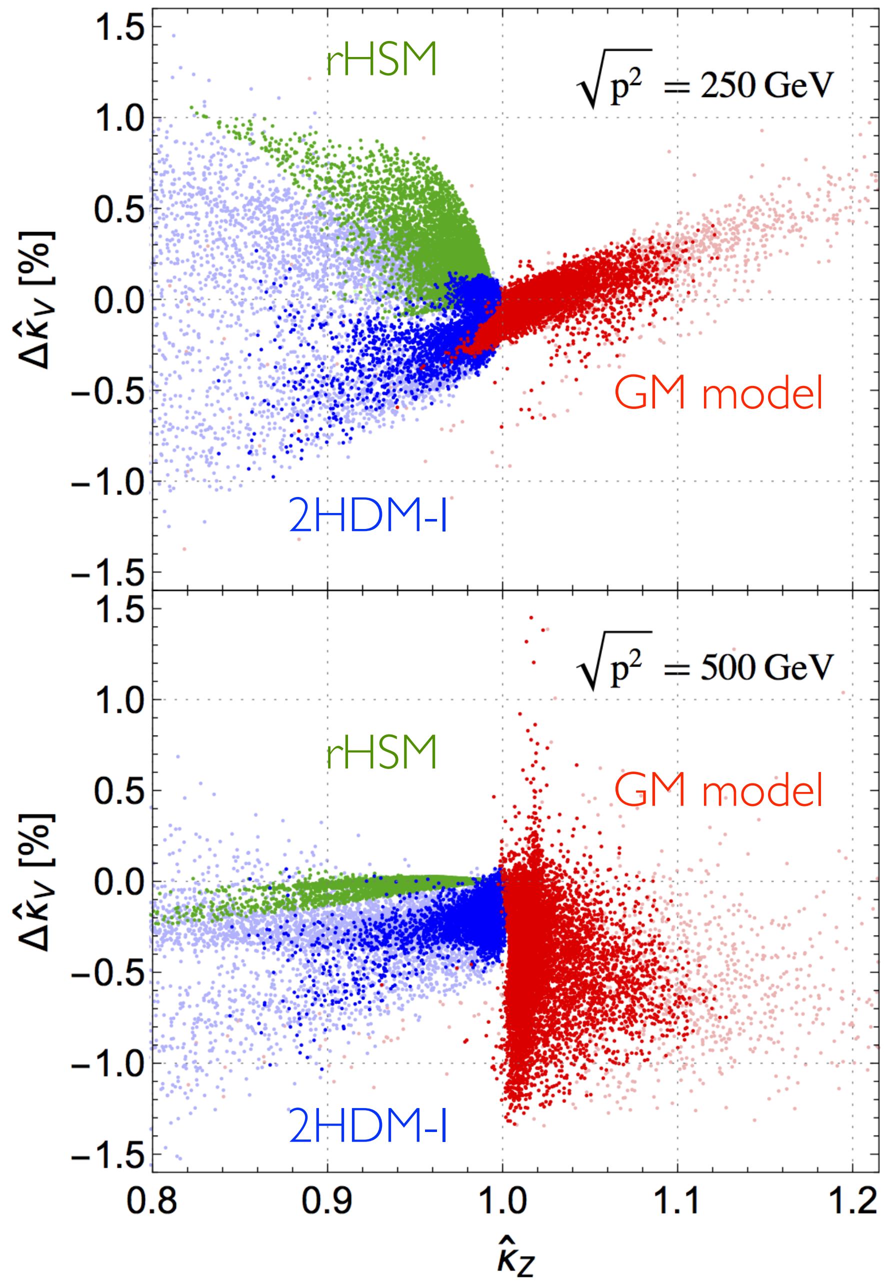
$$\Delta\hat{\kappa}_V \equiv \hat{\kappa}_W - \hat{\kappa}_Z$$



1-Loop Results

- Same parameter sets in the two plots, only different in p.
- Green dots change little after imposing the Higgs data, while 2HDM and GM dots shrink significantly.
- GM prefers $\kappa_Z \in [0.88, 1.12]$, while the others have $\kappa_Z \in [0.8, 1.0]$.
- $\Delta\kappa_V$ may be observable.
- 250-GeV ILC is better than 500-GeV in distinguishing rHSM and 2HDM-I.

$$\Delta\hat{\kappa}_V \equiv \hat{\kappa}_W - \hat{\kappa}_Z$$

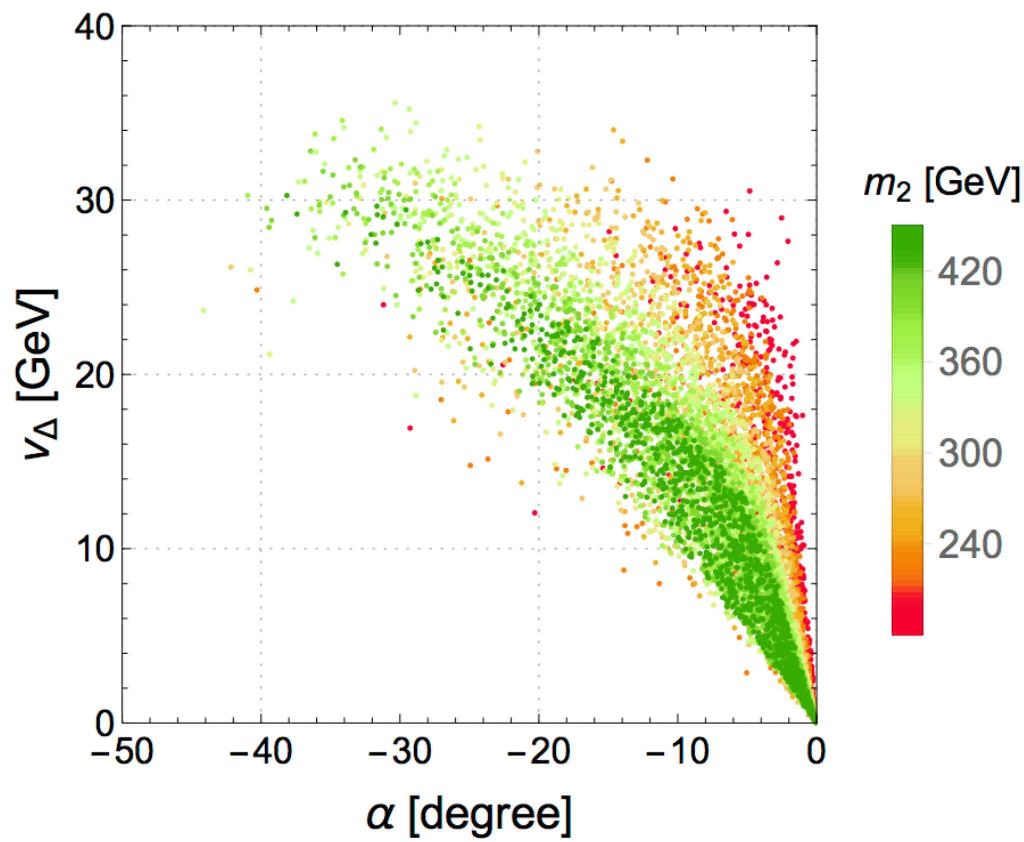


Allowed Space in GM Model

Preliminary

scanned parameter ranges

$$-650 \leq \mu_1 \leq 0 \text{ GeV}$$
$$-400 \leq \mu_2 \leq 50 \text{ GeV}$$
$$180 \leq m_2 \leq 450 \text{ GeV}$$
$$-0628 \leq \lambda_2 \leq 1.57$$
$$-1.57 \leq \lambda_3 \leq 1.88$$
$$-2.09 \leq \lambda_4 \leq 2.09$$
$$-8.38 \leq \lambda_5 \leq 8.38$$

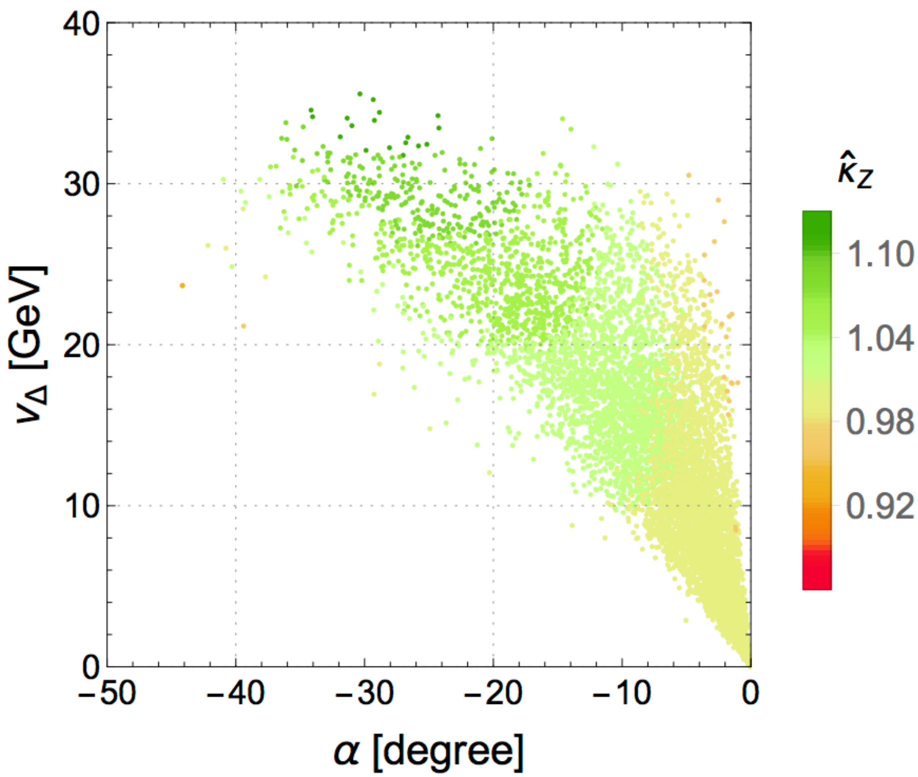


Scaling Factors in GM Model

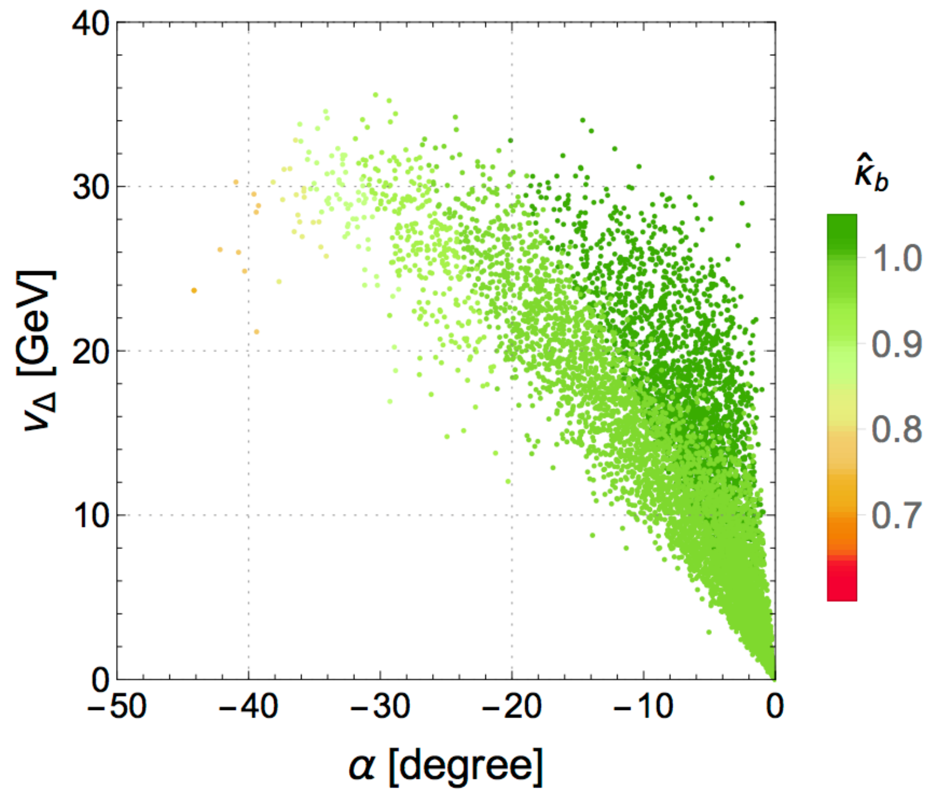
Preliminary

$$\hat{\kappa}_V(p_2^2) \equiv \frac{\hat{\Gamma}^1_{hVV}(m_V^2, p_2^2, m_h^2)_{\rm GM}}{\hat{\Gamma}^1_{hVV}(m_V^2, p_2^2, m_h^2)_{\rm SM}}$$

$$\hat{\kappa}_{b,\tau} \equiv \frac{\hat{\Gamma}^1_{hb\bar{b},h\tau\tau}(m_{b,\tau}^2, m_{b,\tau}^2, m_h^2)_{\rm GM}}{\hat{\Gamma}^1_{hb\bar{b},h\tau\tau}(m_{b,\tau}^2, m_{b,\tau}^2, m_h^2)_{\rm SM}}$$



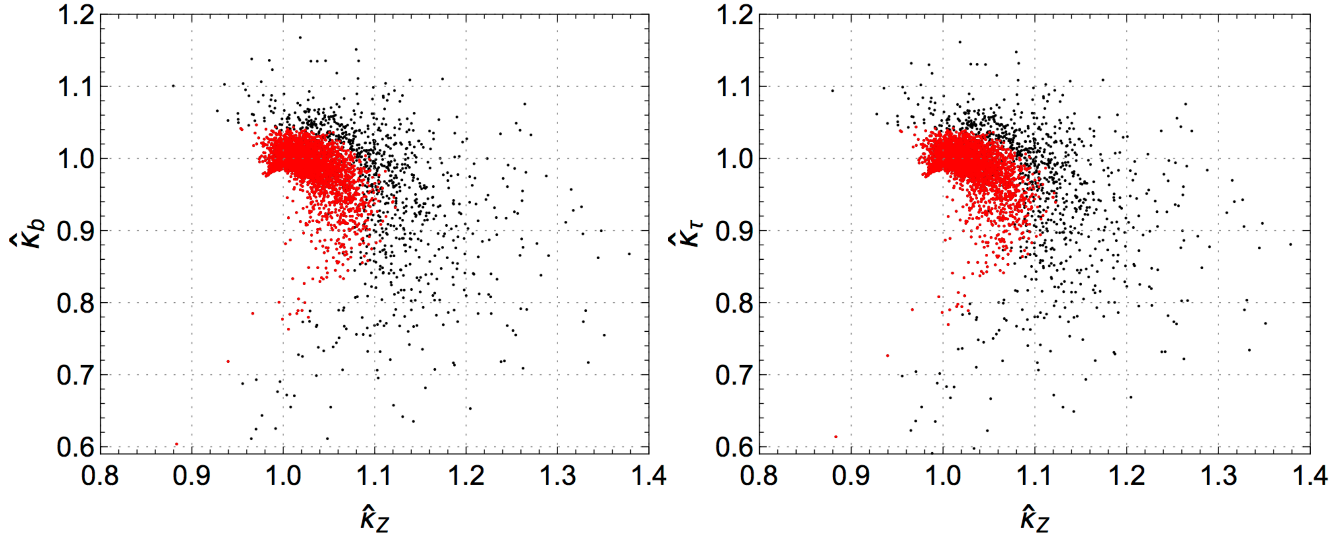
$0.88 \lesssim \hat{\kappa}_Z \lesssim 1.12$



$0.60 \lesssim \hat{\kappa}_b \lesssim 1.05$

κ_f vs κ_Z

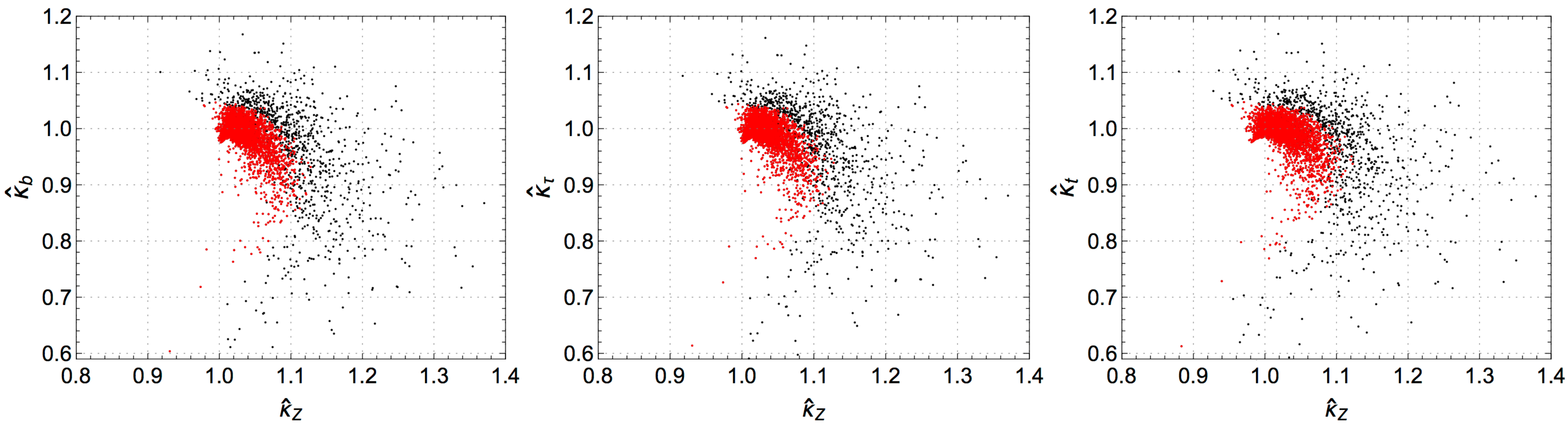
$\sqrt{p_2^2} = 250$ GeV



Preliminary

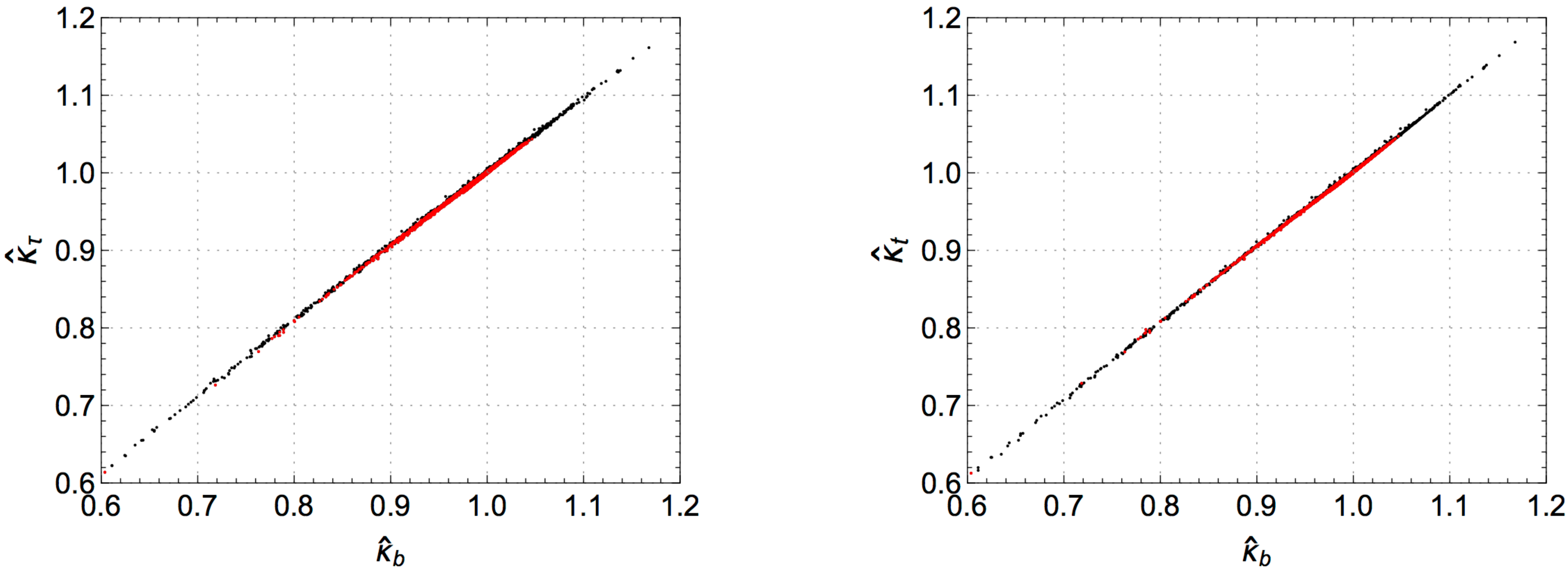
- very little changes in $\kappa_{t,b,\tau}$ and κ_Z in each row
- by definition, no p_2^2 dependence in $\kappa_{t,b,\tau}$

$\sqrt{p_2^2} = 500$ GeV



κ_f Correlations

Preliminary



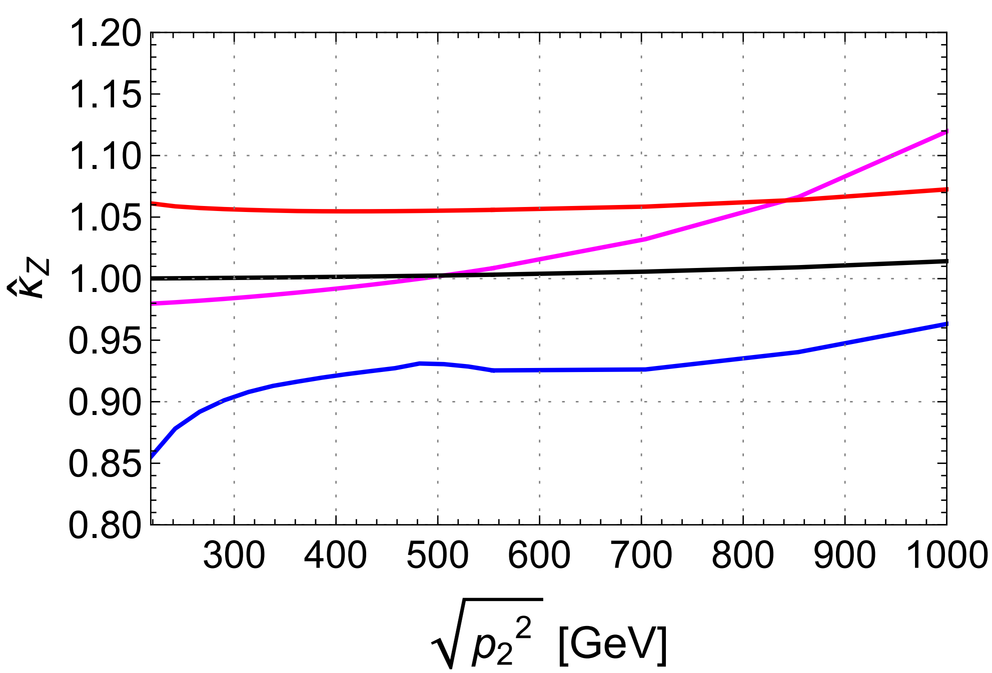
strong correlations among the hff scaling factors
with a range of ~ [0.6,1.2]

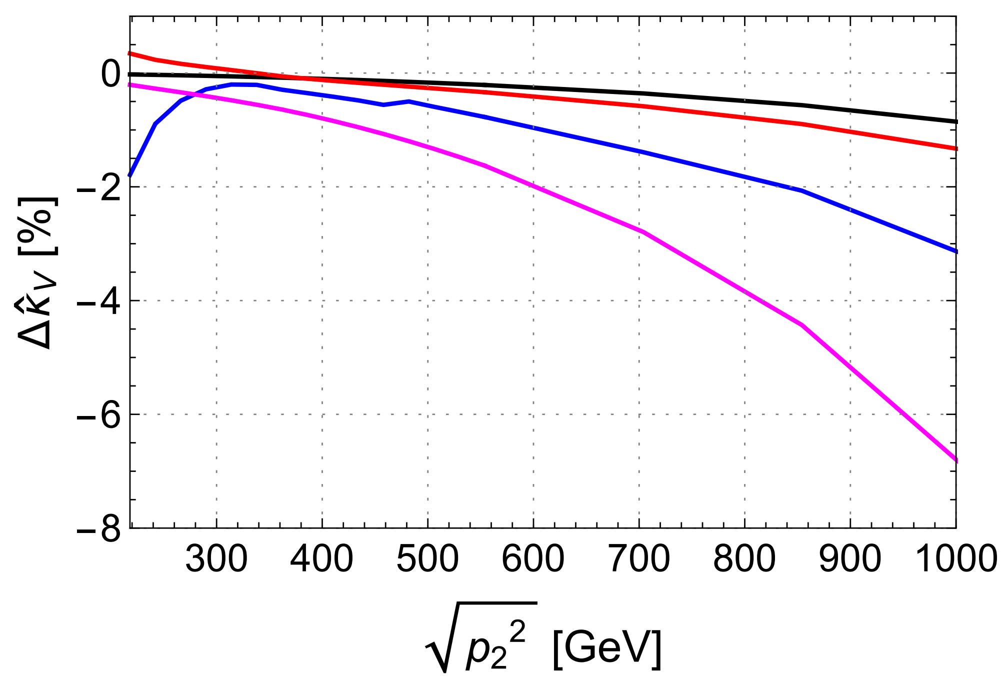
Momentum Dependence

- Examples of a few benchmark points:

Preliminary

v_Δ (GeV)	α	m_{H1} (GeV)	m_{H3} (GeV)	m_{H5} (GeV)	μ_1 (GeV)	μ_2 (GeV)
24.57	-17.21°	609	601	572	-614	-60
0.34	-0.27°	556	562	573	-7	-298
2.08	-0.56°	496	519	562	-34	-32
24.99	-51.68°	389	404	574	-179	-276





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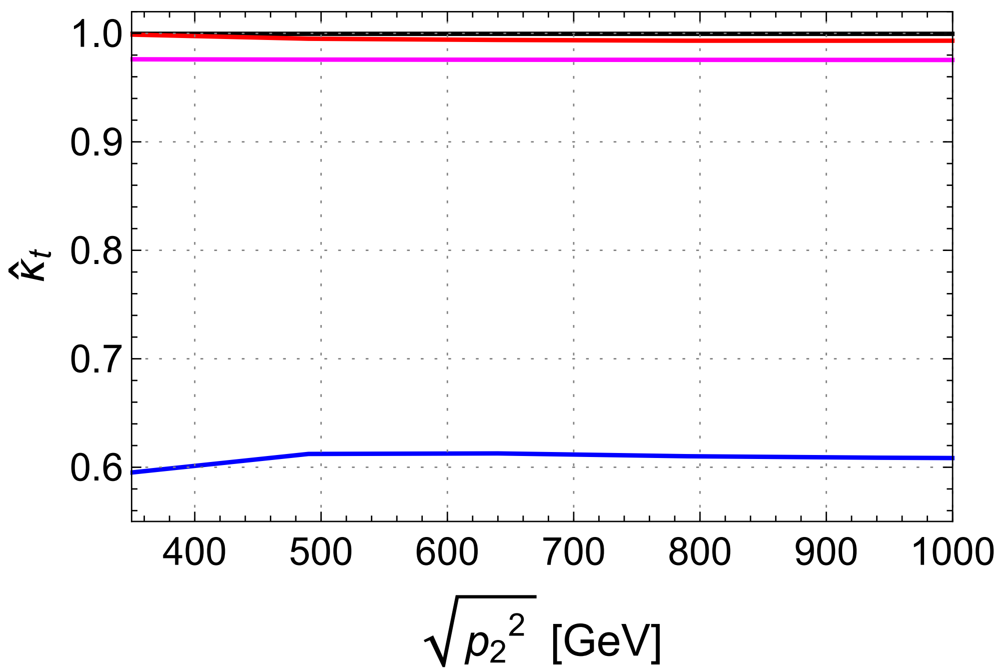

Summary

- We have been doing 1-loop radiative corrections to the Higgs couplings in the GM model, and made a comparison among the three simplest custodial Higgs models (rHSM, 2HDM-I and GM).
- Theoretical (unitarity, stability, perturbativity, and oblique parameters) and experimental (Higgs signal strengths) constraints have been imposed to find viable parameter spaces.
- We have presented numerical results for the hVV couplings in the rHSM, 2HDM-I and GM models at 250-GeV and 500-GeV ILC, showing power to discriminate among the models.
- We have presented preliminary results of hff couplings in the GM model, showing their correlations among themselves and with hVV and the momentum dependence of these couplings.

- Radiative corrections to the hhh coupling are in progress.
- More phenomenological analyses will follow.

Thank You!