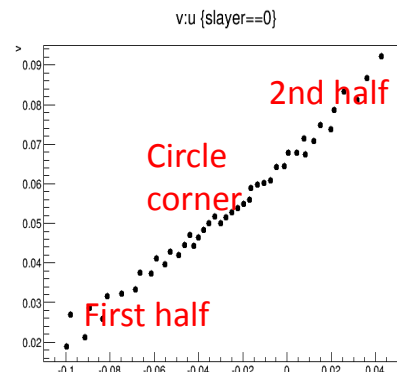
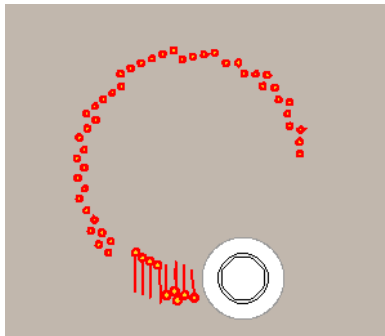


# Introduction of updated Hough Transform

- Conformal transform
- Hough transform
- Modified method
- CFS->HS ( Conformal Space->Hough Space)
- Differ half circle
- Bin method on new Hough space
- Test for bin method idea

# Conformal transform

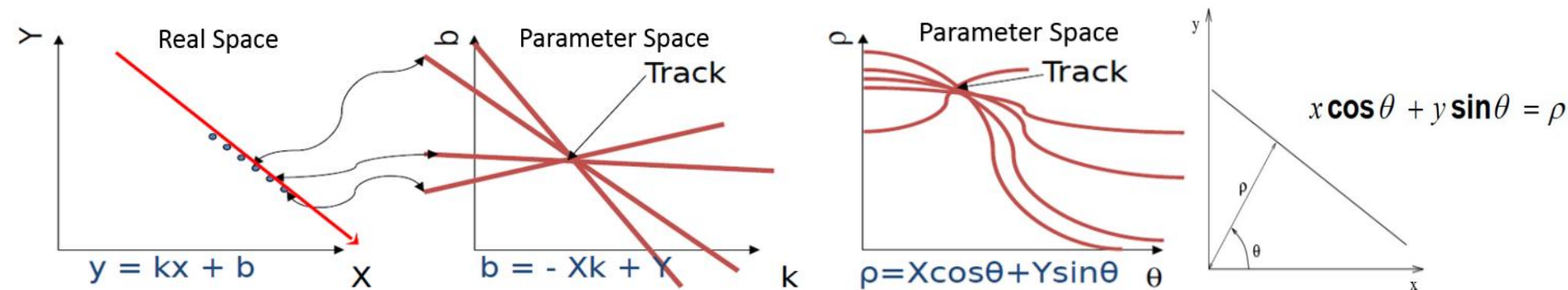
- Conformal transform:
  - Circle  $\rightarrow$  straight line
  - Like to stretching a “bend rope” to straight
- Previous conformal transform
  - $u = x/(x^2 + y^2)$
  - $v = y/(x^2 + y^2)$
  - $(x, y)$  midpoint of hits in real space
  - $(u, v)$  points in CF space
- use only mid point of axial hits
- every points have big error due to neglect drift distance



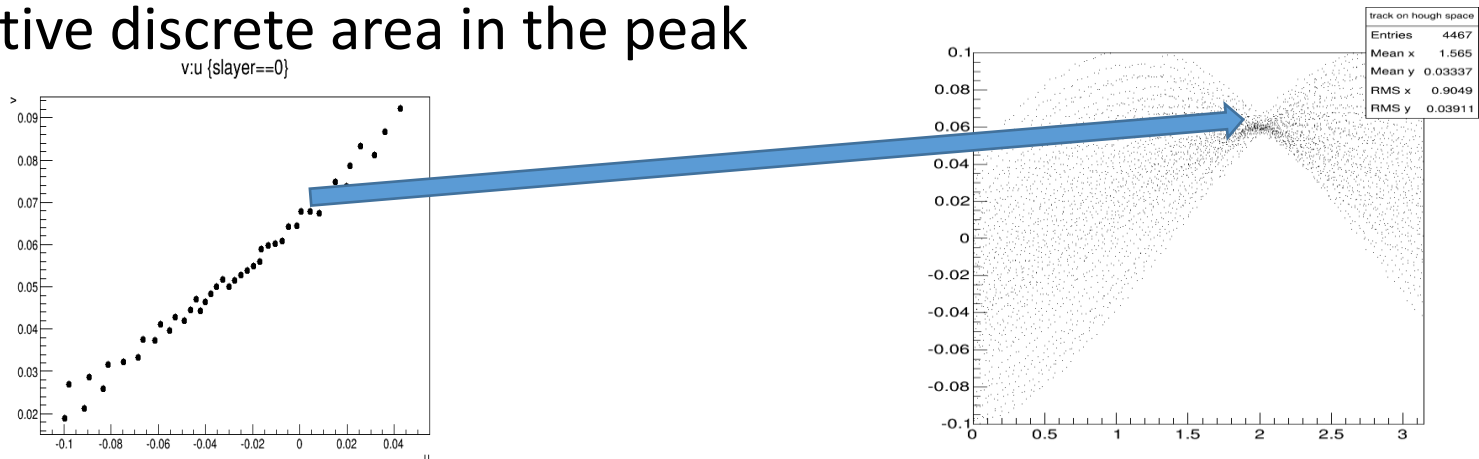
# Line performed on Hough Space

- As we use hough transform

Explain of parameters



- The error of drift distance bring about a relative discrete area in the peak



Fix pt 60 MeV for example

# Modified CFT and HT method

Circle not passed (0,0) will still be circle by modified CFT

For circle  $(x_c, y_c, r_c)$  in real space :

$$X_c = 2x_c / (x_c^2 + y_c^2 - r_c^2)$$

$$Y_c = 2y_c / (x_c^2 + y_c^2 - r_c^2)$$

$$R_c = 2r_c / (x_c^2 + y_c^2 - r_c^2)$$

$$\left( X - \frac{2x_c}{x_c^2 + y_c^2 - R^2} \right)^2 + \left( Y - \frac{2y_c}{x_c^2 + y_c^2 - R^2} \right)^2 = \left( \frac{2R}{x_c^2 + y_c^2 - R^2} \right)^2,$$

Circle  $(x_c, y_c, r_c)$  turned to  $(X_c, Y_c, R)$  in CF space

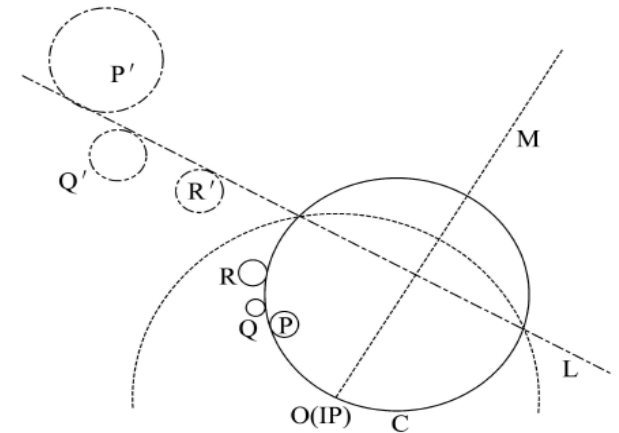


Fig. 2. After conformal transformation, the circle C which passes O(IP) is transferred into line L; the circles P, Q and R which do not pass O are transferred into new circles P', Q' and R'.

Proved by legendre transform , circle tangent to the same circle will tangent to the straight line in CF space

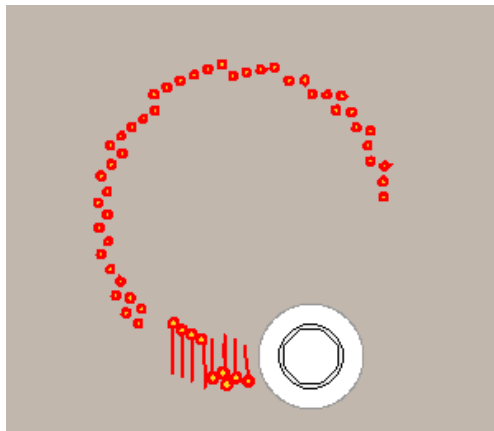
# Perform on Hough Space

- Every drift circle is transformed by CFT to CF space

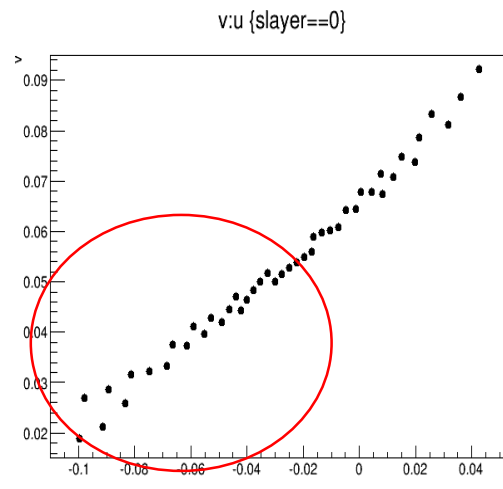
$$X_c = x / (x^2 + y^2 - \text{drift}^2)$$

$$Y_c = y / (x^2 + y^2 - \text{drift}^2)$$

$$\text{CFdrift} = \text{drift} / (x^2 + y^2 - \text{drift}^2)$$

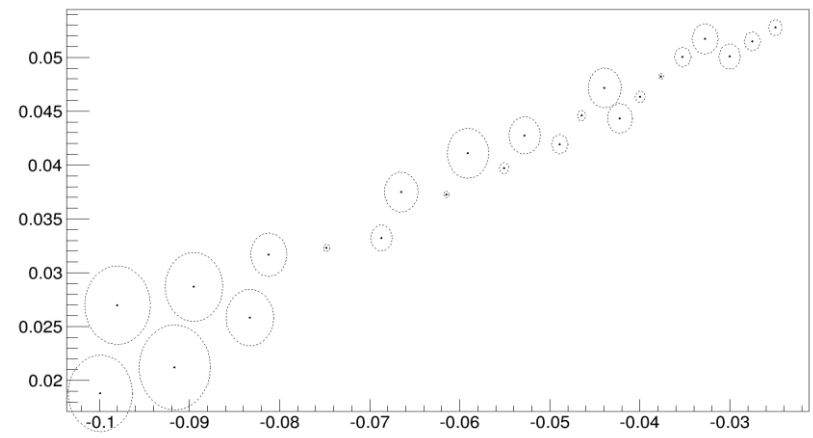


Real Space



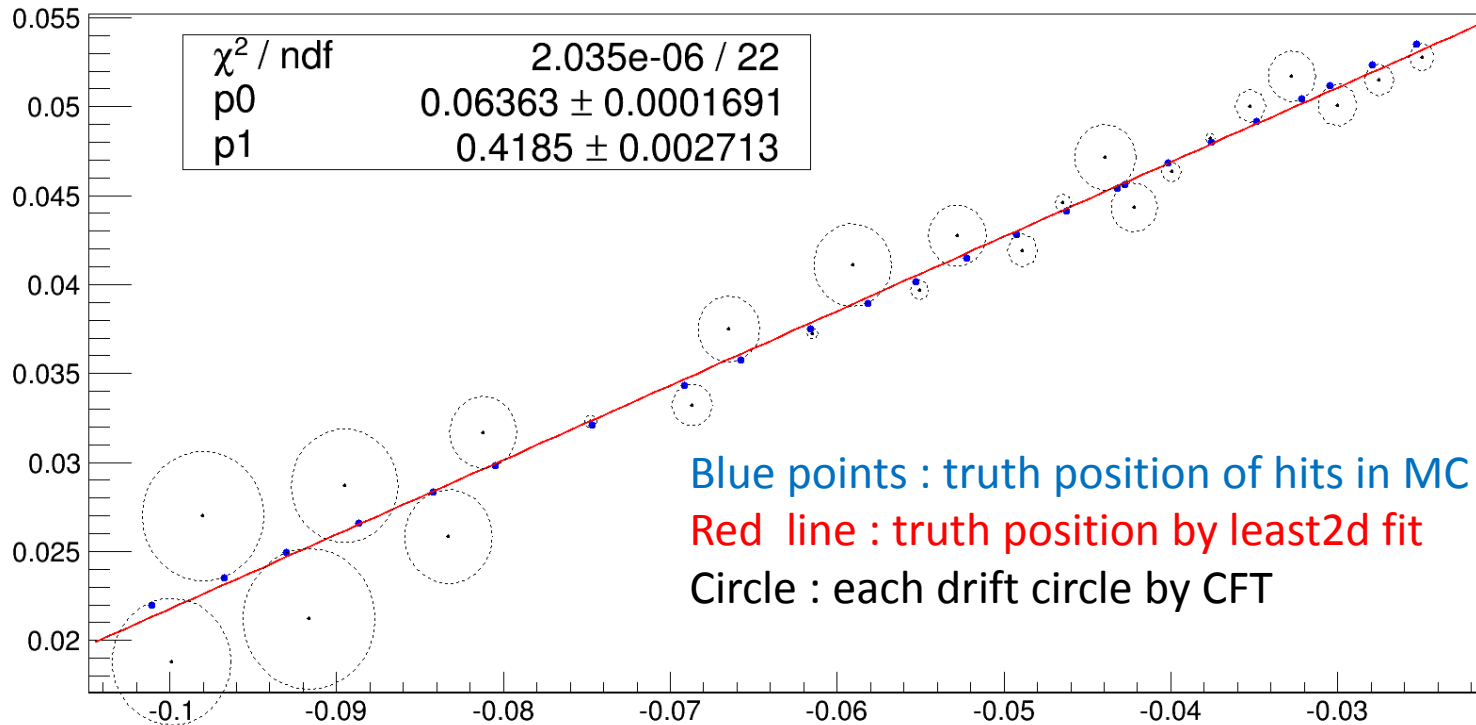
Old CFT

Only use 1<sup>st</sup> half circle hits



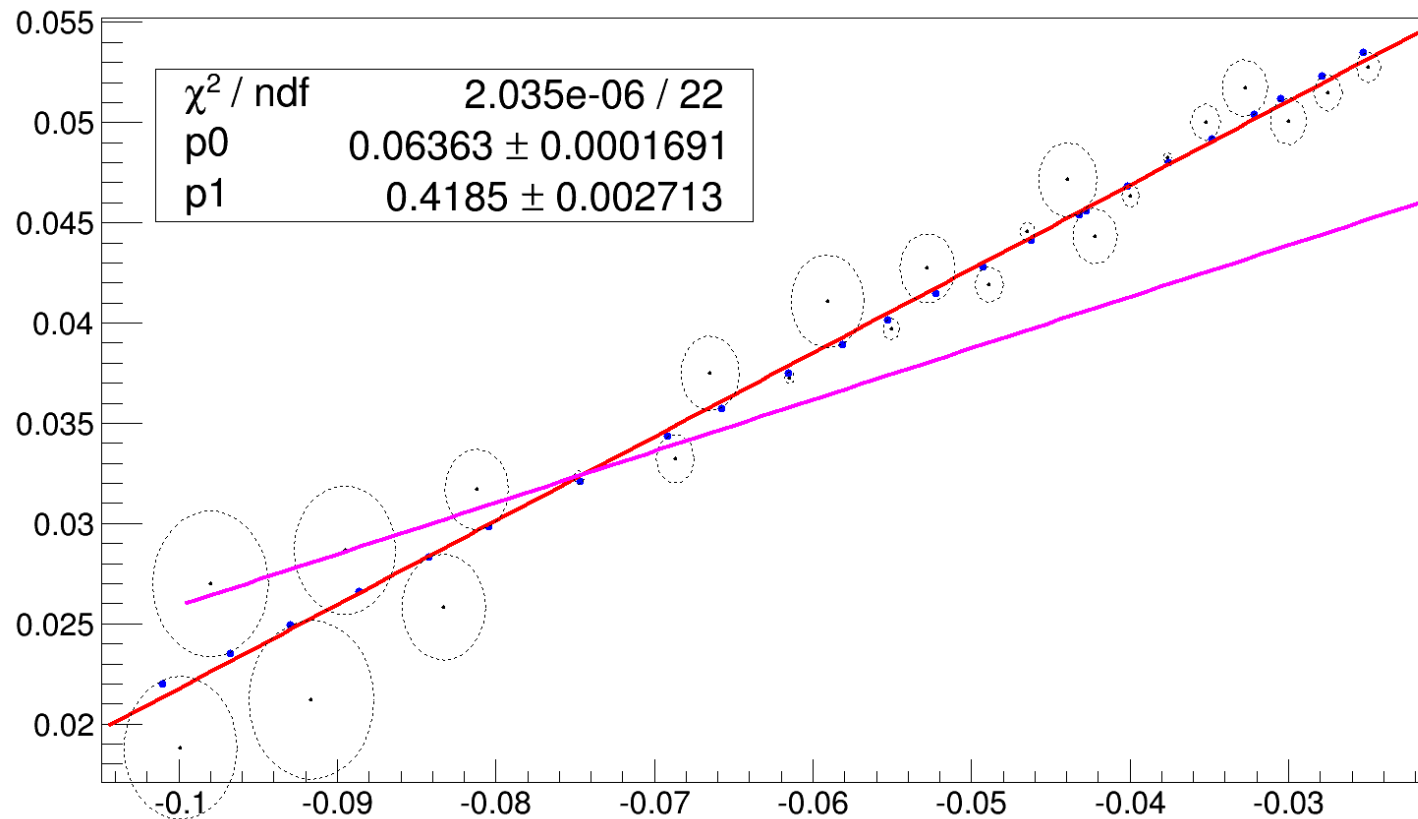
new CFT

# Truth point on CF space



- Hit truth position is close on drift circle in CF space
- Aim is changed to look for a straight line tangent to each drift circle
- Each tangent point is the position of the track passing the MDC wire

# Why use the truth position of hits



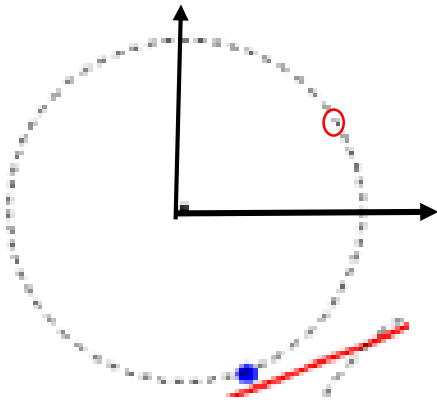
Pink line is the truth particle , think of energy loss , far from the real circumstance



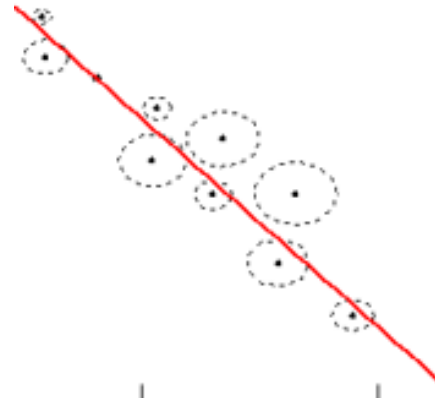
# CF Space -> Hough Space

The straight line is tangent to each drift circle on CFS

Split the drift circle along alpha , for example (0~360) to 60 points



Method 1: straight line will pass through the drift circle

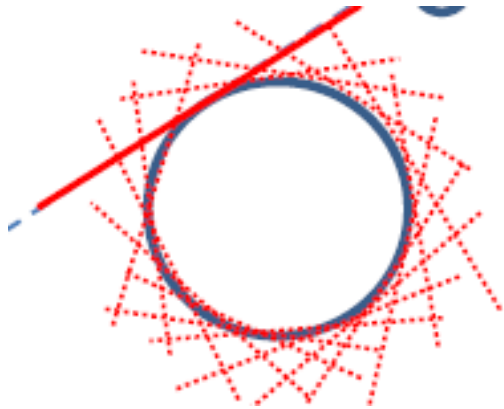


Two method for hough transform

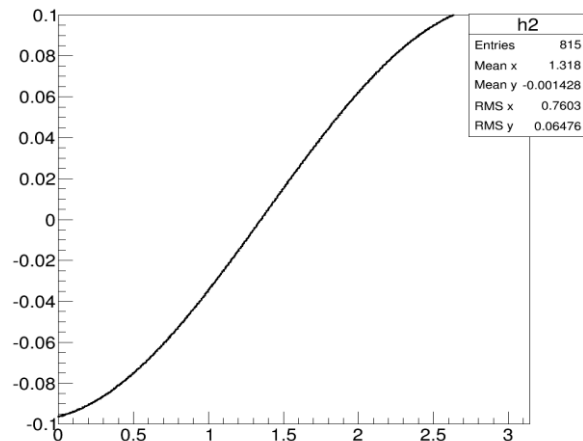
- Each points to a sino curve line to Hough space
- tangent line to circle at each points to Hough space

# CF Space -> Hough Space

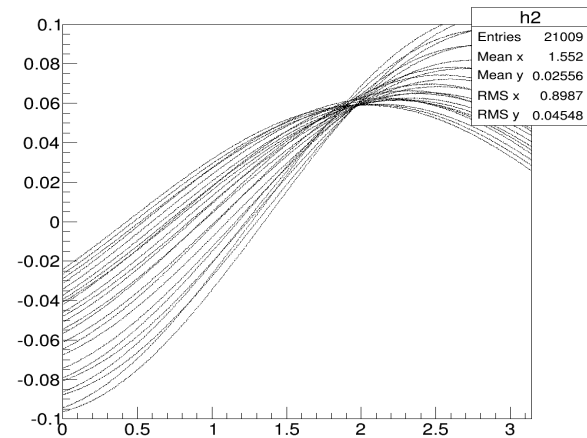
Calculate tangent line of each points on the drift circle  
Convert parameters to (rho,theta) , and fill Hough Space



A drift circle on Hough Space



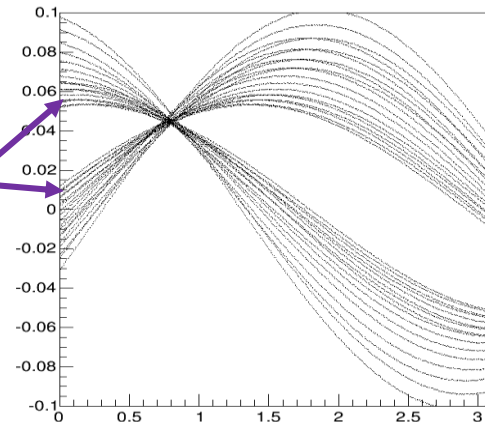
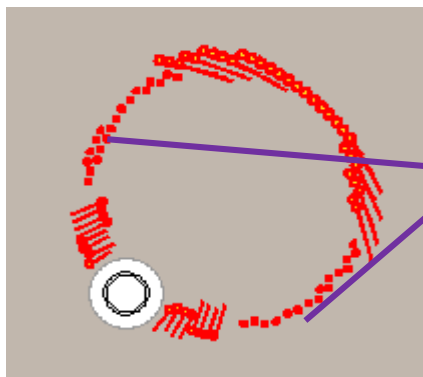
all drift circles on Hough Space



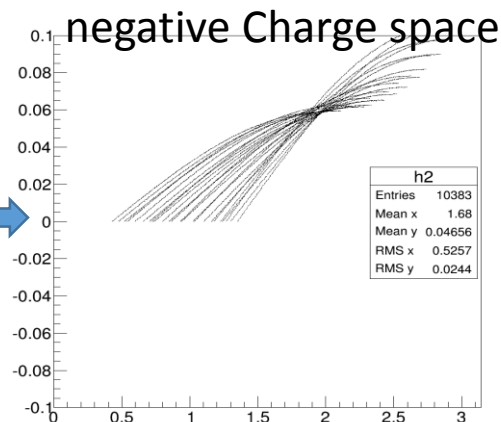
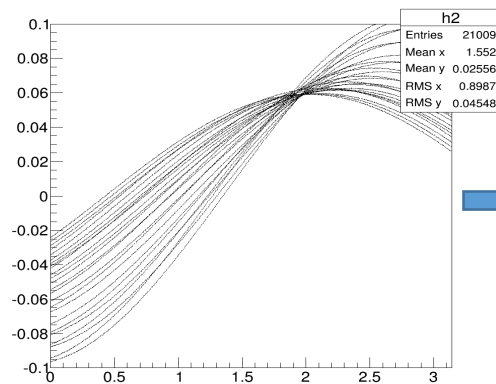
# Differ half circle hits

Same with the previous method

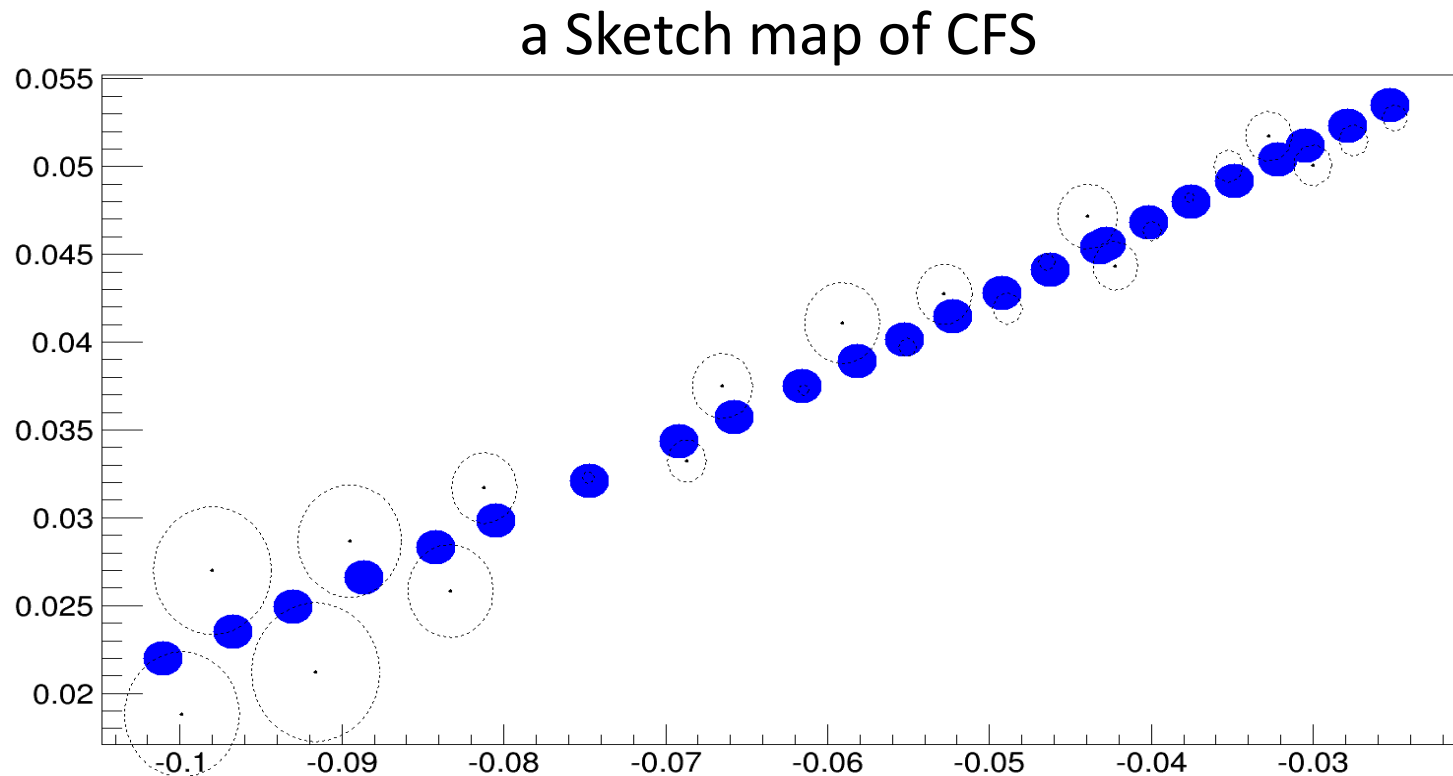
Determine by **sign of slant & sign of rho**  
**(sign of slant) \* (sign of rho)**



Hits on 1<sup>st</sup> half and 2<sup>nd</sup> half have different **sign** of slant when there appears a track

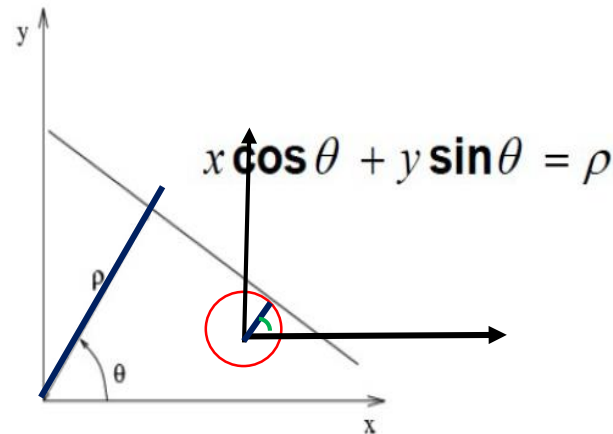


# What is the range to determine line?

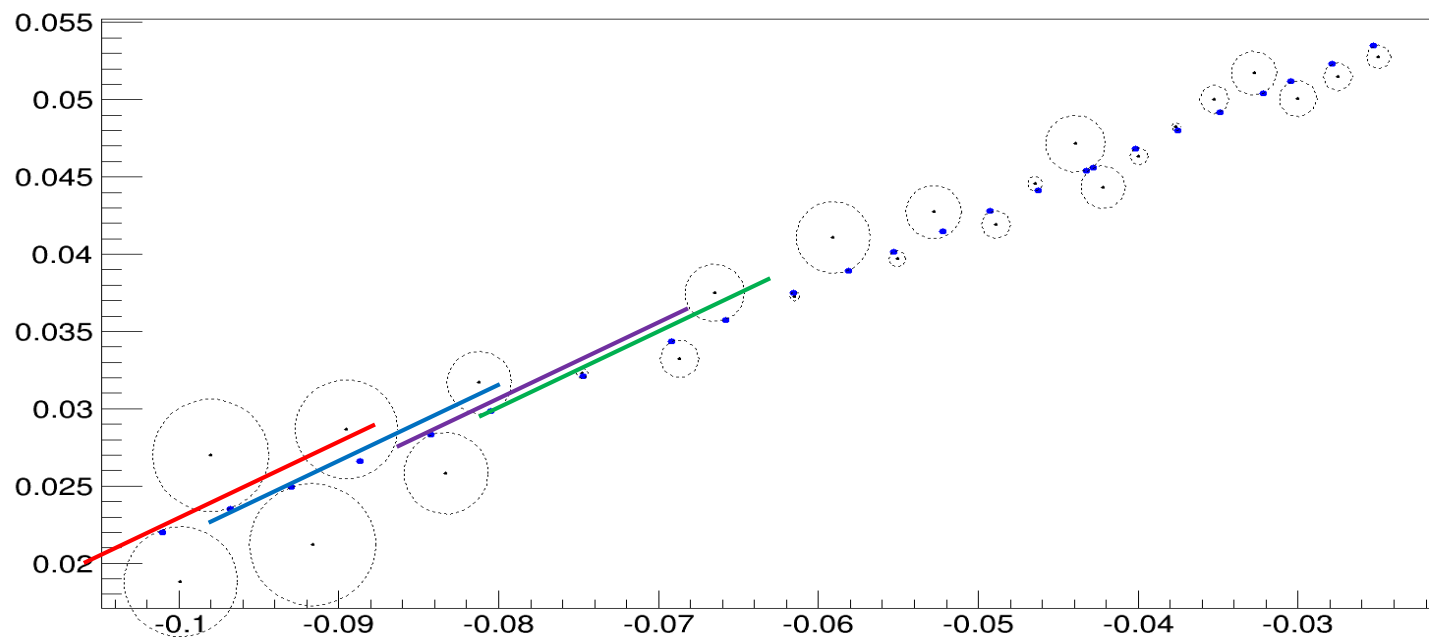
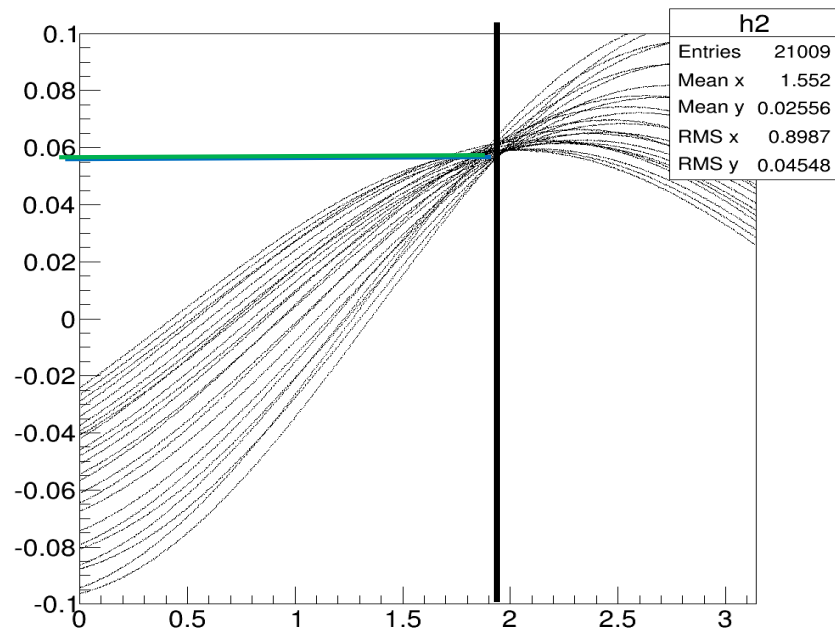


- Actually , think of the position differentiate , truth position can be transformed into a list of circles in CFS
- the range of straight line is contain all the maximum & minmum (rho,theta)

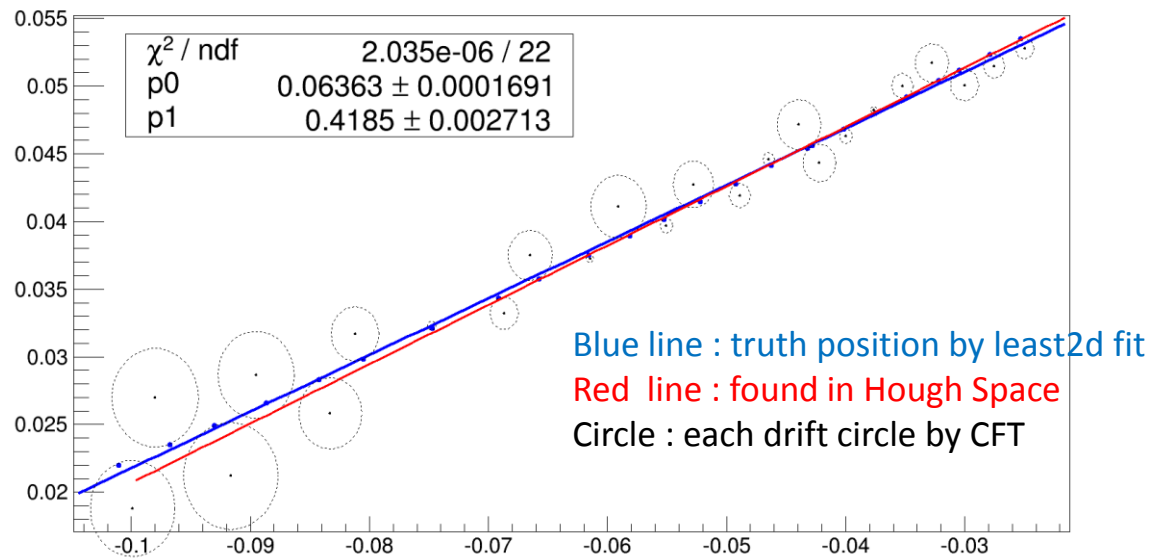
# Bin method along theta



- Alpha  $\rightarrow$  theta one to one match
- How to split the drift circle determine the theta bin on Hough map
- By this method :
- Each bin along theta axis of HS represent one possible position of the straight line in CFS



- More bins we split along theta axis, more accurate of the straight line
- Split bin into 1000\*1000 of HoughSpace
- Select the max bin as the “found track”

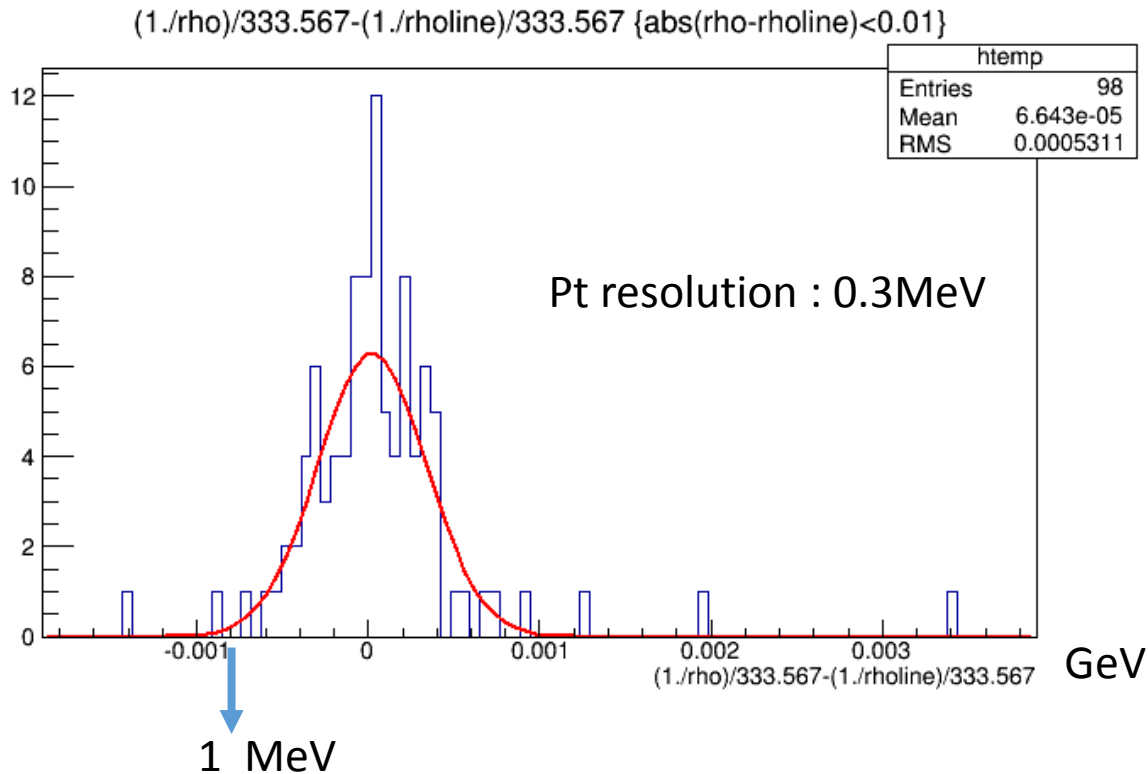


# Test 100 fixpt 60 MeV events

Pt1: Truth pt : truth position of all hits fit by least2d

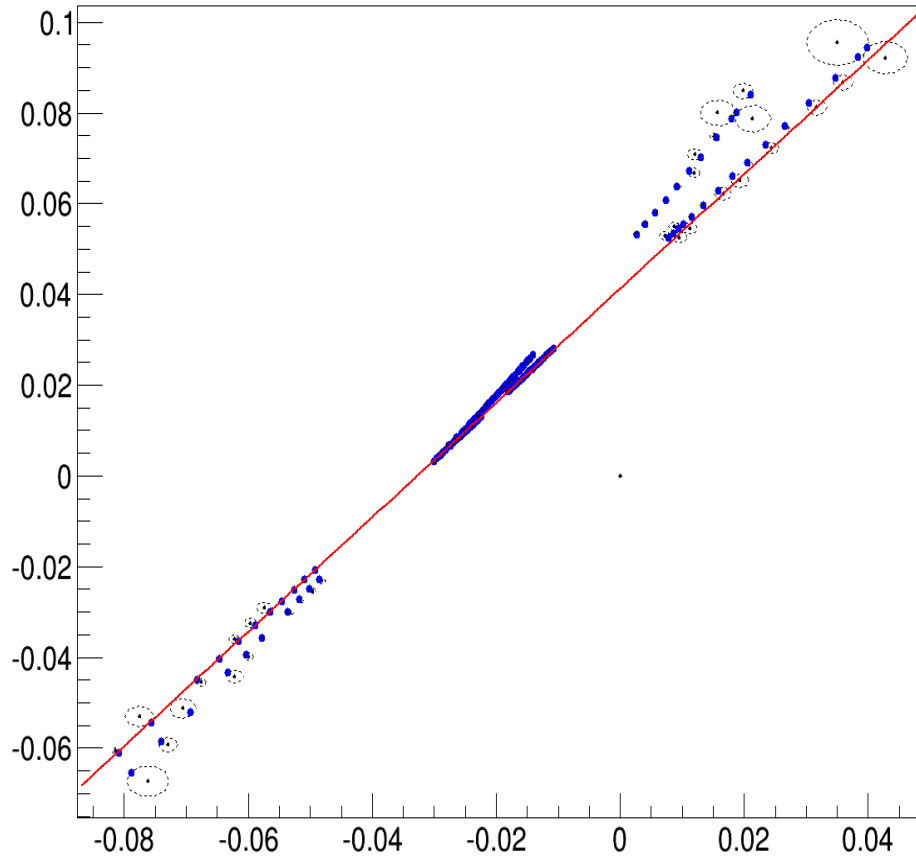
Pt2: Pt of “found track “ : found in Hough Space by maximum bin

Pt resolution : pt1-pt2

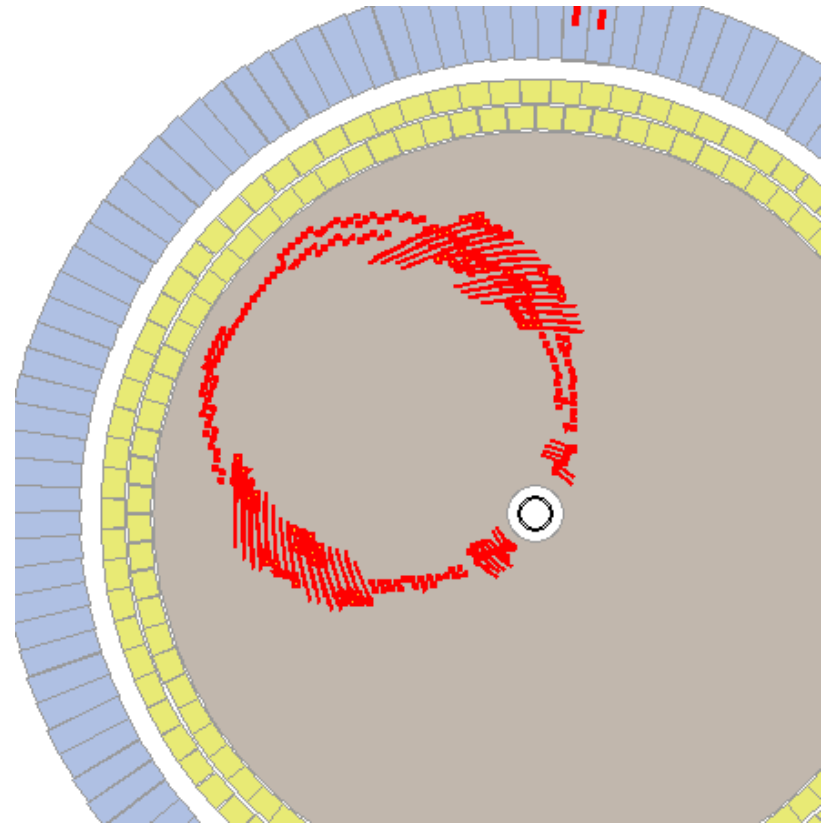




# Deal with multi turn tracks ?



Red line : found in Hough Space



# Now finding the method how to deal with the rho axis ...

Idea is to calculate the width of bin along rho axis **at maximum bin**

