Calculation of Atmospheric Neutrino Flux.

July 4, 2017 M. Honda

	Order of talk
1. Elements of Calculation	
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iv. Geomagnetic field and Rigidity cutoff	2
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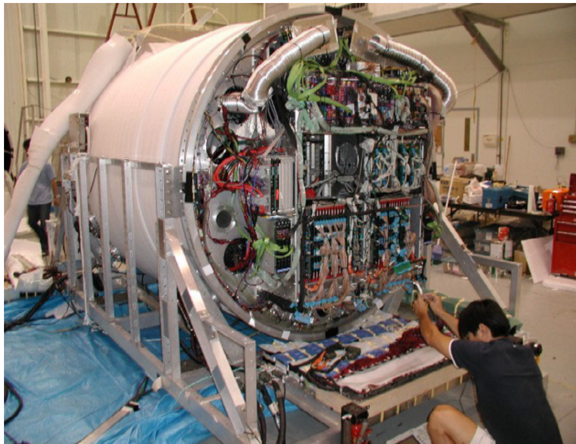
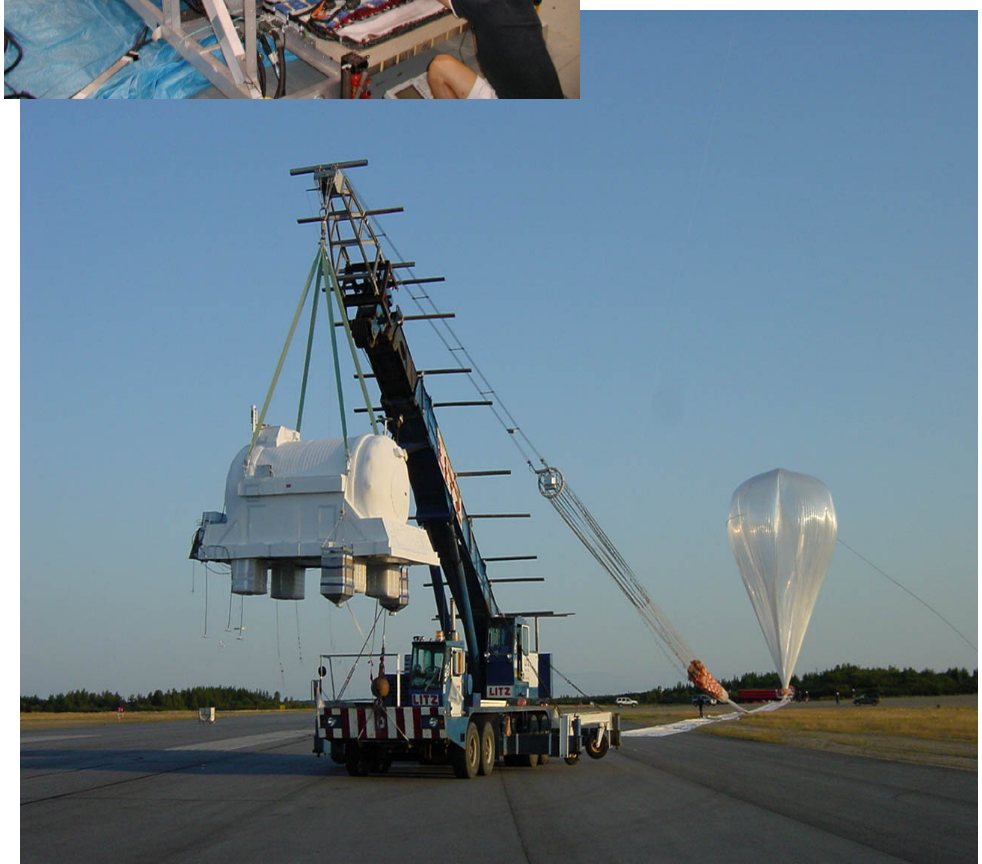
2. 1D vs. 3D

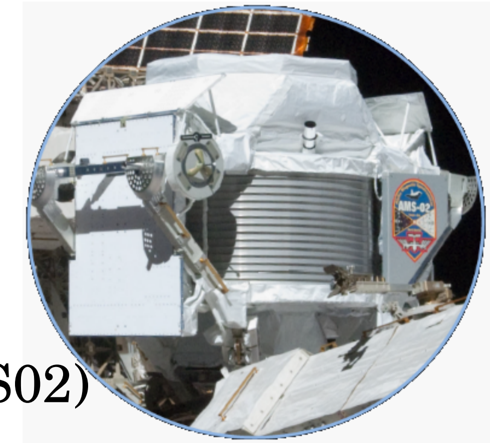
3 Our calculation, Scheme and results.

Direct Observation

Balloon Borne (BESS)

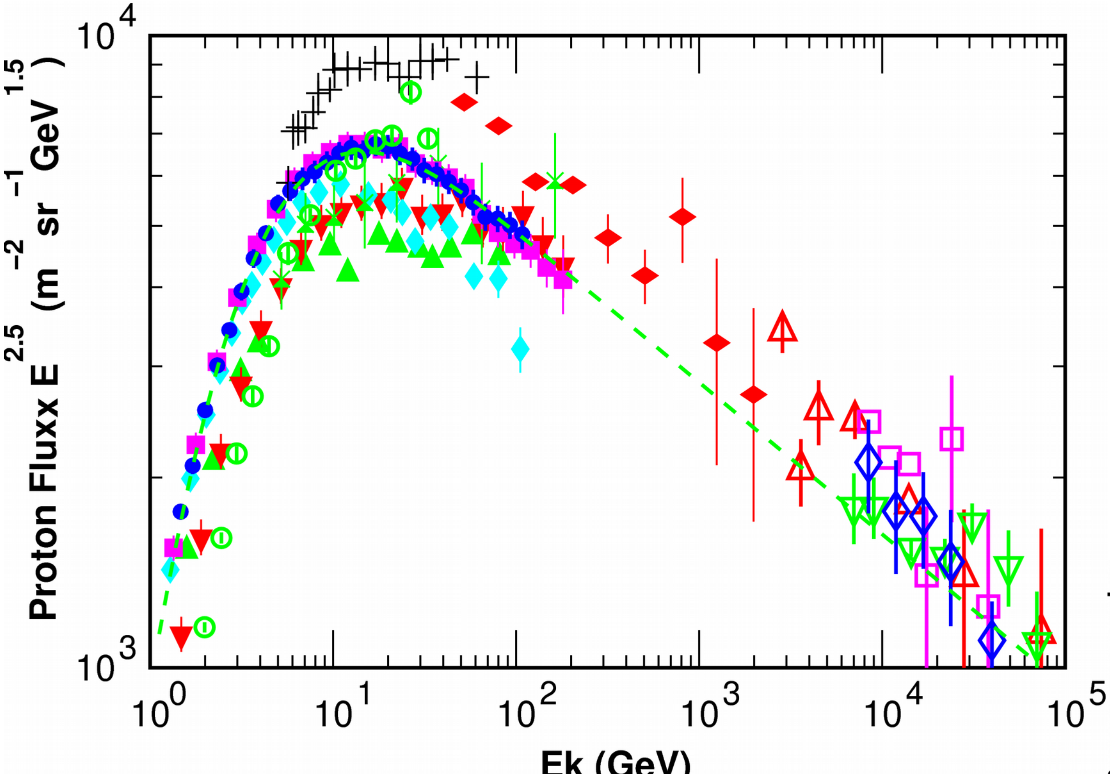


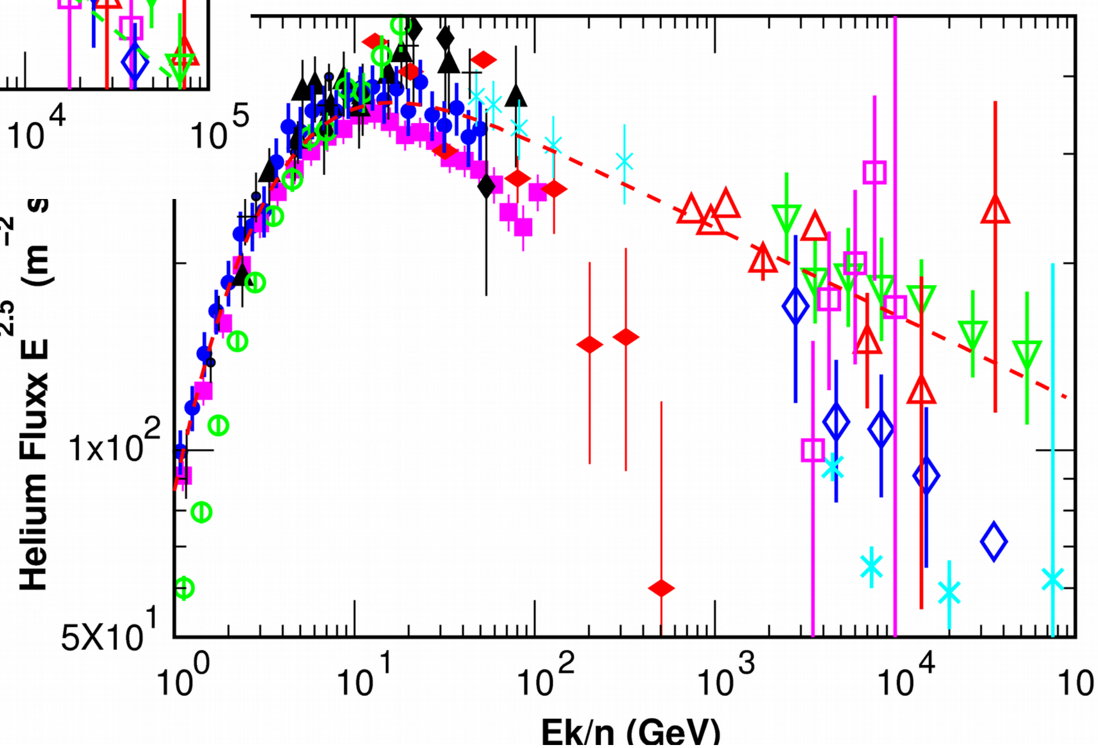




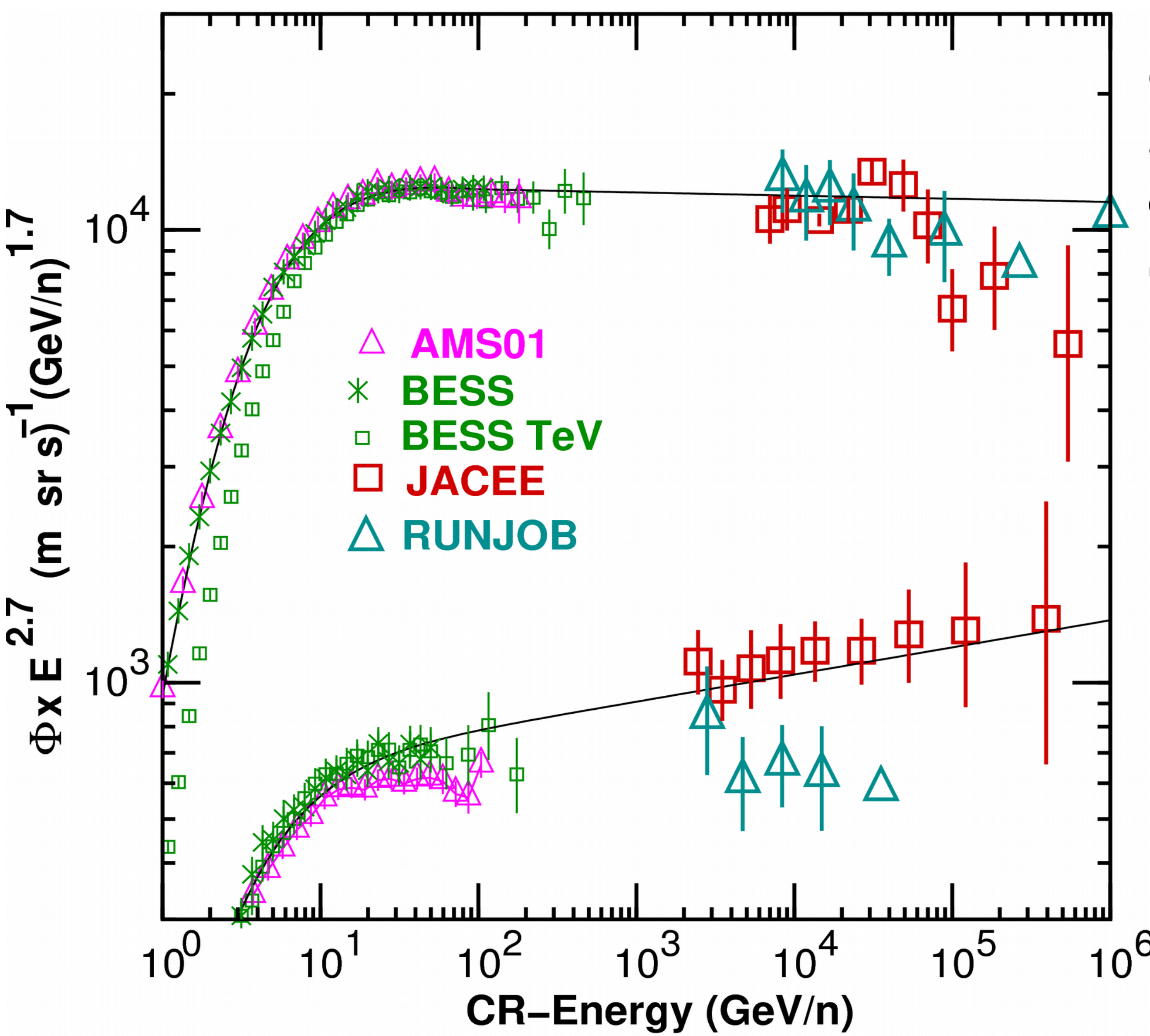
Satellite (ISS, AMS02)

Primary Cosmic Ray Models (Gaisser Honda 2002)



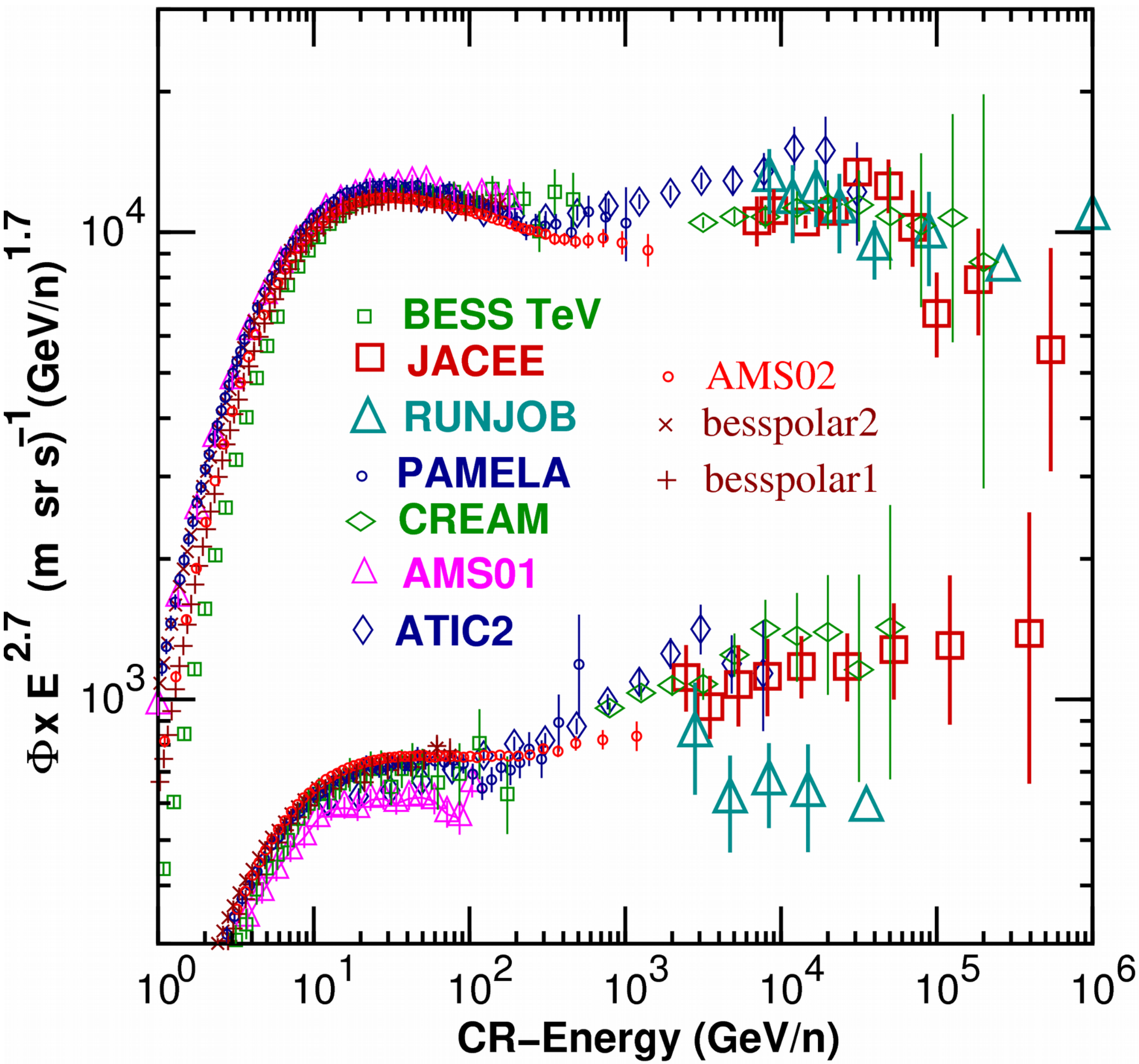


Primary Cosmic Ray Model and referred data (2004)

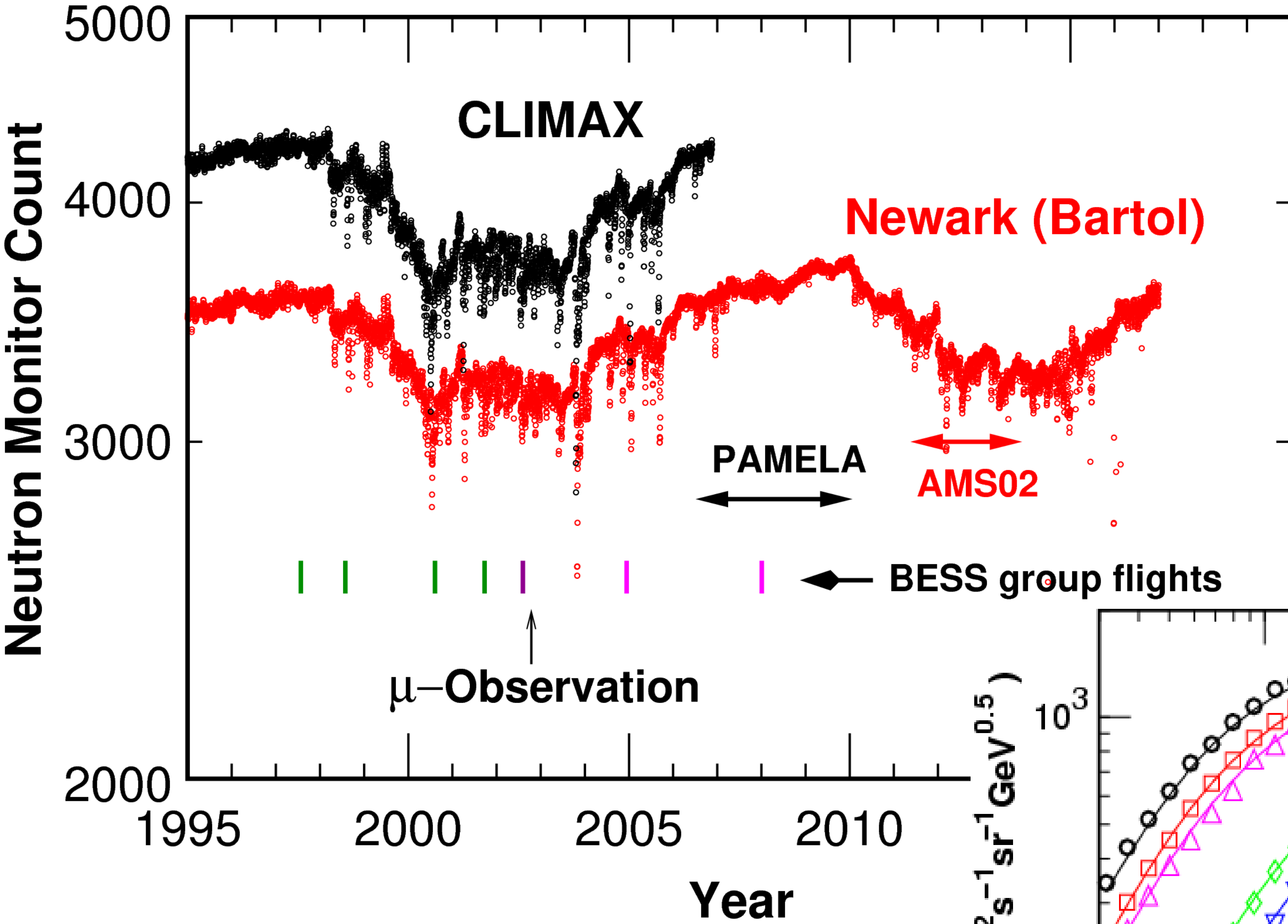


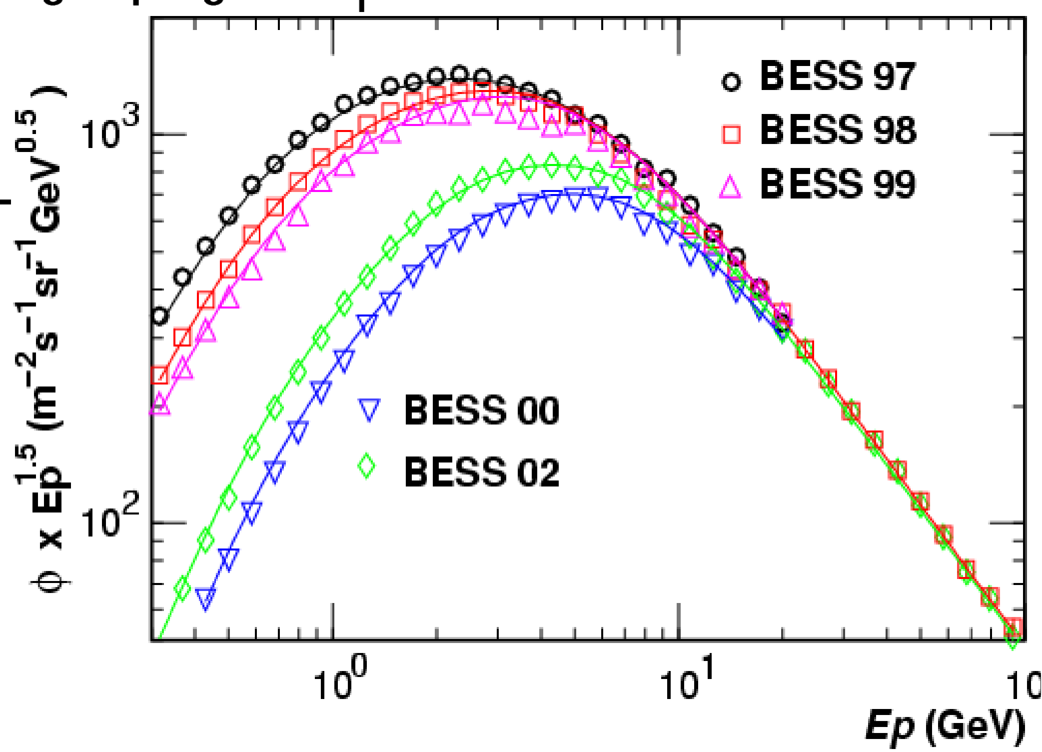
Other chemical compositions are also considered in the calculation, but they give small contributions.

Recent Cosmic Ray observation and available High Energy data

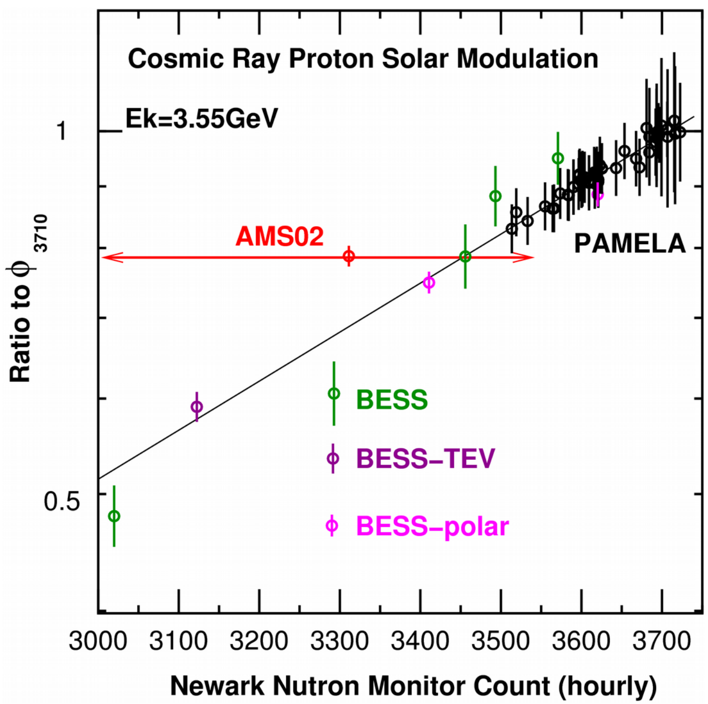


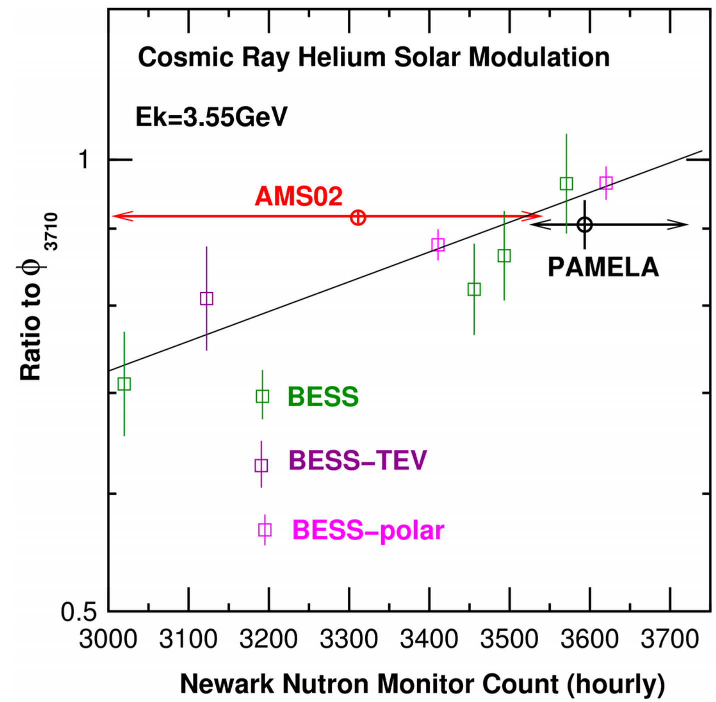
Solar Modulation and Neutron Monitor





Solar Modulated Flux at Fixed Energy

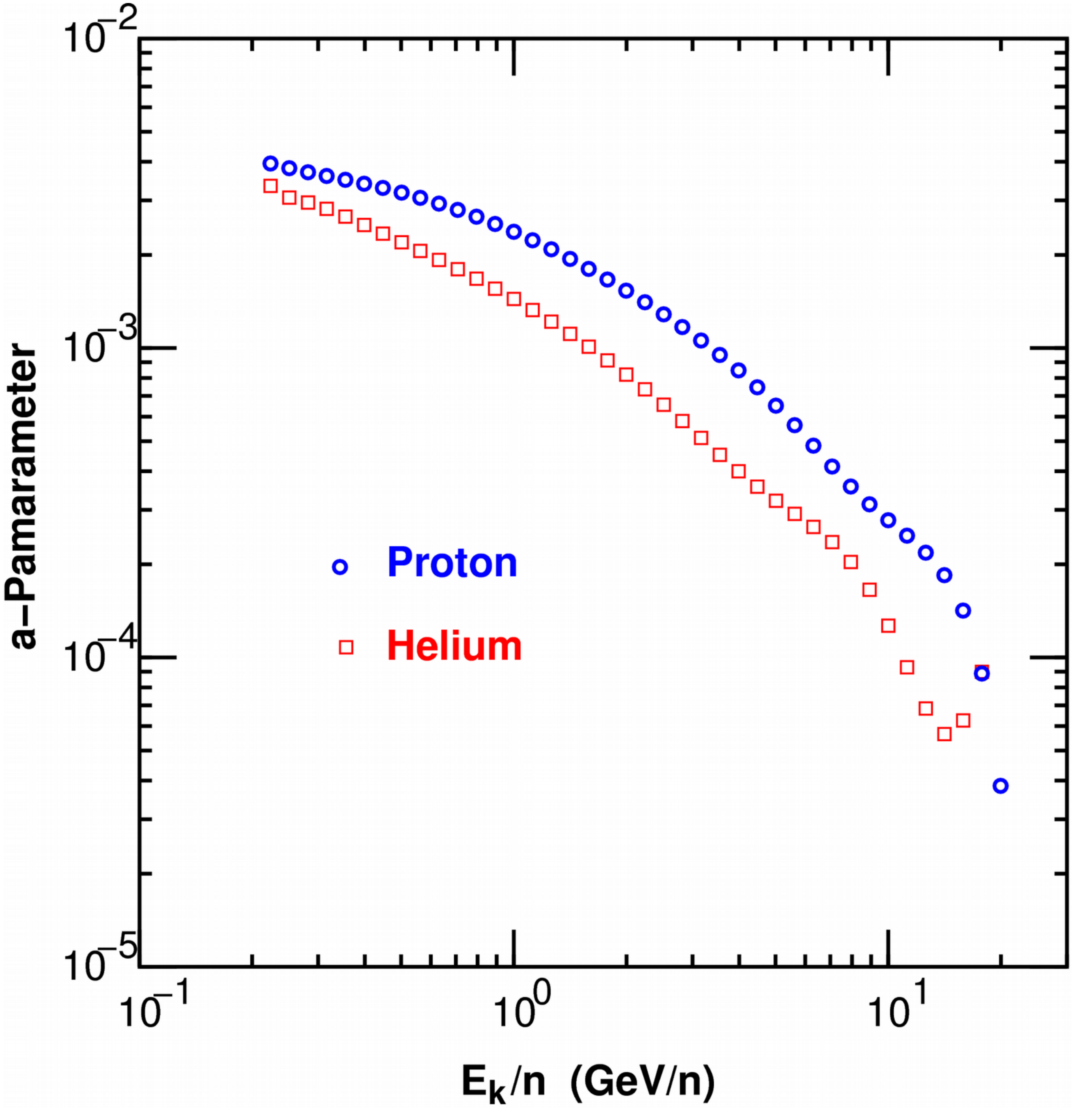




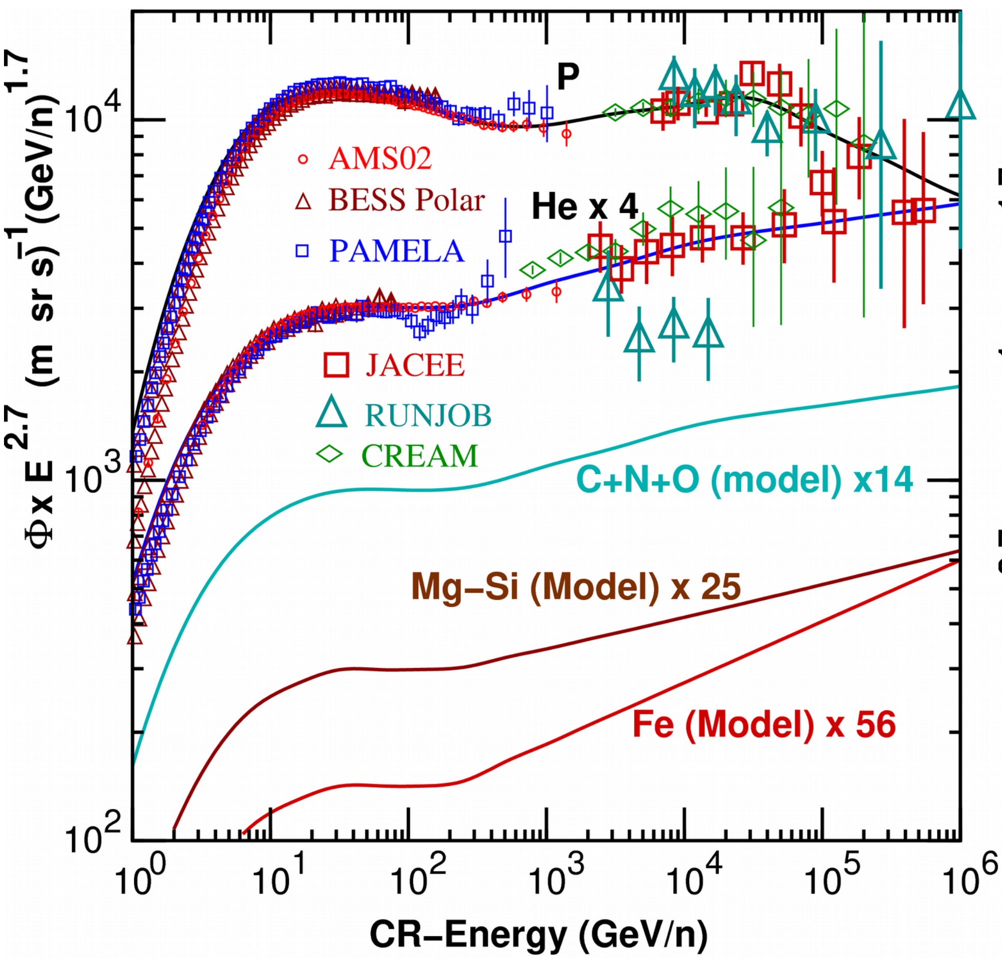
Practical Formula for Solar Modulation

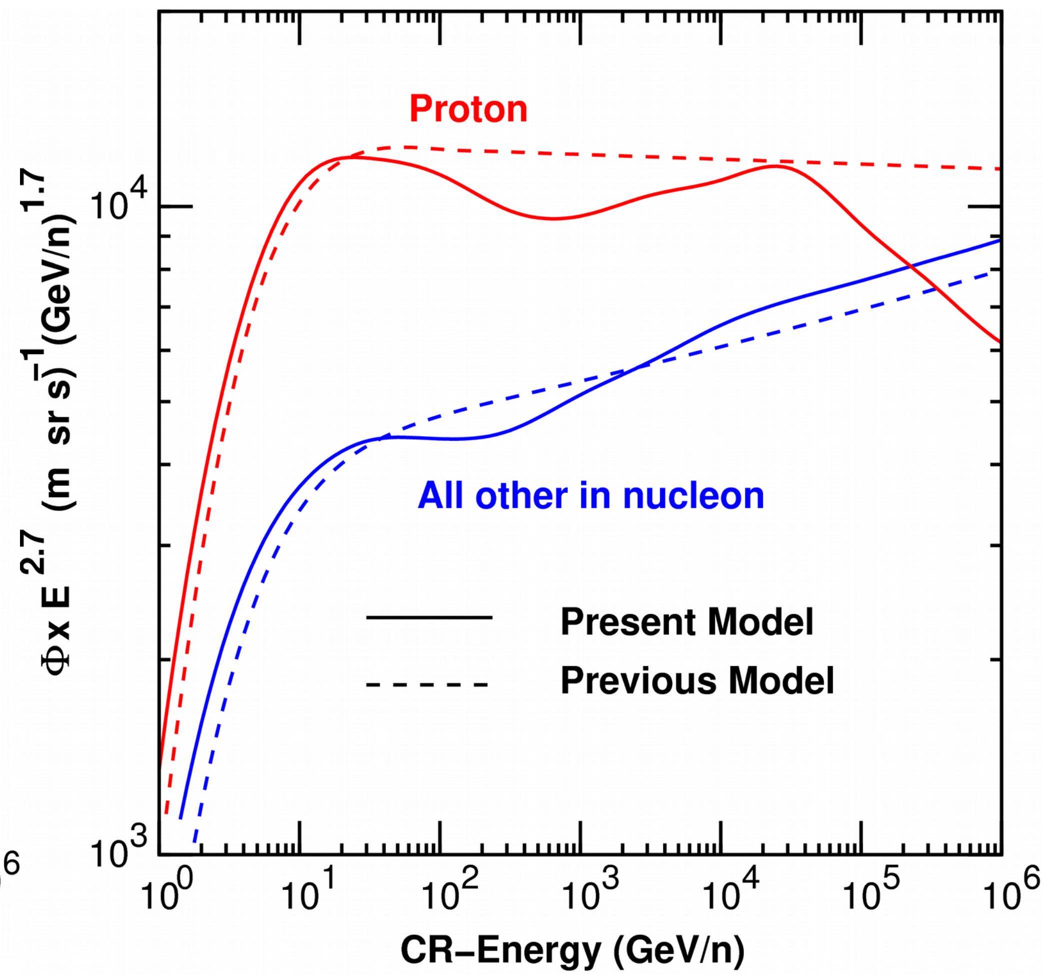
$$\phi(E,N) = \begin{cases} \phi(E,3710)\exp(a\cdot(N-3710)) \\ \phi(E,3710) \end{cases}$$

Where a is from right figure, and N is the Count of Newark Neutron Monitor.



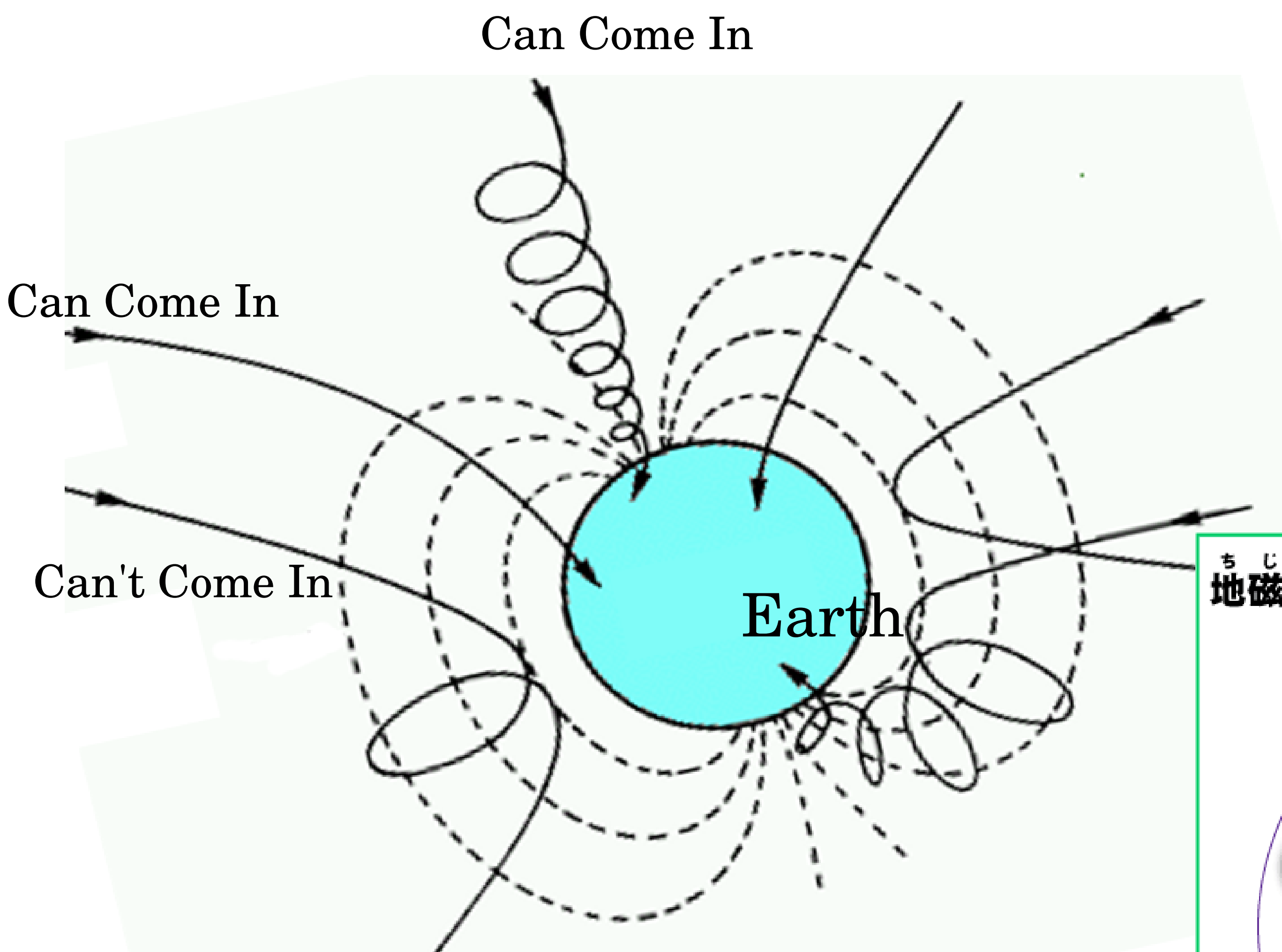
Cosmic Ray Spectra Model Based on AMS02 Observation

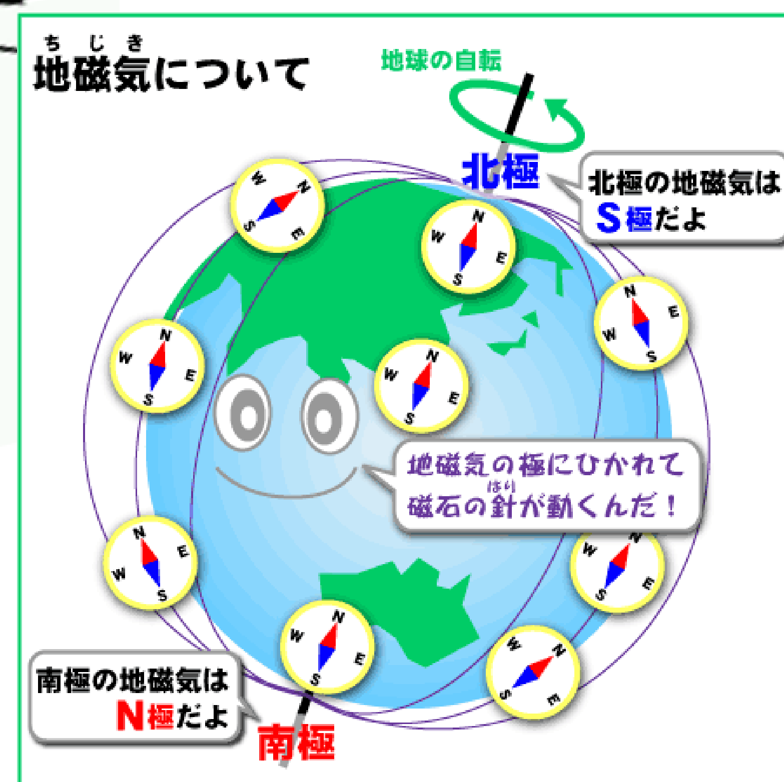




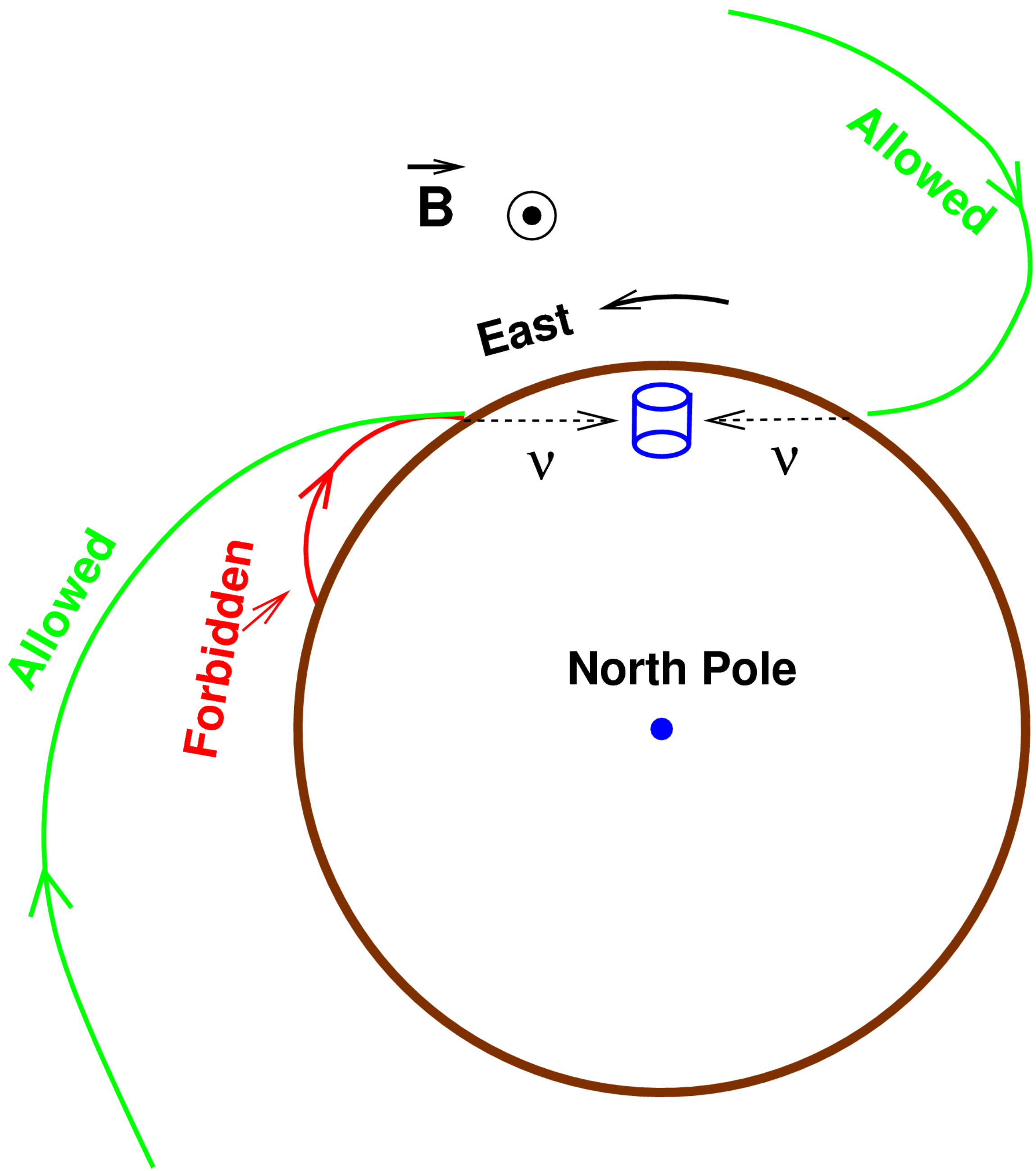
Rigidity Cutoff and Geomagnetic Field

Rigidity Cutoff and Geomagnetic Field (cartoon)





Rigidity Cut Off



Assuming

1. Dipole magnetic field with the Same axis as the Earth rotation

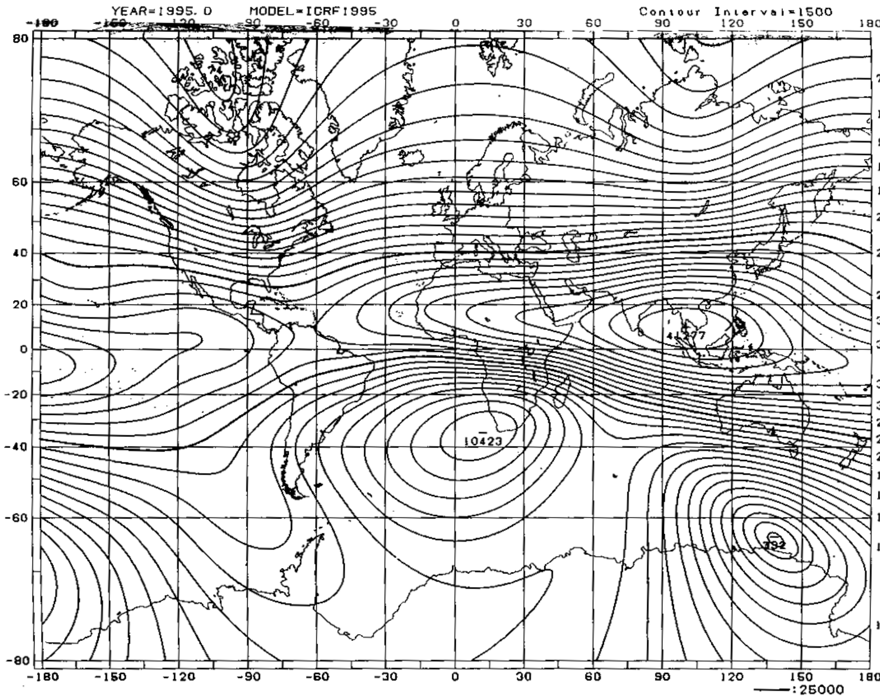
2. $B_h - 0.3\times10^{-4}$ at the Equator

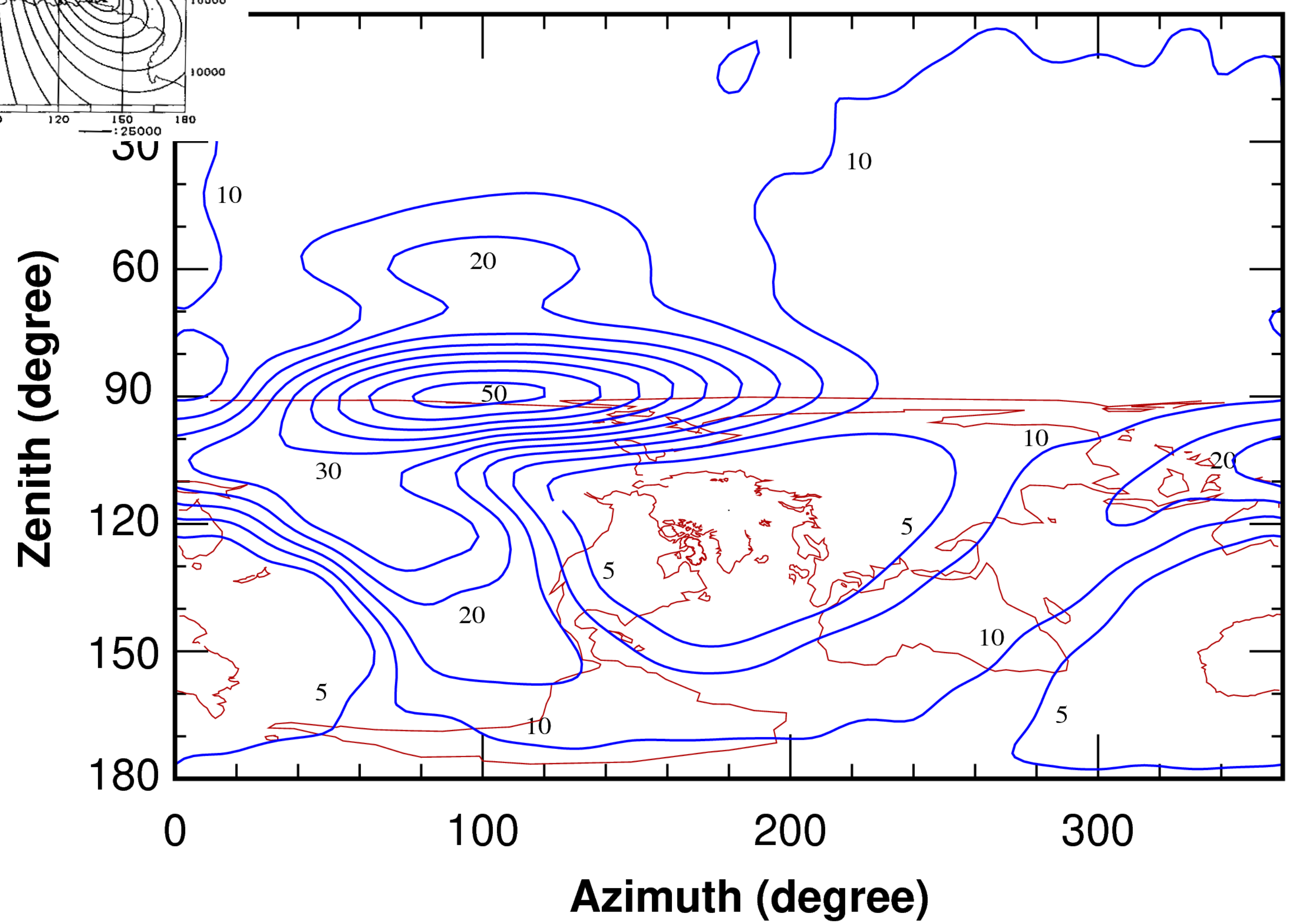
Calculate the rigidity cutoff for East directio at equator.

You may use the formula

$$\left(\frac{E}{1\,GeV}\right) \leq 0.3\times\left(\frac{r_g}{1\,m}\right)\left(\frac{B}{1\,T}\right)$$

Geomagnetic field (horizontal, IGRF2000)



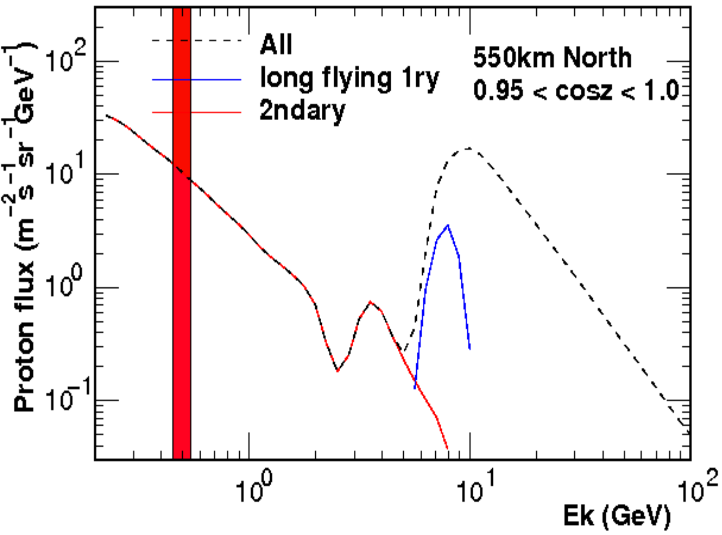


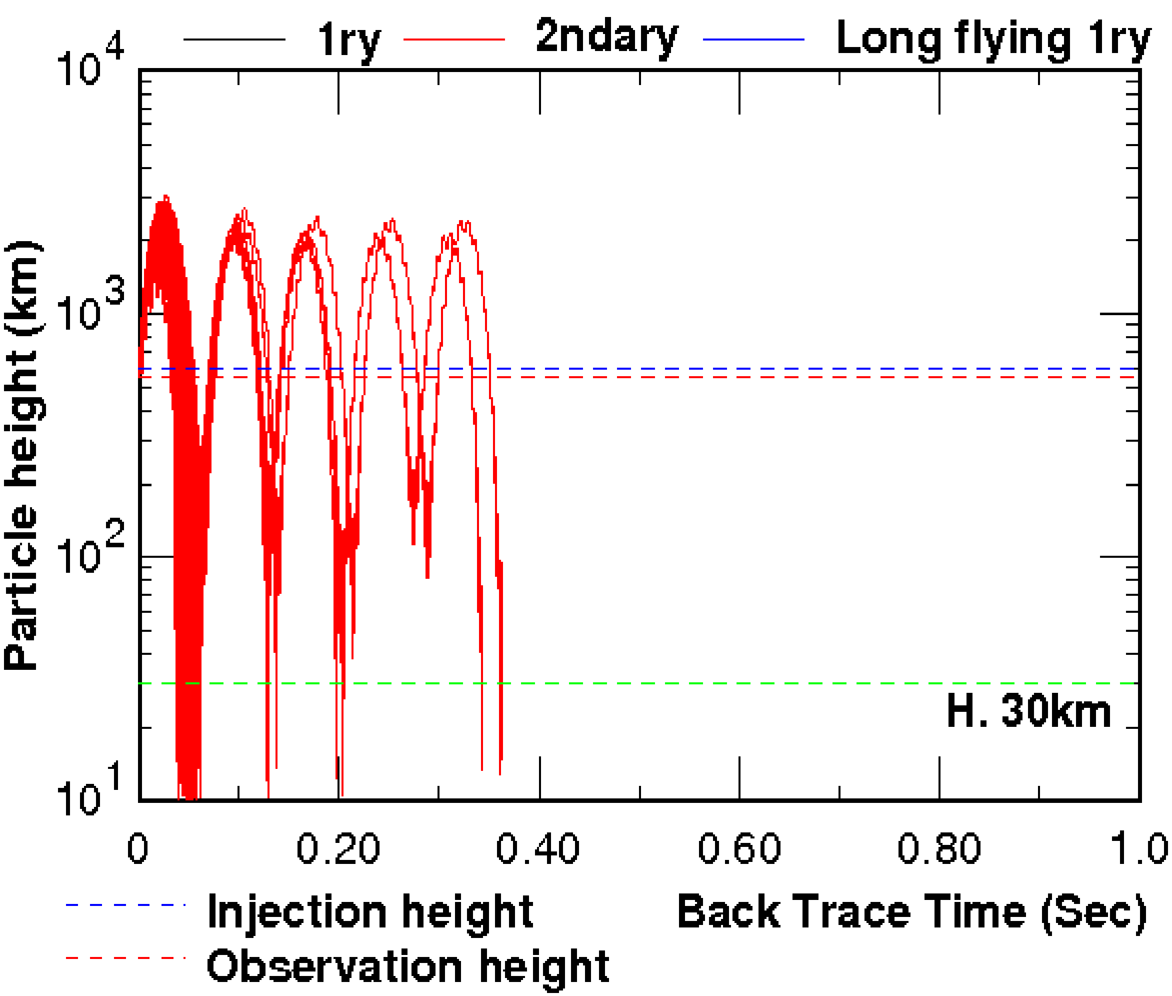
Rigidity Cutoff
For SK direction

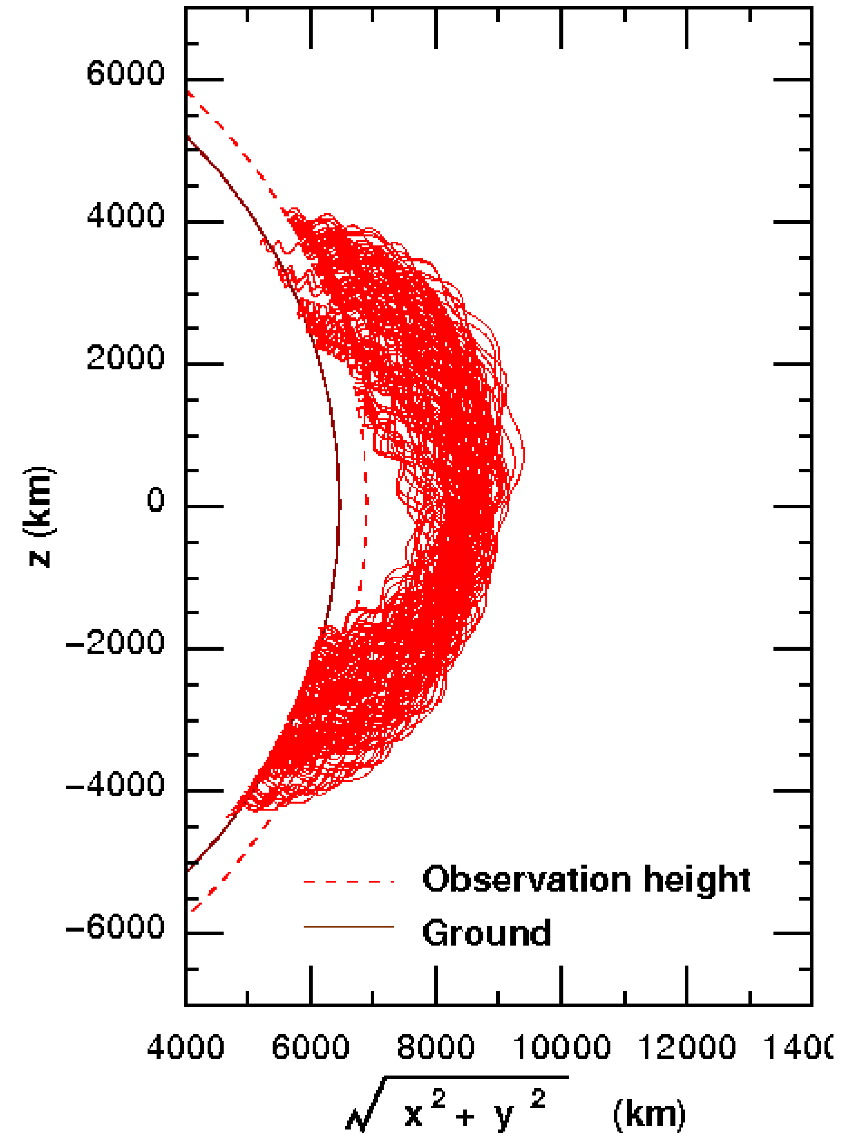
Back Trace of Observed Particles

1.000 < P < 1.122 GeV/c

0.454 < Ek < 0.544 GeV



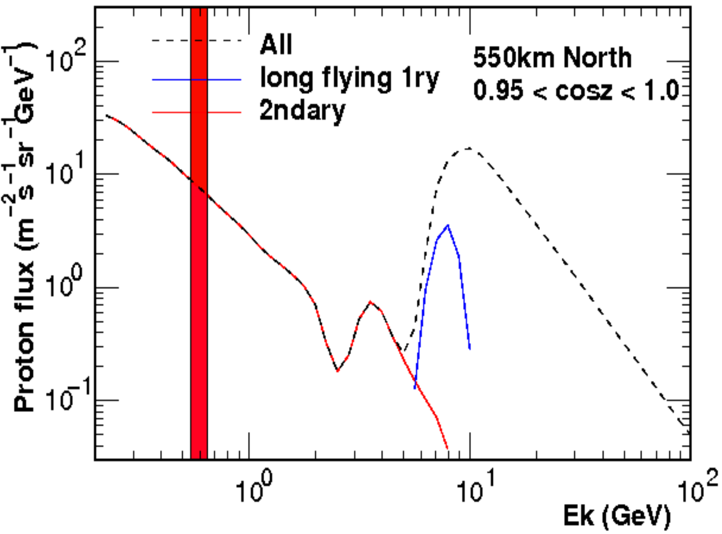


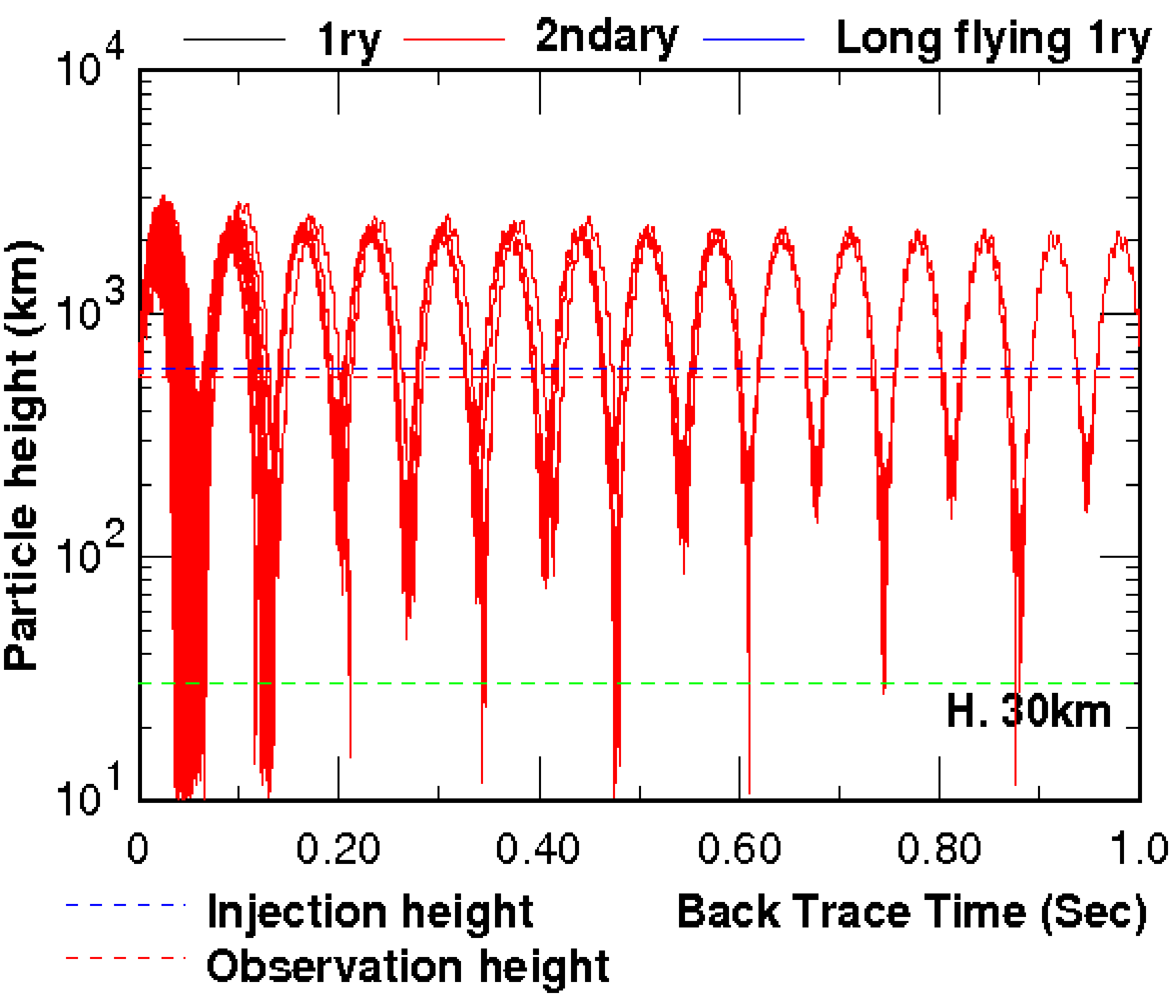


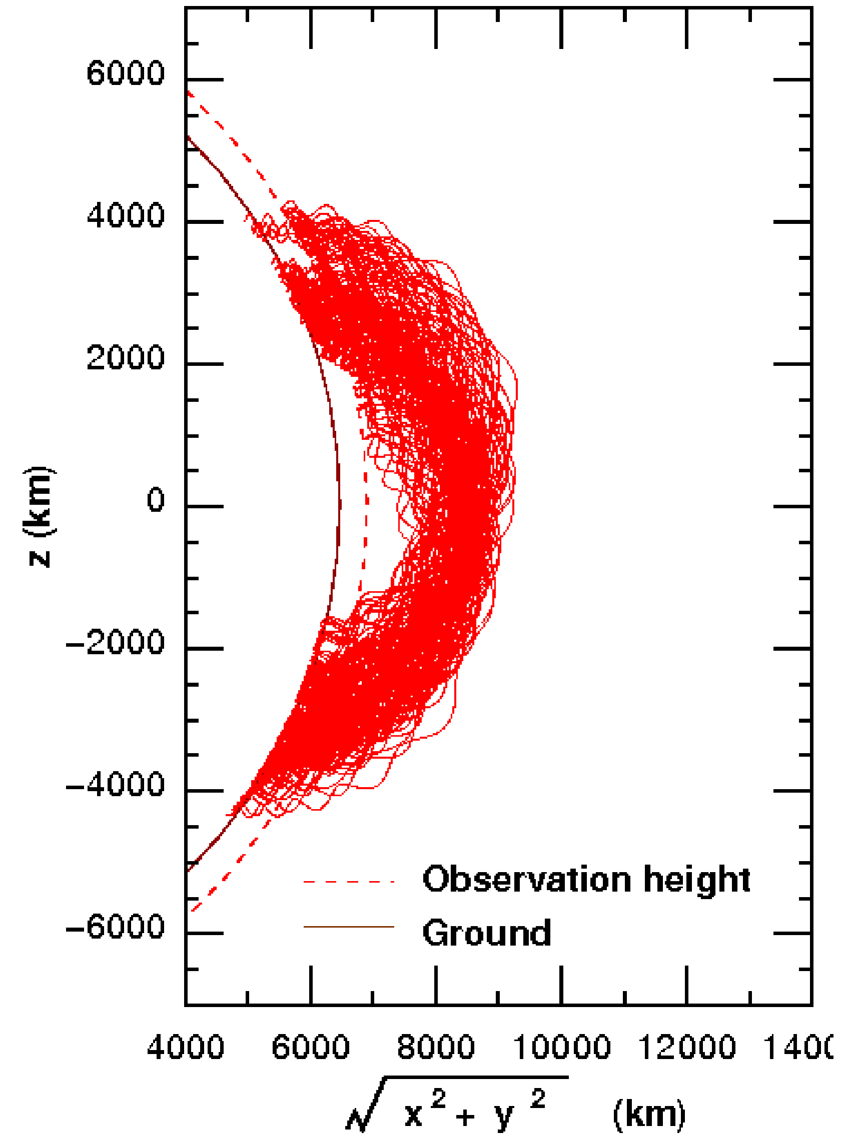
Back Trace of Observed Particles

1.122 < P < 1.259 GeV/c

0.544 < Ek < 0.650 GeV



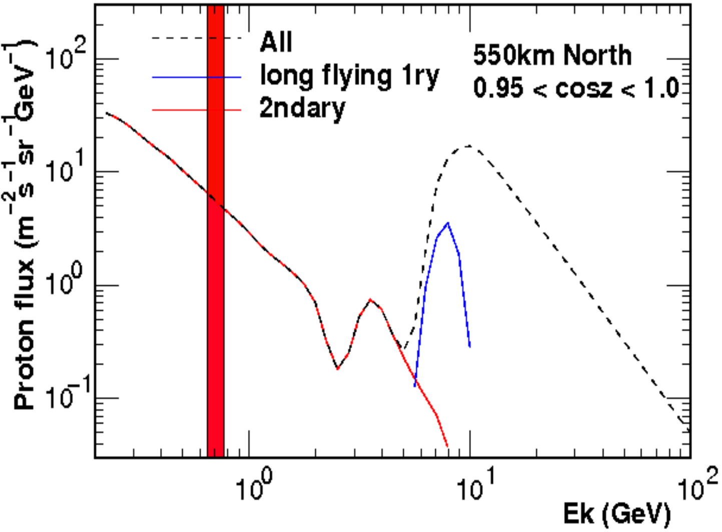


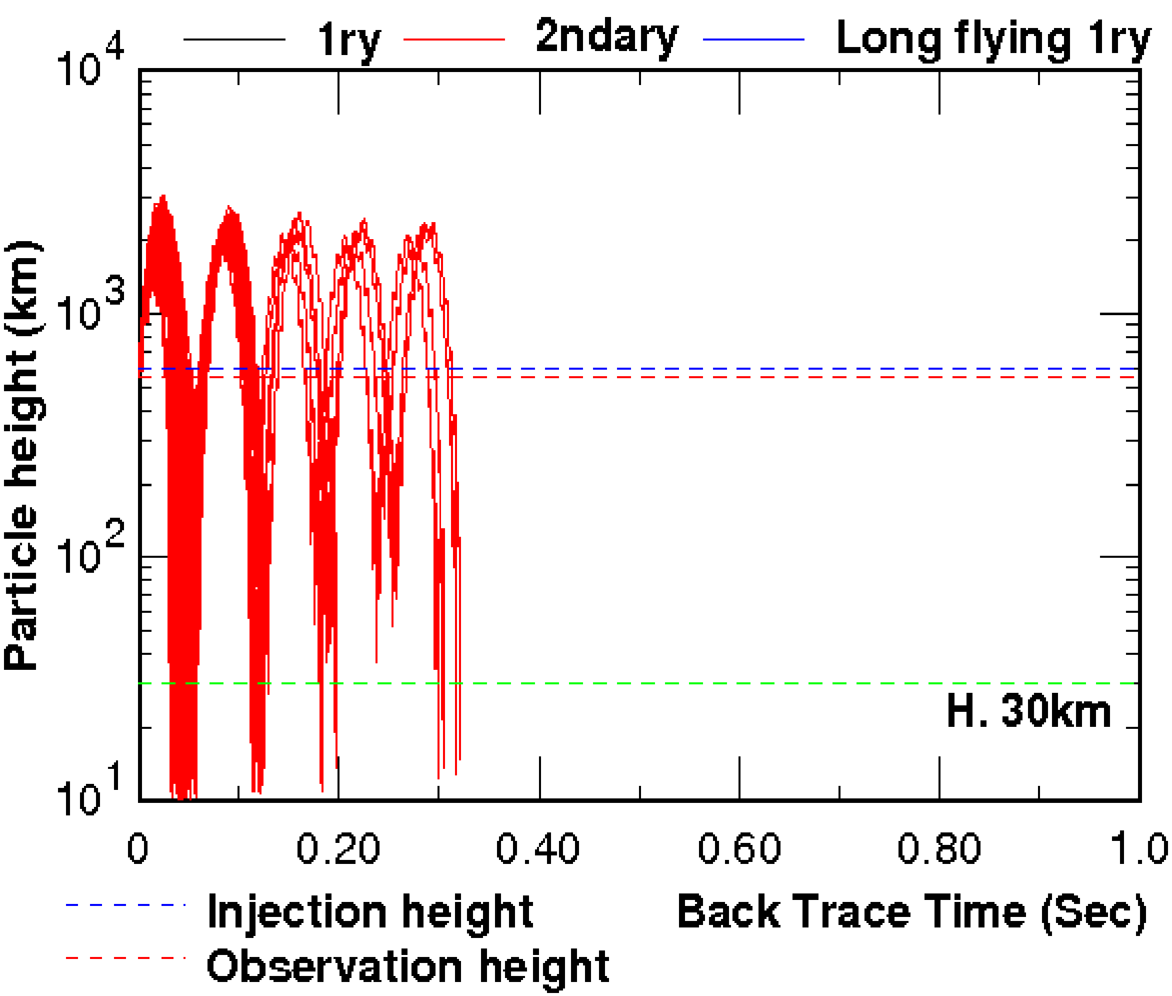


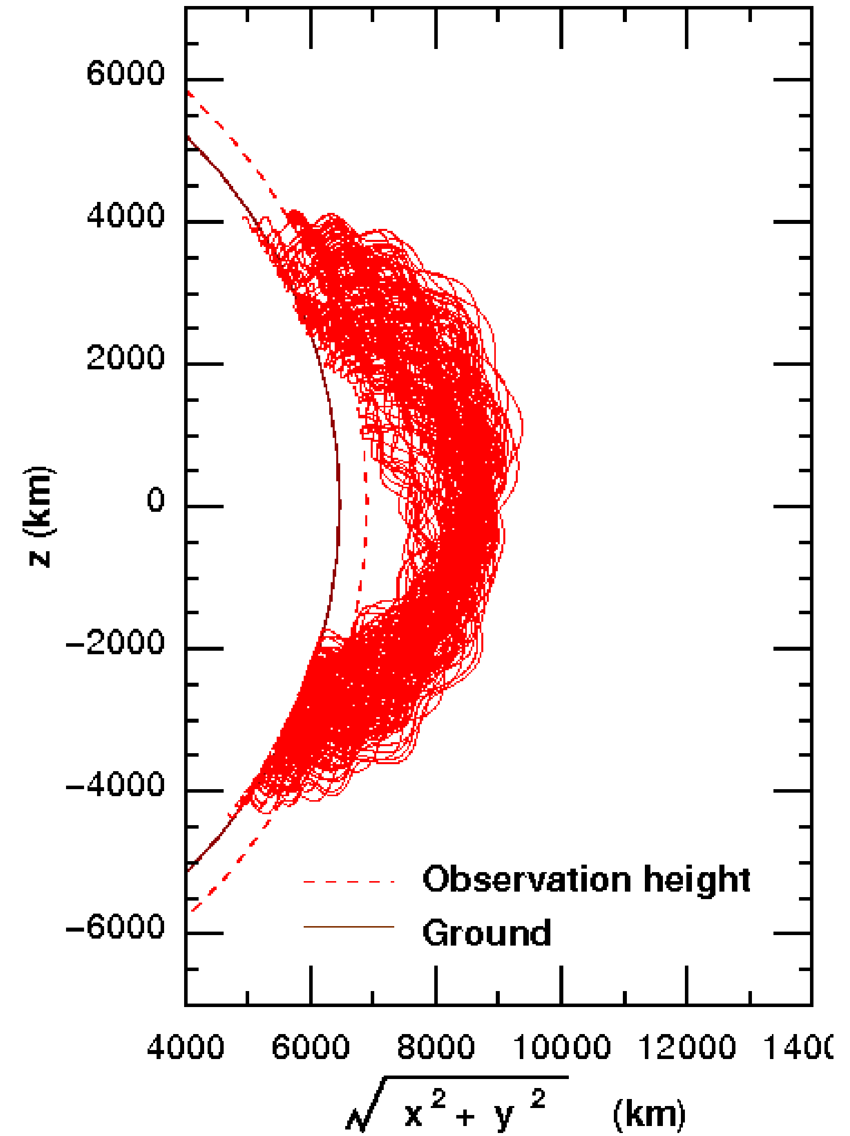
Back Trace of Observed Particles

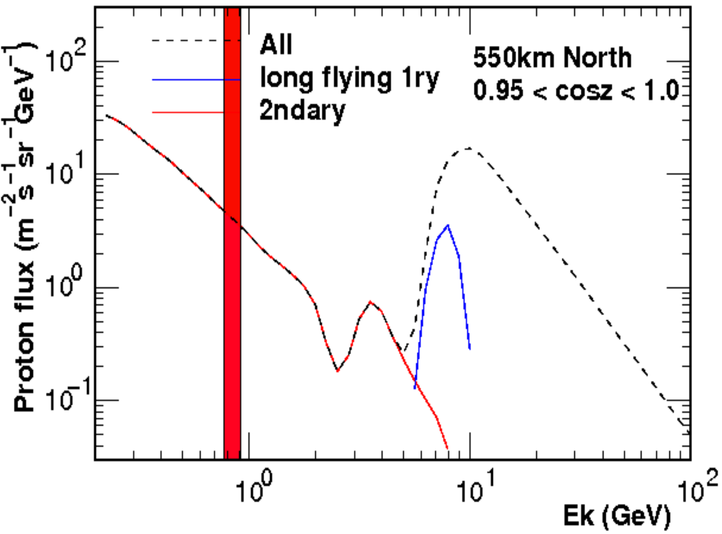
1.259 < P < 1.413 GeV/c

0.650 < Ek < 0.774 GeV





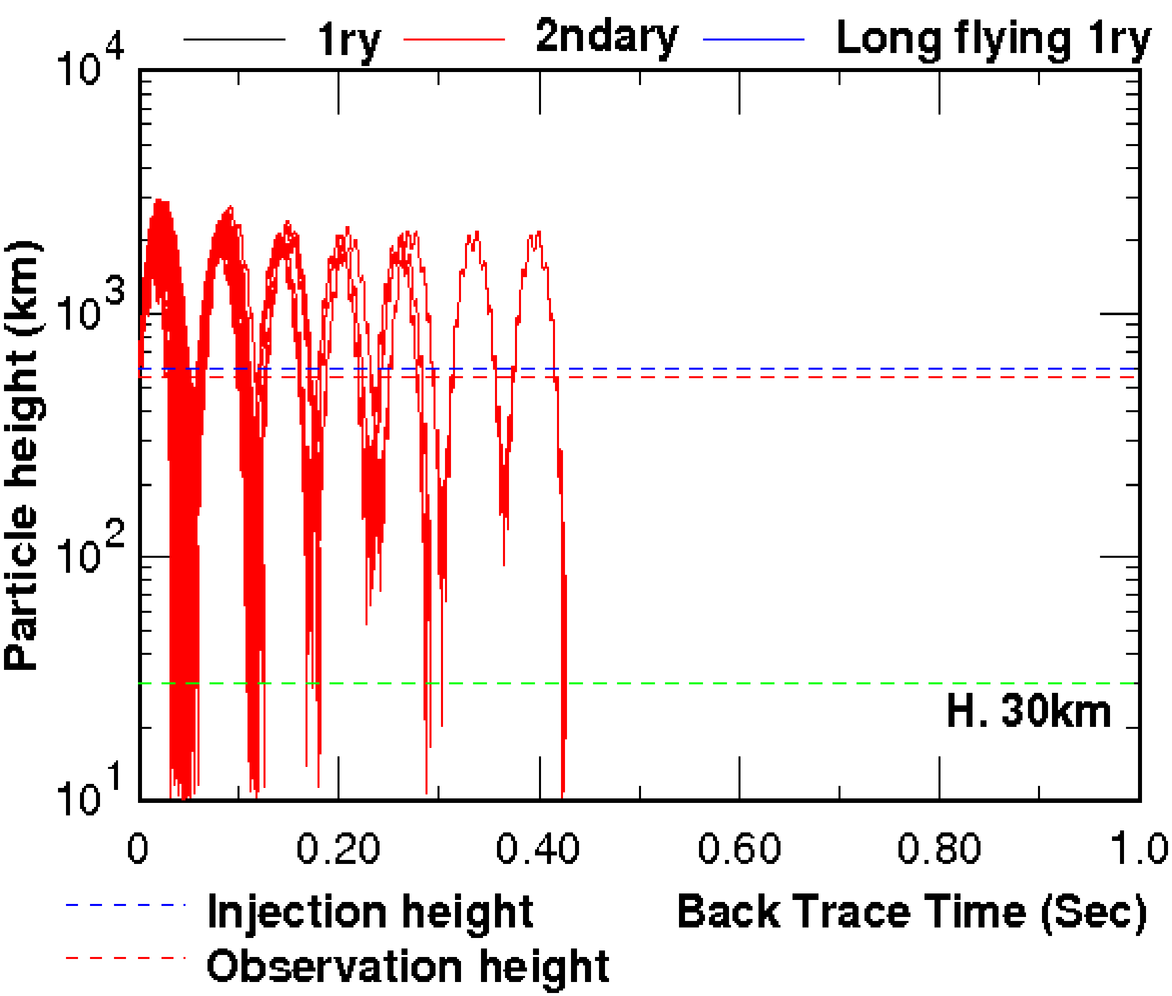


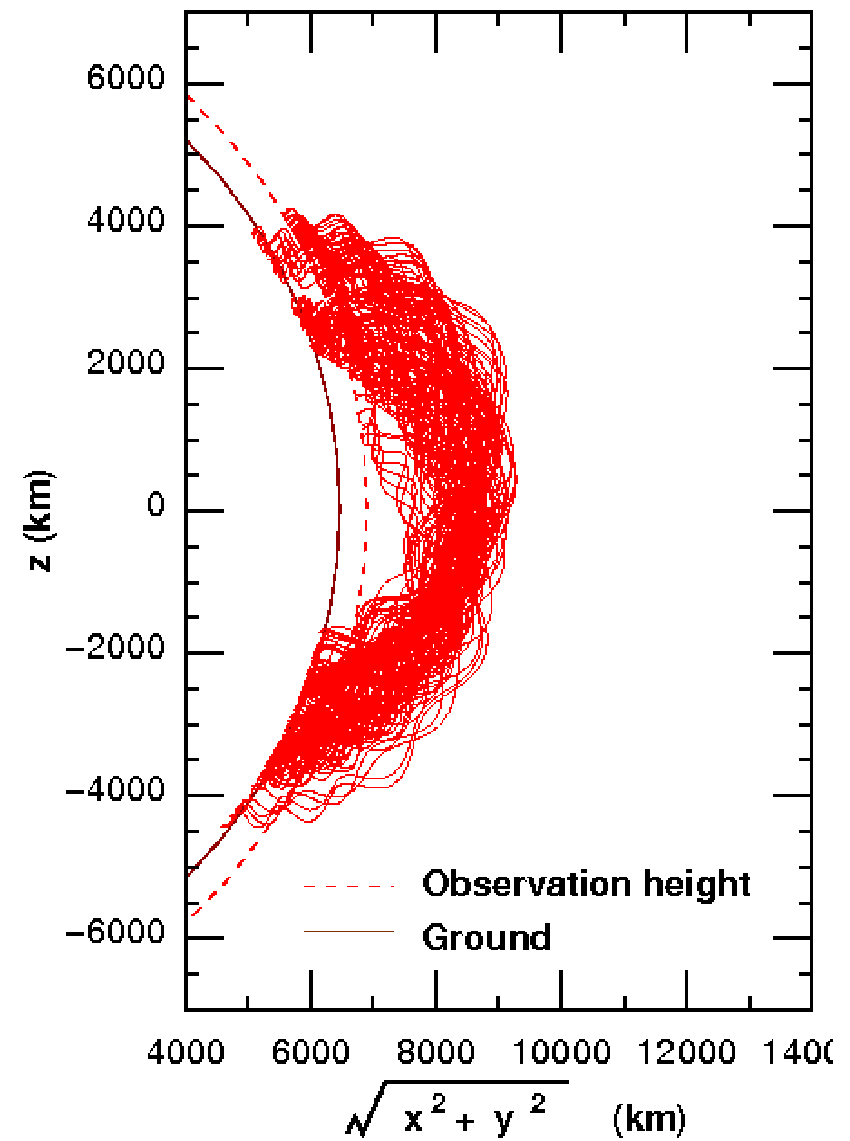


Back Trace of Observed Particles

$1.413 < P < 1.585$ GeV/c

$0.774 < Ek < 0.919$ GeV

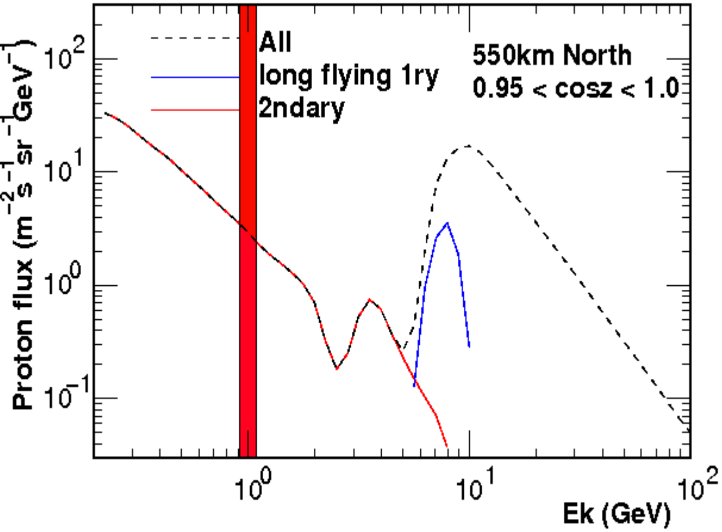


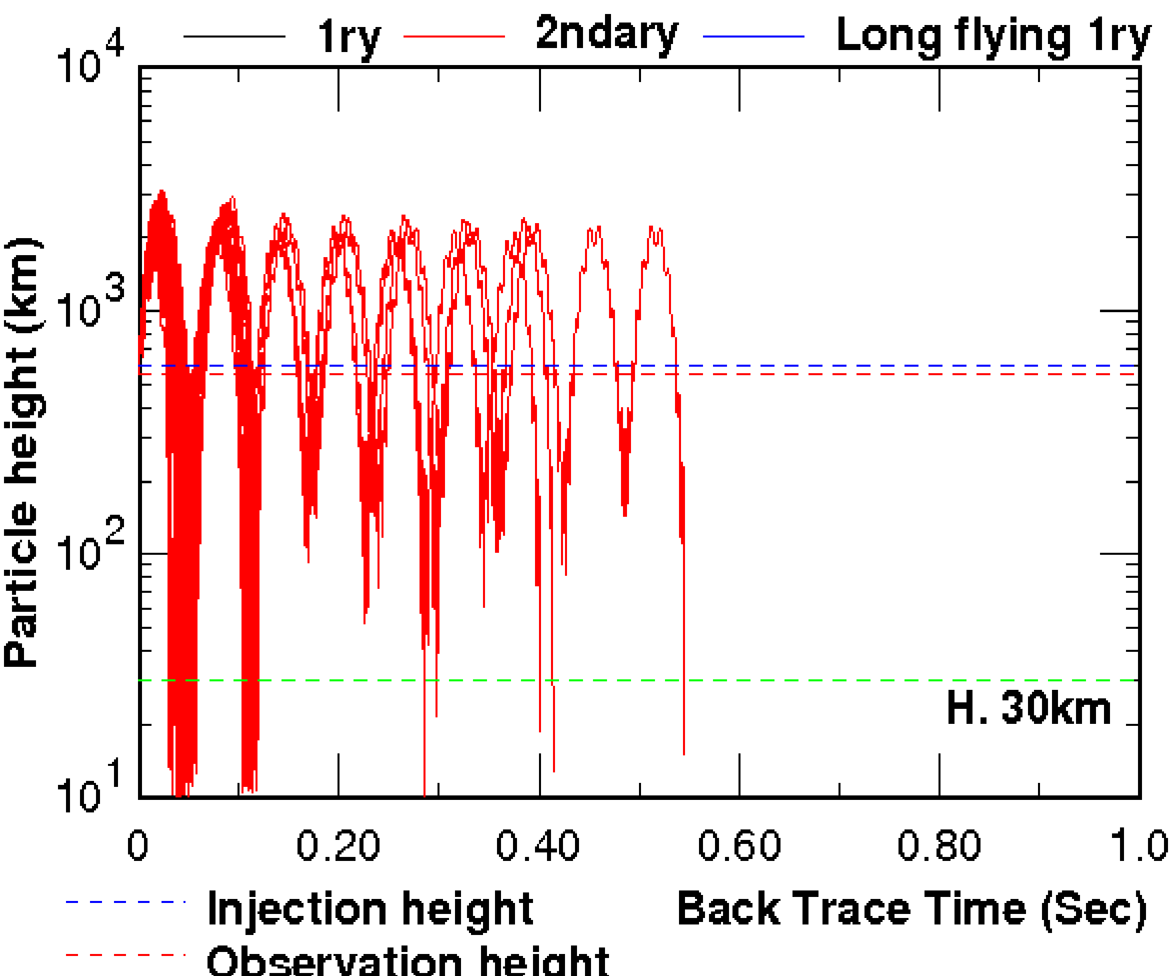


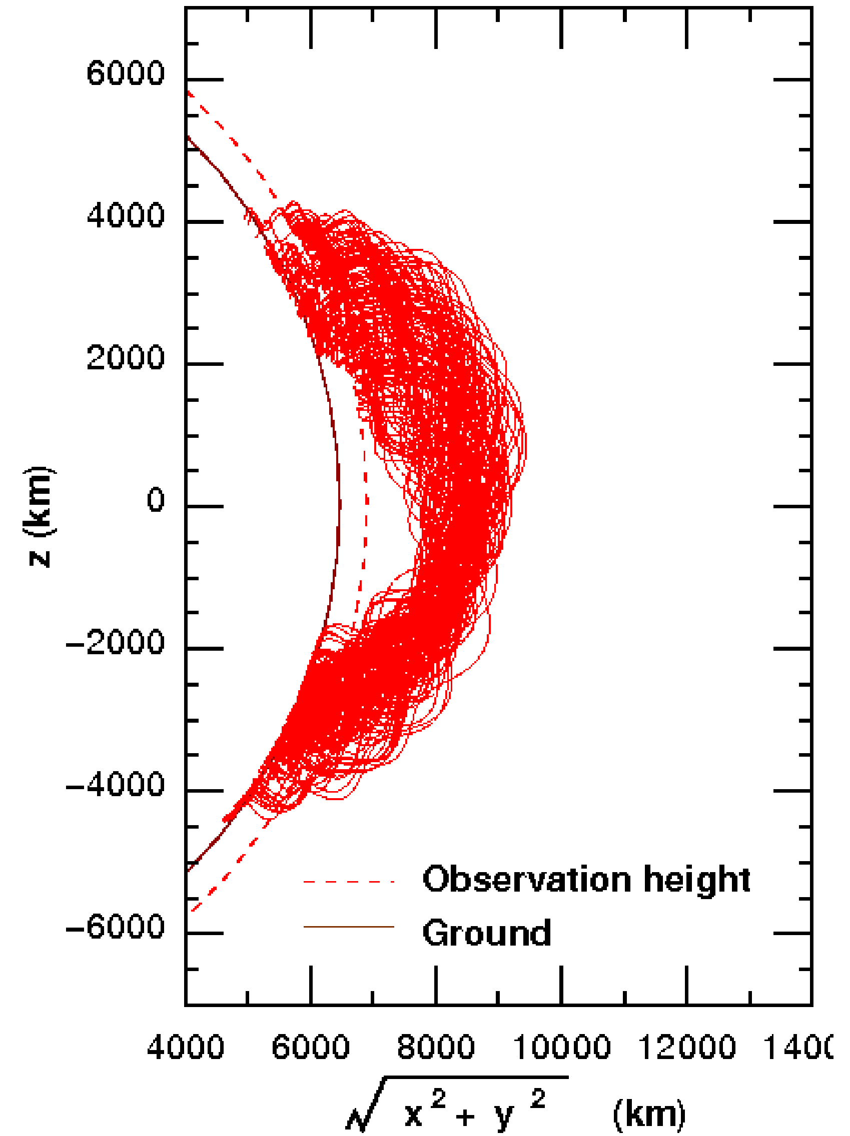
Back Trace of Observed Particles

$1.585 < P < 1.778$ GeV/c

$0.919 < Ek < 1.086$ GeV



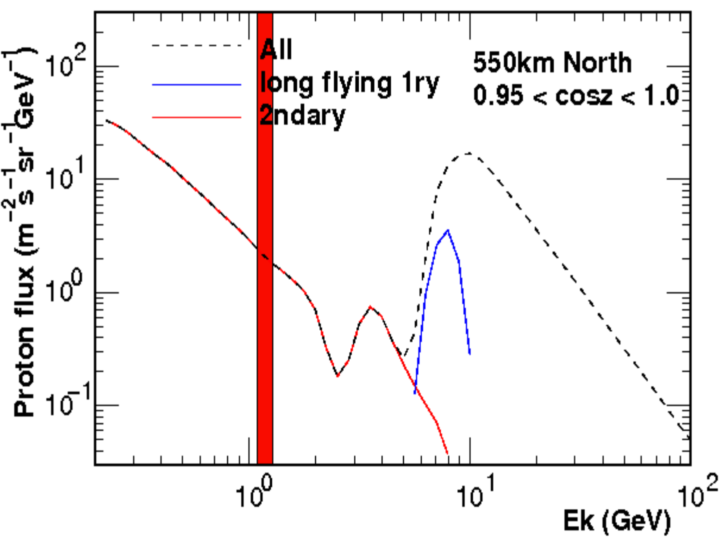


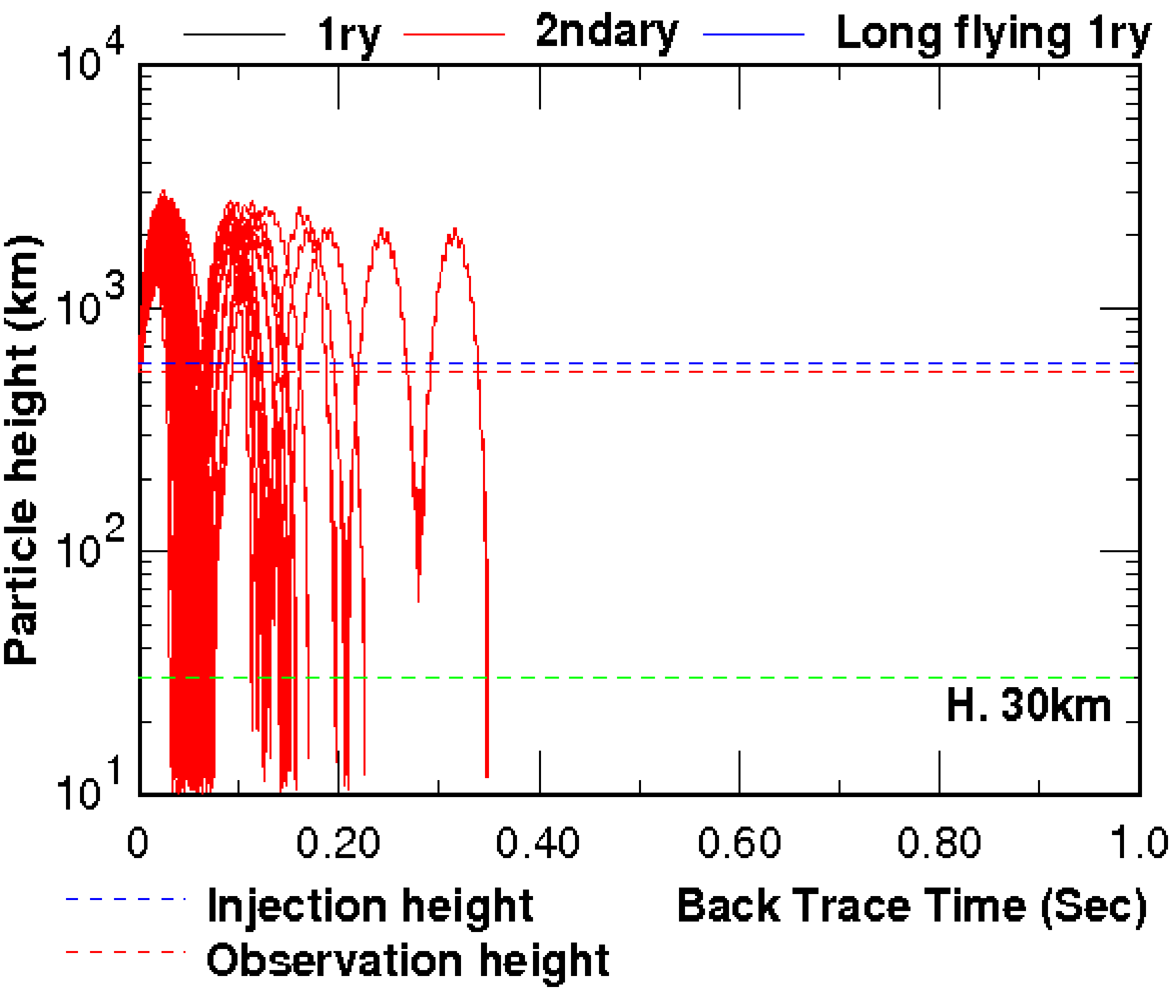


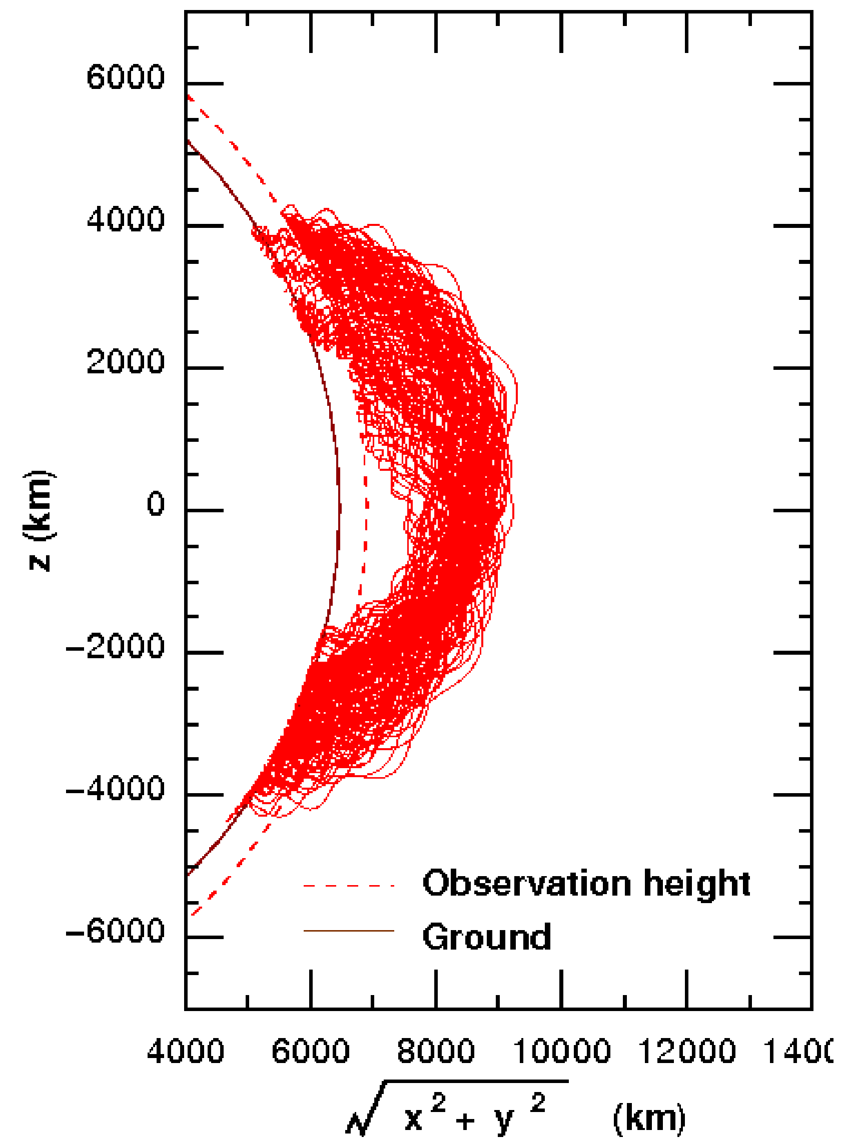
Back Trace of Observed Particles

1.778 < P < 1.995 GeV/c

1.086 < Ek < 1.280 GeV



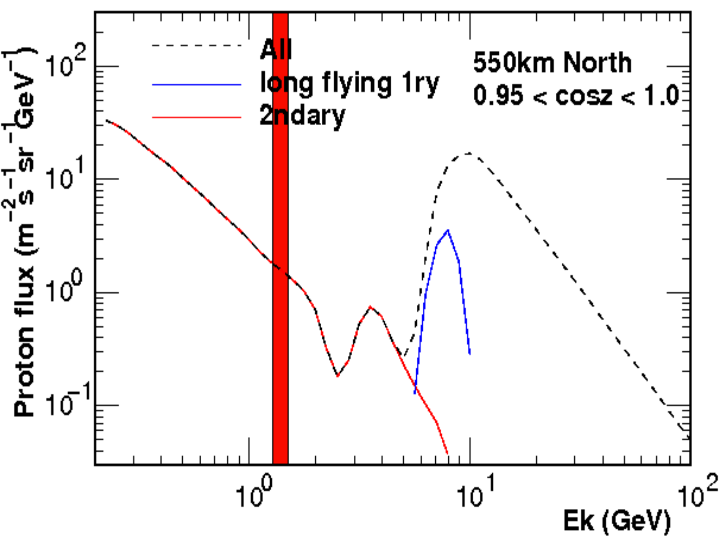


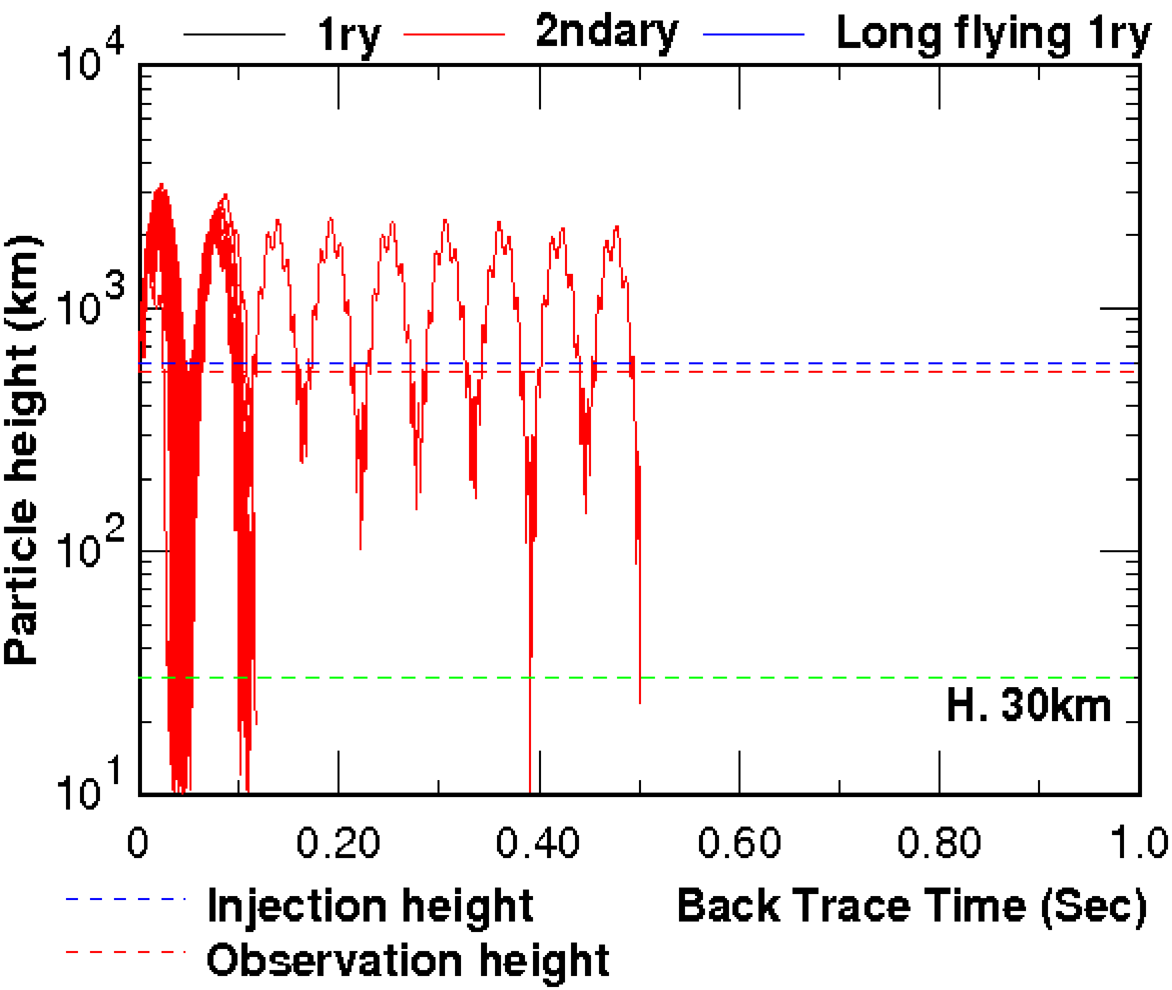


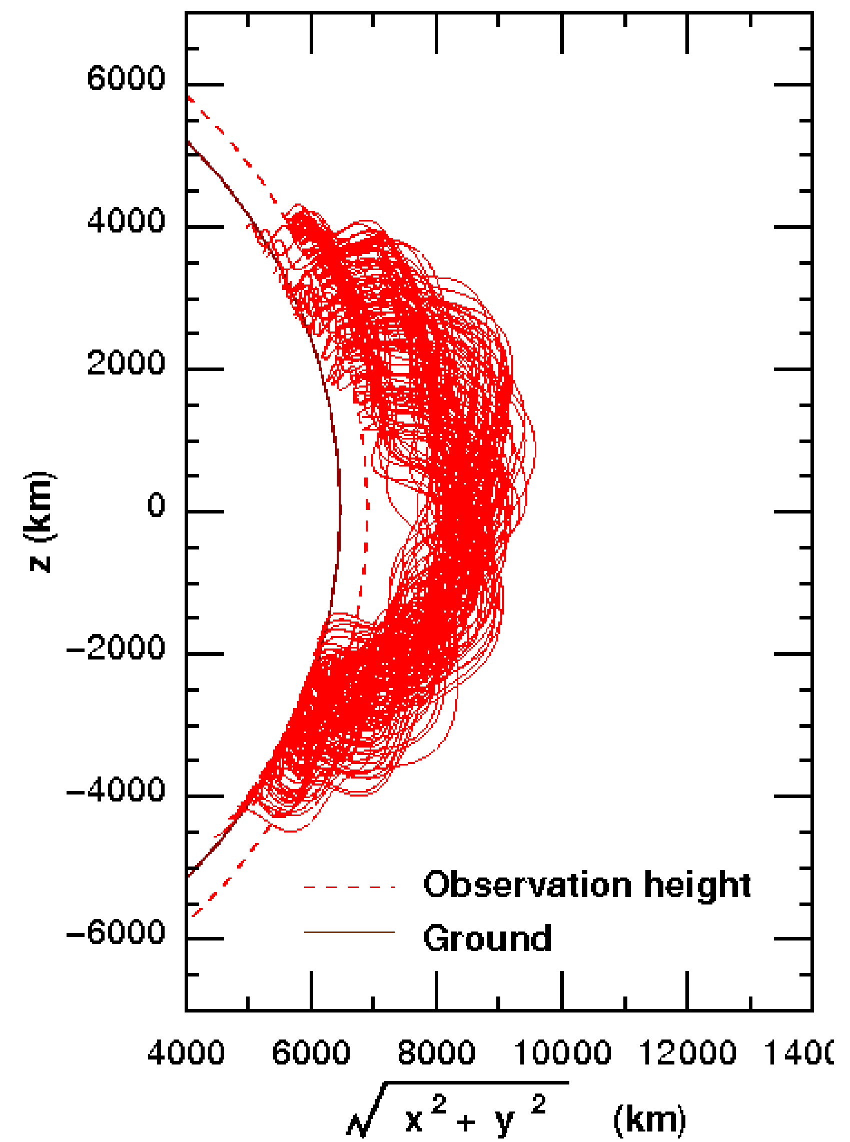
Back Trace of Observed Particles

1.995 < P < 2.239 GeV/c

1.280 < Ek < 1.501 GeV



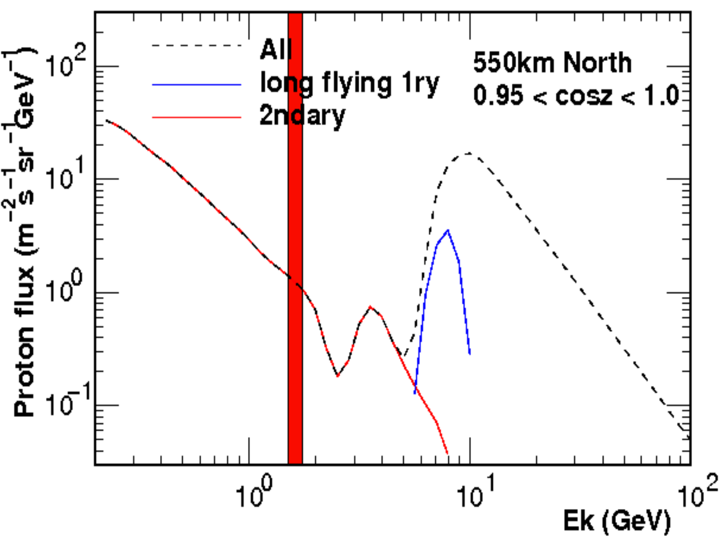


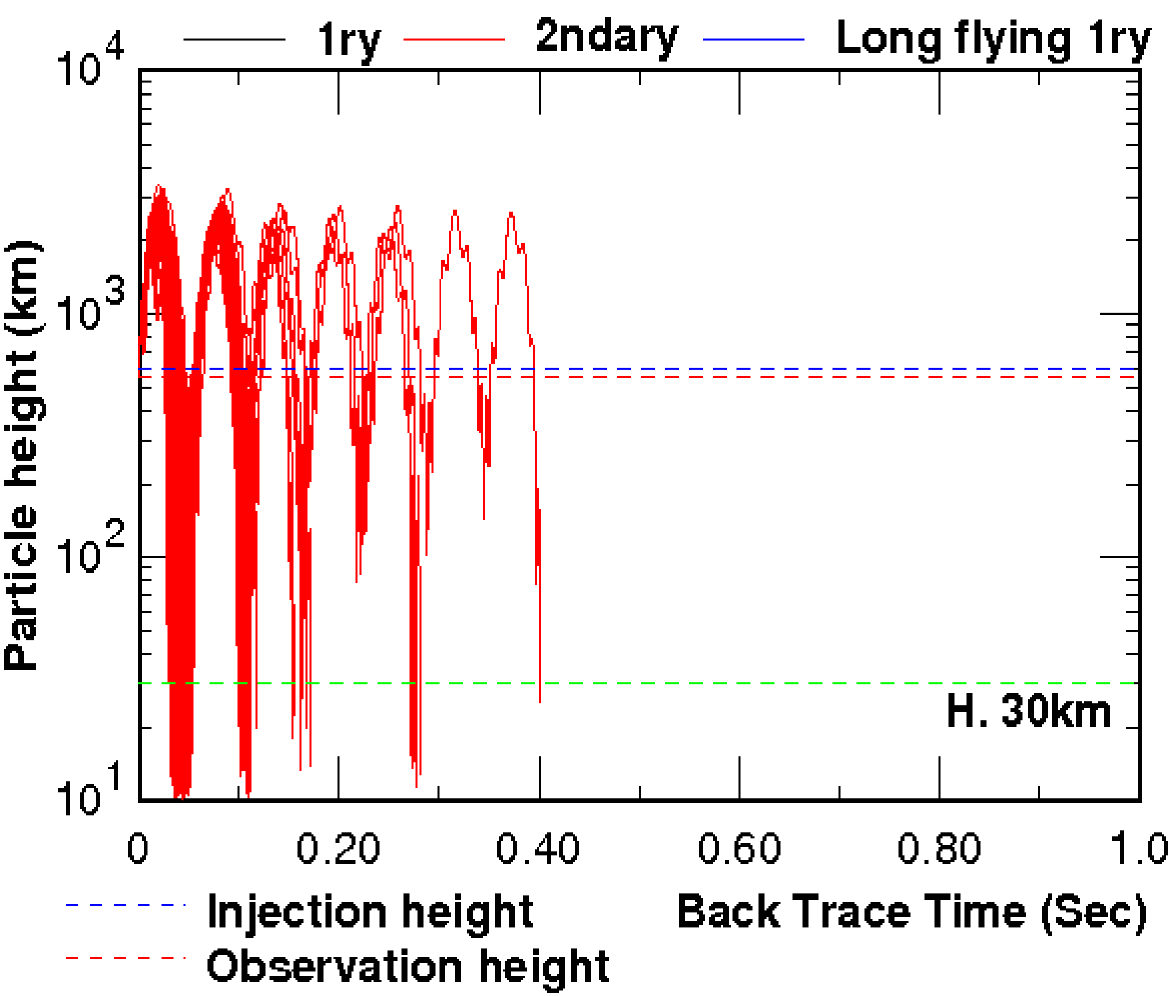


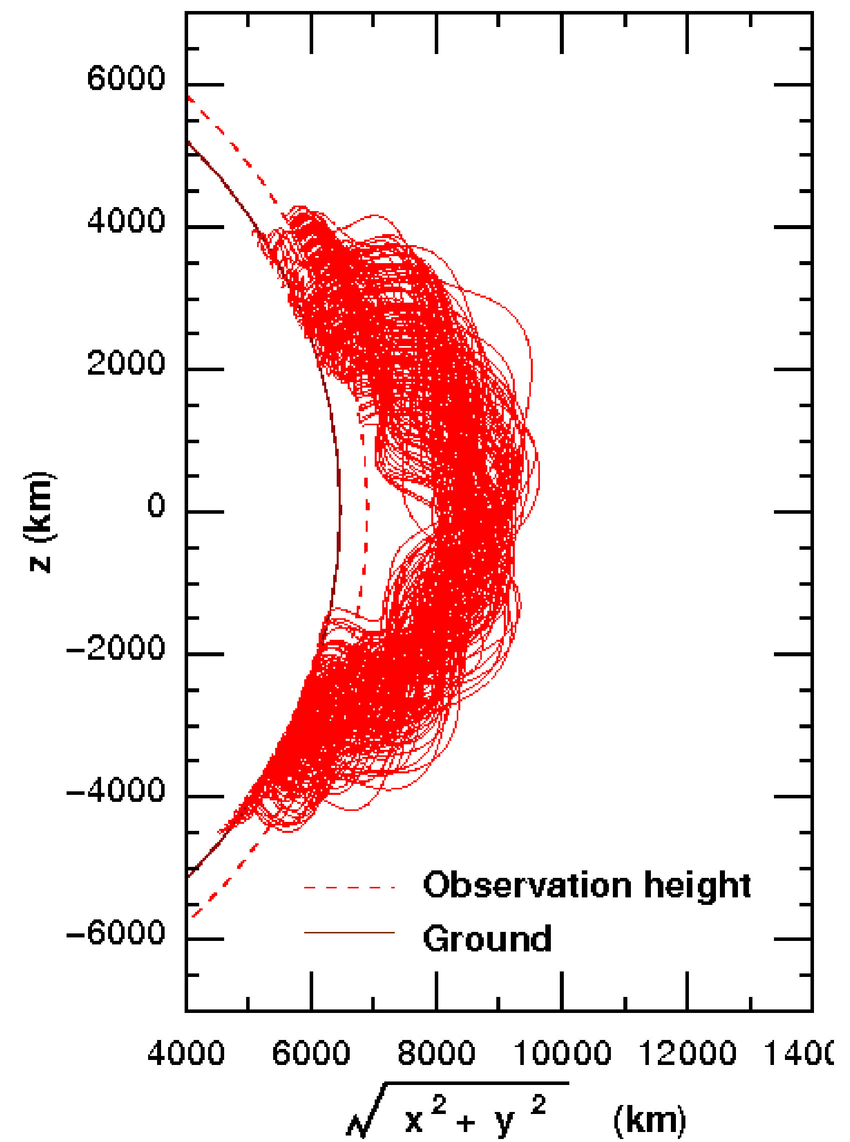
Back Trace of Observed Particles

2.239 < P < 2.511 GeV/c

1.501 < Ek < 1.754 GeV



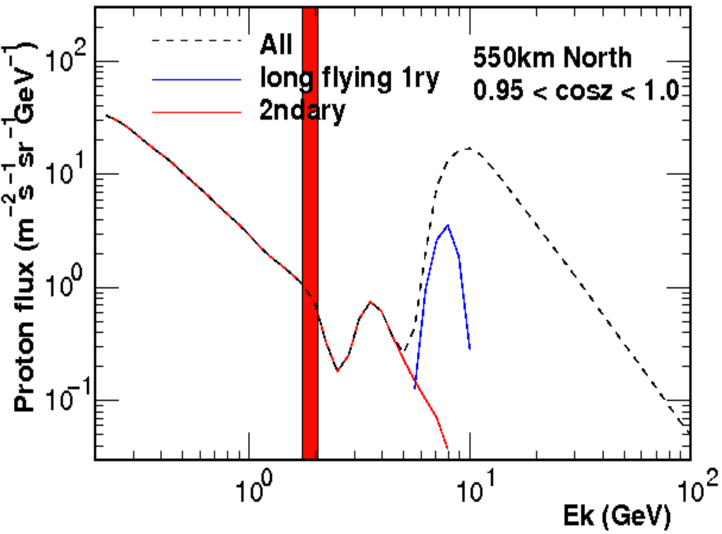


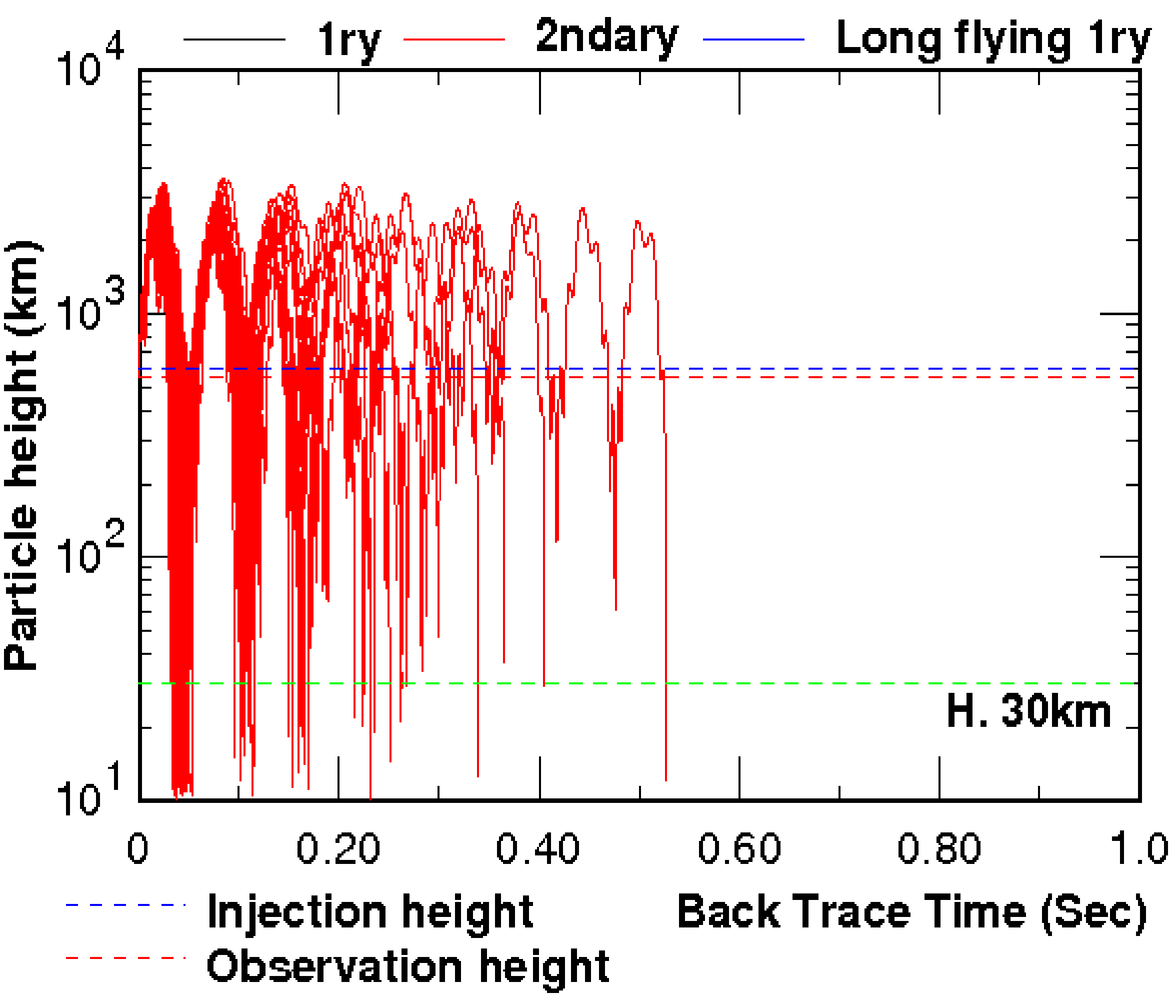


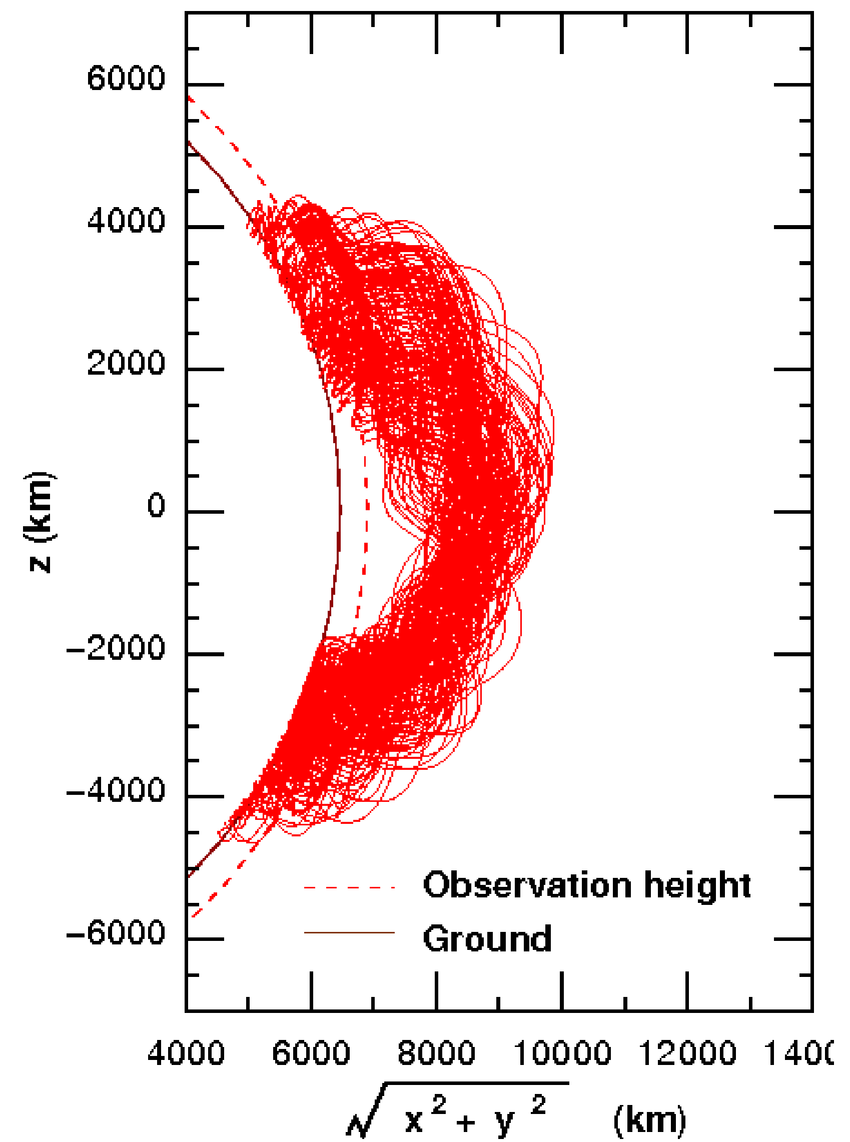
Back Trace of Observed Particles

2.512 < P < 2.818GeV/c

1.754 < Ek < 2.042 GeV



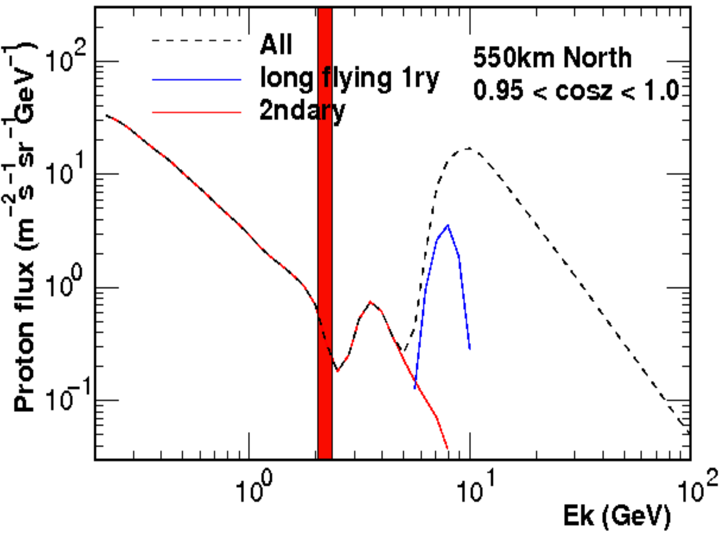


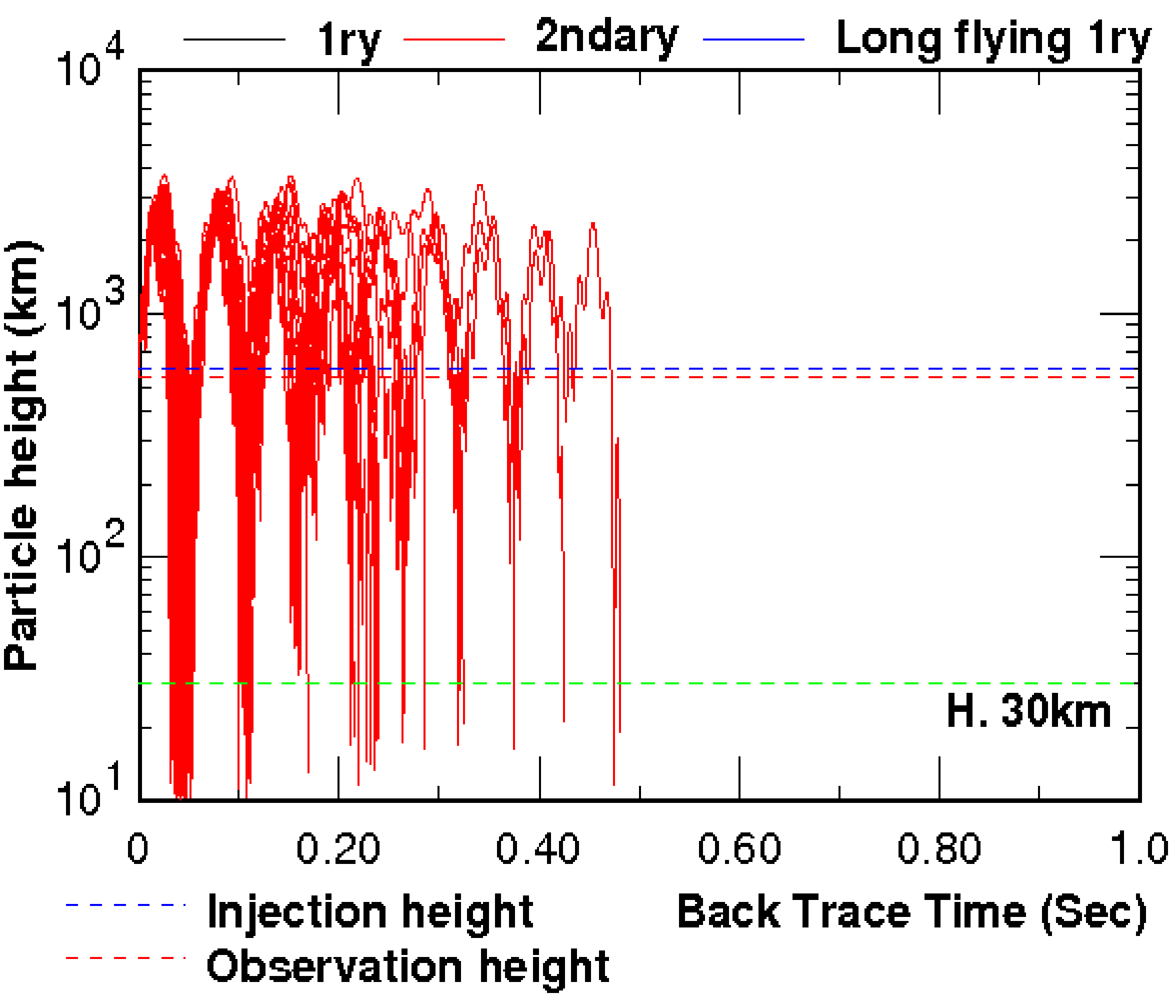


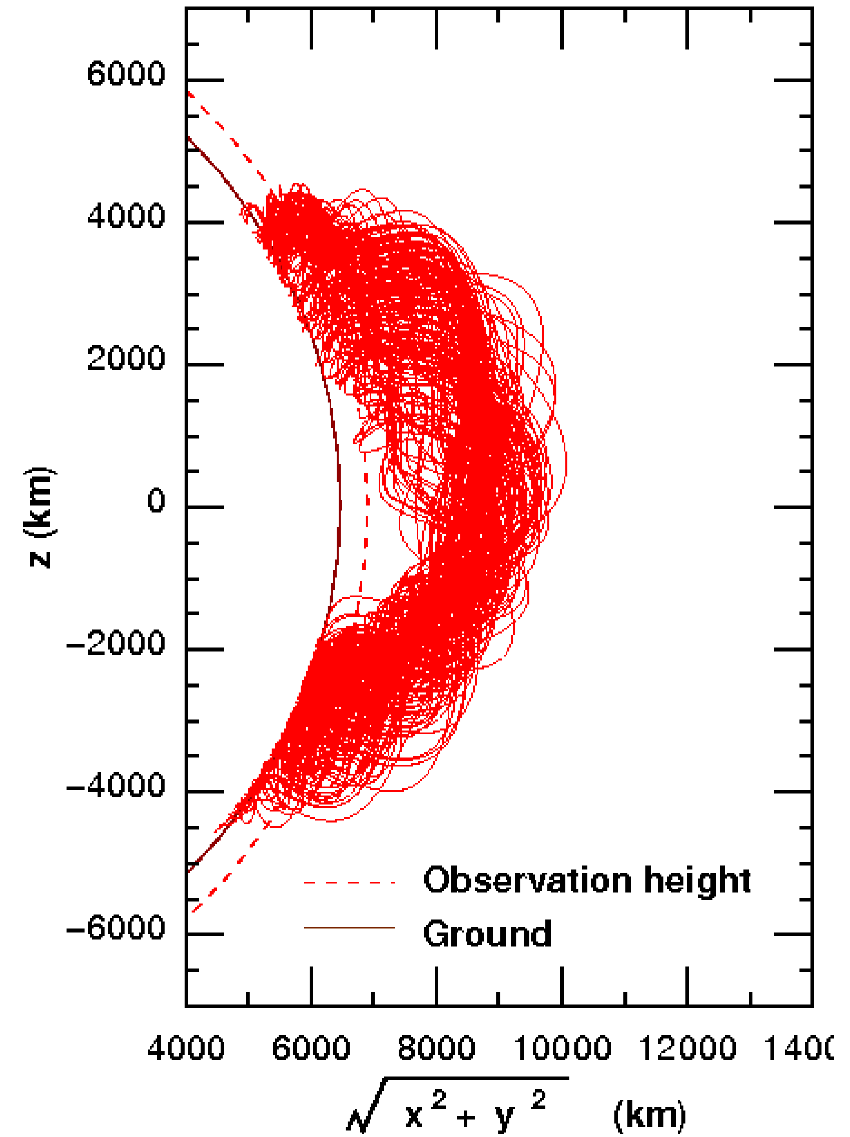
Back Trace of Observed Particles

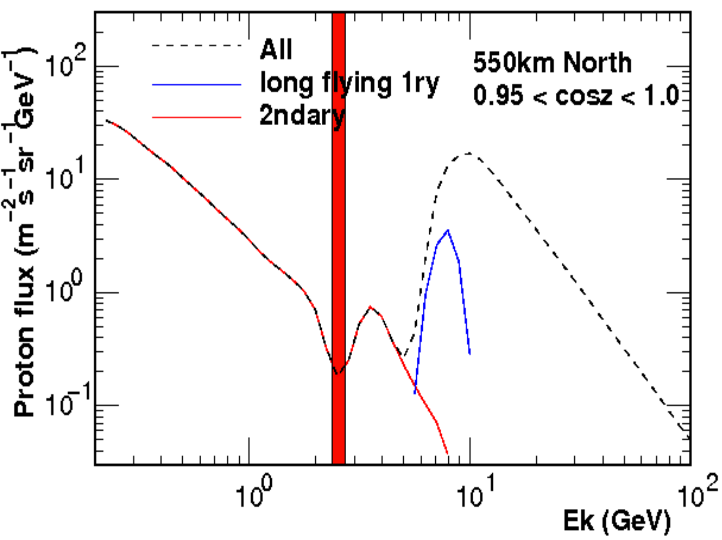
2.818 < P < 3.162 GeV/c

2.042 < Ek < 2.369 GeV





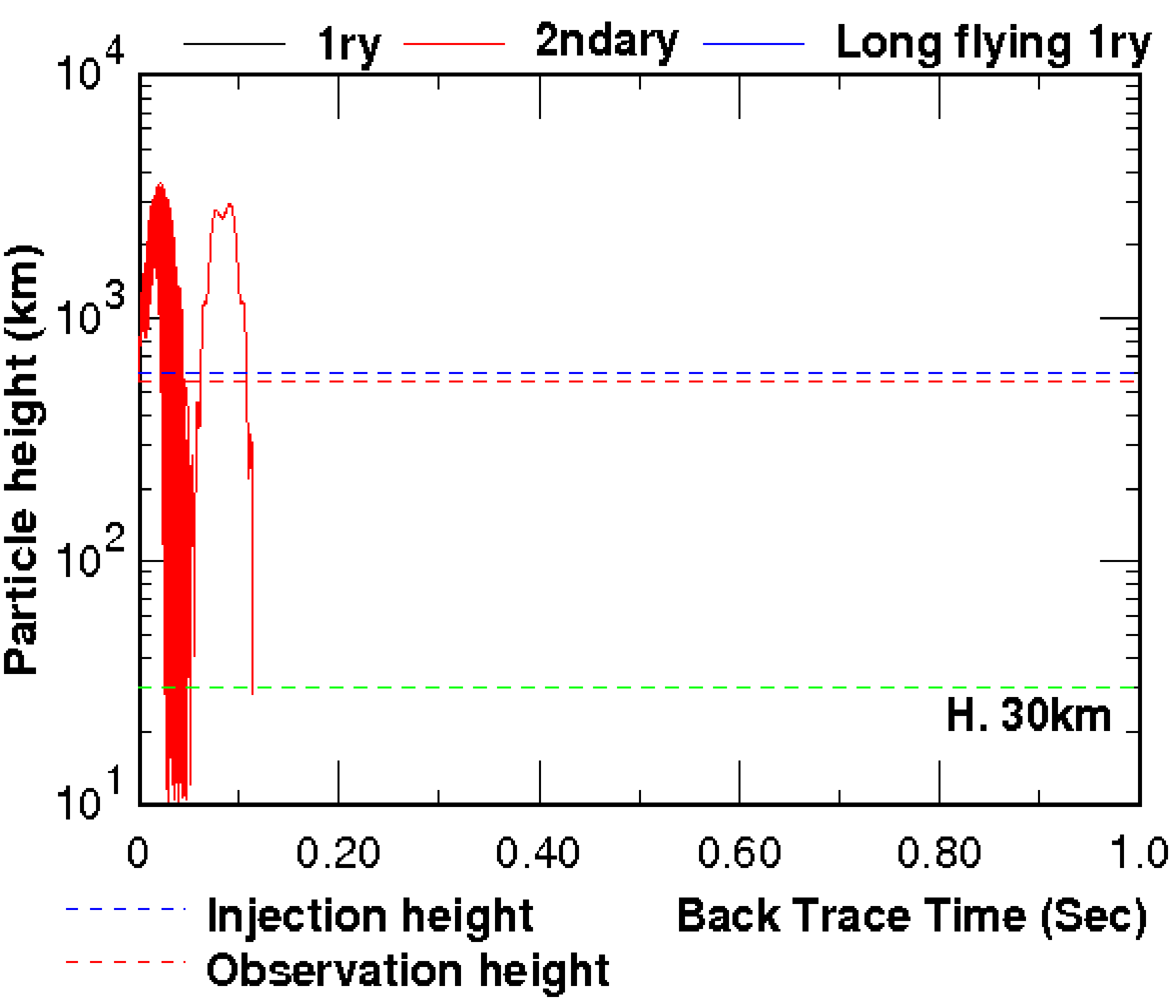


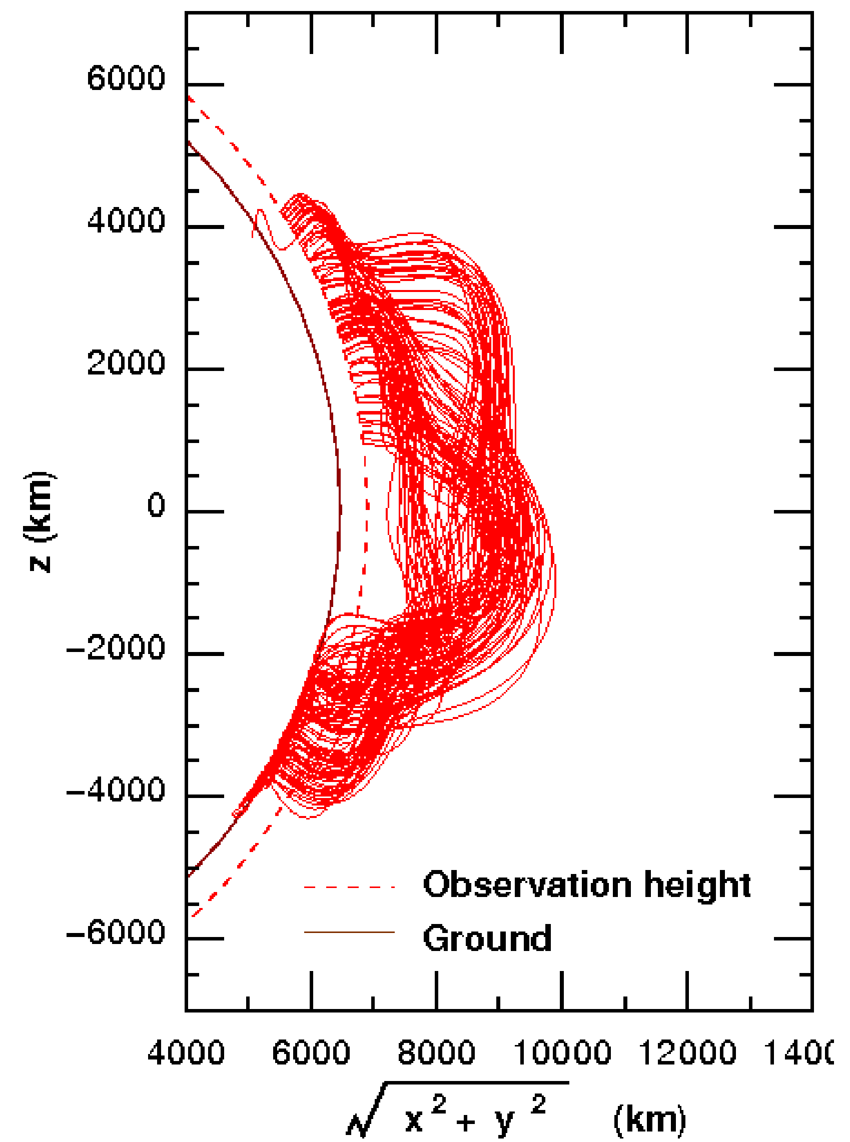


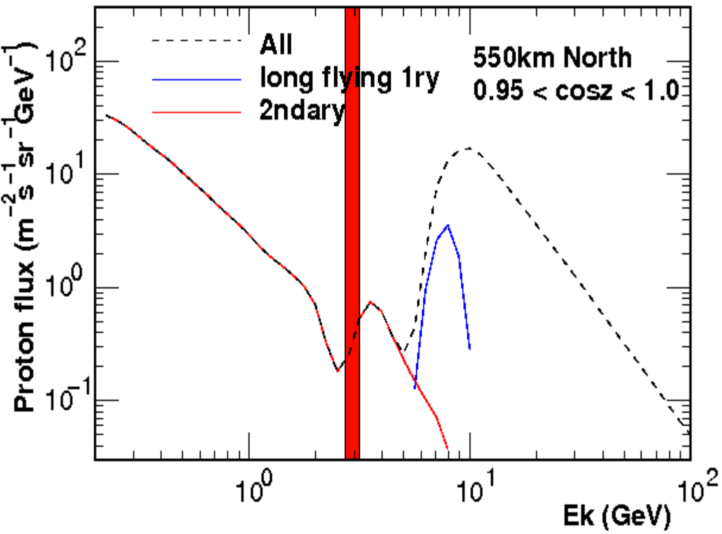
Back Trace of Observed Particles

3.162 < P < 3.548 GeV/c

2.369 < Ek < 2.739 GeV



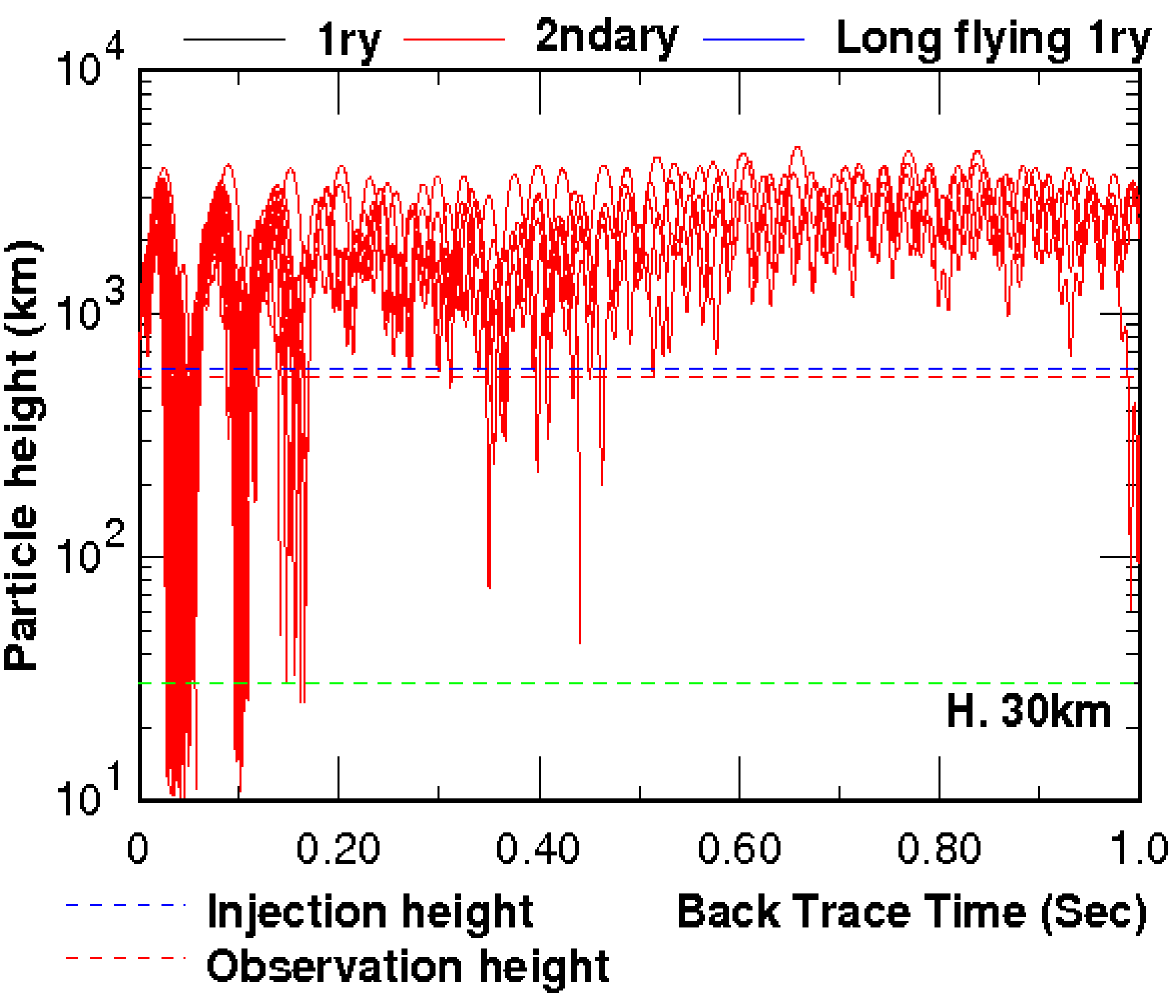


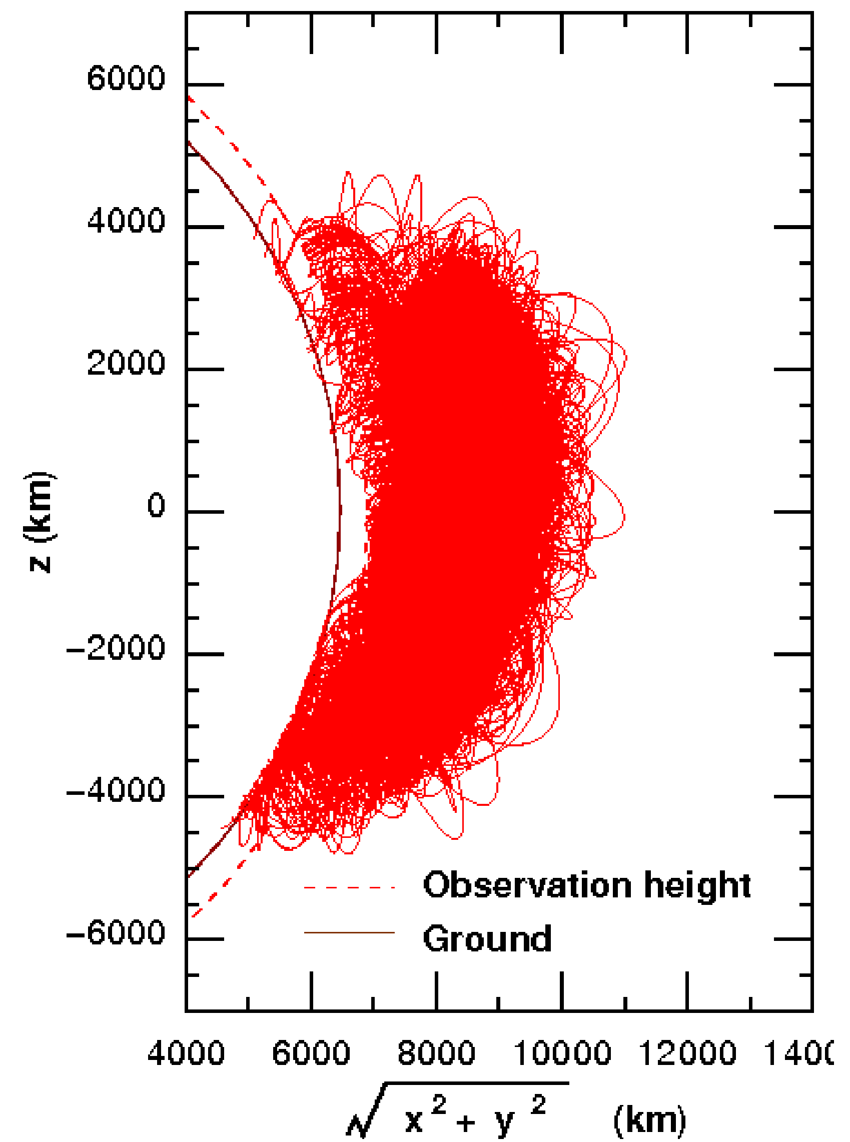


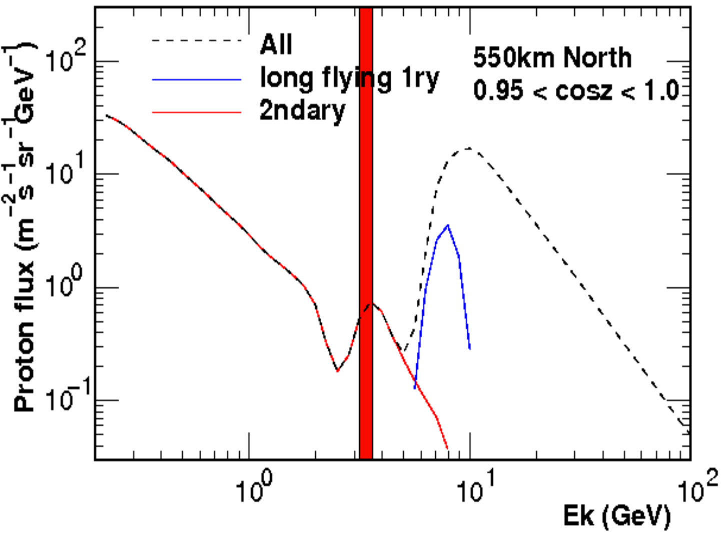
Back Trace of Observed Particles

3.548 < P < 3.981 GeV/c

2.739 < Ek < 3.159 GeV



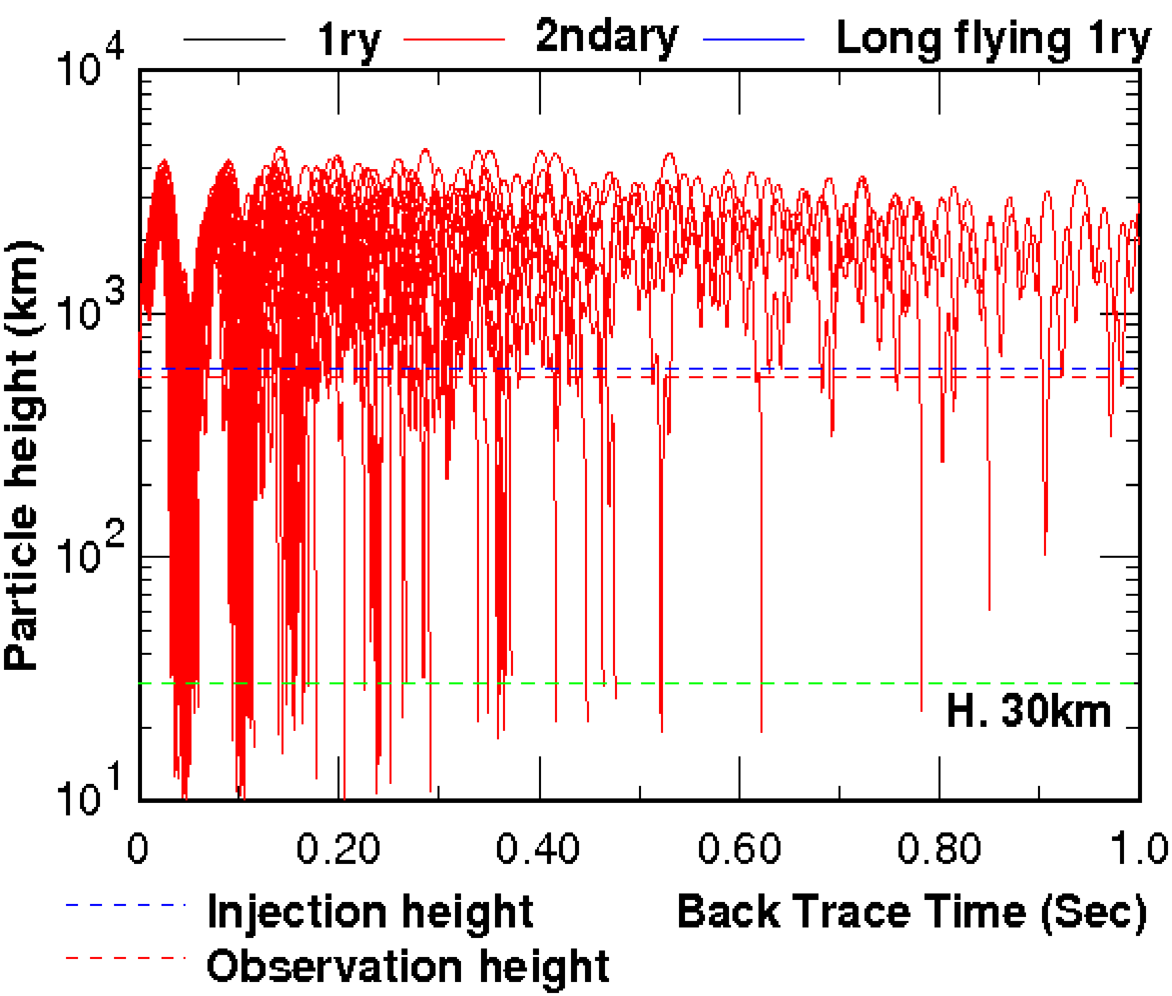


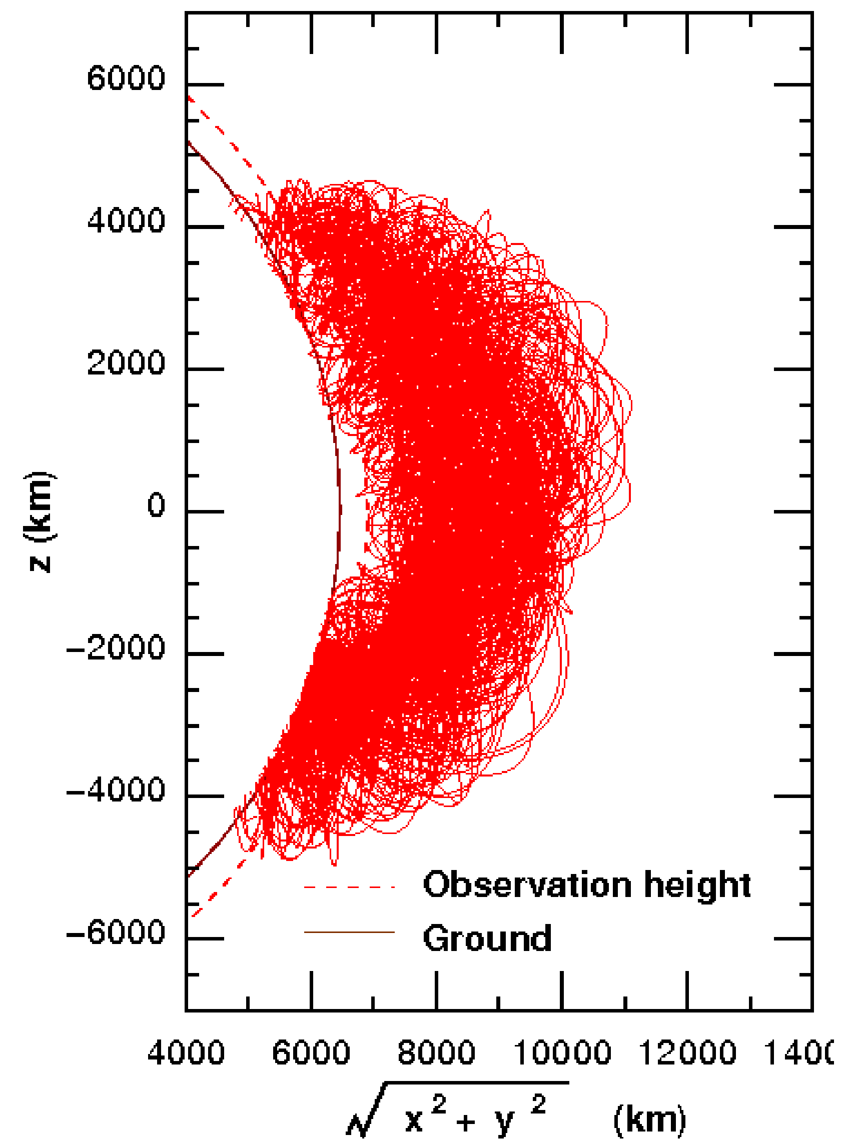


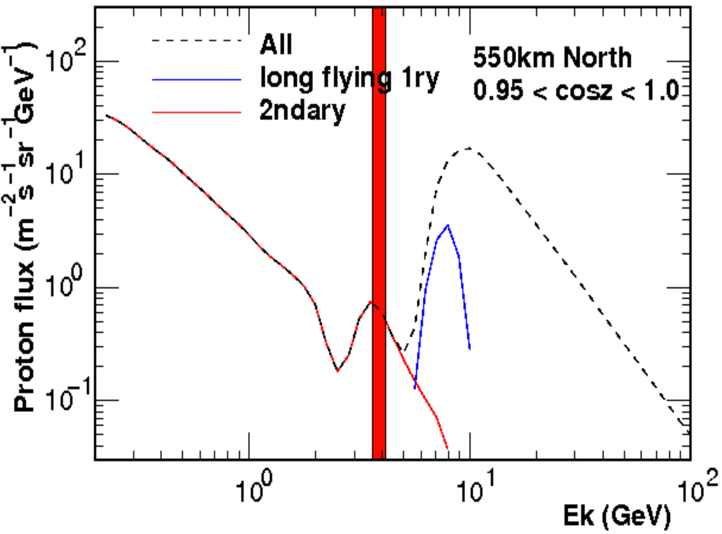
Back Trace of Observed Particles

3.981 < P < 4.467 GeV/c

3.159 < Ek < 3.632 GeV



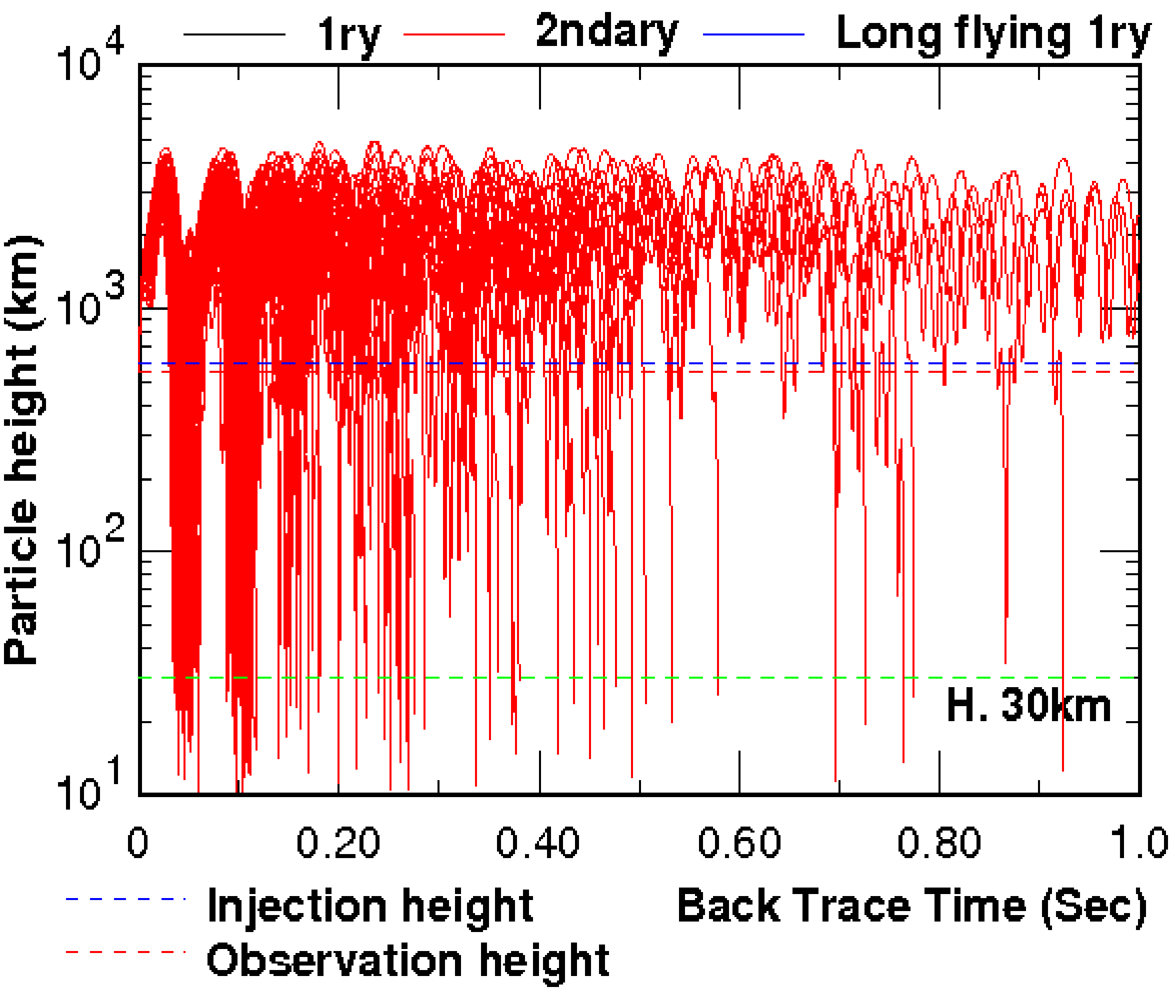


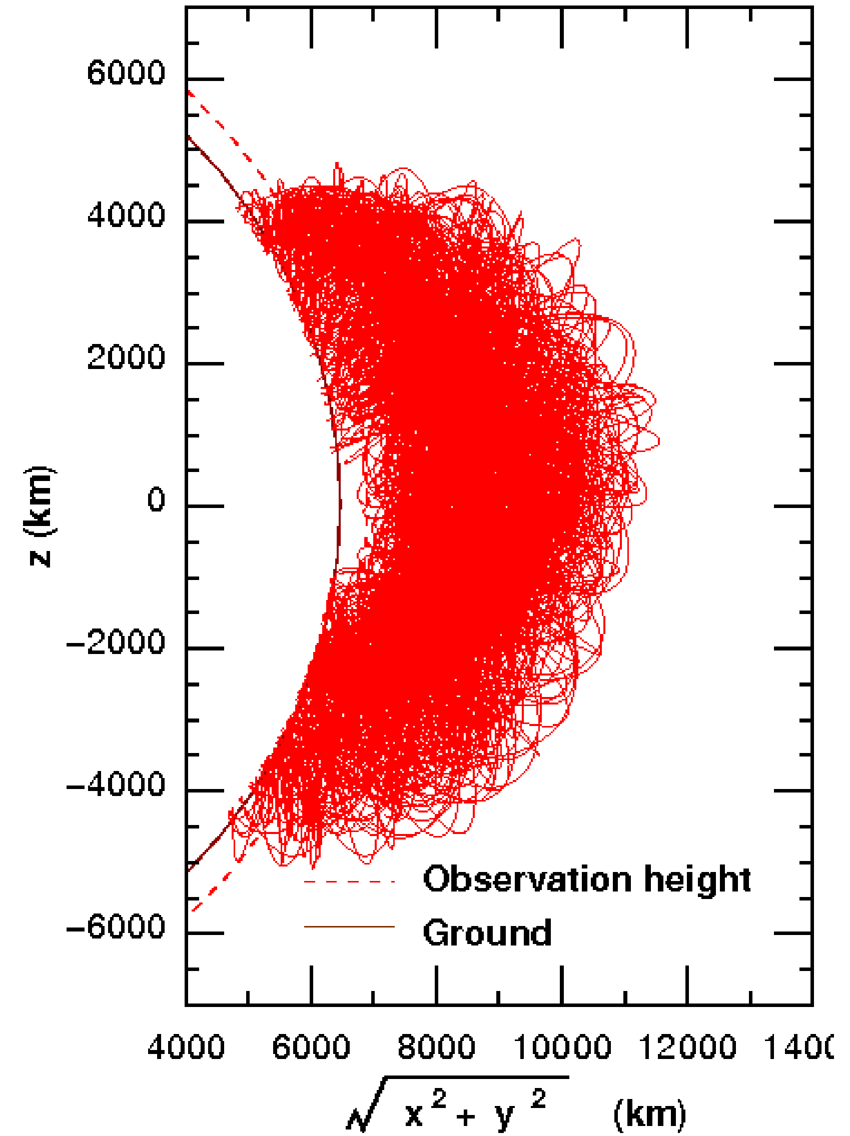


Back Trace of Observed Particles

4.467 < P < 5.011 GeV/c

3.632 < Ek < 4.167 GeV

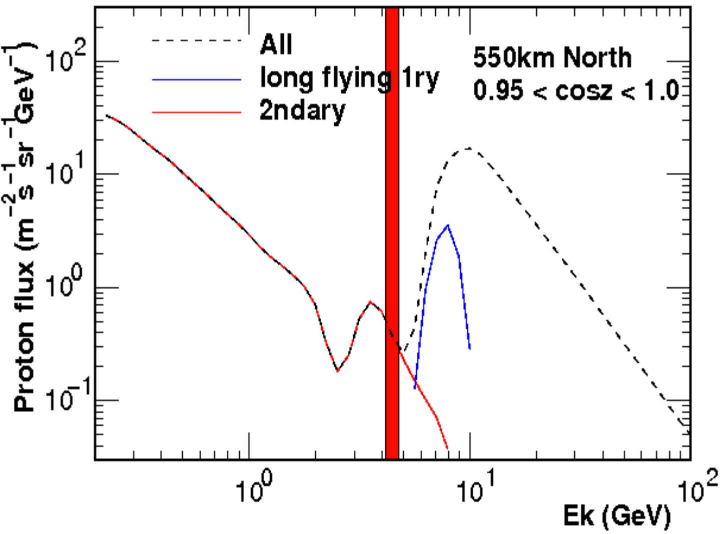


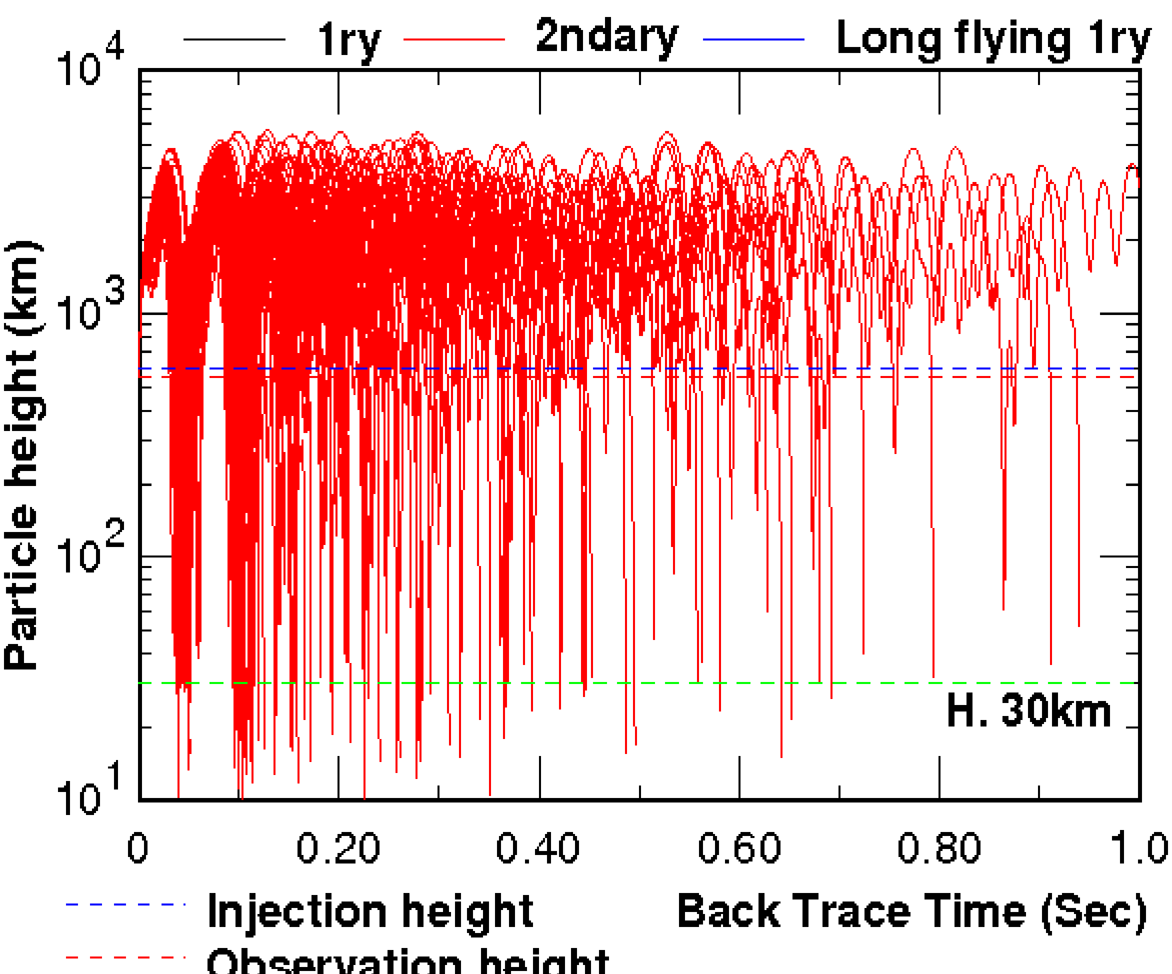


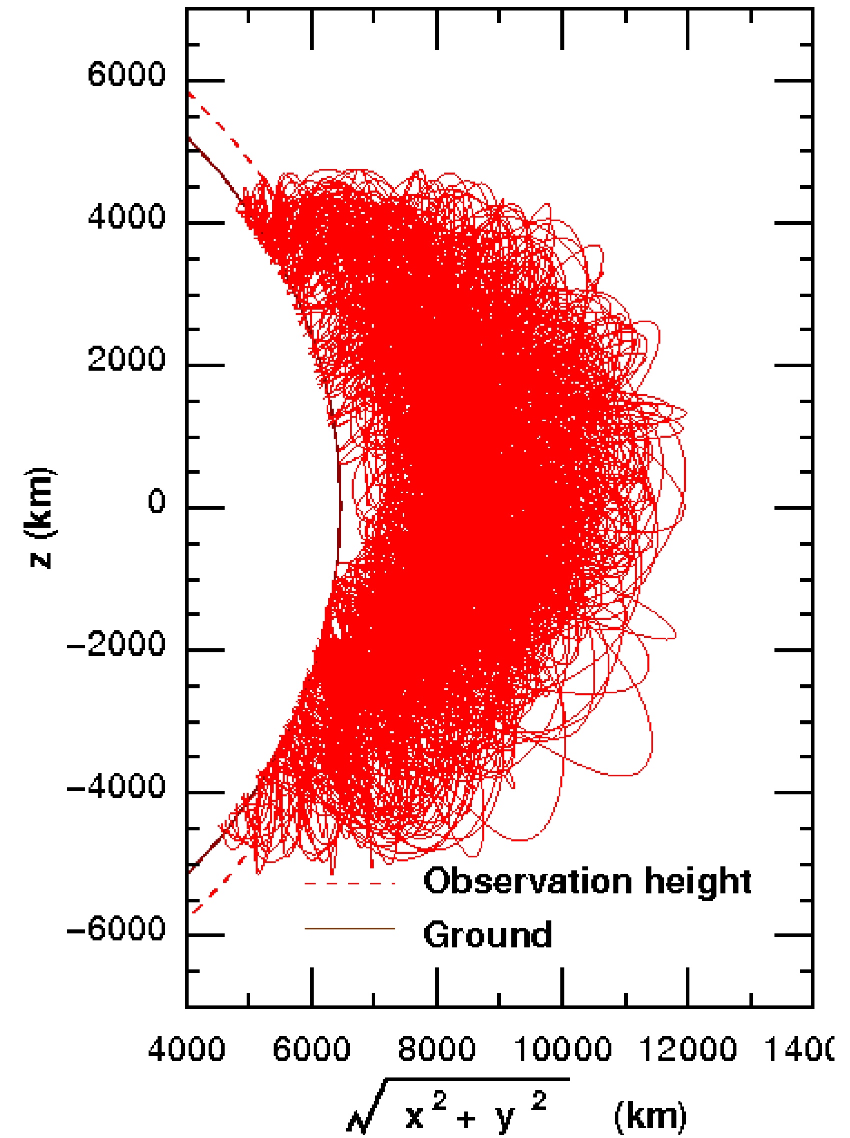
Back Trace of Observed Particles

5.011 < P < 5.623 GeV/c

4.167 < Ek < 4.768 GeV



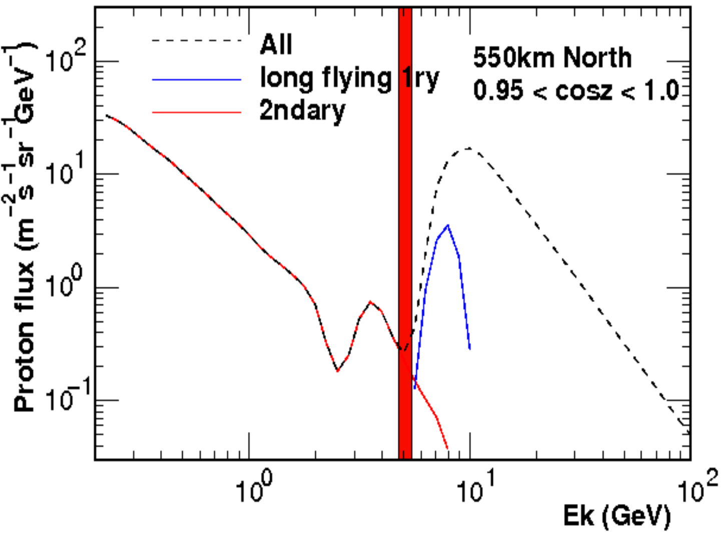


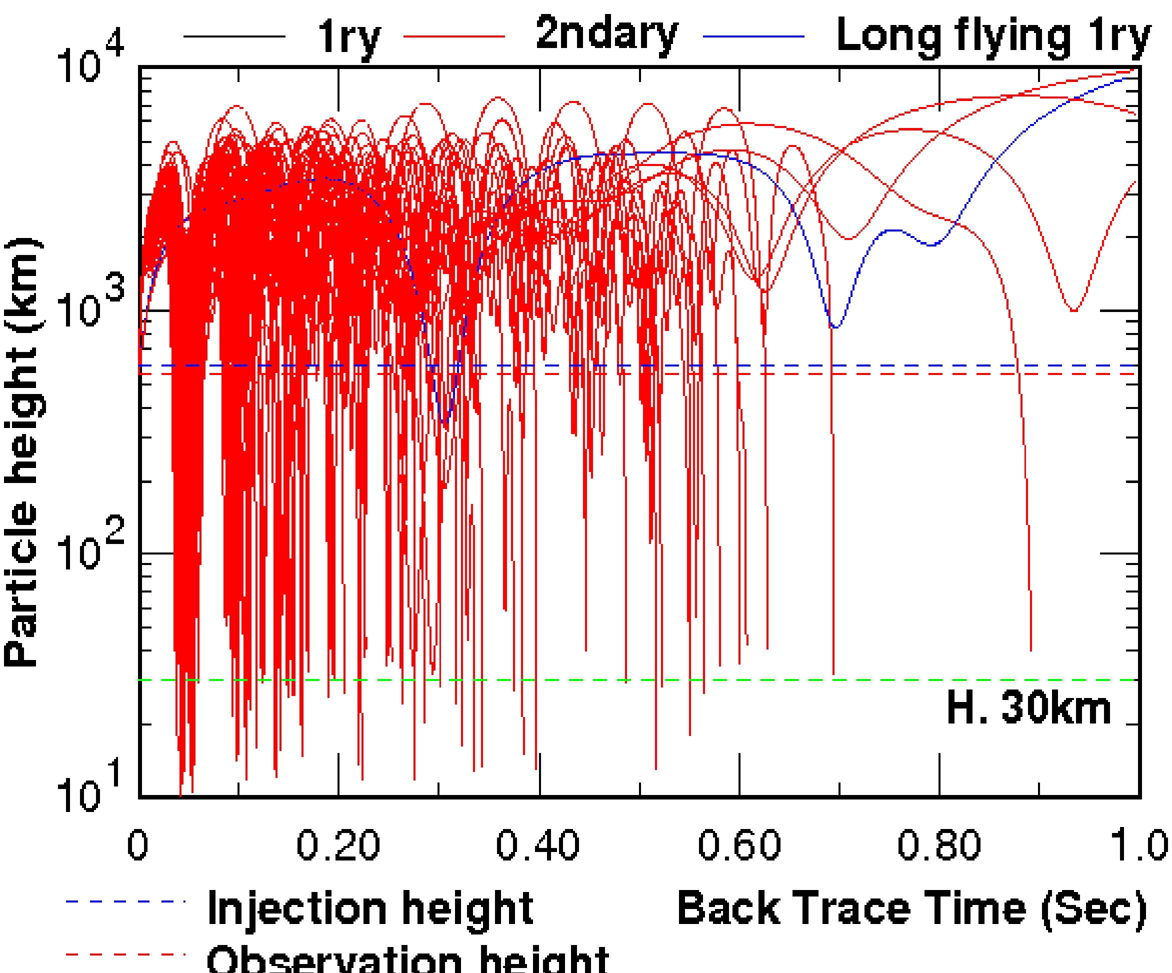


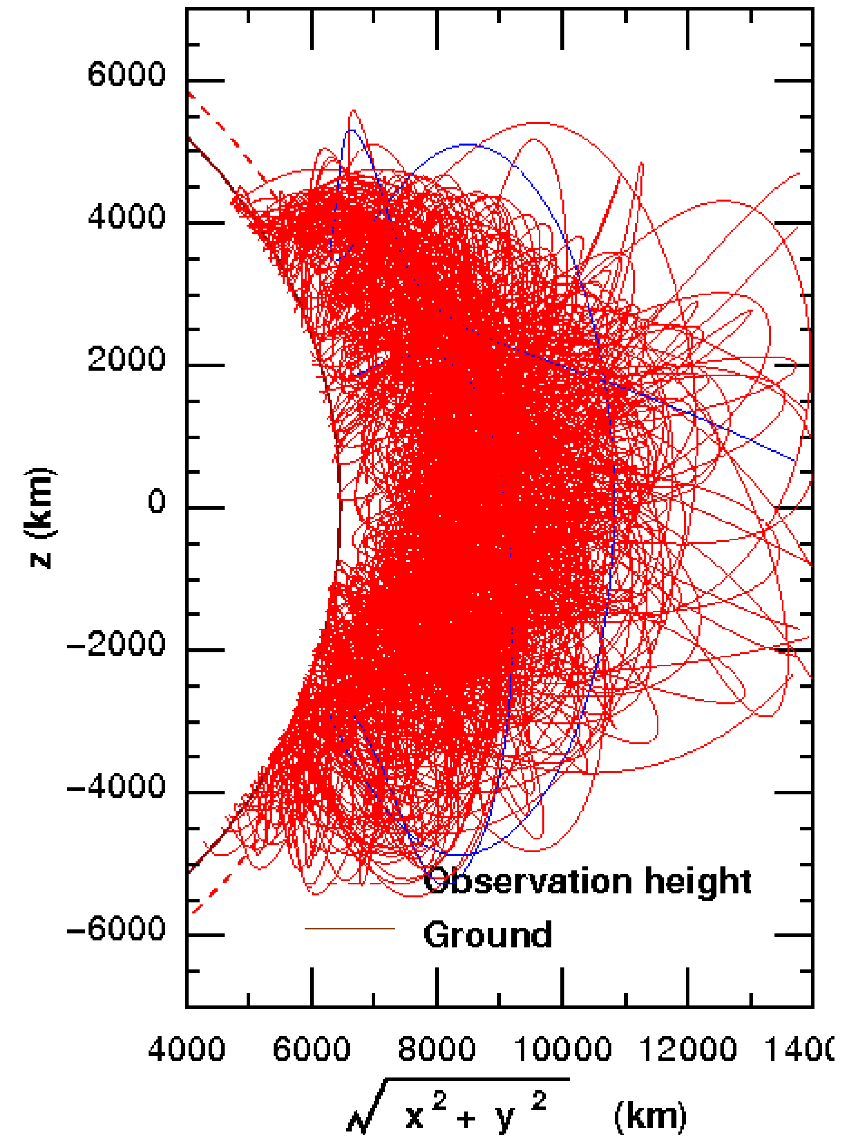
Back Trace of Observed Particles

5.623 < P < 6.310 GeV/c

4.768 < Ek < 5.445 GeV



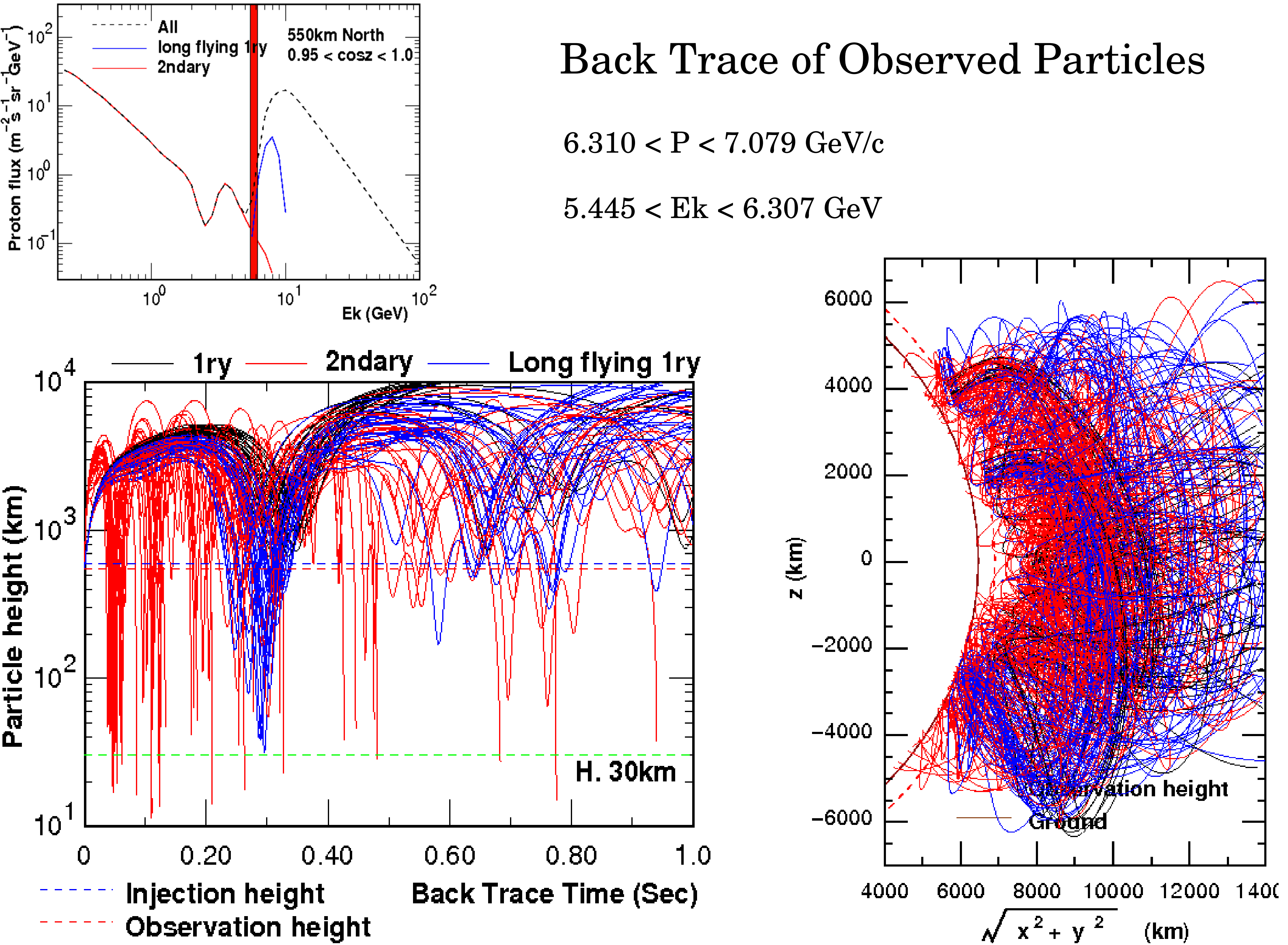


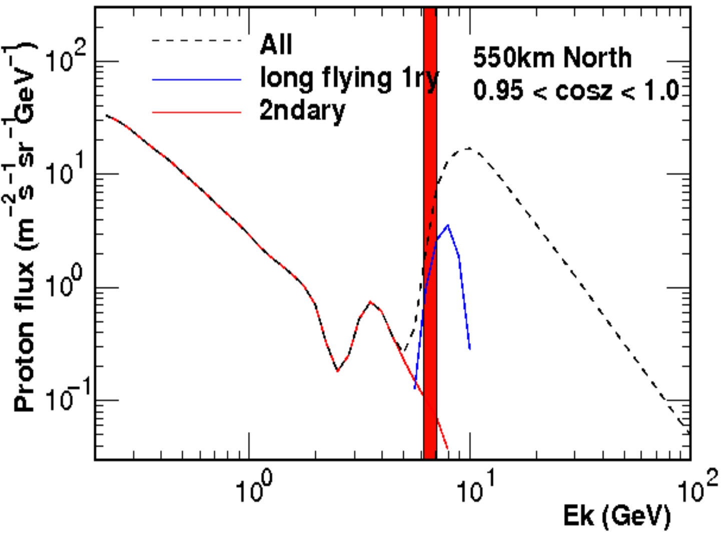


Back Trace of Observed Particles

6.310 < P < 7.079 GeV/c

5.445 < Ek < 6.307 GeV

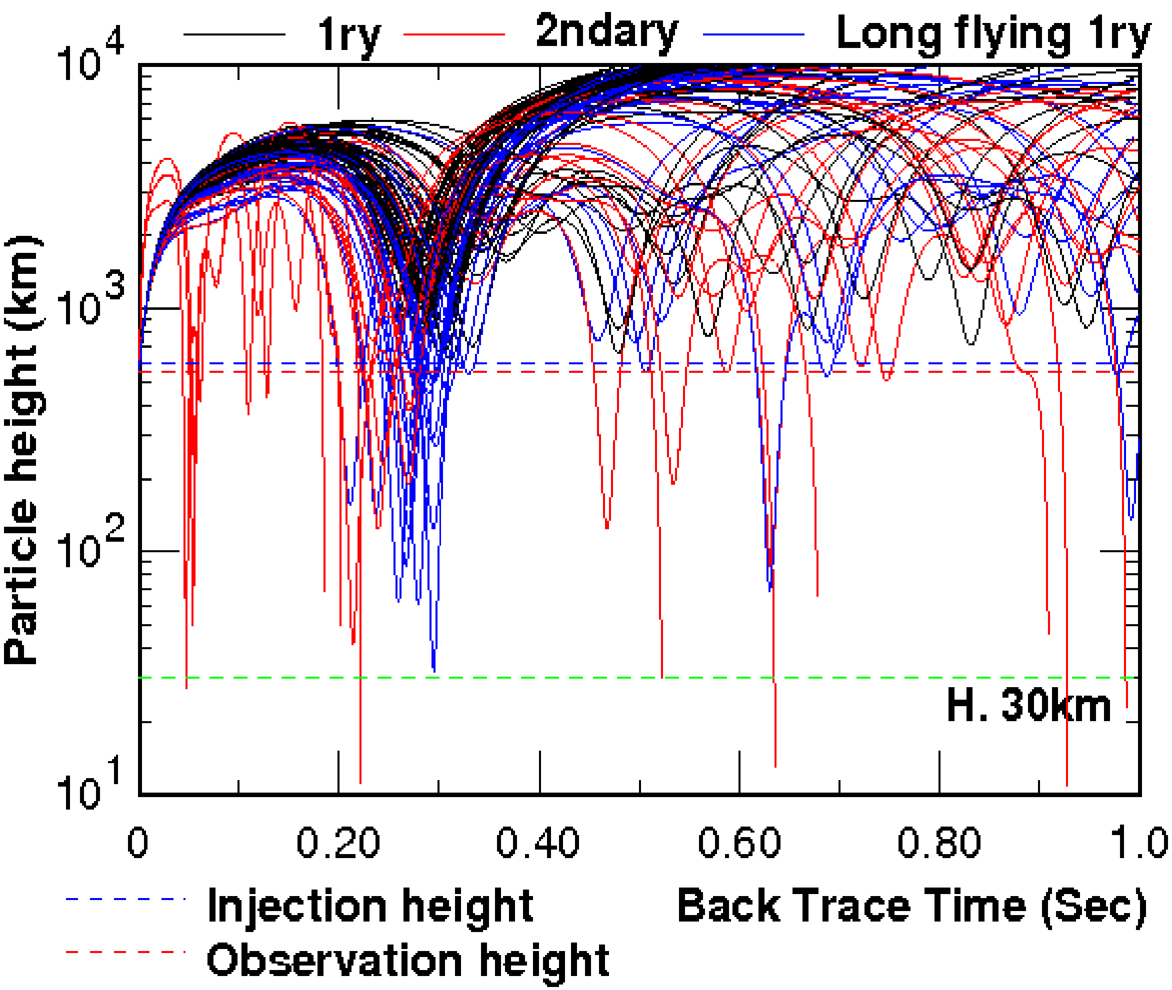


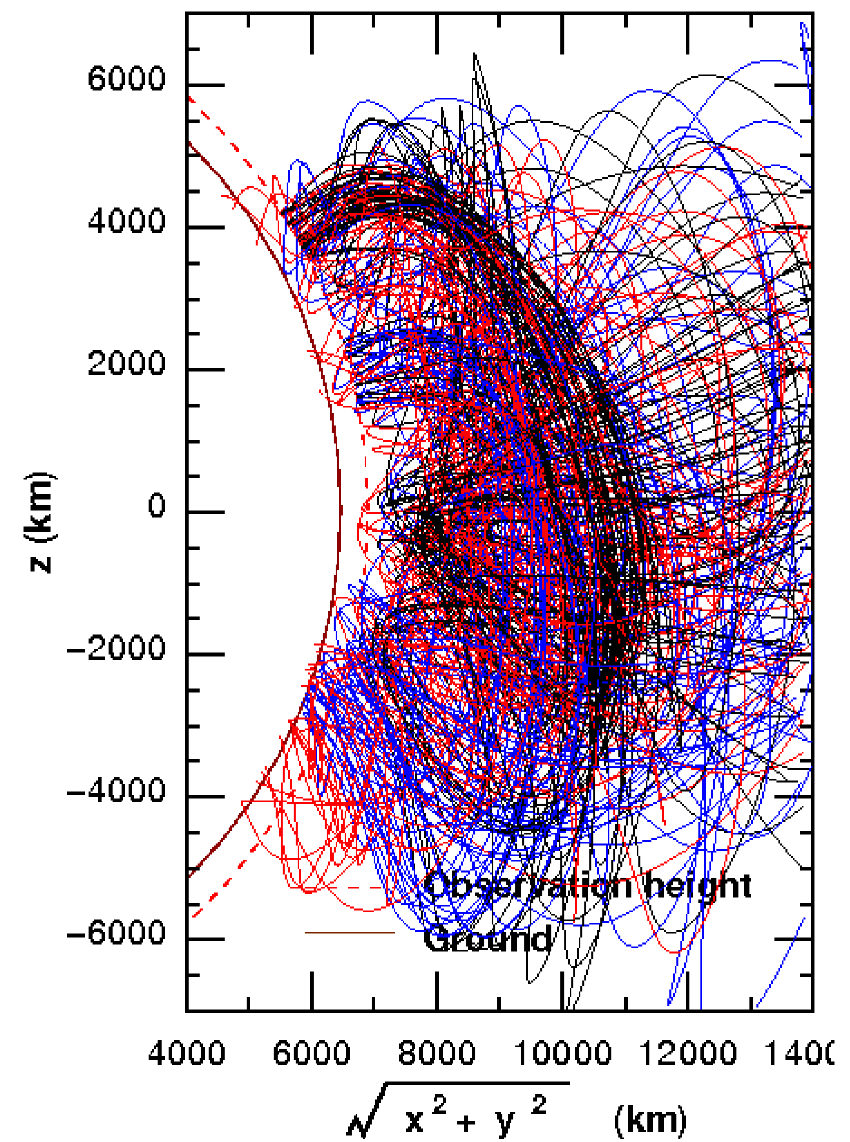


Back Trace of Observed Particles

$7.079 < P < 7.943$ GeV/c

$6.207 < Ek < 7.064$ GeV

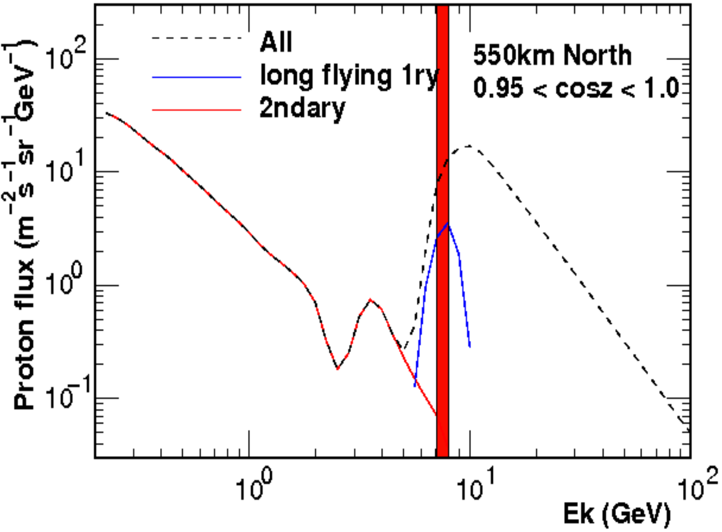


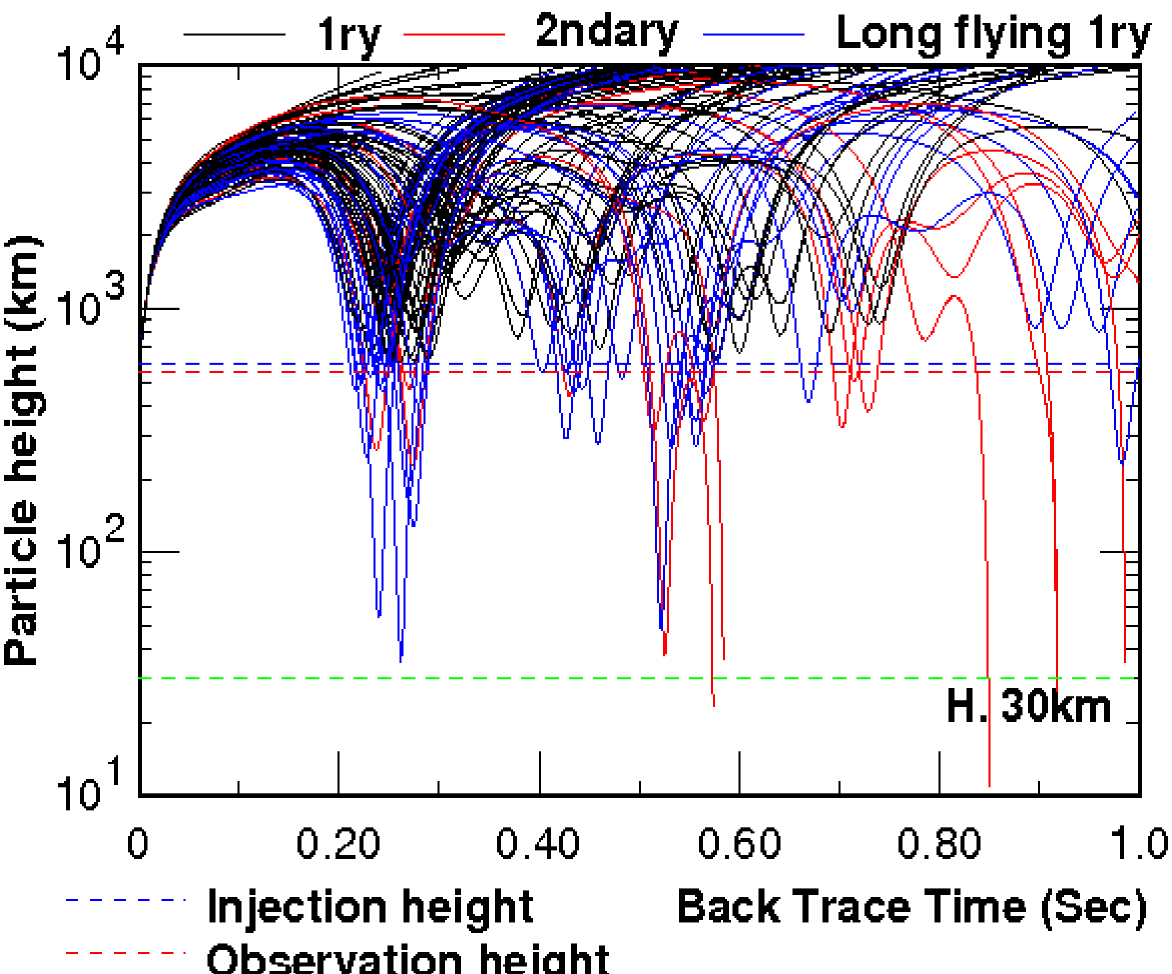


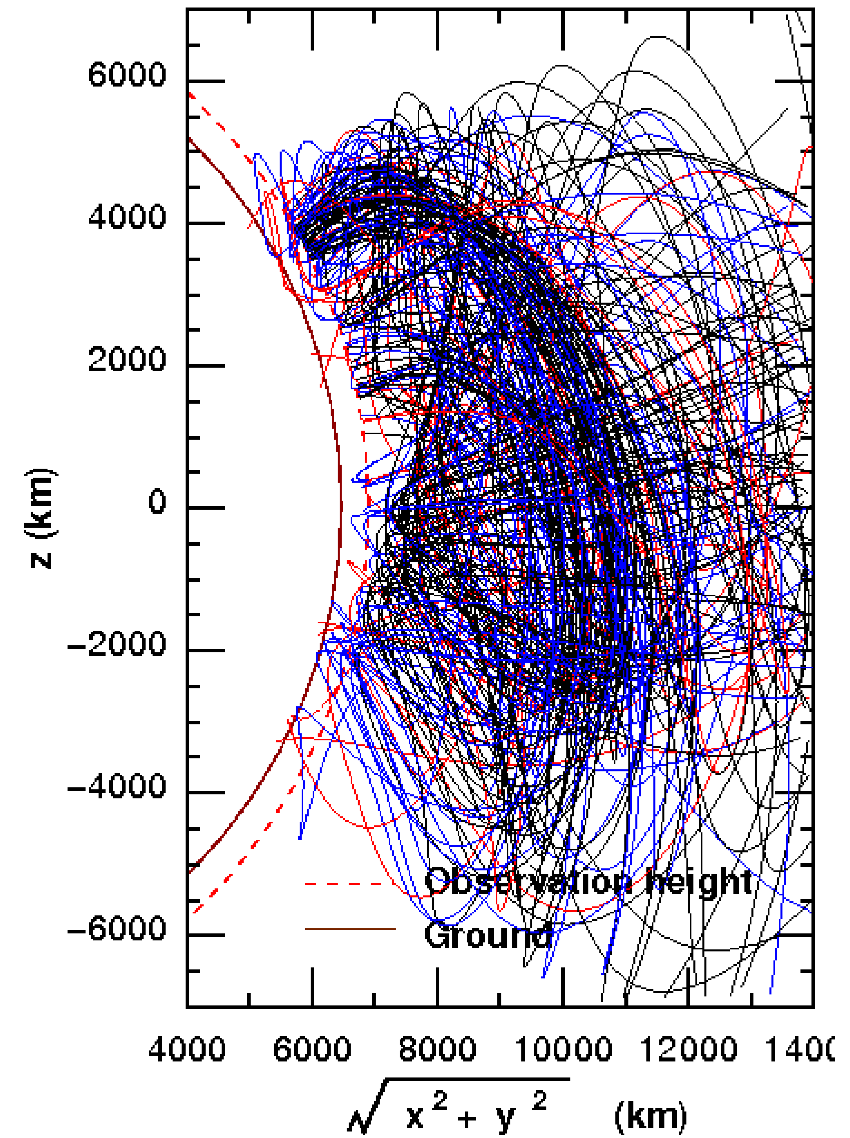
Back Trace of Observed Particles

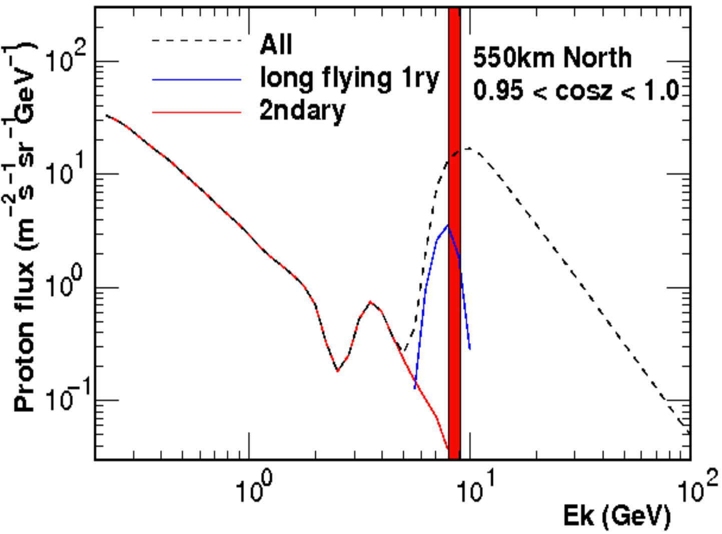
7.953 < P < 8.912 GeV/c

7.063 < Ek < 8.026 GeV





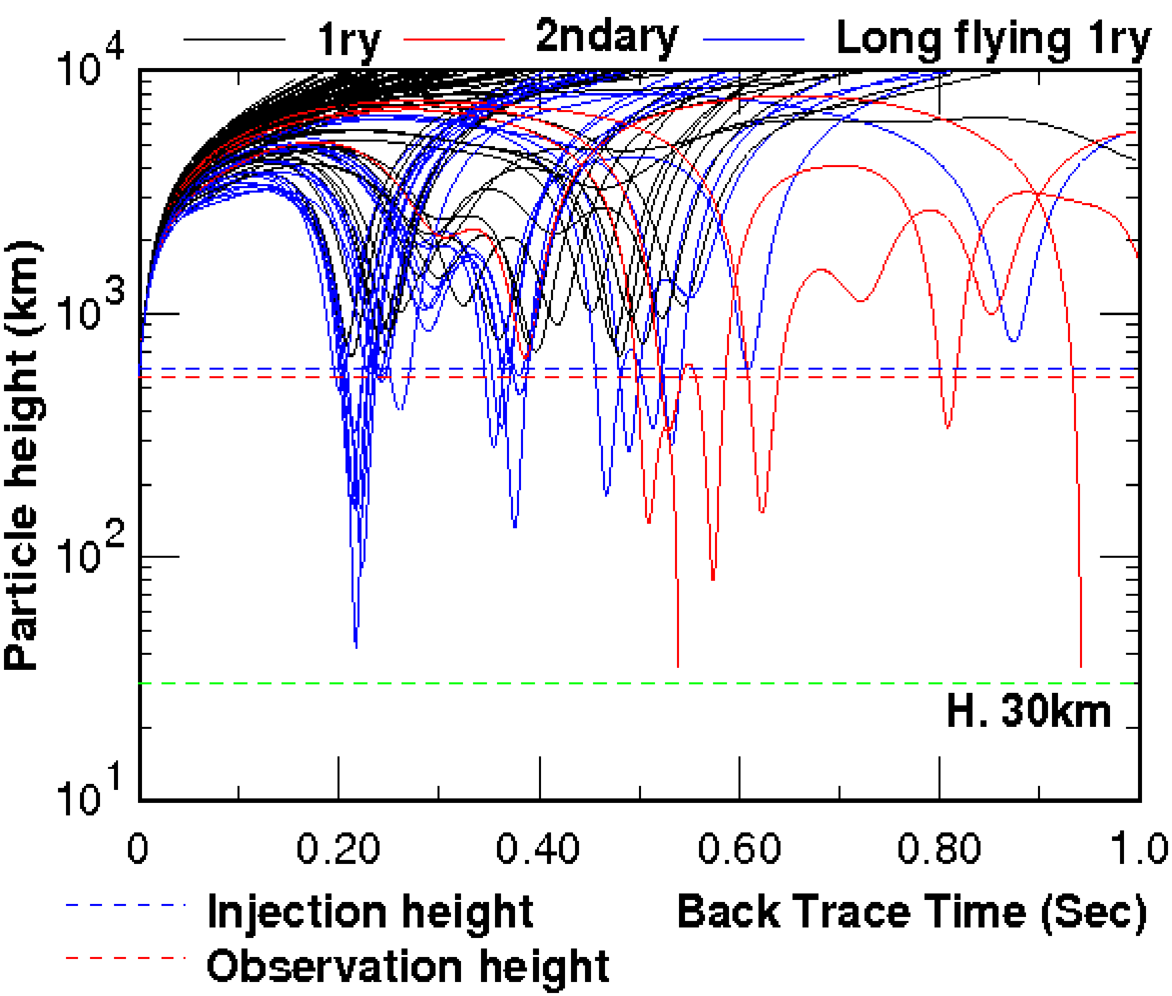


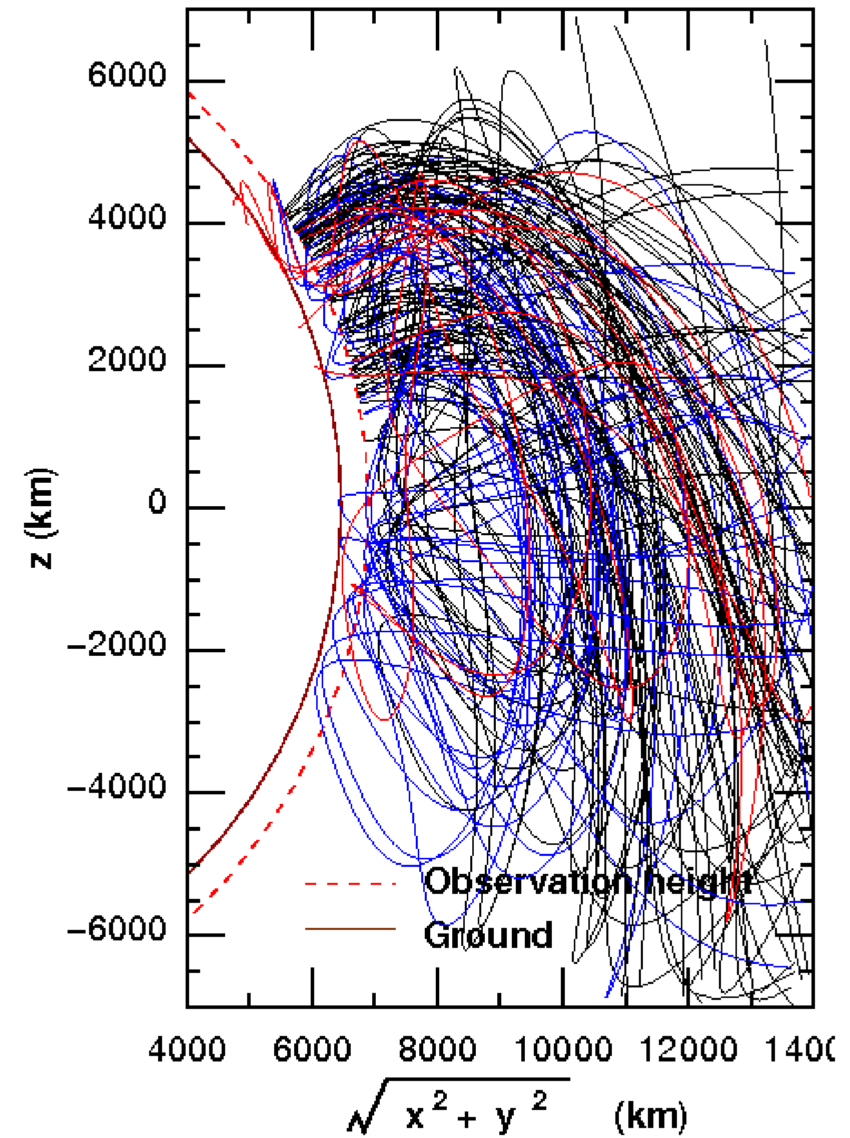


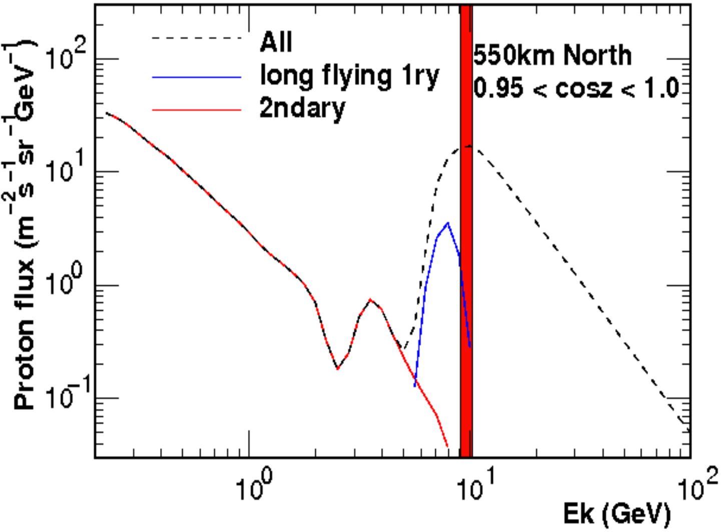
Back Trace of Observed Particles

8.912 < P < 10.000 GeV/c

8.026 < Ek < 9.109 GeV



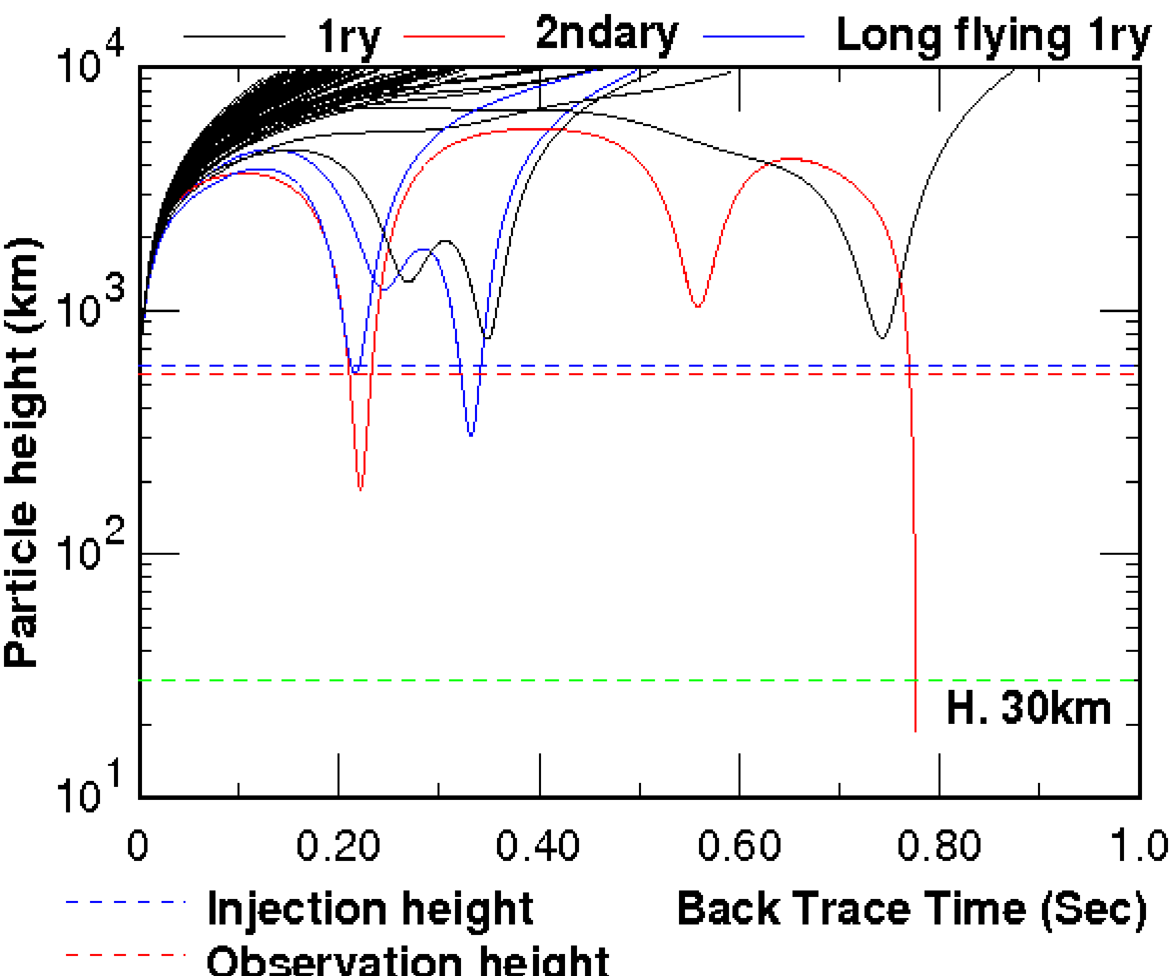


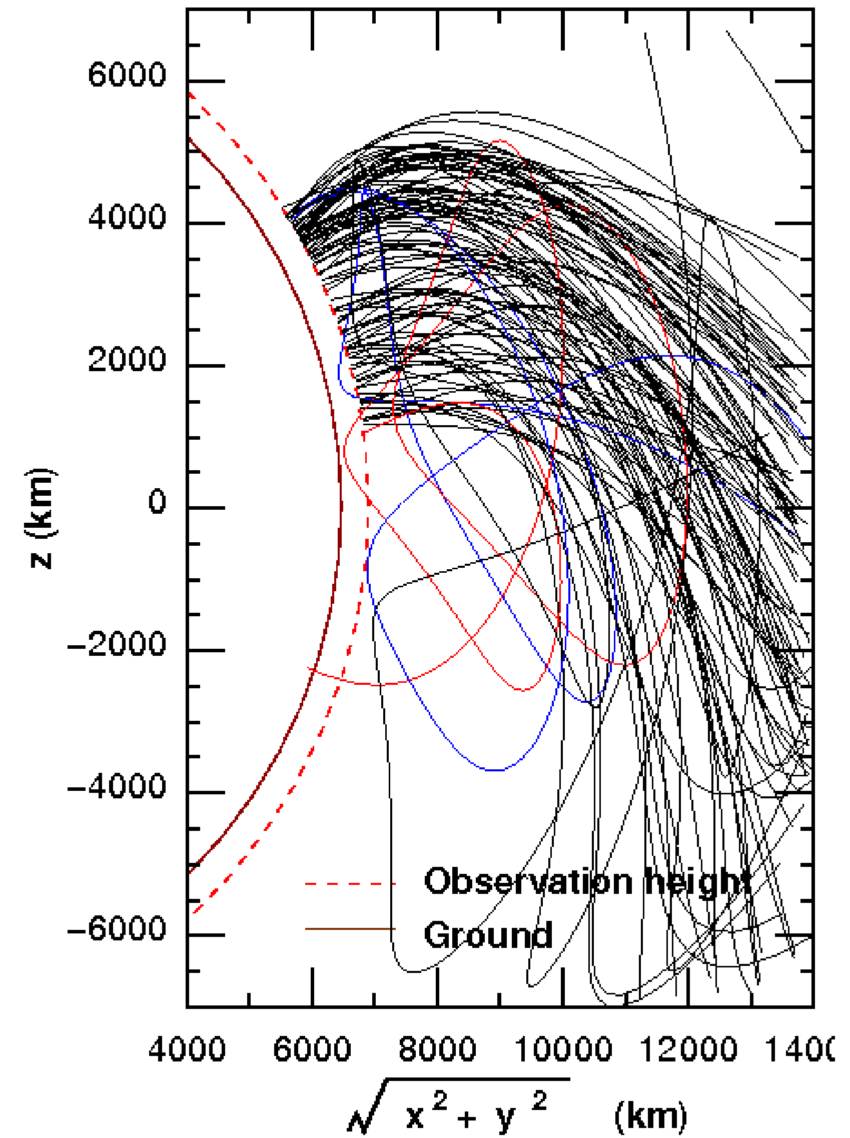


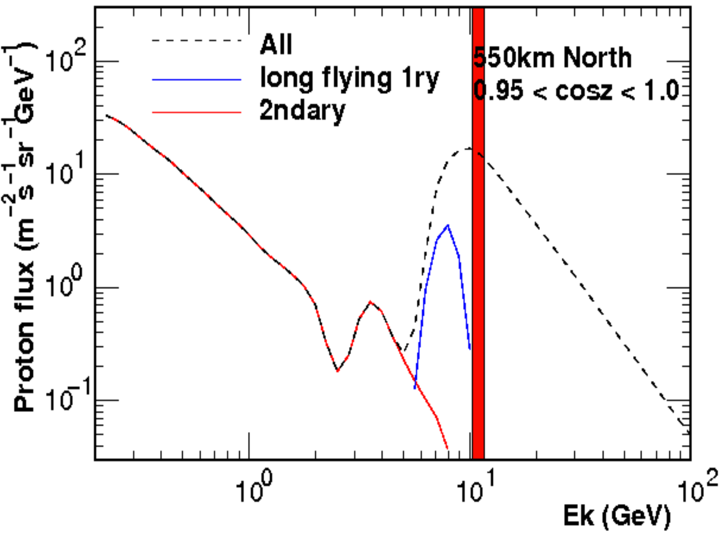
Back Trace of Observed Particles

10.00 < P < 11.22 GeV/c

9.109 < Ek < 10.32 GeV



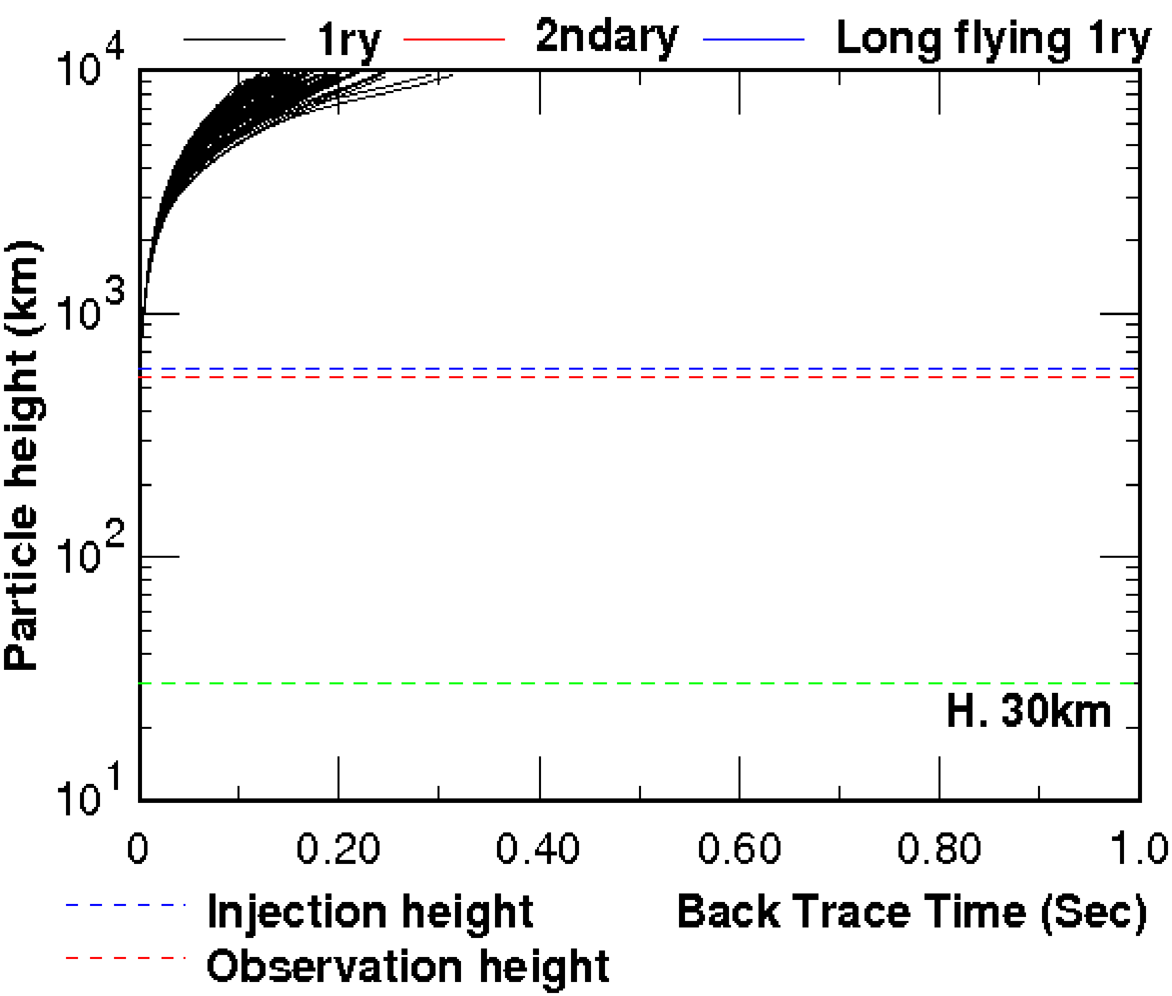


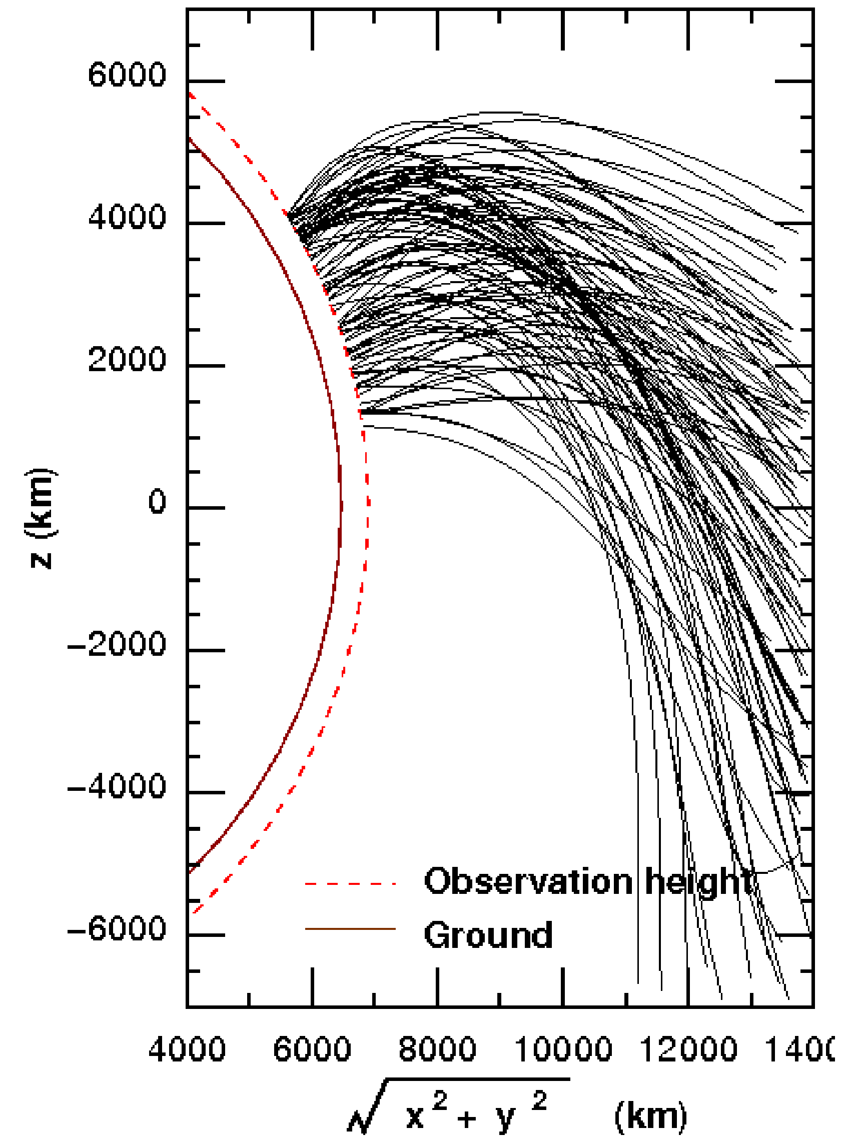


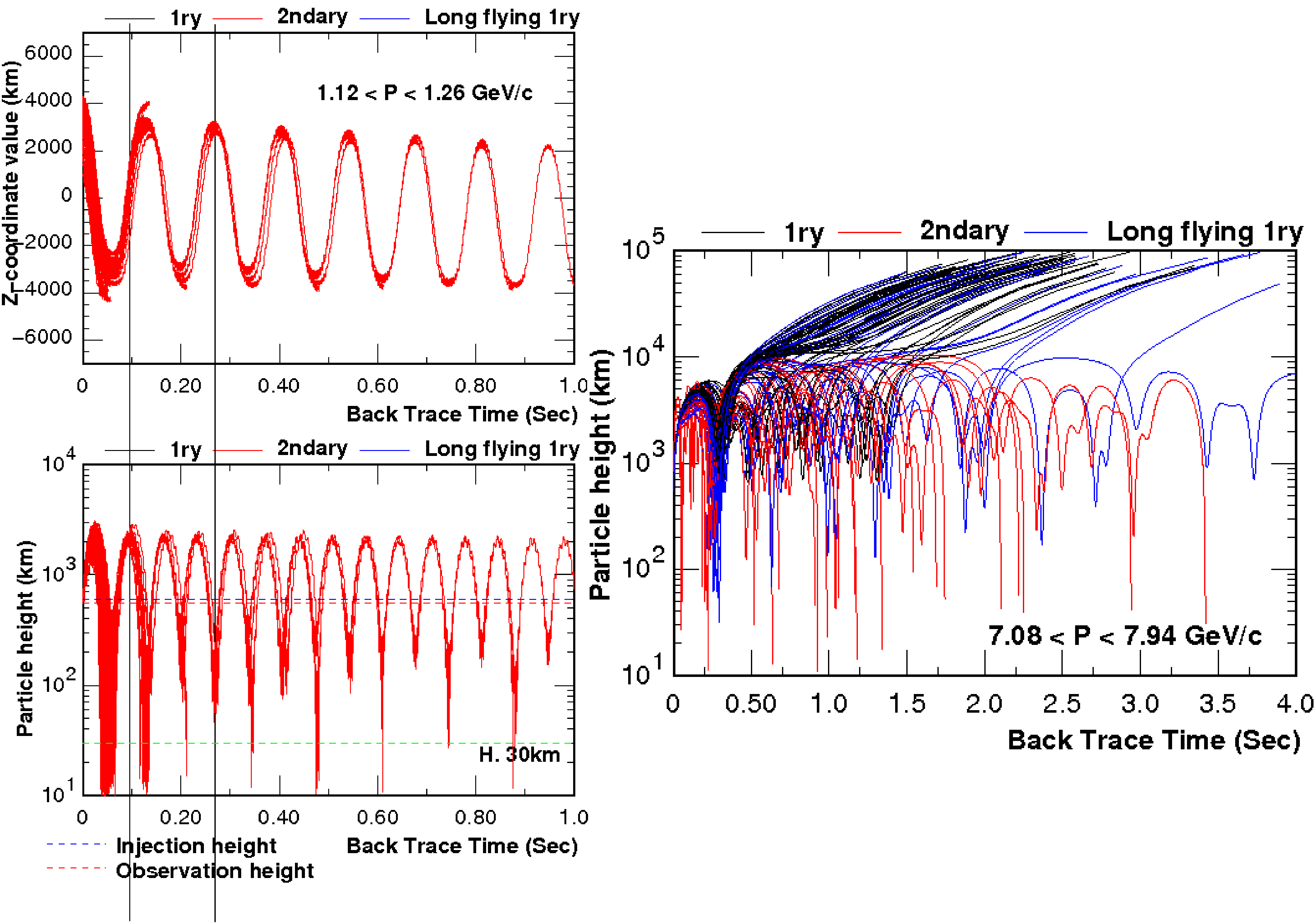
Back Trace of Observed Particles

$11.22 < P < 12.59$ GeV/c

$10.32 < Ek < 11.69$ GeV



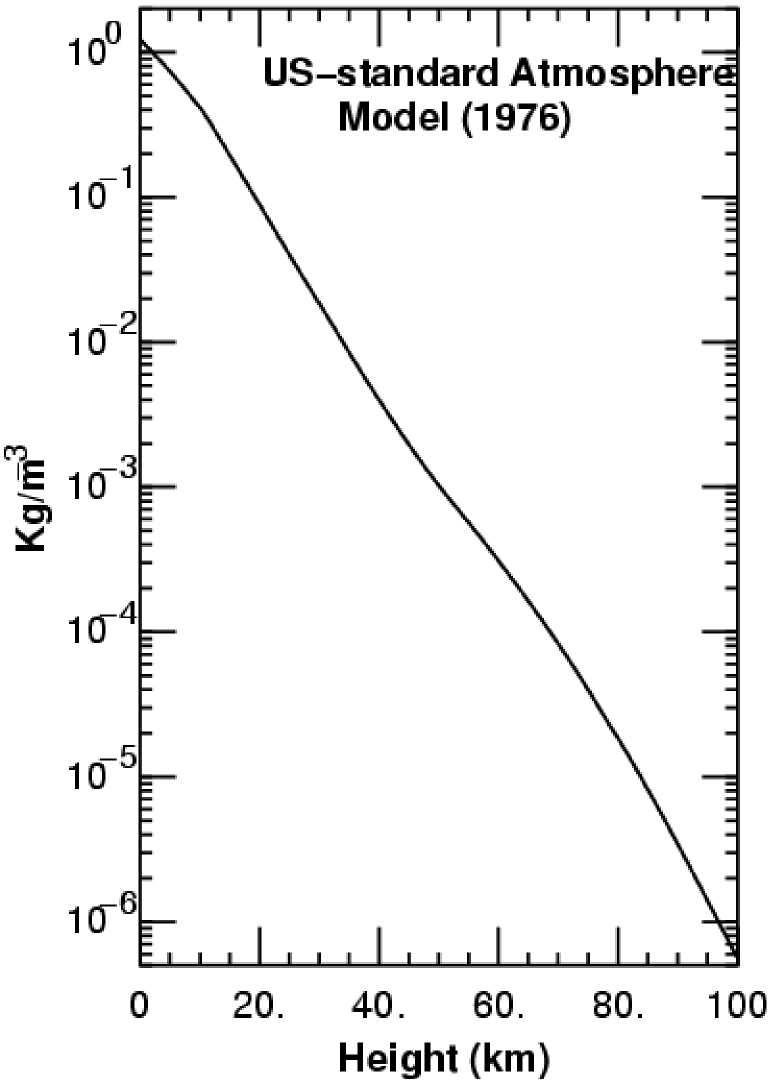




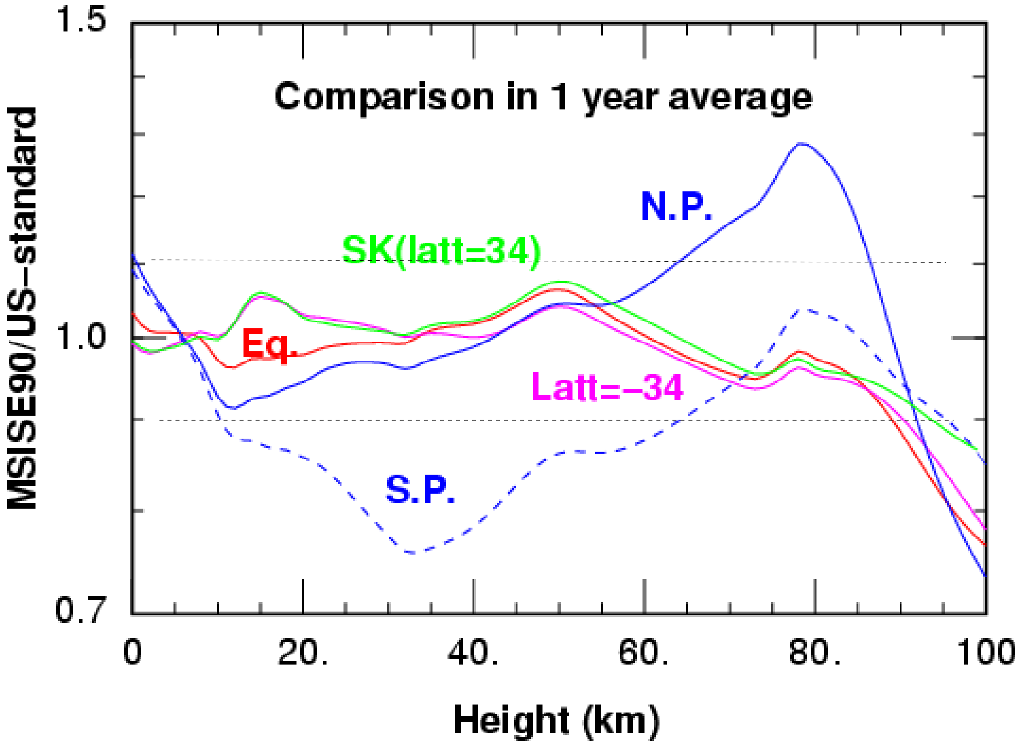
1ry
2ndary
Long flying 1ry
1.12 < P < 1.26 GeV/c
Z-coordinate value (km)
6000
4000
2000
0
-2000
-4000
-6000
0
0.20
0.40
0.60
0.80
1.0
Back Trace Time (Sec)
1ry
2ndary
Long flying 1ry
Particle height (km)
10^4
10^3
10^2
10^1
H. 30km
0
0.20
0.40
0.60
0.80
1.0
Back Trace Time (Sec)
Injection height
Observation height
1ry
2ndary
Long flying 1ry
Particle height (km)
10^5
10^4
10^3
10^2
10^1
7.08 < P < 7.94 GeV/c
0
0.50
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Back Trace Time (Sec)


Atmosphere
NISHIKAWA

Atmosphere Model

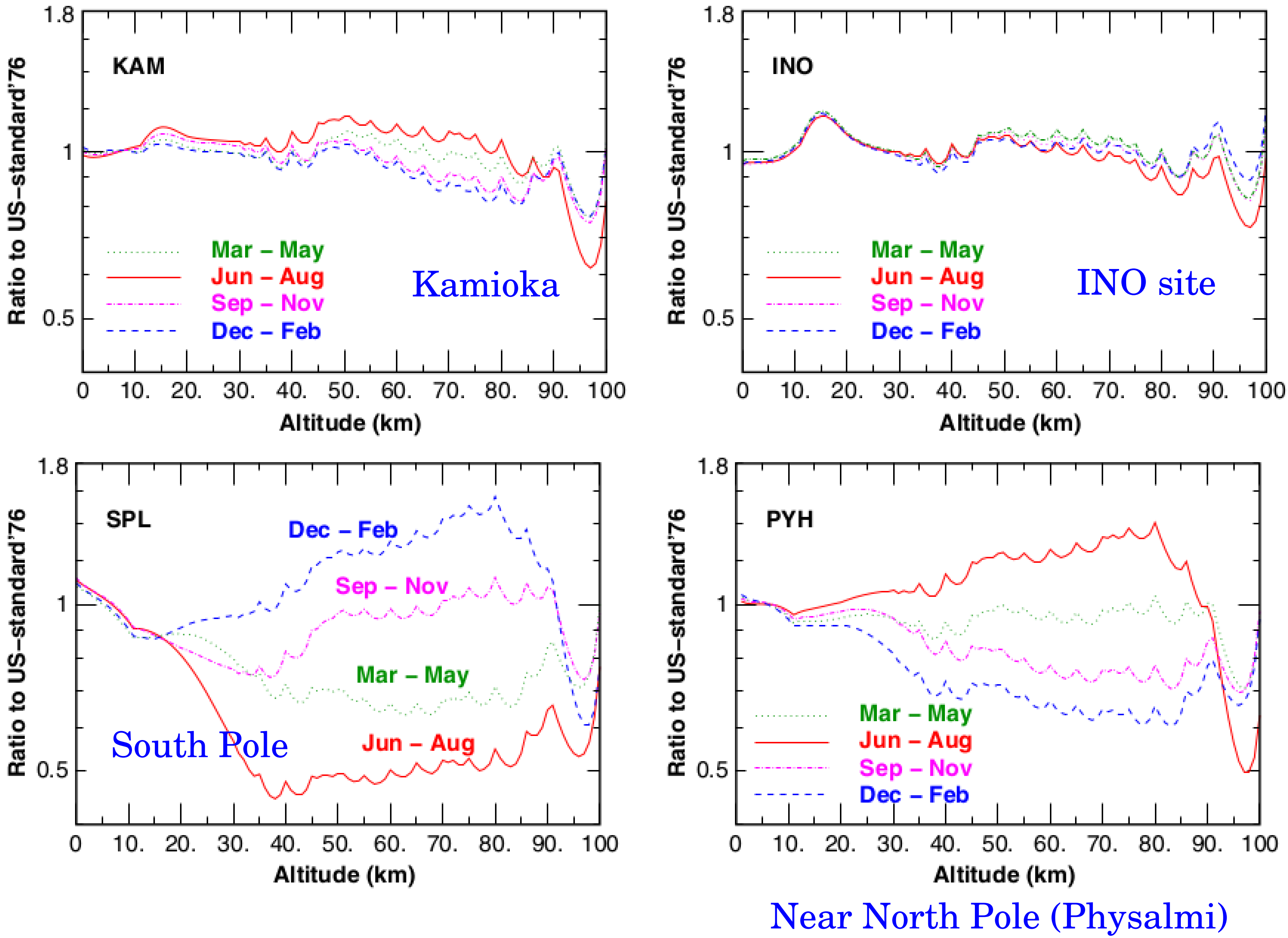


Air density comparison with MSISE90

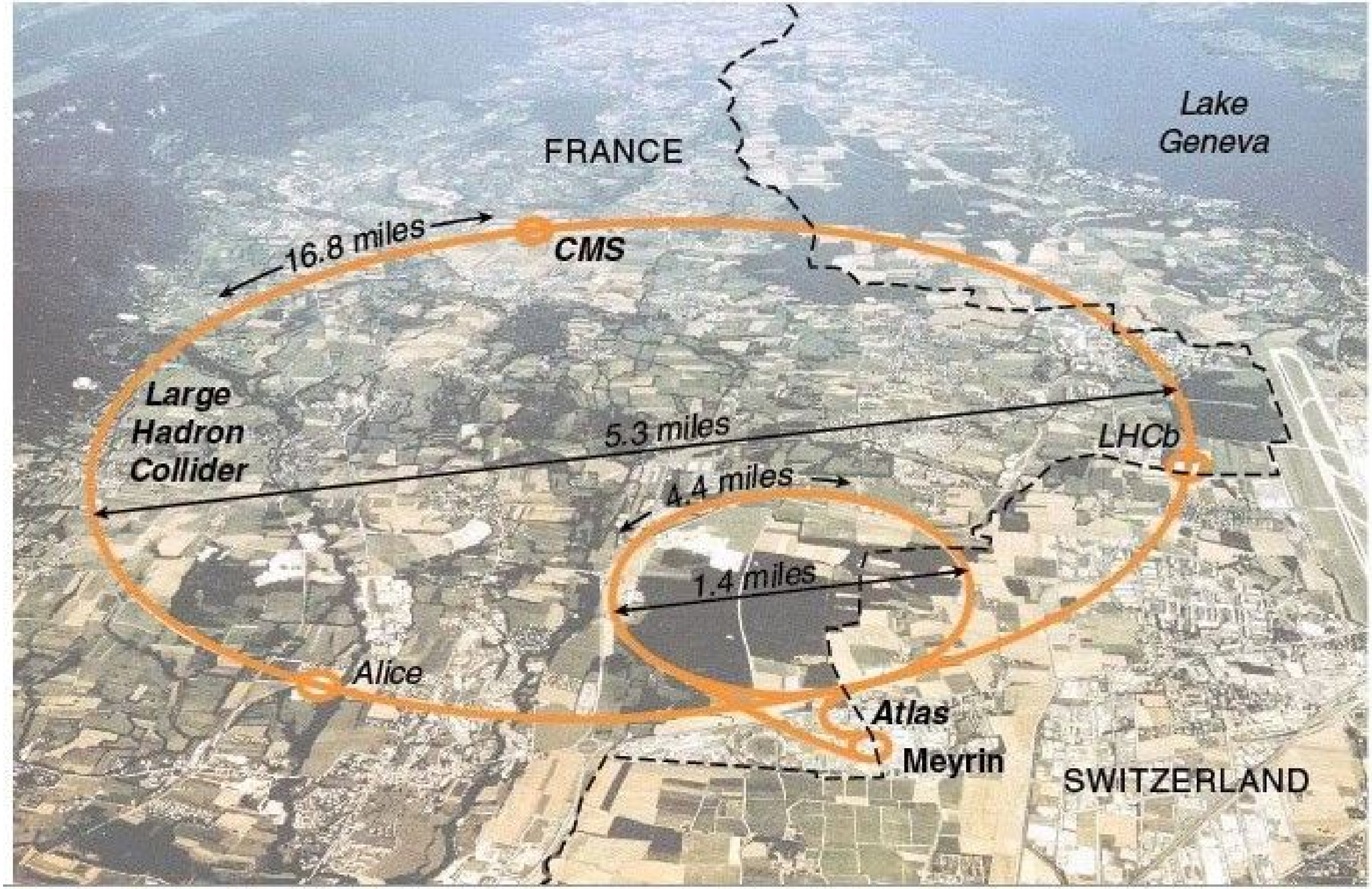


US-starndard'76 may be used as the global approximation of the Atmosphere.

Atmosphere model (NRLMSISE-00) and seasonal variations

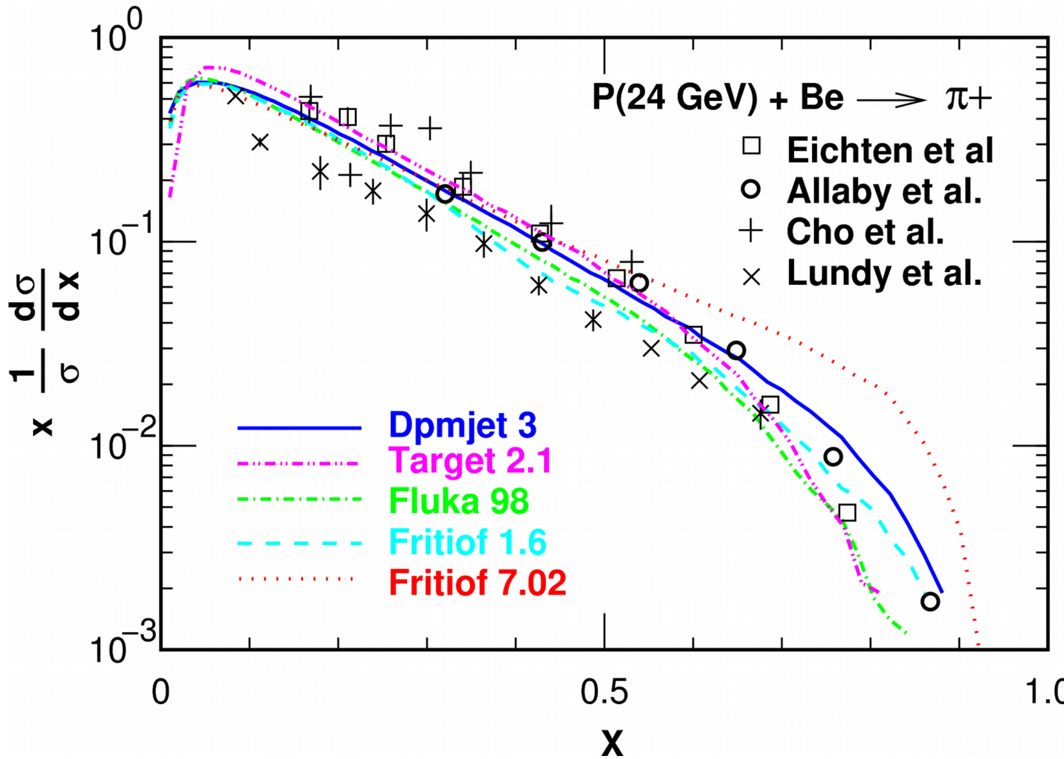


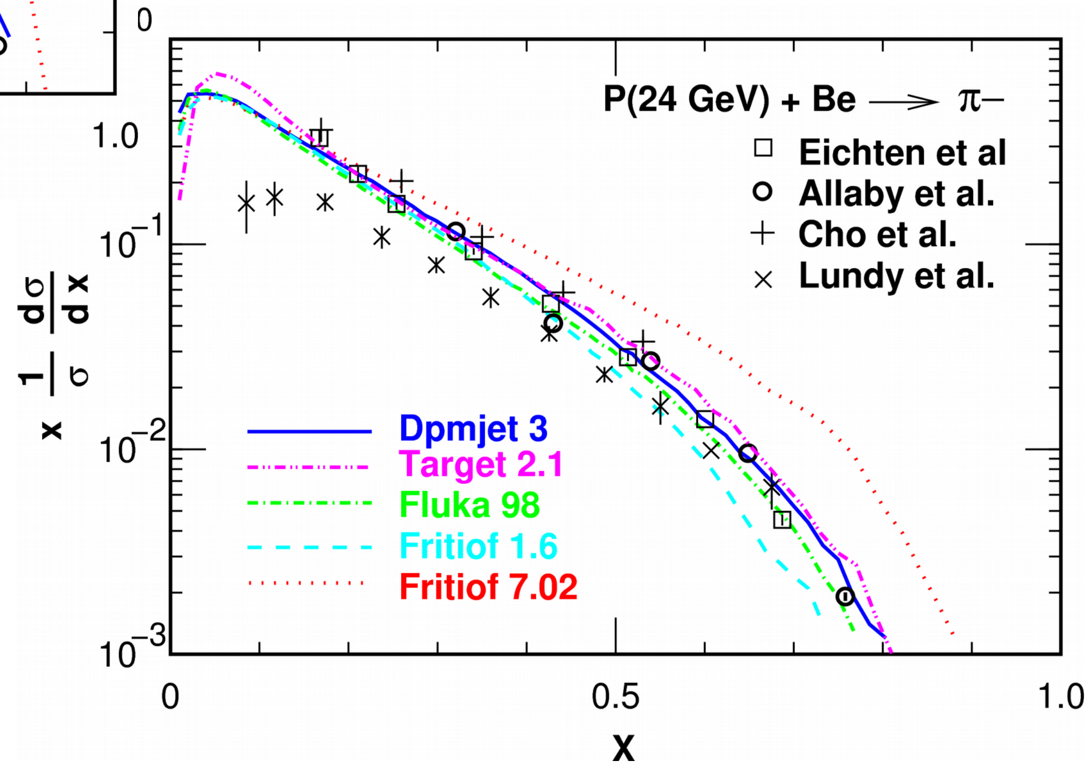
Hadronic Interaction Model



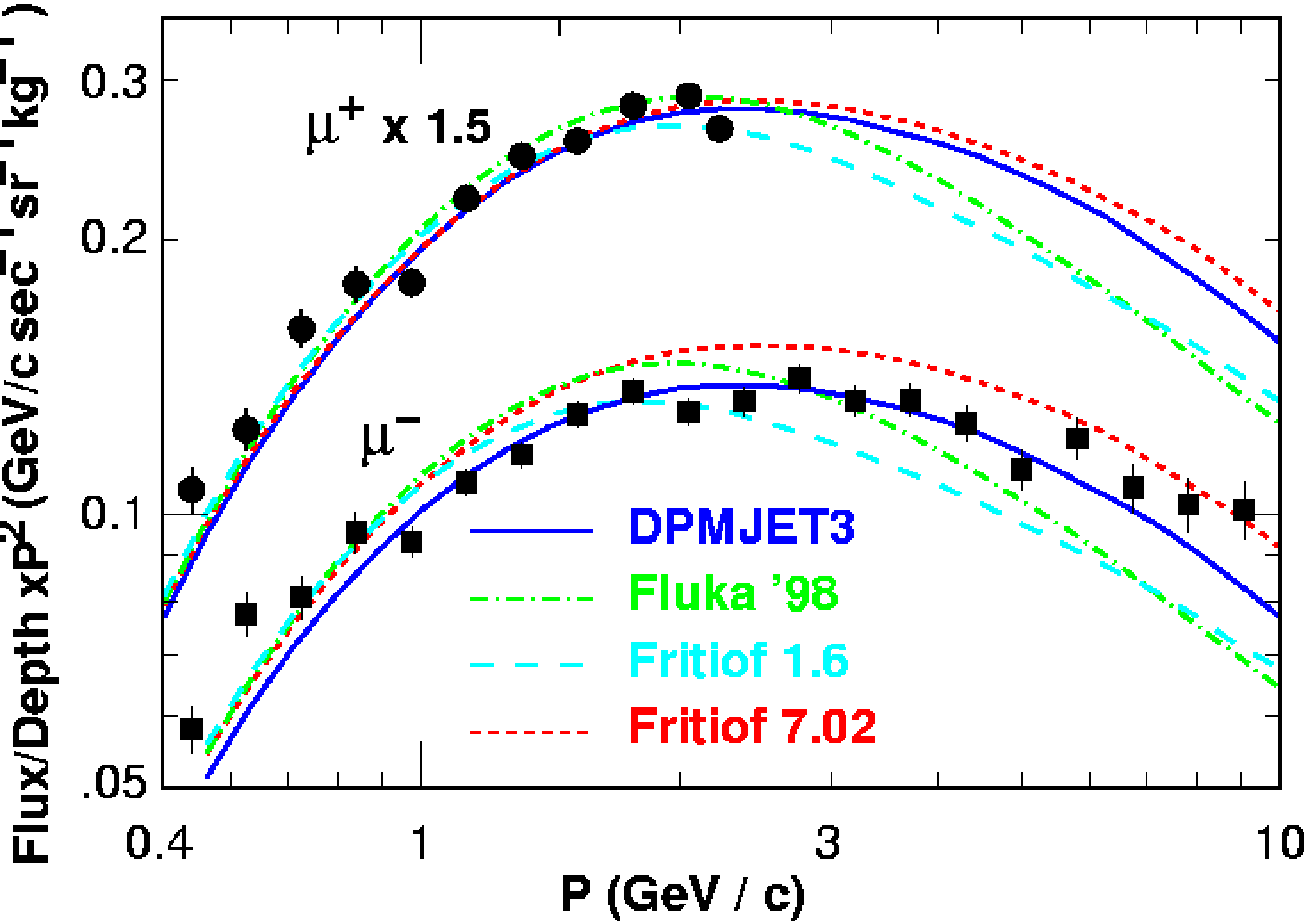
LHC

Hadronic Interaction Models (Gaisser Honda 2002)





Comparison in [Flux/depth]

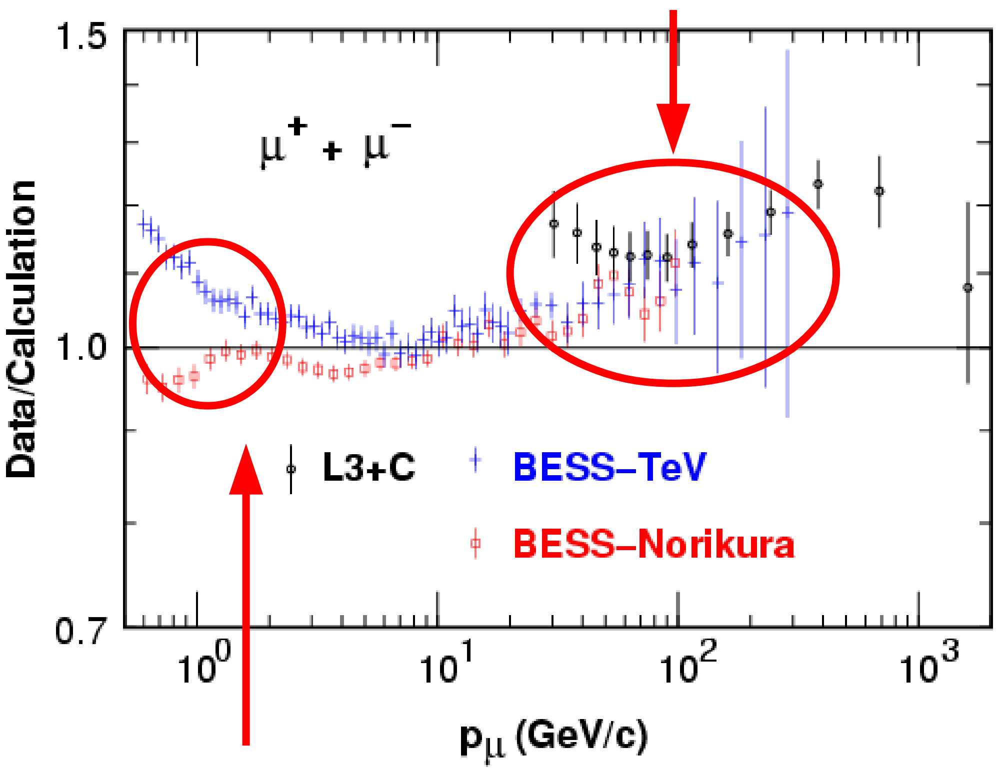


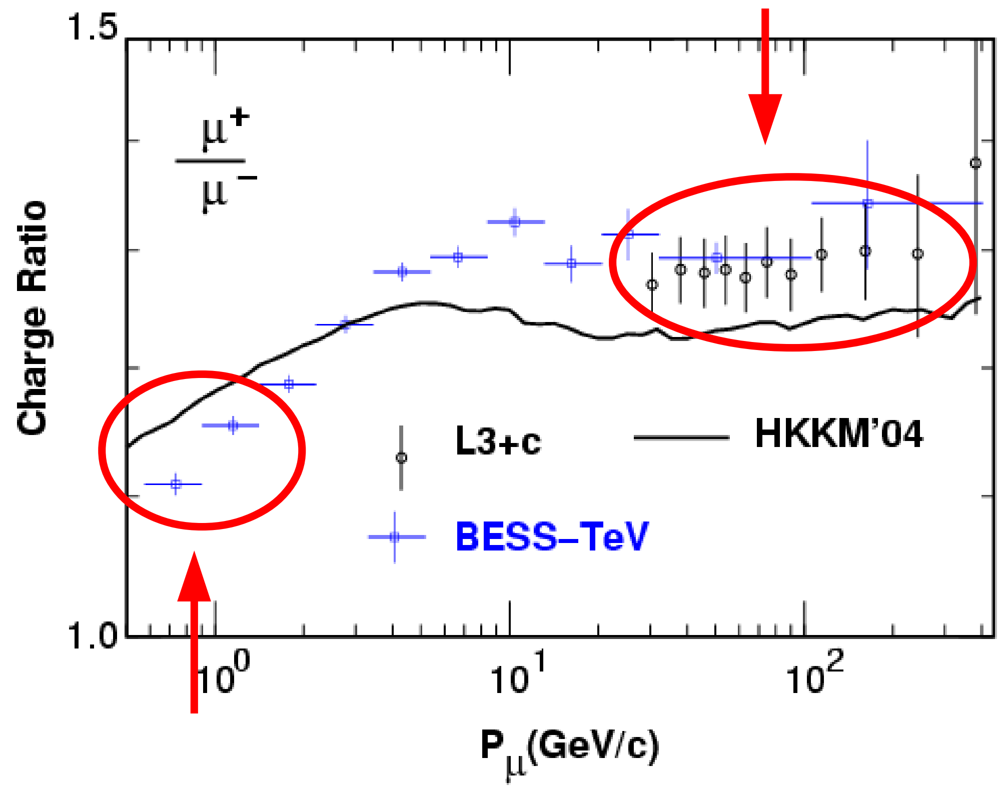
DPMJET-III show the best agreement

Muon Calibration of inclusive DPMJET-III

Data are larger by ~15%

Data are larger by ~0.05





~15% scatter ?

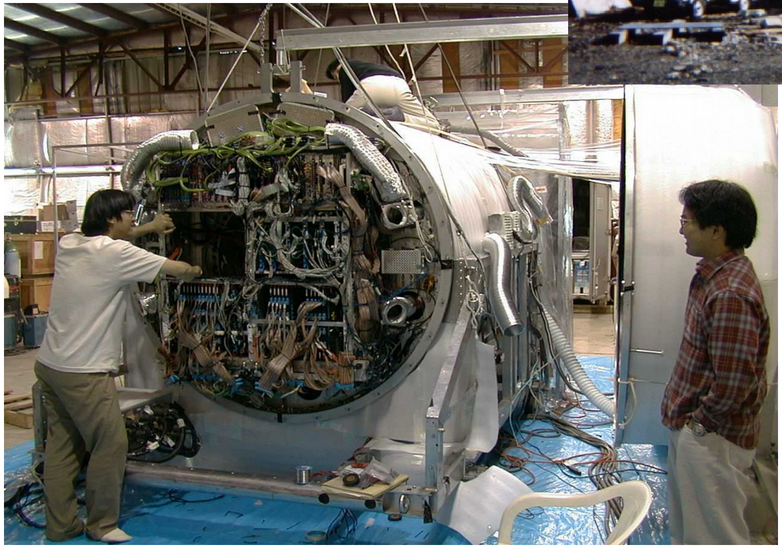
Data are smaller by ~0.05

==> DPMJET-III Should be Modified

Muon Observations

Balloon
Altitude

L3+C



BESS

Mt Norikura

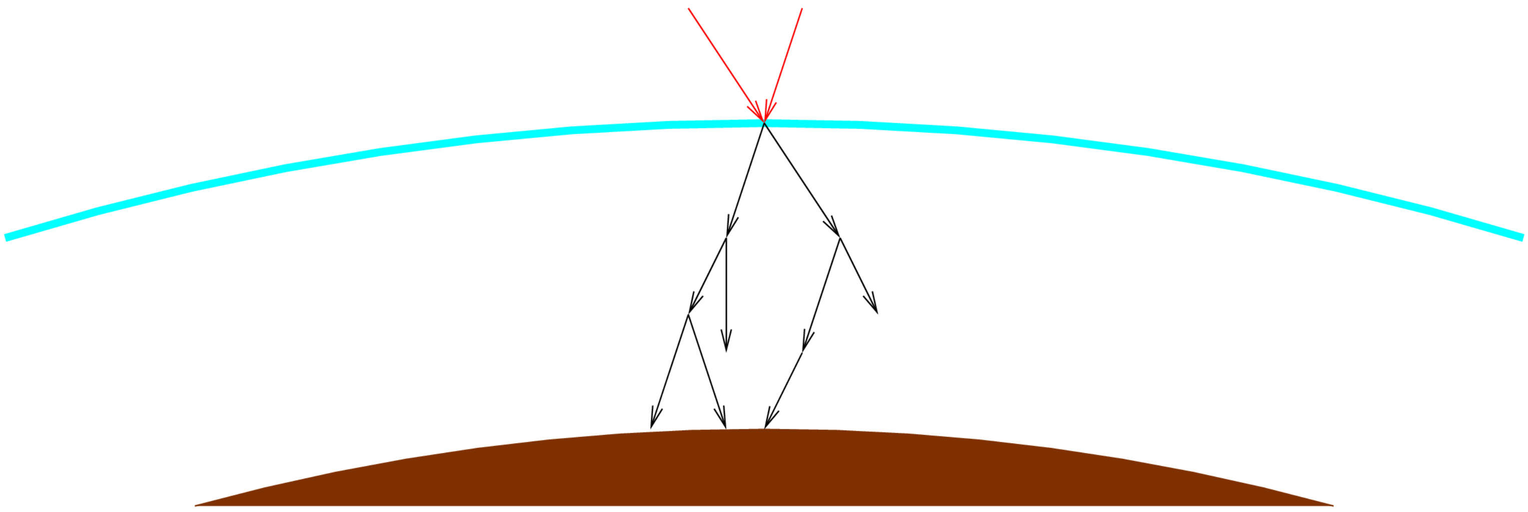
Tsukuba
(KEK)

Muon Calibration of Interaction Model

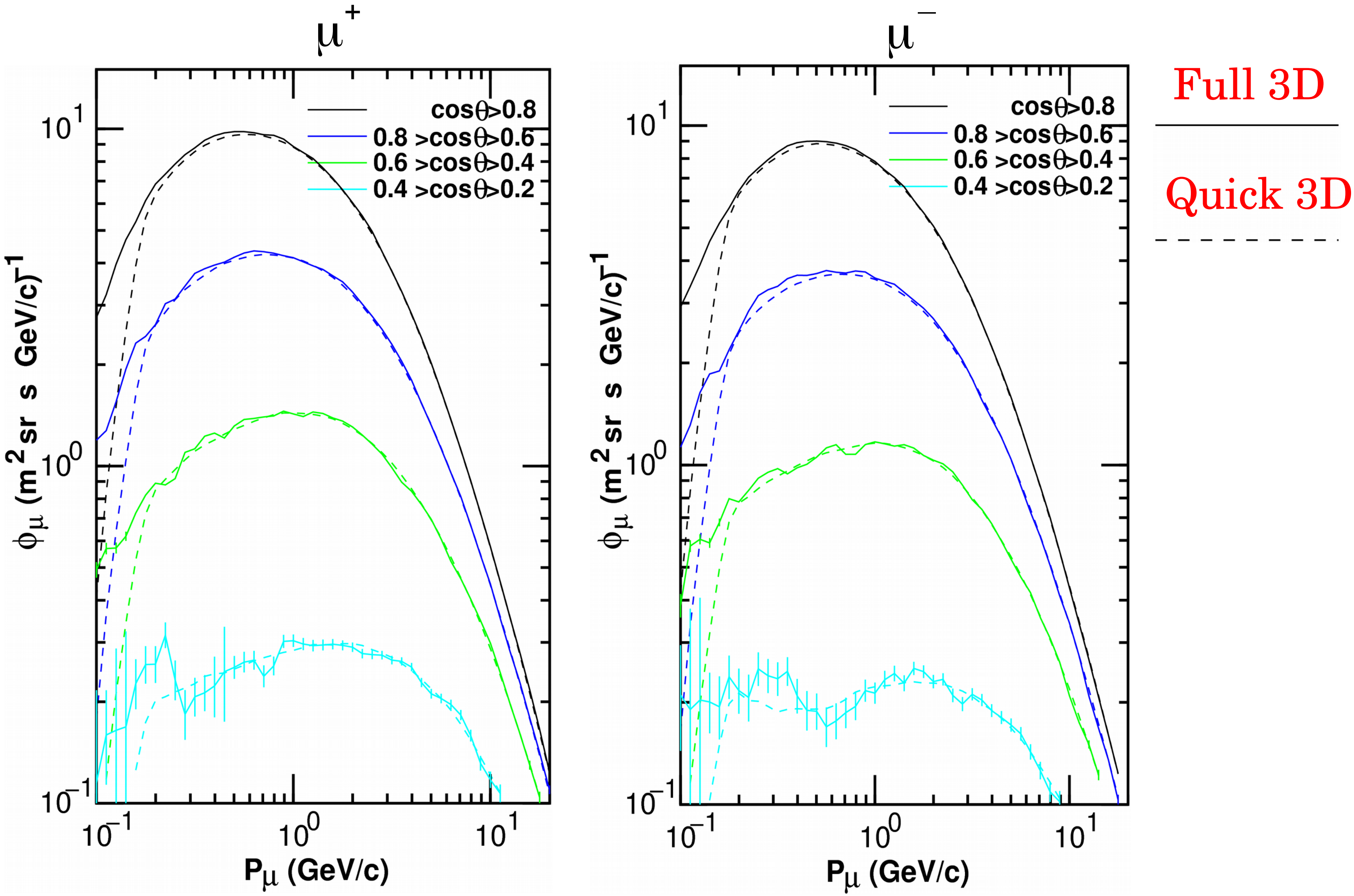
Quick 3D calculation of muon flux.

As the muon flux is a “local quantity” ($\gamma ct \sim$ 60km at10 GeV),
We can calculate it in a quick calculation method:

1. Inject cosmic rays just above the observation point,
2. Analyze all muons reach the surface of Earth.

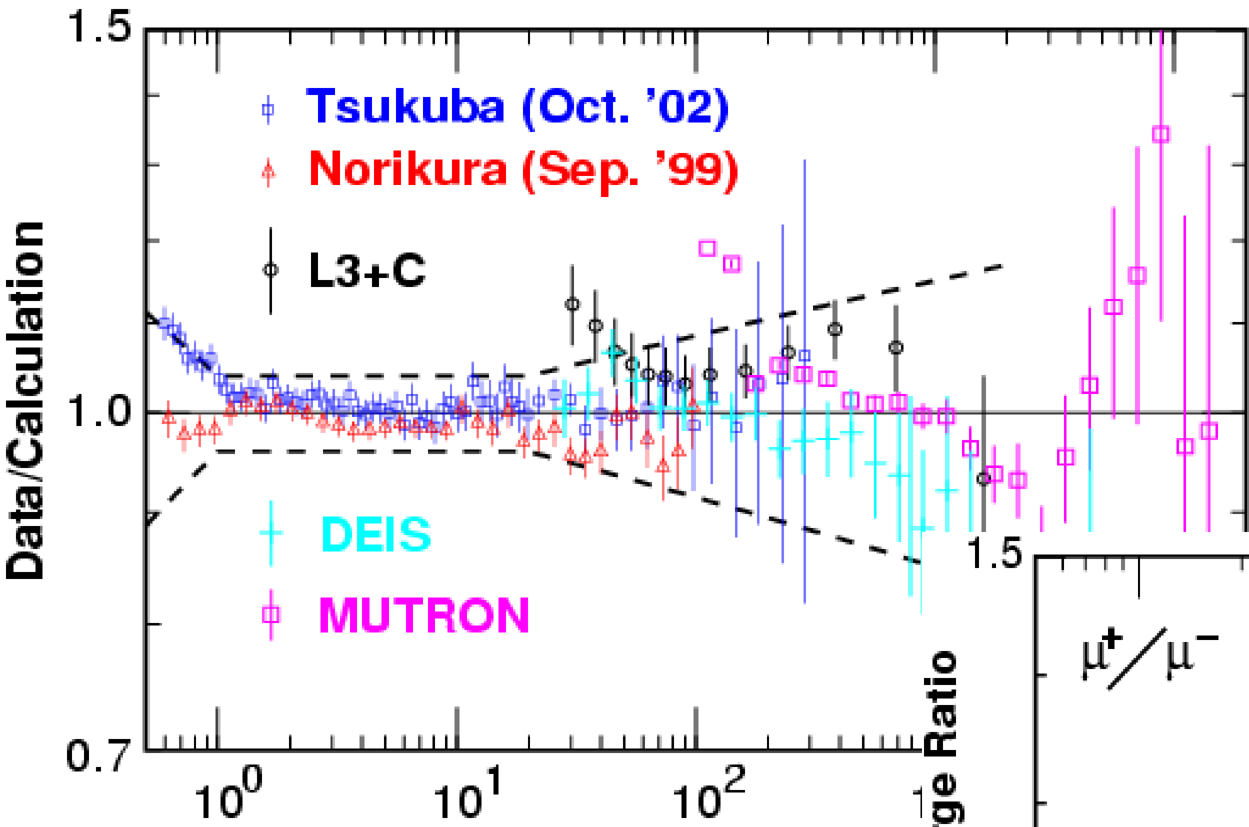


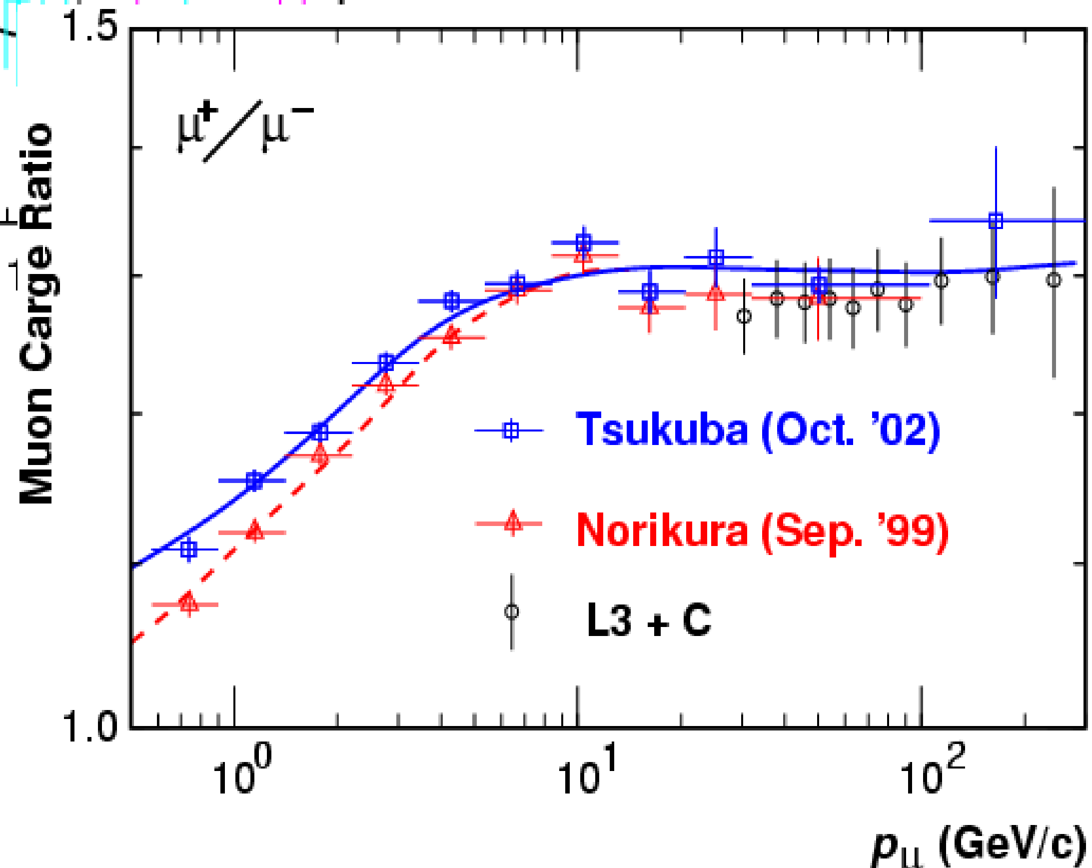
Comparison of Quick 3D calculation with Full 3D calculation



This method works above 0.2 GeV/c.

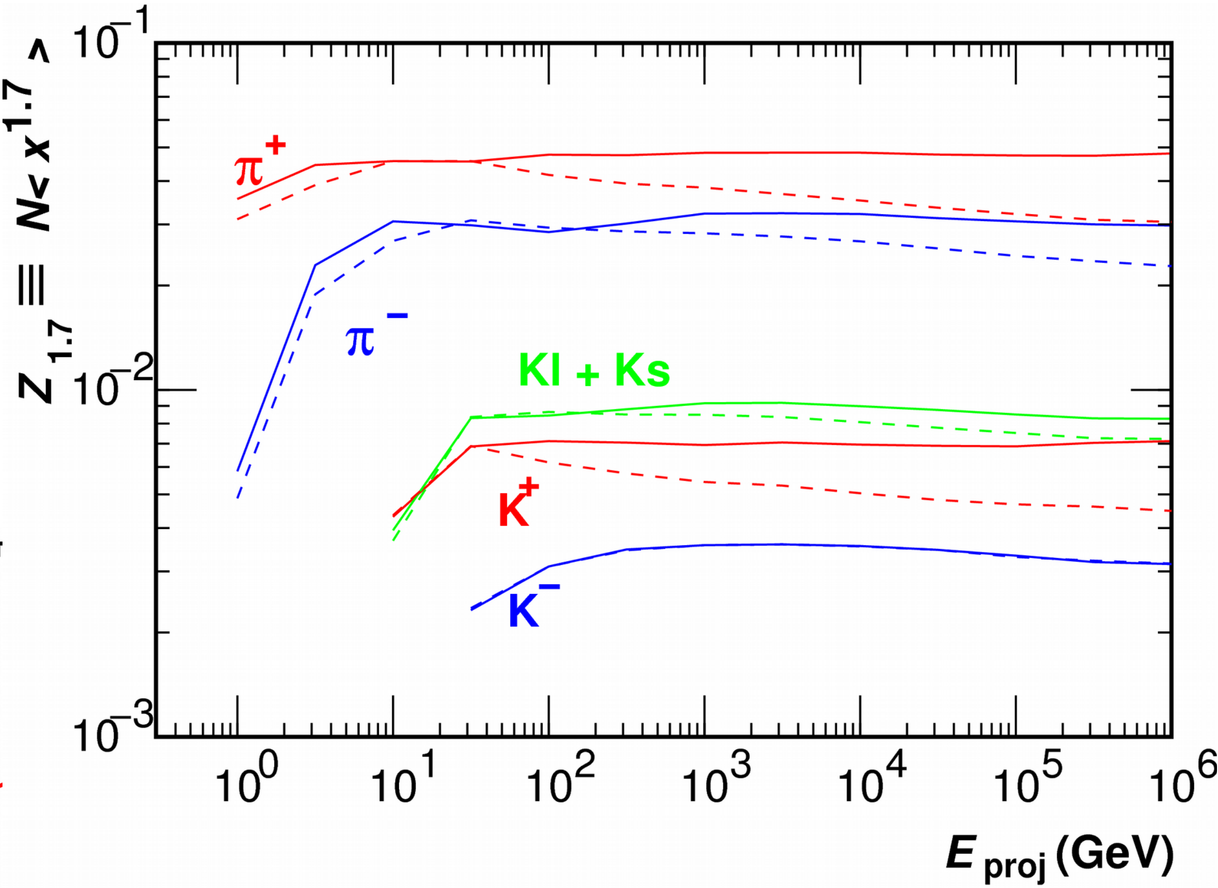
Comparison AFTER the modification

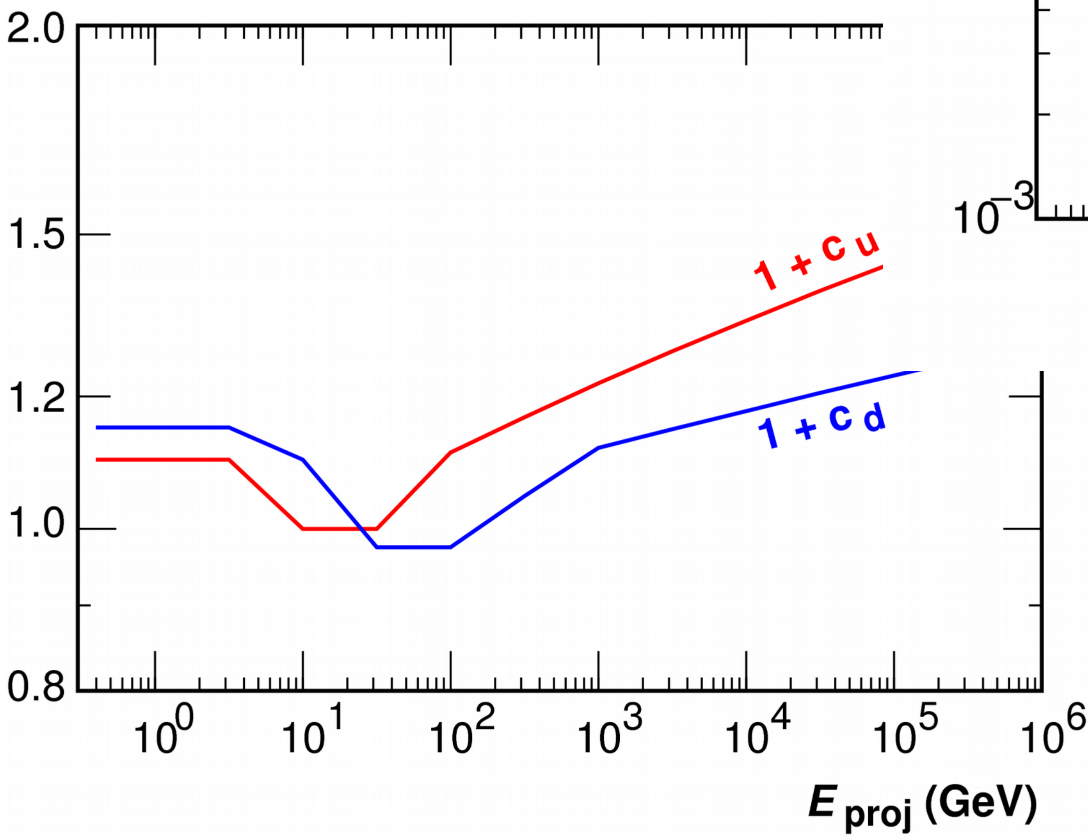




Amplitude of Modification (SHKKM 2006)

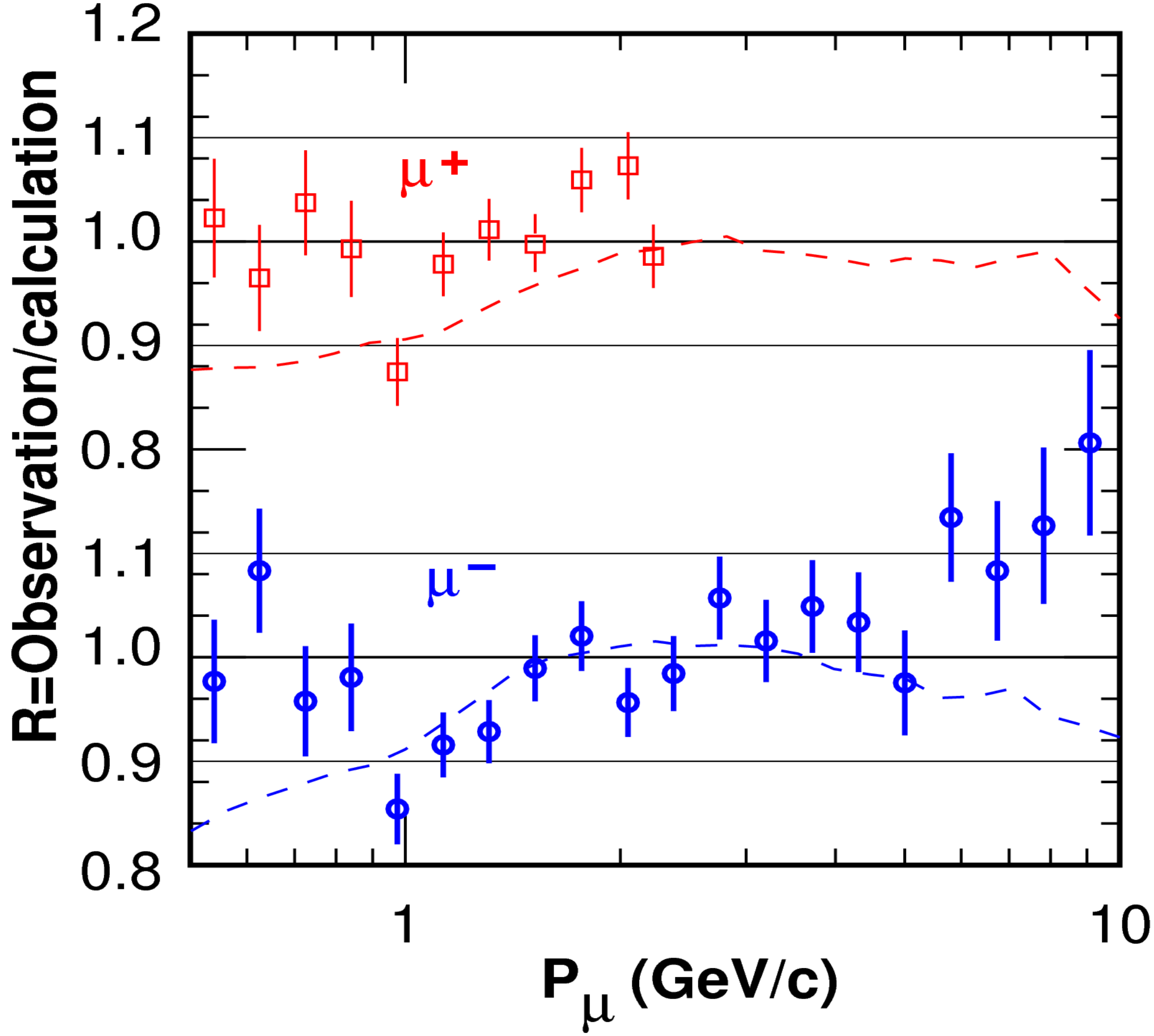
Z-Factor →





<--- Modification parameter in code

JAM + Modified DPMJET-II vs Muons at the Balloon altitude
(HKKM2011)

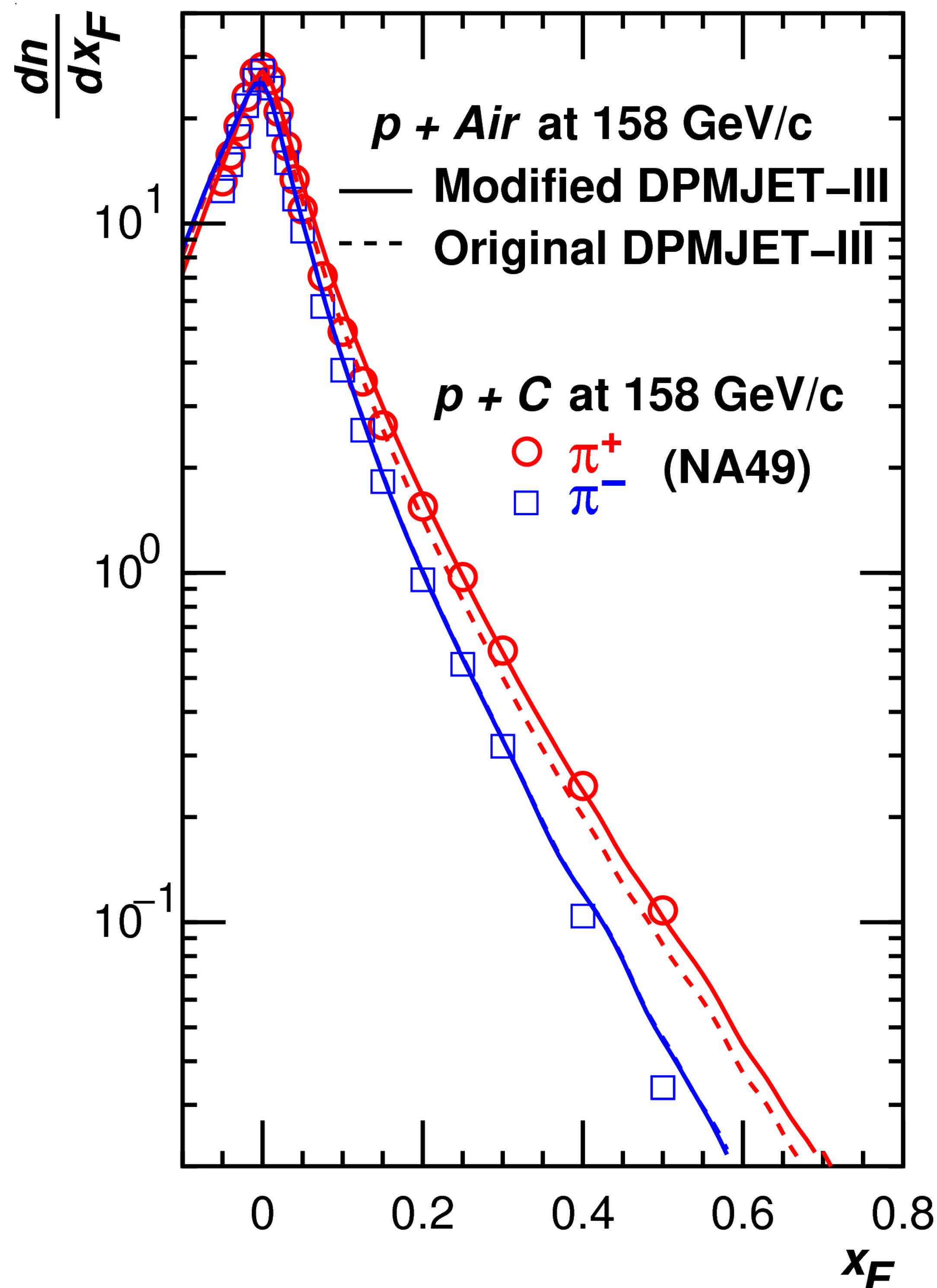


Good agreement !

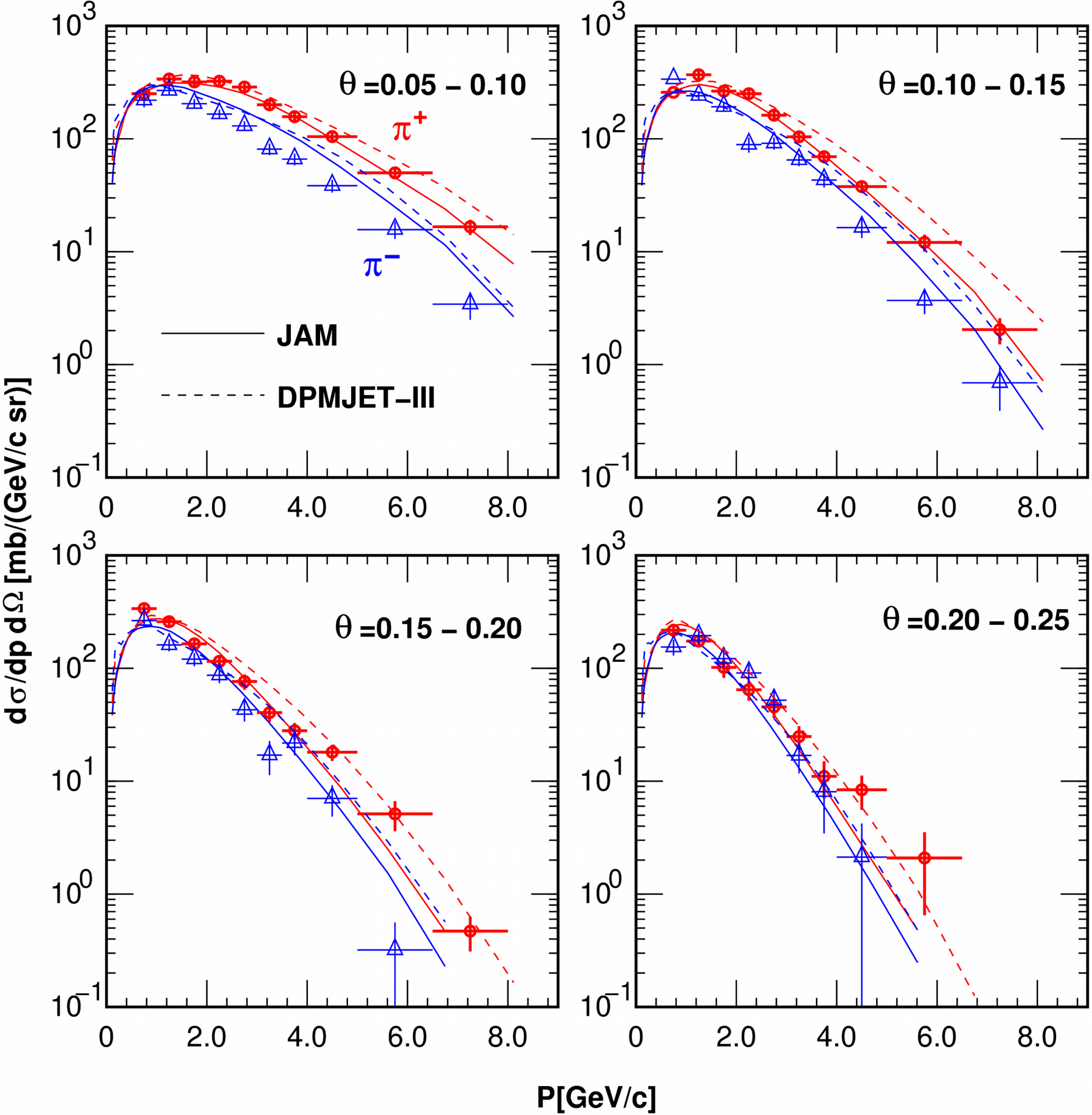
Use DPMJET-III above 32 GeV
and JAM below 32 GeV

Comparison with Accelerator data

DPMJET-III vs NA49

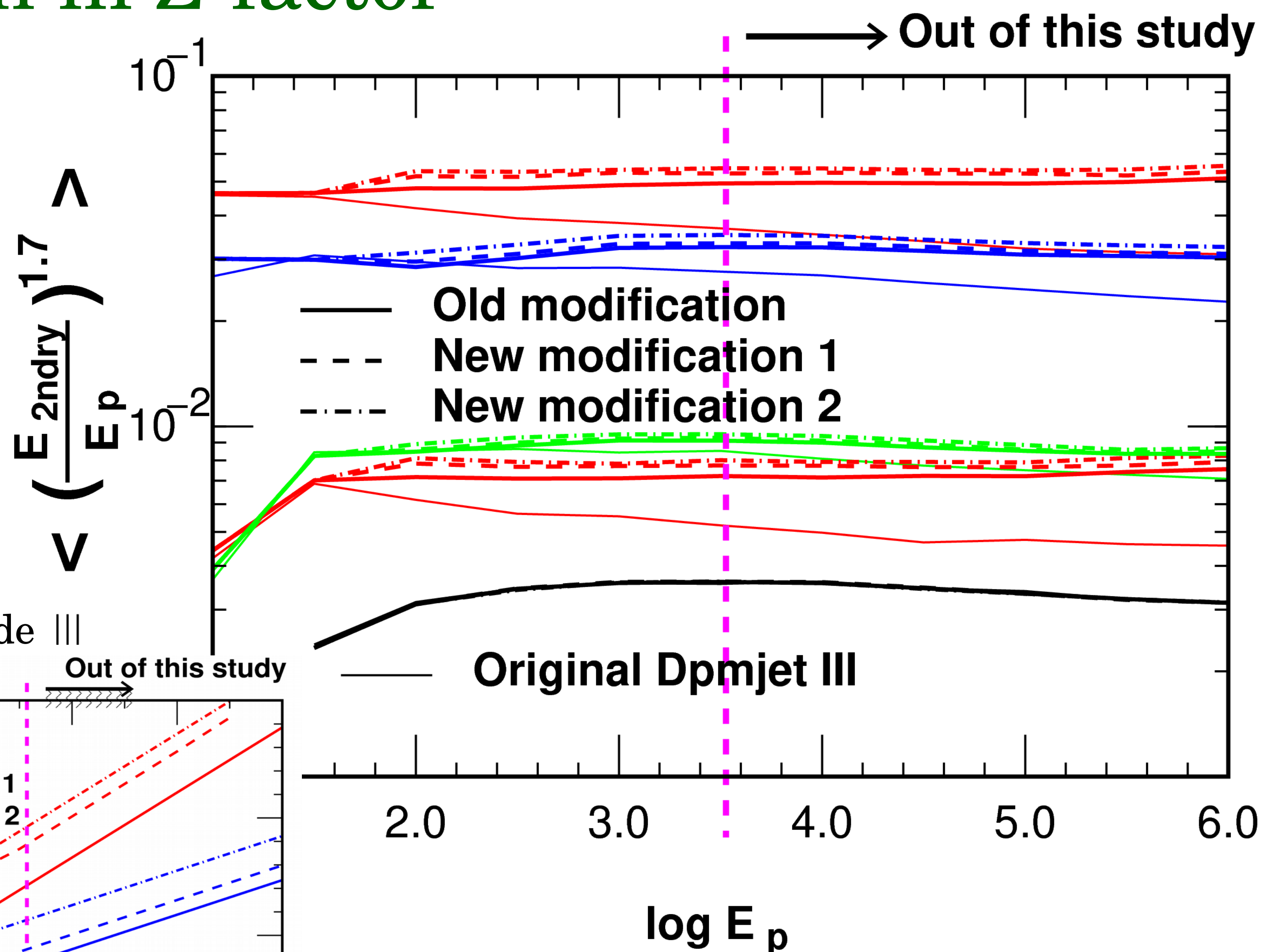


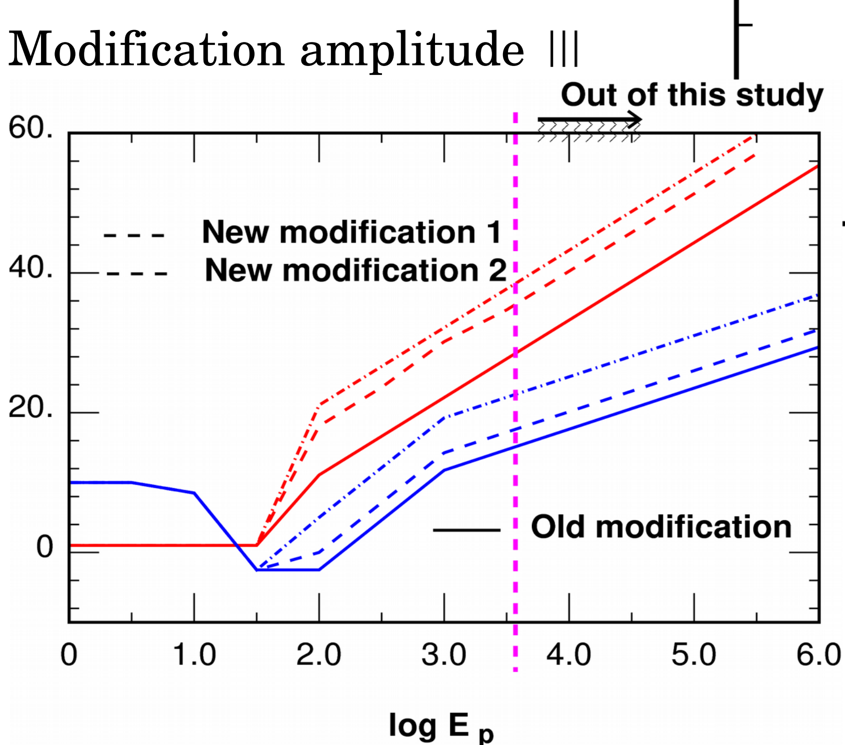
vs HARP



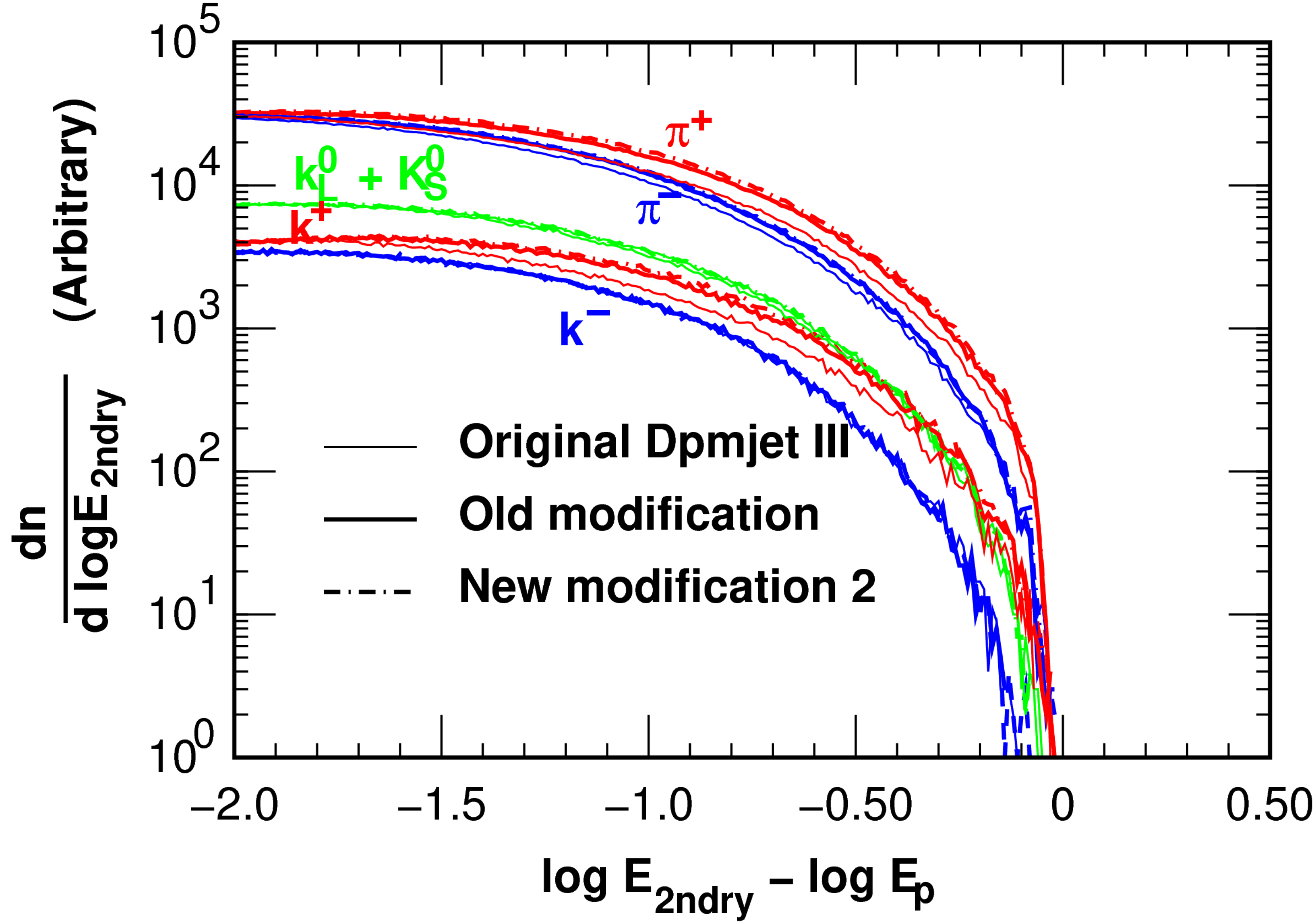
θ =0.05 – 0.10
θ =0.10 – 0.15
θ =0.15 – 0.20
θ =0.20 – 0.25
π+
π−
JAM
DPMJET–III
dσ/dp dΩ [mb/(GeV/c sr)]
P[GeV/c]

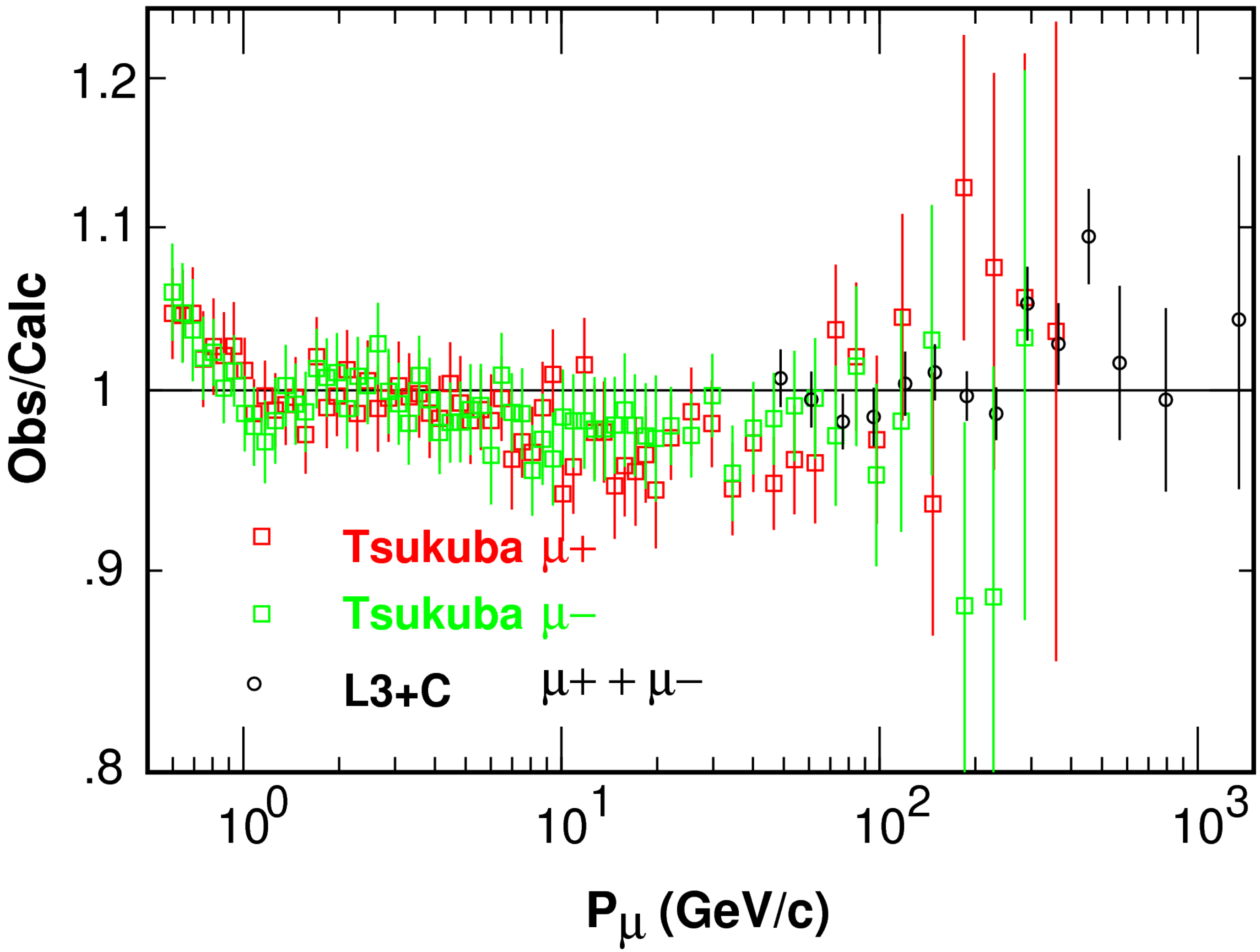
Comparison in Z-factor





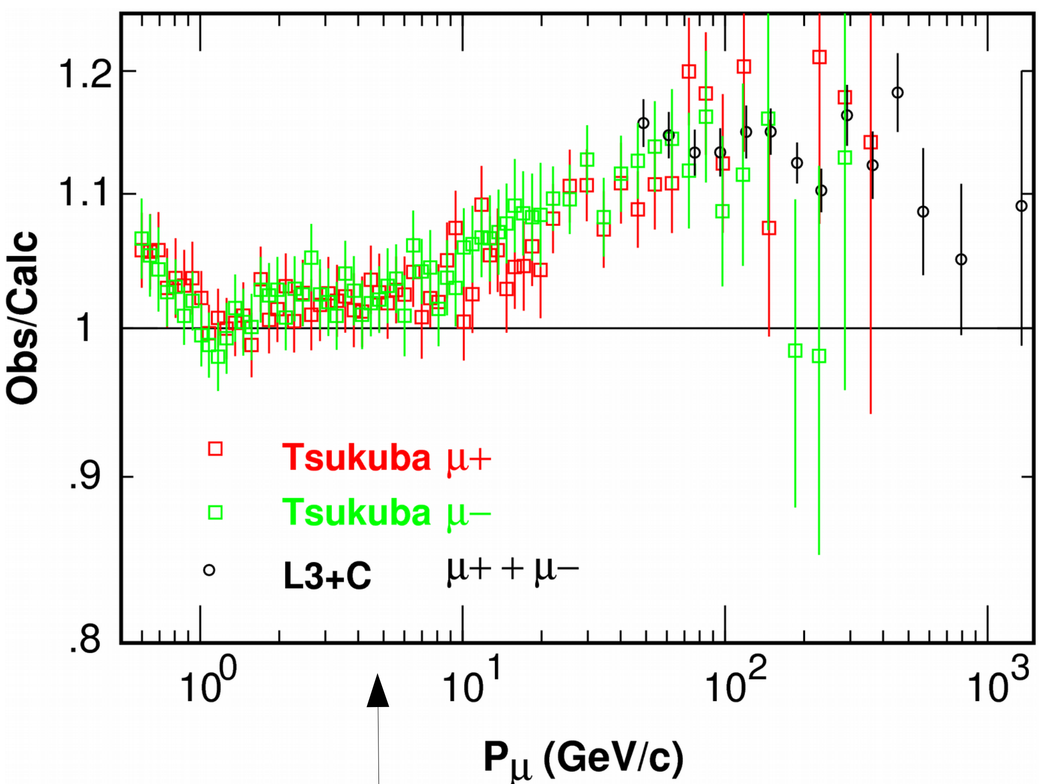
Comparison of secondary spectra of interaction models at 1 TeV





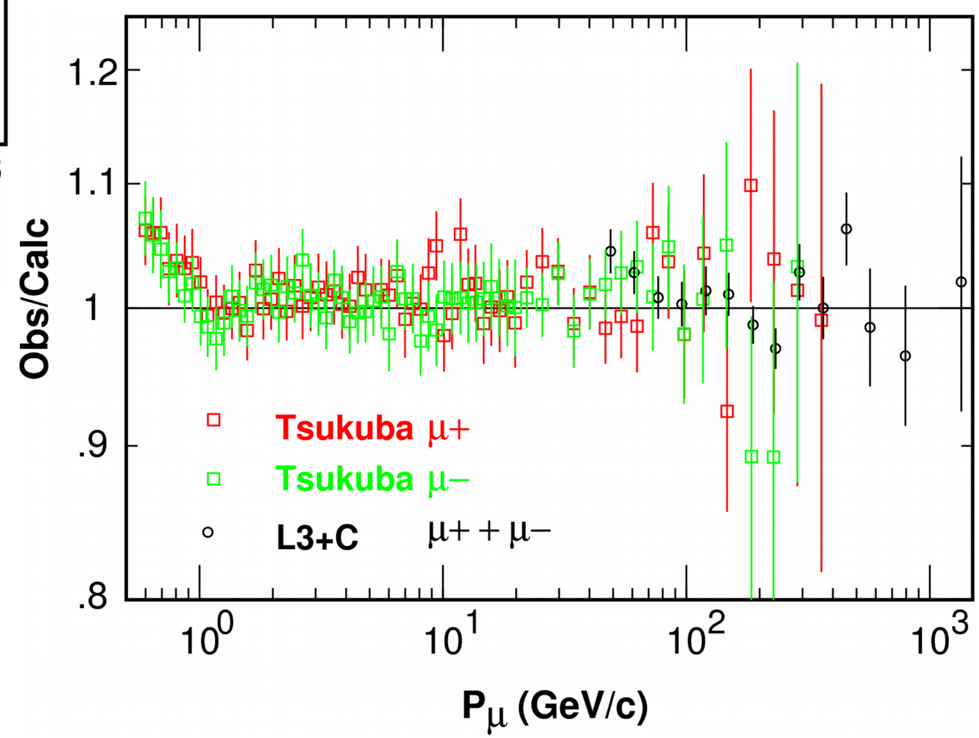
Obs/Calc
1.2
1.1
1
.9
.8
10^0
10^1
10^2
10^3
P_μ (GeV/c)
Tsukuba μ+
Tsukuba μ−
L3+C
μ+ + μ−

With Cosmic ray spectra model based on AMS02 observation

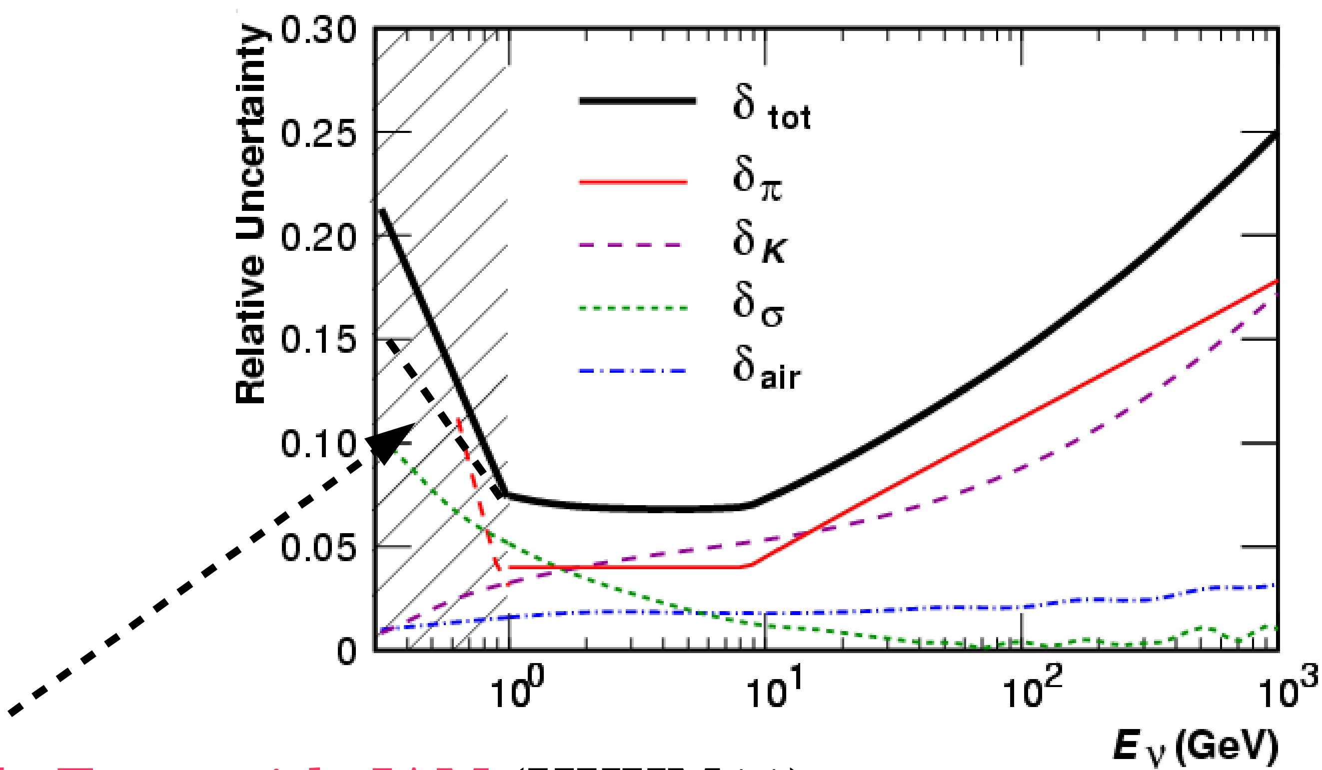


Before muon re-calibration

After muon re-calibration



Estimated Error in Atmospheric ν-flux Calculation (HKKMS07)



Possible Error with JAM (HKKM11)

δ_{π} μ -observation error + Residual of reconstruction

δ_{K} Kaon production uncertainty

δ_{σ} Mean free path (interaction crossection) uncertainty

δ_{air} Atmosphere density profule uncertainty

Decay's Need to Consider

$\pi^{\pm} \to \mu^{\pm} + \nu_{\mu} (\bar{\nu}_{\mu})$ (100%)

$\mu^{\pm} \to e^{\pm} + \nu_{e} (\bar{\nu}_{e}) + \bar{\nu}_{\mu} (\nu_{\mu})$ (100%)

$K^{\pm} \to \mu^{\pm} + \nu_{\mu} (\bar{\nu}_{\mu})$ (63.5%)

$\to \pi^{\pm} + \pi^{0}$ (21.2%)

$\to \pi^{\pm} + \pi^{+} + \pi^{-}$ (5.6%)

$\to \pi^{0} + \mu^{\mp} + \nu_{\mu} (\bar{\nu}_{\mu})$ (3.2%)

$\to \pi^{0} + e^{\mp} + \nu_{e} (\bar{\nu}_{e})$ (4.8%)

$\to \pi^{\pm} + \pi^{0} + \pi^{0}$ (1.73%)

$K_{l}^{0} \to \pi^{0} + \pi^{+} + \pi^{-}$ (12.37%)

$\to \pi^{\pm} + \mu^{\mp} + \nu_{\mu} (\bar{\nu}_{\mu})$ (27.0%)

$\to \pi^{\pm} + e^{\mp} + \nu_{e} (\bar{\nu}_{e})$ (38.6%)

$K_{s}^{0} \to \pi^{+} + \pi^{-}$ (68.6%)

$\to \pi^{\pm} + \mu^{\mp} + \nu_{\mu} (\bar{\nu}_{\mu})$ (0.0469%)

$\to \pi^{\pm} + e^{\mp} + \nu_{e} (\bar{\nu}_{e})$ (0.0704%)

Cosmic rays in atmosphere

$$p_{CR} + [Air] \rightarrow \begin{pmatrix} n^{\pm} \cdot \pi^{\pm} \\ m \cdot \pi^{0} \end{pmatrix} + X(p, n, K, ...)$$

$$\pi^{0} \rightarrow 2\gamma$$

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$$\mu^{\pm} \rightarrow \nu_{e}(\bar{\nu}_{e}) + \bar{\nu}_{\mu}(\nu_{\mu}) + e^{\pm}$$

Atmospheric Neutrino

$$\nu_{\mu} : \nu_{e} \approx 2:1$$

γ, $e^{\pm} \rightarrow$ EM-cascade → Air Shower

Other p's, n's, and sometimes π's repeat above interactions.

Even K/π ratio is small (~0.1), the importance of kaon increase at High Energies

1. Competition of Interaction and decay

$$c\tau \frac{E}{mc^2} \sim \frac{1}{\sigma n}$$

$$E \sim \frac{mc^2}{c\tau\sigma n} = \begin{Bmatrix} 12 \text{ (GeV, for } \pi^{\pm}) \\ 22 \text{ (GeV, for } K_L^0) \\ 90 \text{ (GeV, for } K^{\pm}) \end{Bmatrix} \times \frac{\rho_{[sea\ level]}}{\rho}$$

2. Kinematices

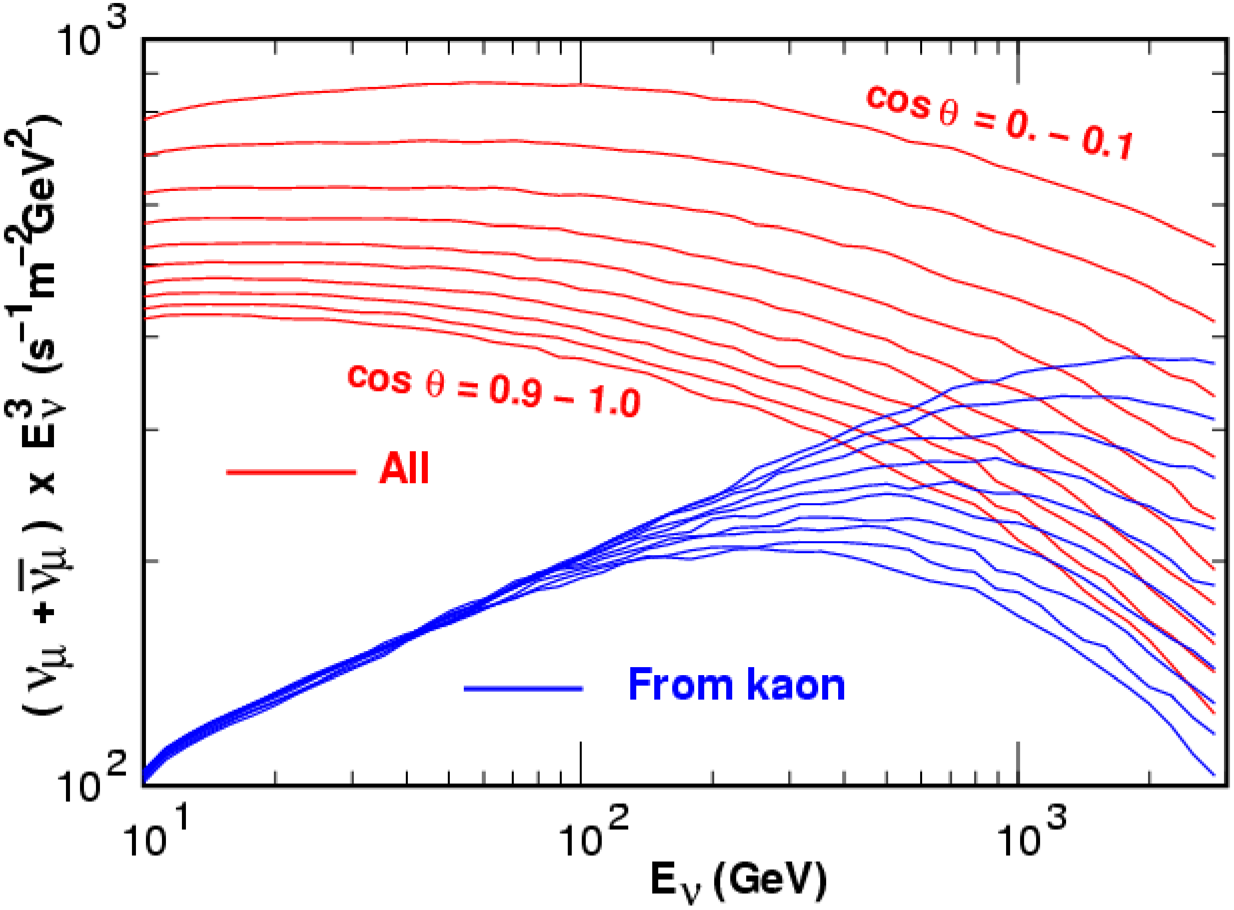
$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$$K^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$$\langle E_{\nu} \rangle = \frac{M^2 - m_{\mu}^2}{2M^2} E_0 = \begin{cases} 0.21\, E_0 \\ 0.48\, E_0 \end{cases}$$

Where M is the mass of Kaon or Pion.

Contribution of Kaon



Why 1D calculation is so preferred ?

1. $$\frac{[3D-efficiency]}{[1D-efficiency]} \sim \frac{[Area\ of\ virtual\ detector]}{[Area\ of\ the\ surface\ of\ Earth]}$$

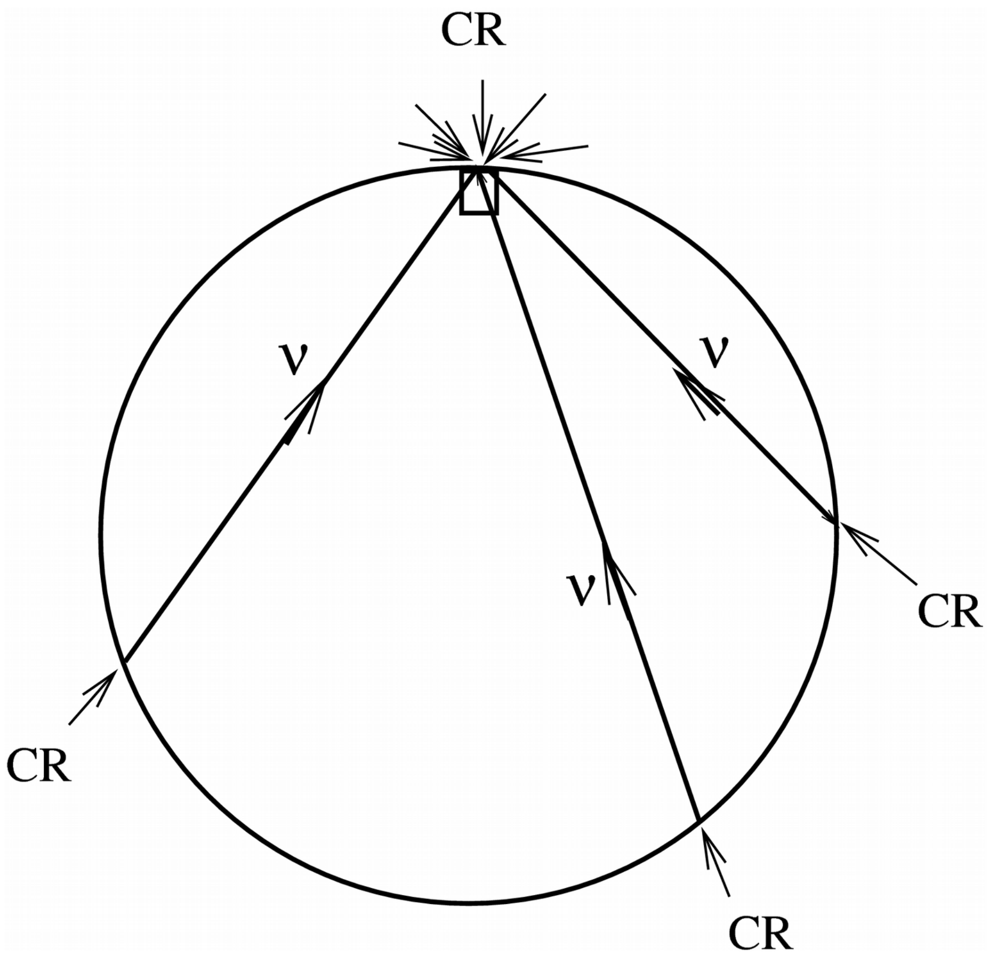
2. Angles in Hadronic Interactions

$$\Delta\theta \sim \frac{p_t}{E_\pi} \sim \frac{0.3}{E_\pi/1\mathrm{GeV}} \sim \frac{0.1}{E_\nu/1\mathrm{GeV}}$$

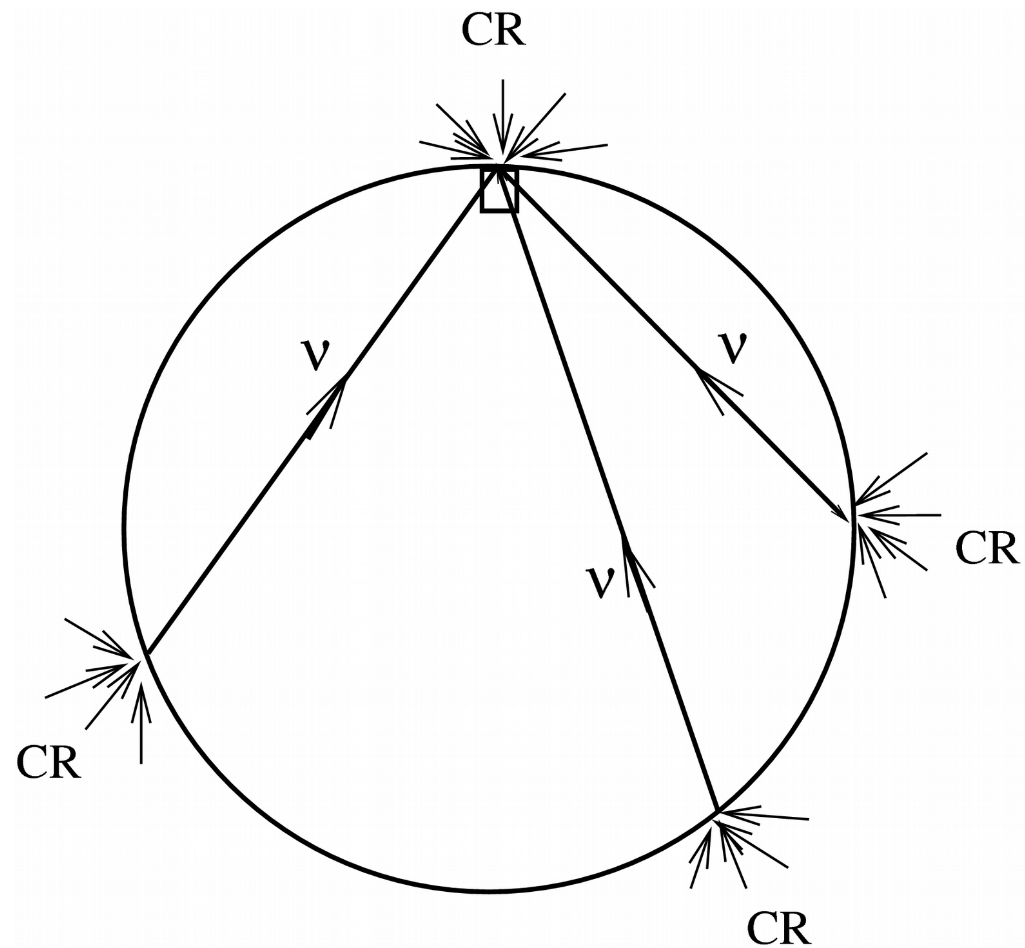
3. Muon Curvature is energy independent ~ 5 degree

General understanding *before* Fluka group 3D calculation was, *2 and 3 are not important*

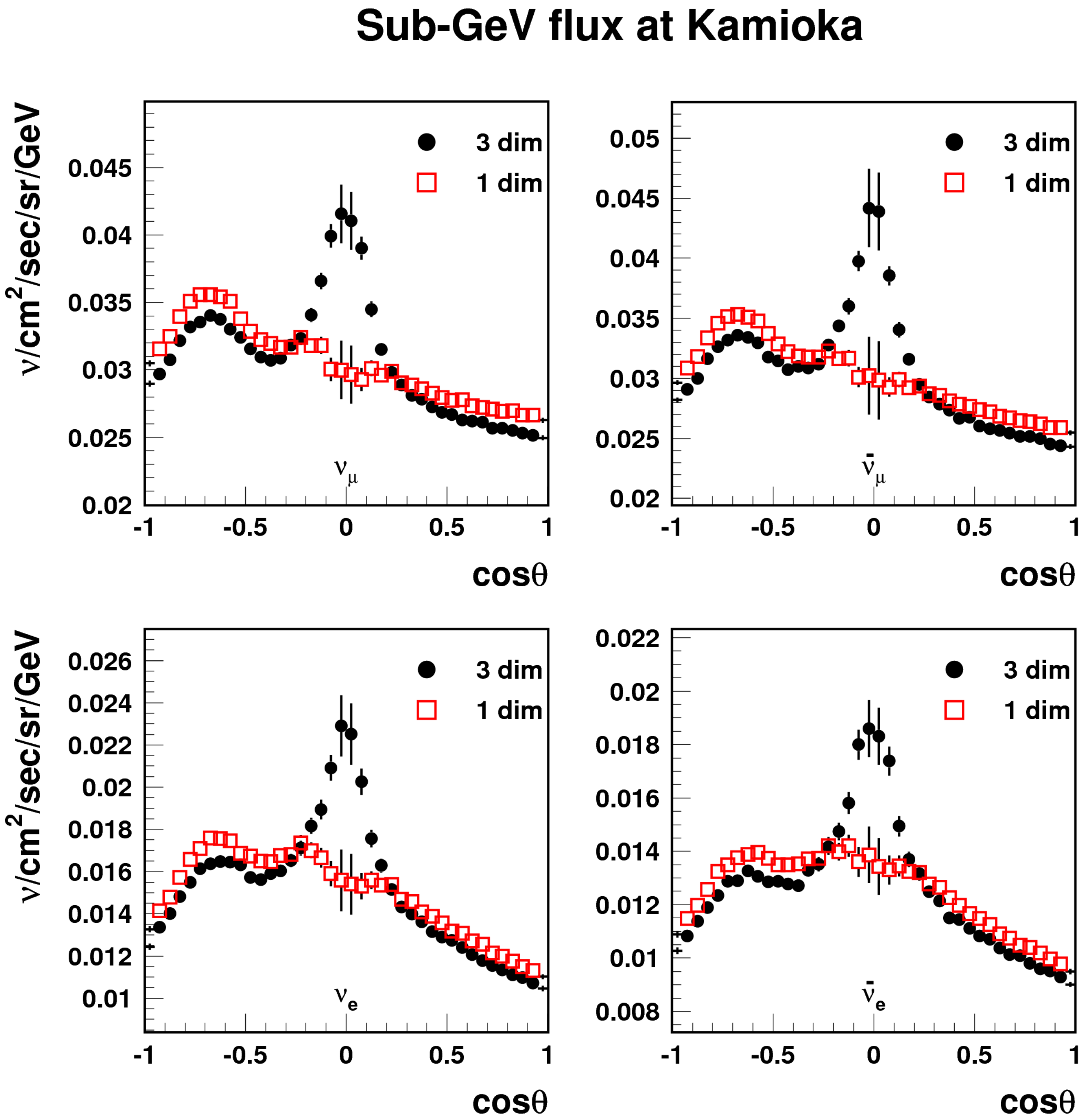
1D-calculation



3D-calculation

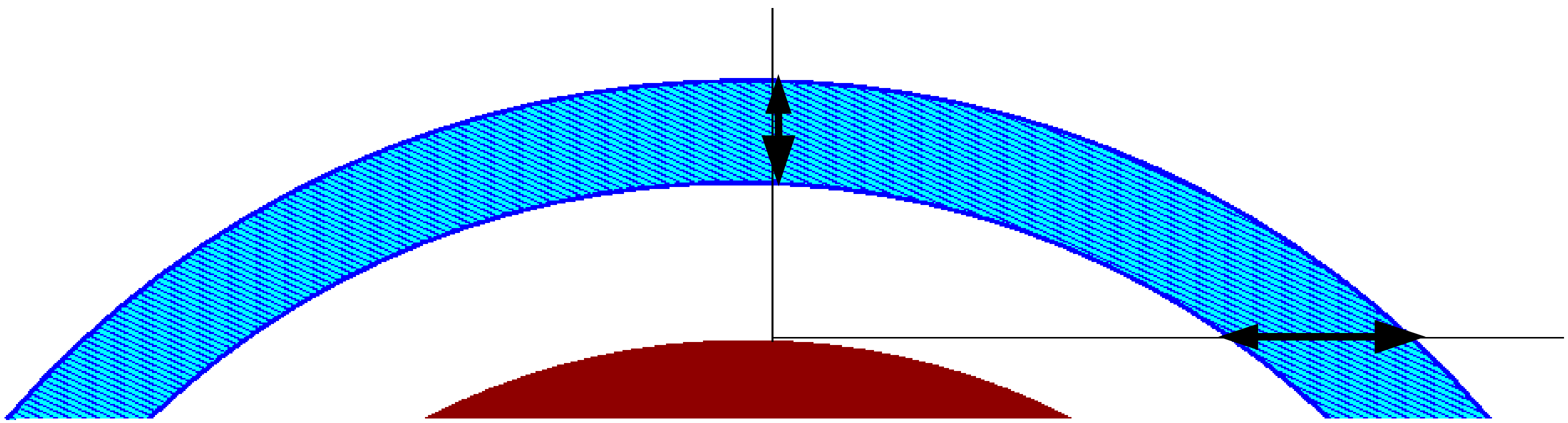


Horizontal enhancement of neutrino flux



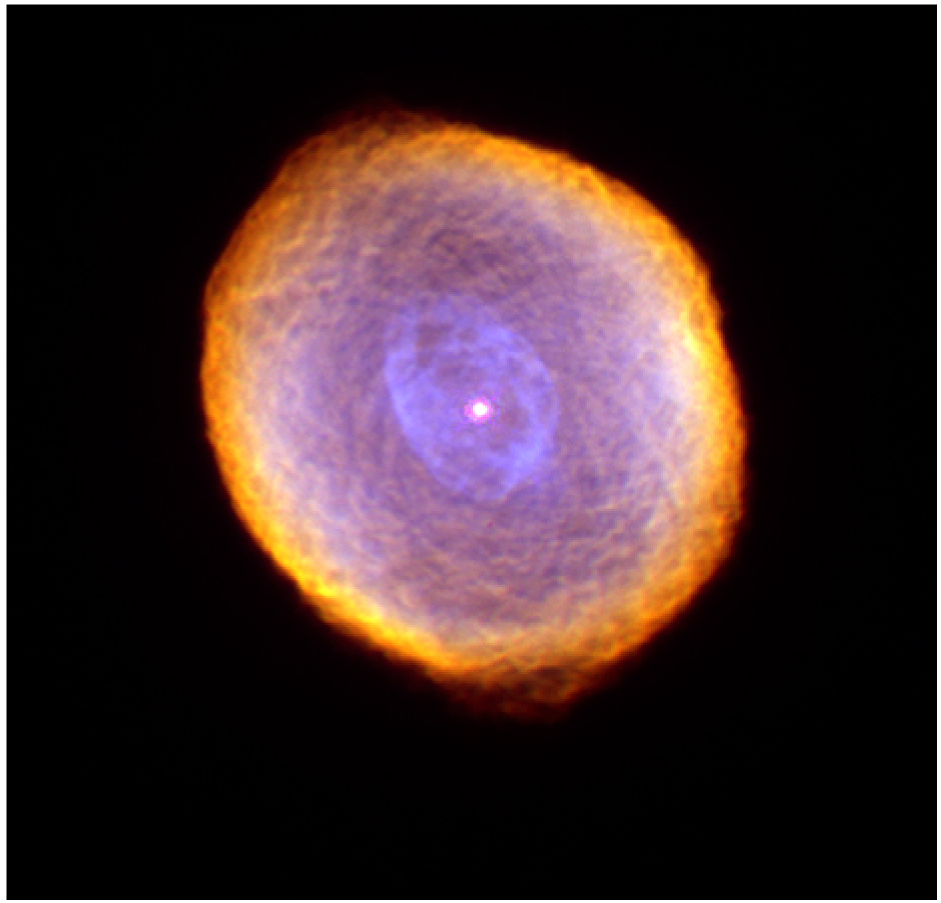
(Battistoni et al. Astropart. Phys 1999)

Interpretation of horizontal enhancement



Longer integration length in the neutrino production zone for horizontal directions

100MeV neutrino image of Earth



OR

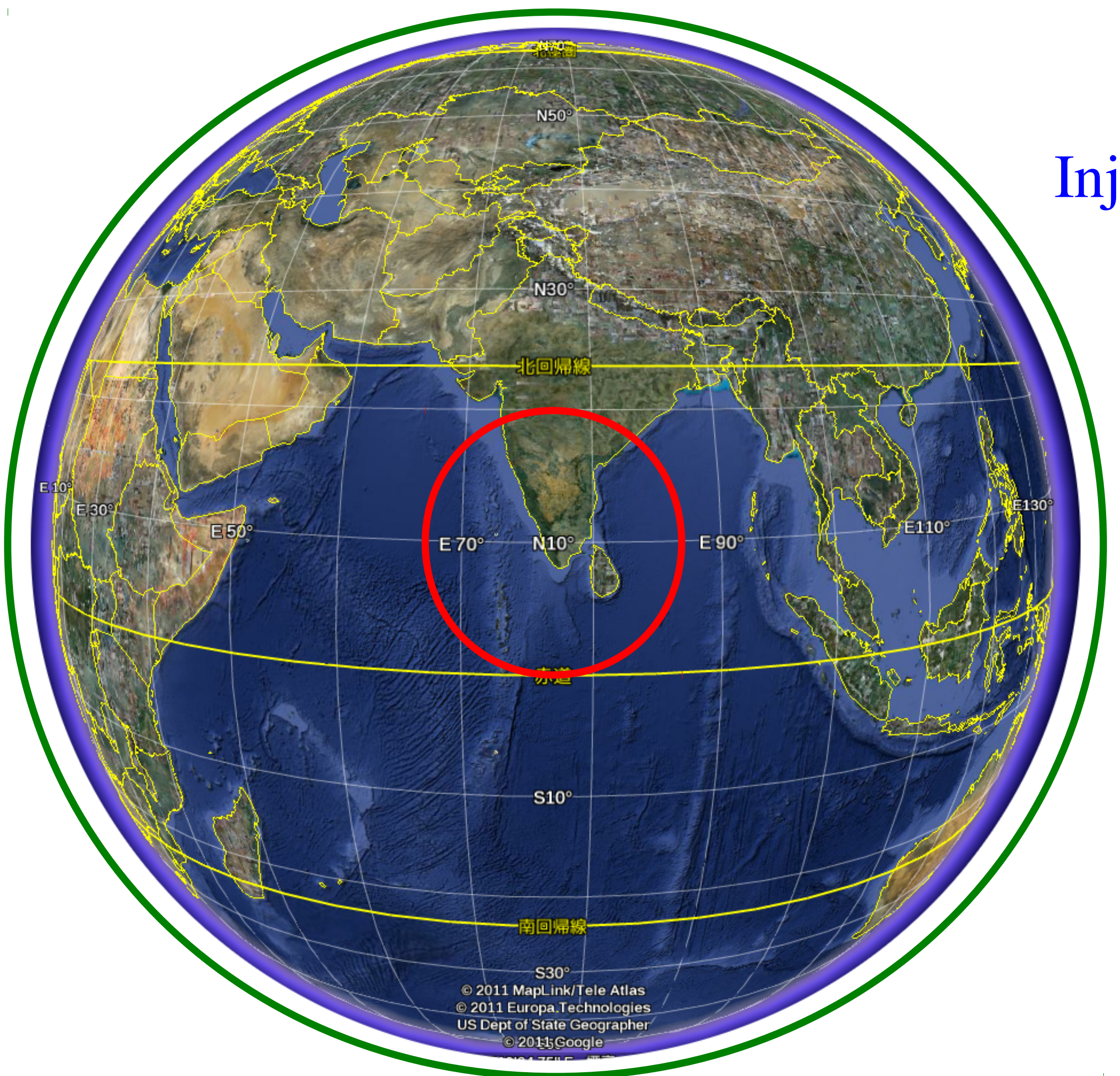

Our Calculation, Scheme and Results

3D-Calculation Geometry

Re = 6378km

Simulation Sphere (Rs = 10 x Re)

Cosmic ray go out this sphere are discarded.
Cosmic rays go beyond are pass the rigidity cutoff test



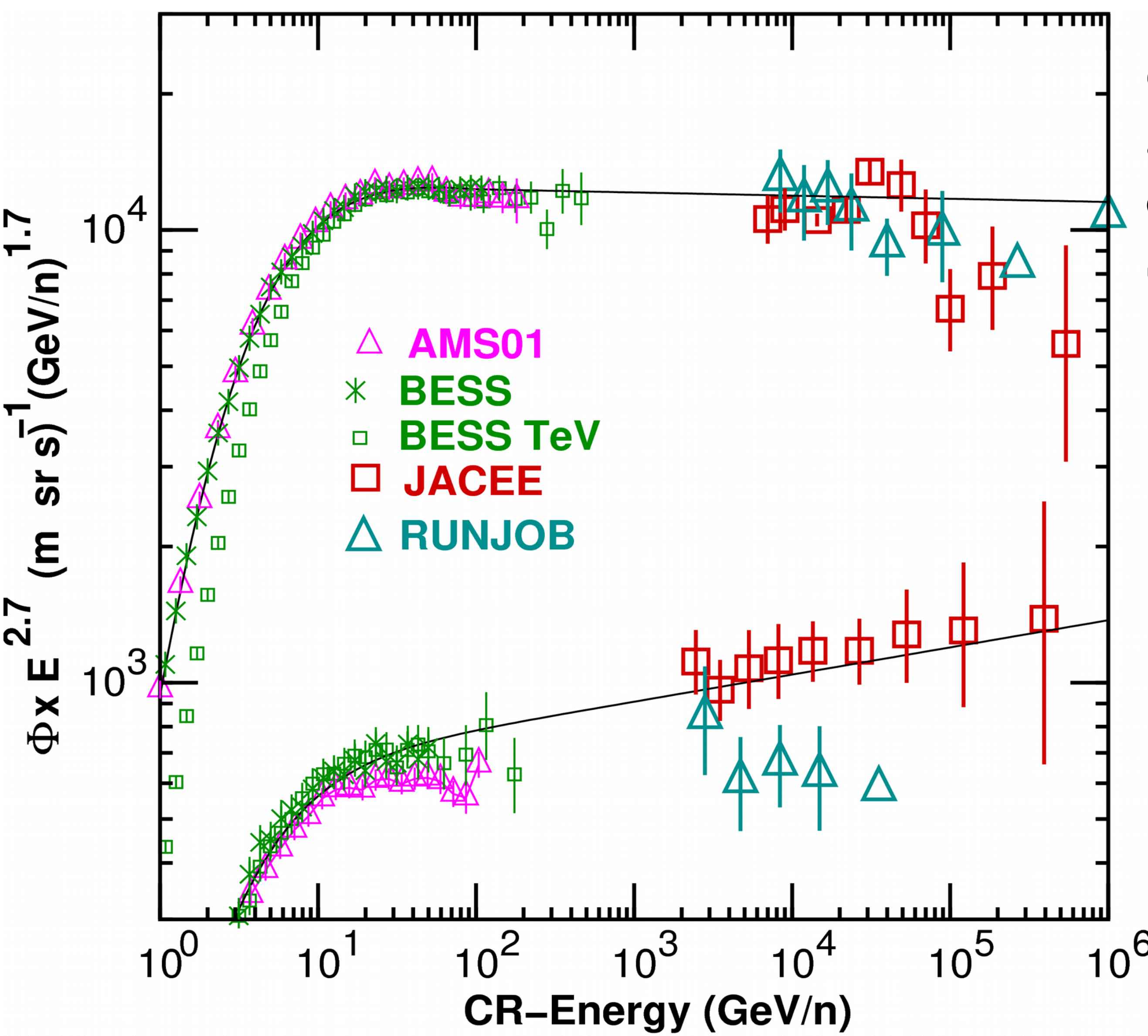
Injection Sphere (Re +100lm)

Cosmic Rays are sampled and injected here

Virtual Detector

All neutrinos path through are recorded

Primary Cosmic Ray Model and referred data (2004)



Other chemical compositions are also considered in the calculation, but they give small contributions.

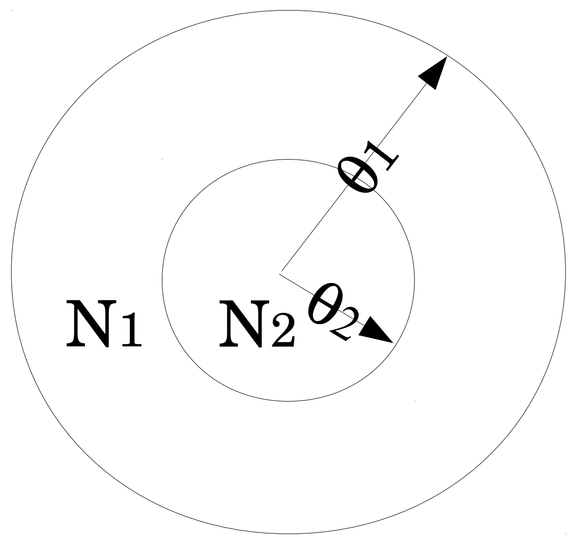
Virtual detector correction

Averages in $\theta<\theta_1$ and $\theta<\theta_2$ can be written with the central value φ_0 as

$$\varphi_1 \simeq \phi_0 + \phi' \theta_1^2$$

$$\varphi_2 \simeq \phi_0 + \phi' \theta_2^2$$

where ϕ' is a constant.



Then we can calculate the central flux value as

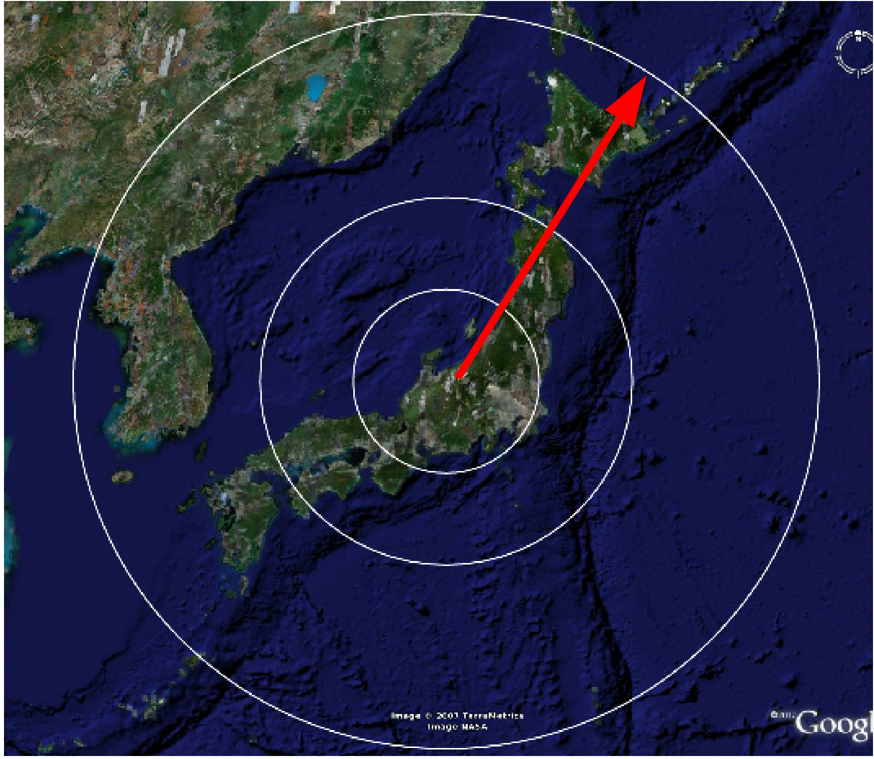
$$\phi_0 \simeq \frac{\theta_1^2 \varphi_2 - \theta_2^2 \varphi_1}{\theta_1^2 - \theta_2^2} = \frac{\varphi_2 - r^2 \varphi_1}{1 - r^2} \quad \text{for} \quad r = \left(\frac{\theta_2}{\theta_1}\right),\ r<1$$

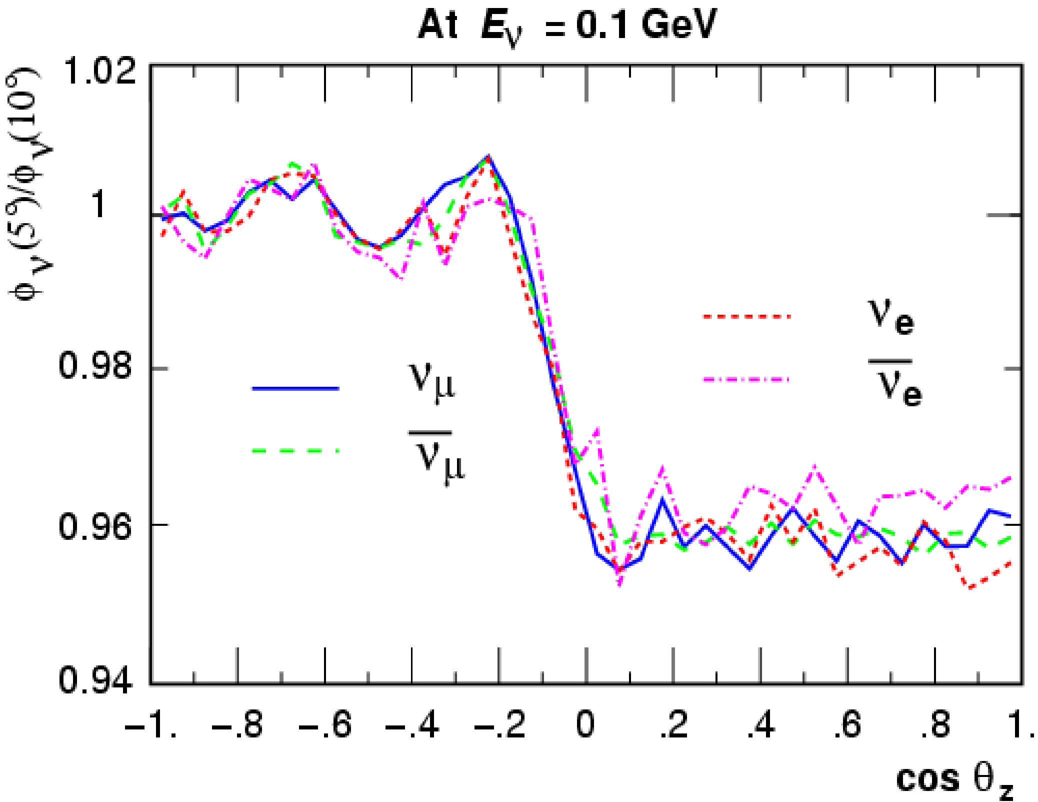
Apply this relation to the MC results

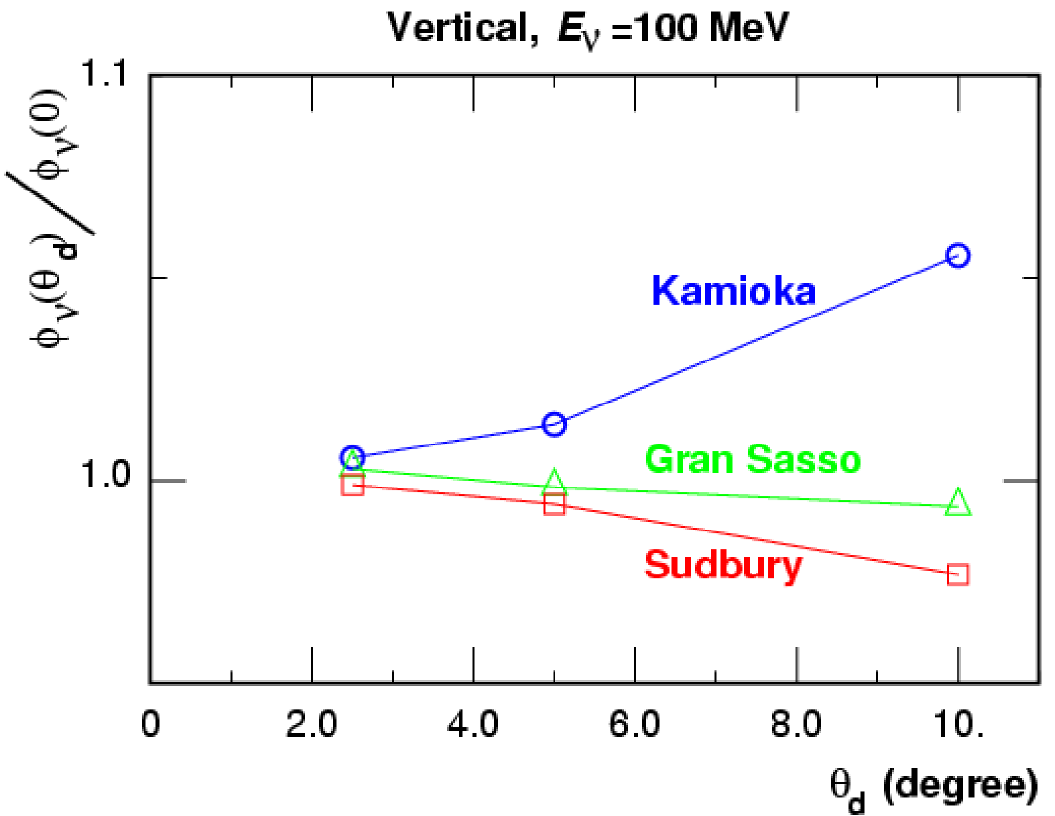
$$\phi_1 = \frac{N_1}{T\pi\theta_1^2},\quad \phi_2 = \frac{N_2}{T\pi\theta_2^2}$$

Example in HKKM06 (PRD 2007) with

$$\phi_\nu(0) \simeq -\frac{1}{3}\phi_\nu(10) + \frac{4}{3}\phi_\nu(5)$$

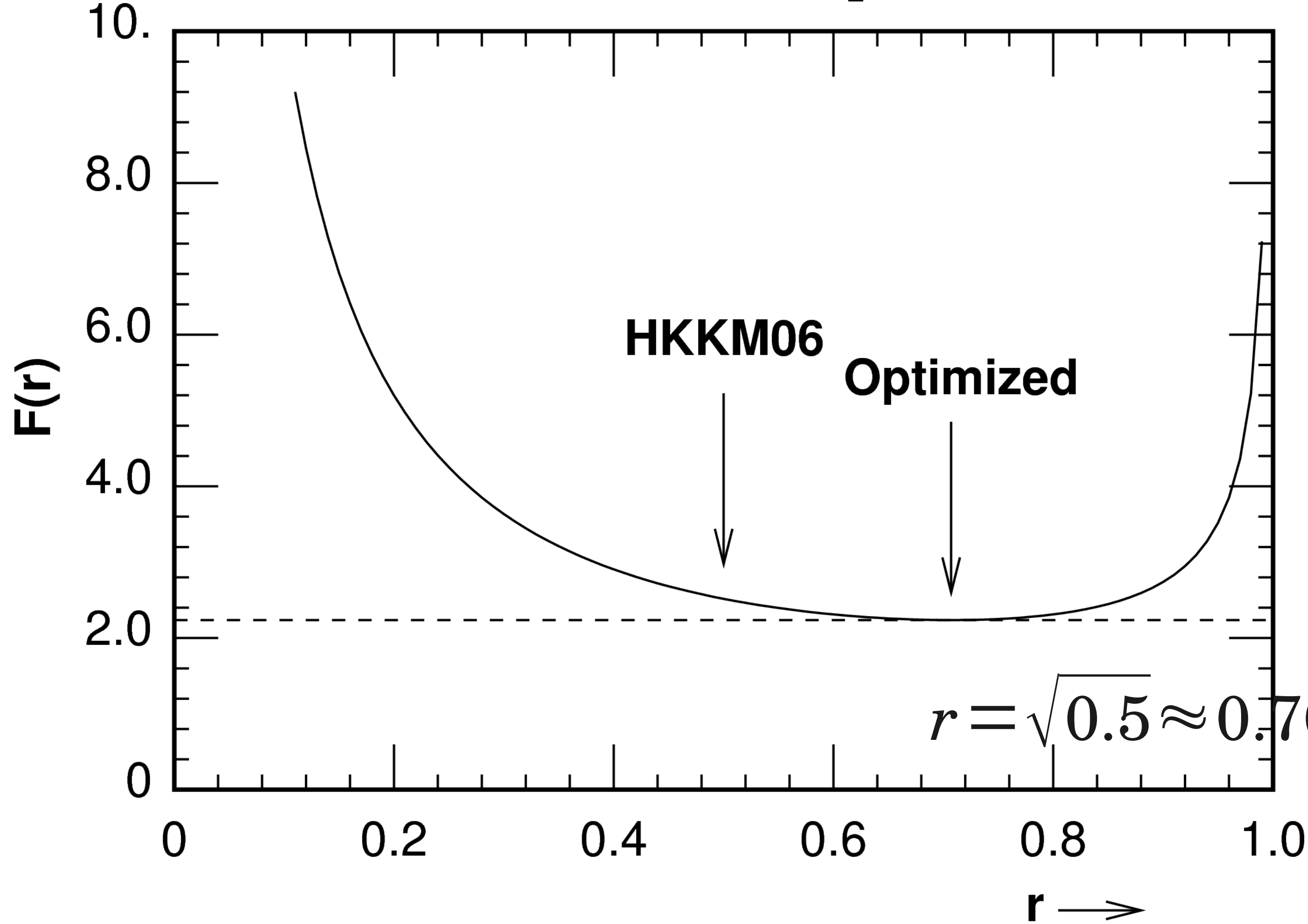






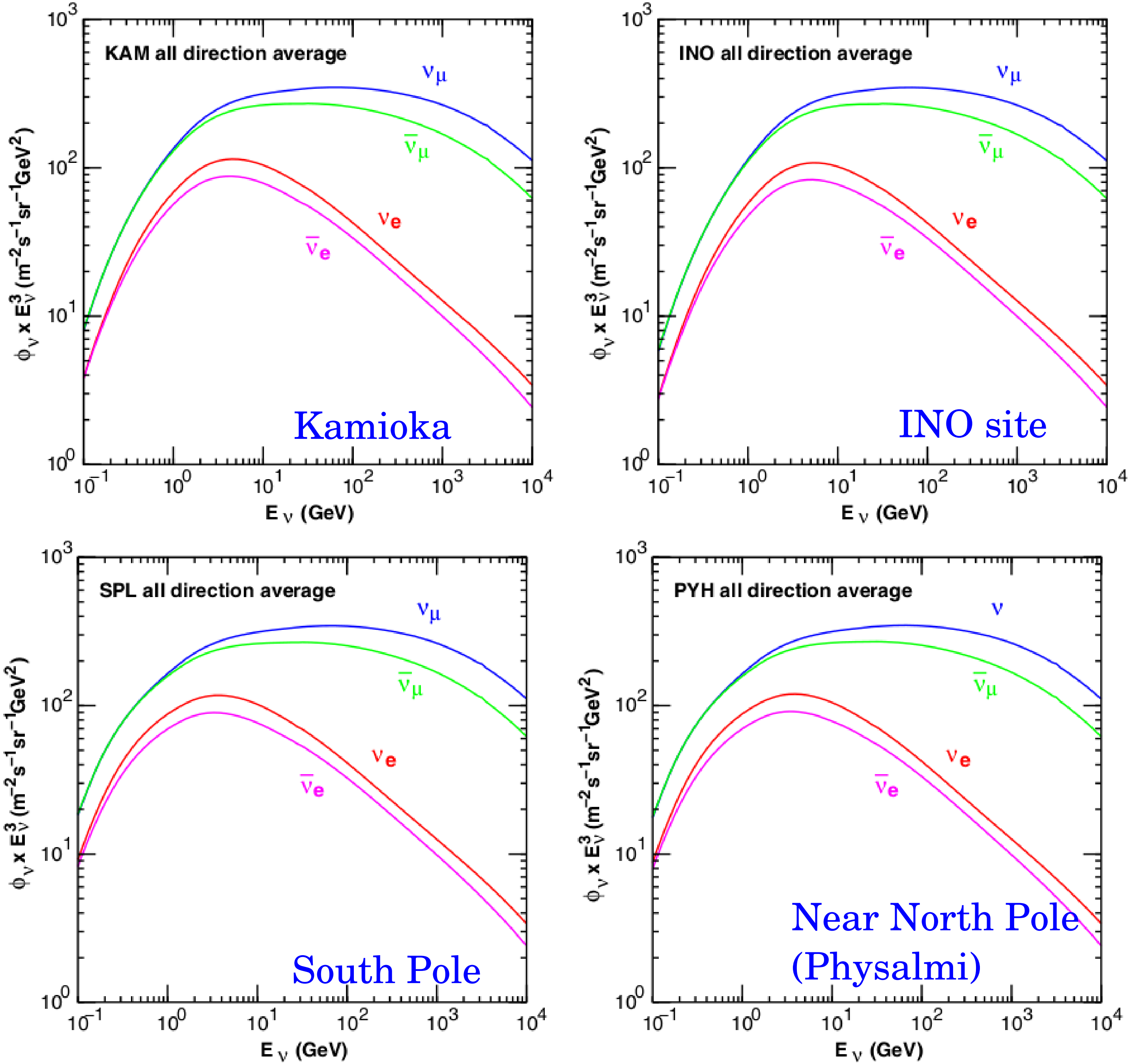
Optimization of $r = (\frac{\theta_2}{\theta_1})$ to minimize the statistical error.

$$\frac{\Delta\phi_0}{\phi_0} = F(r) \cdot \frac{\Delta\varphi_1}{\varphi_1}$$

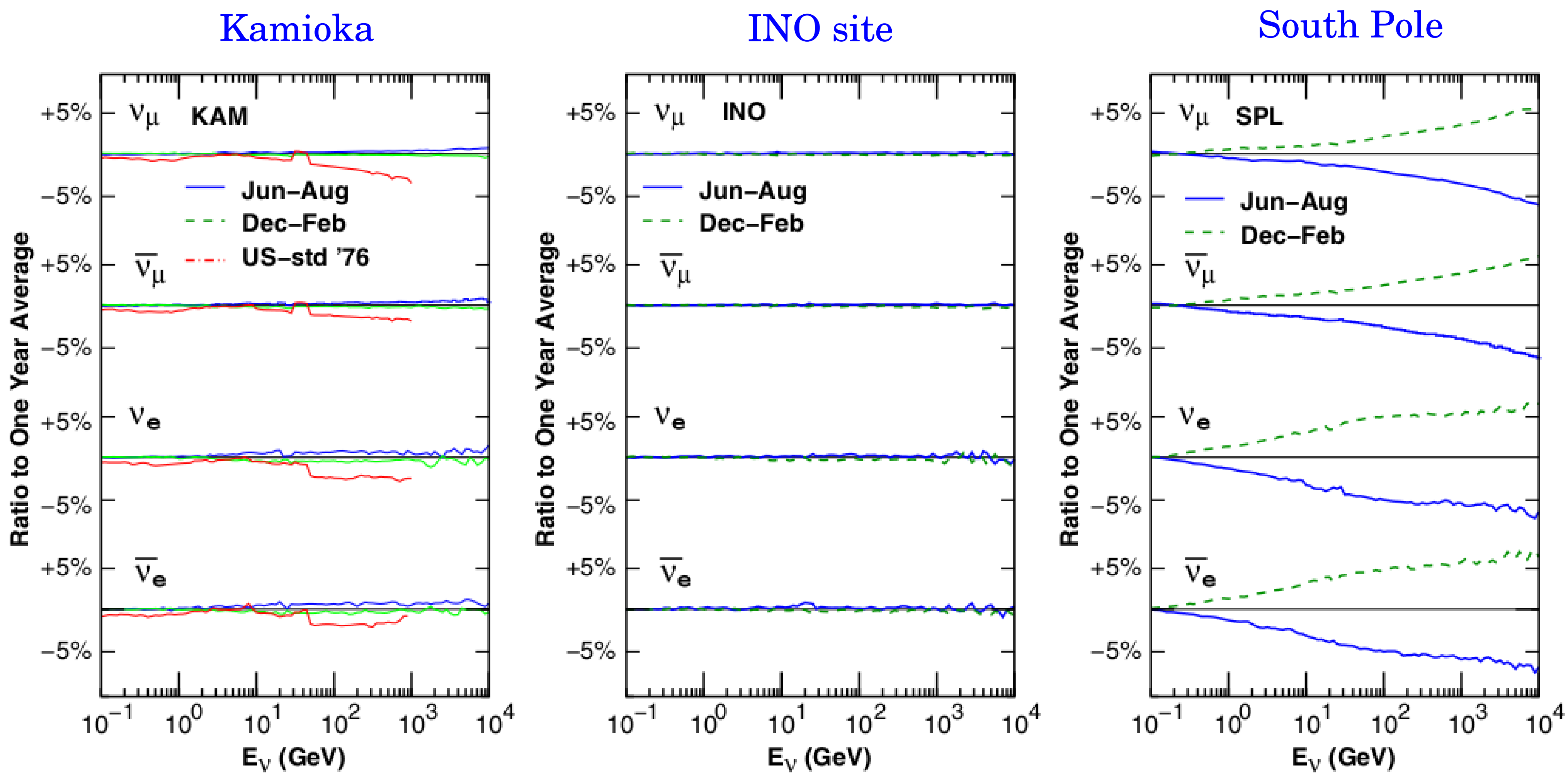


$r = \sqrt{0.5} \approx 0.7071$

Calculated Atmospheric Neutrino Flux averaged over all directions

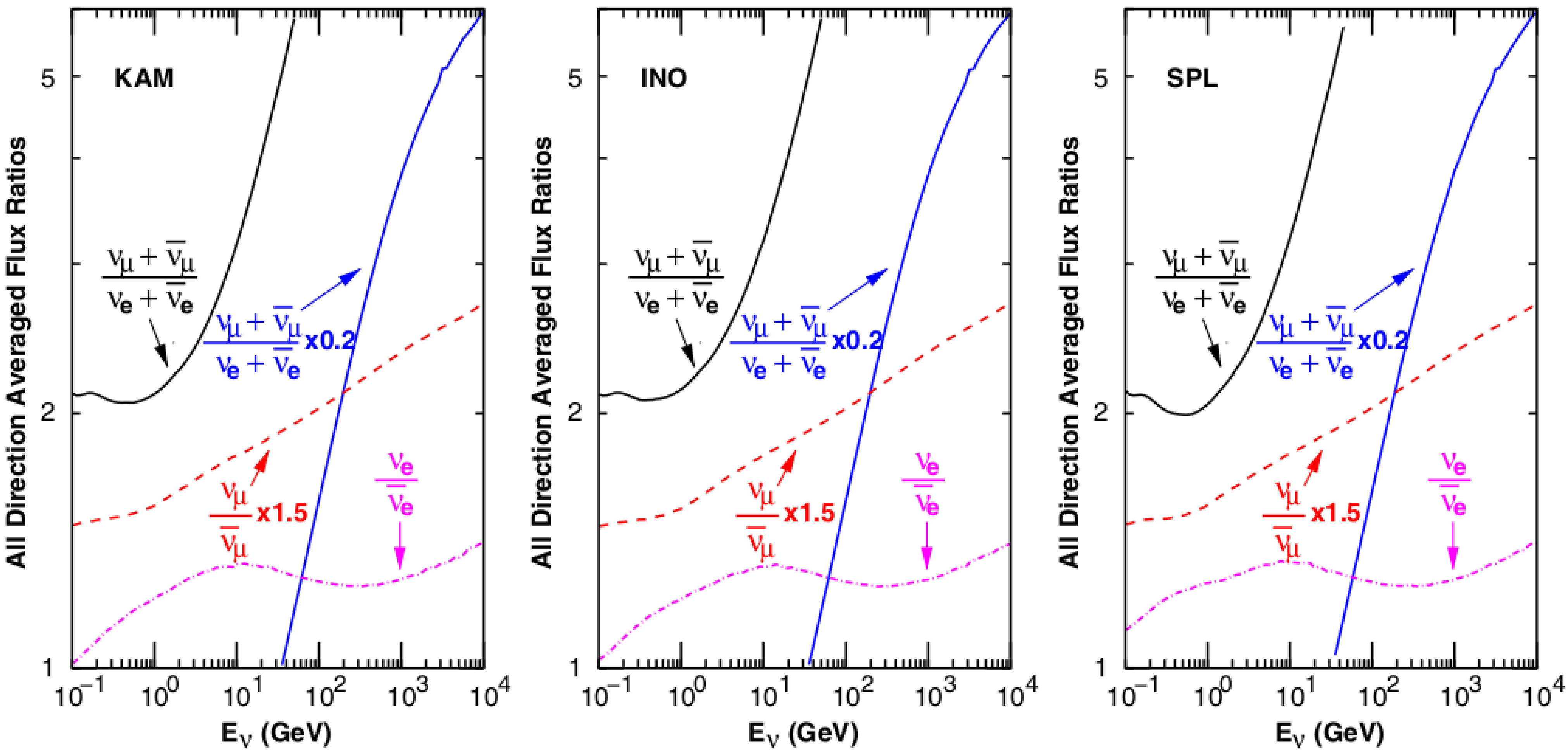


Seasonal Variation of Atmospheric Neutrino flux

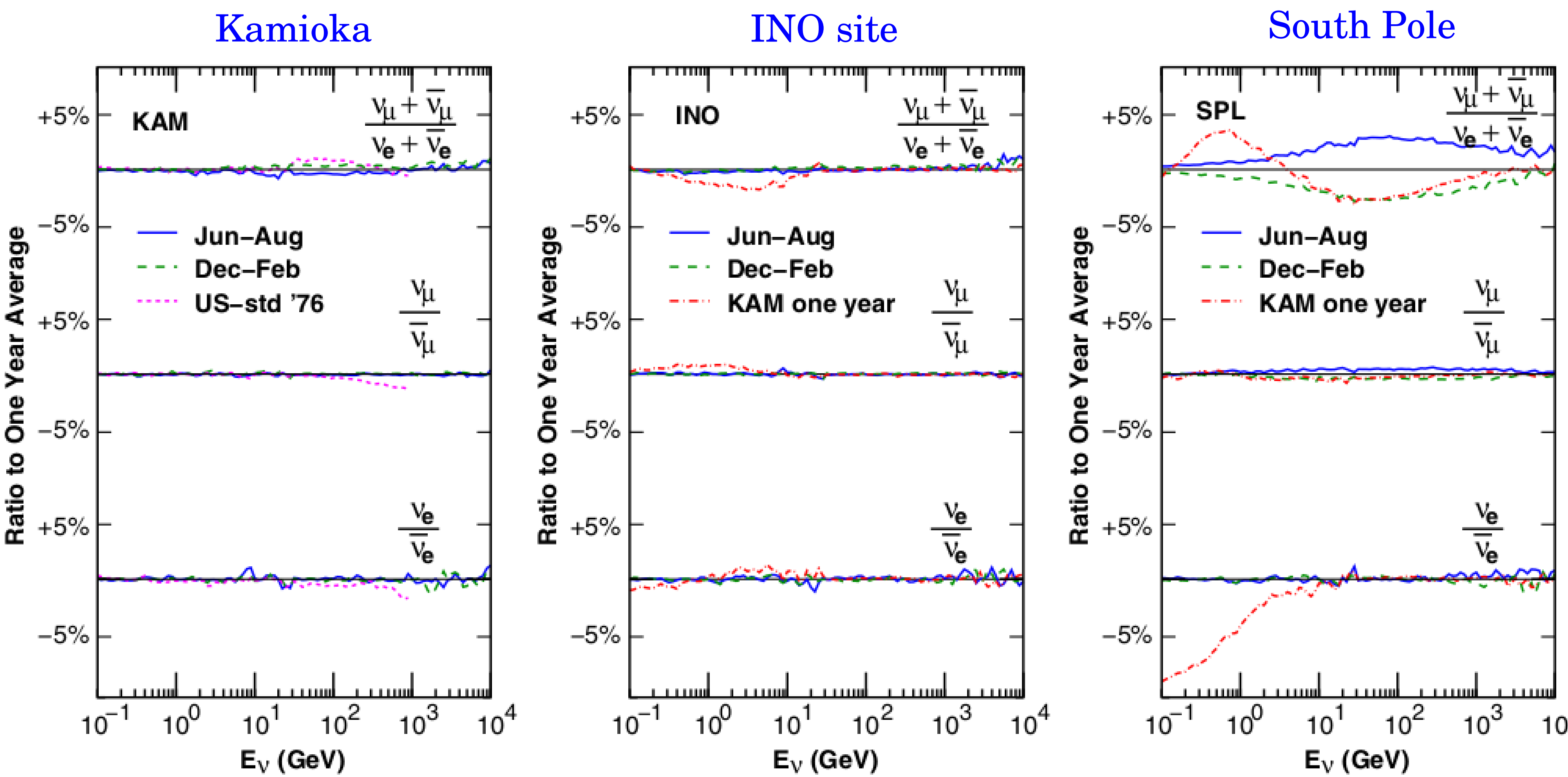


Flavor Ratios of Atmospheric Neutrino Flux

Kamioka | INO site | South Pole

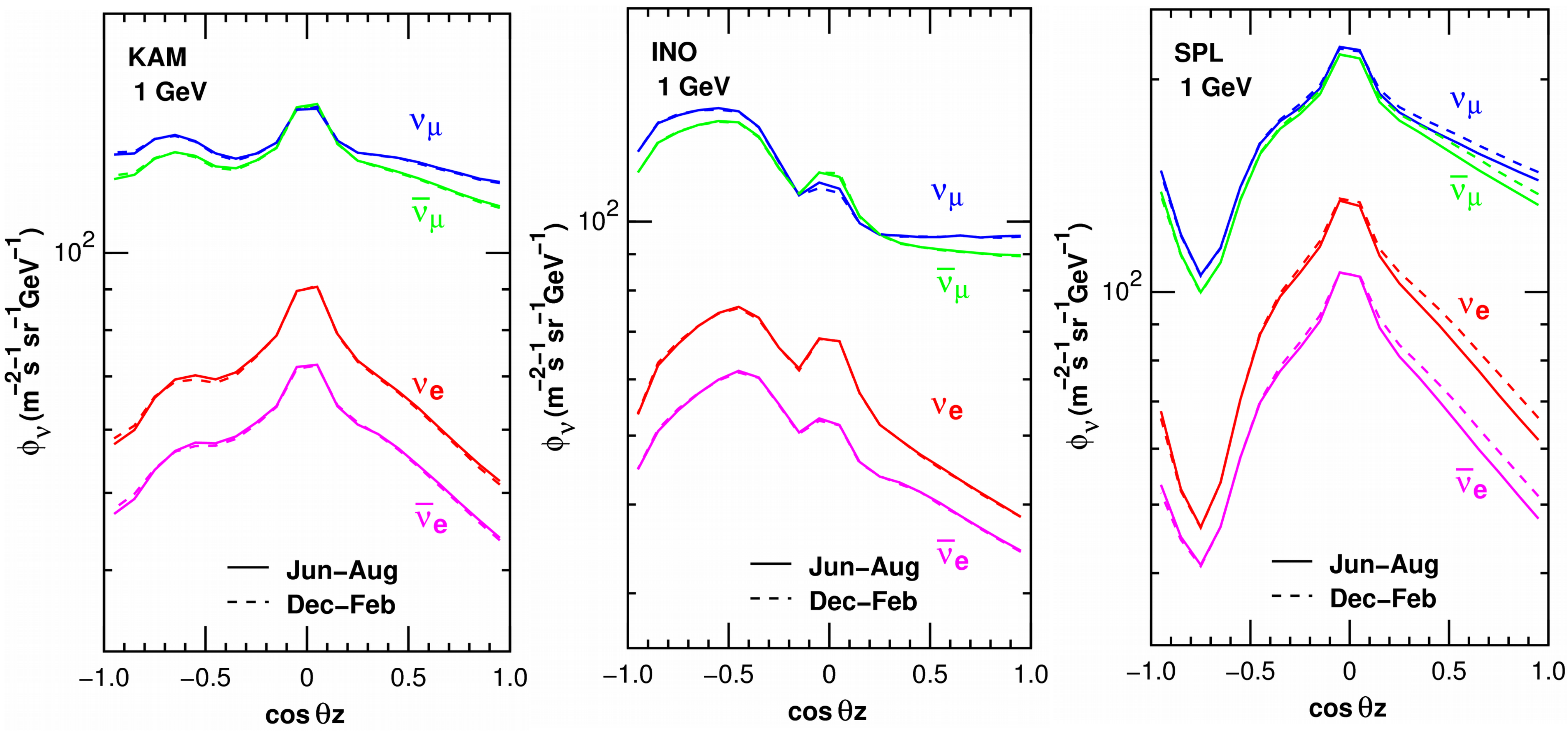


Seasonal and Site Variation of Atmospheric Neutrino Flavor Ratios

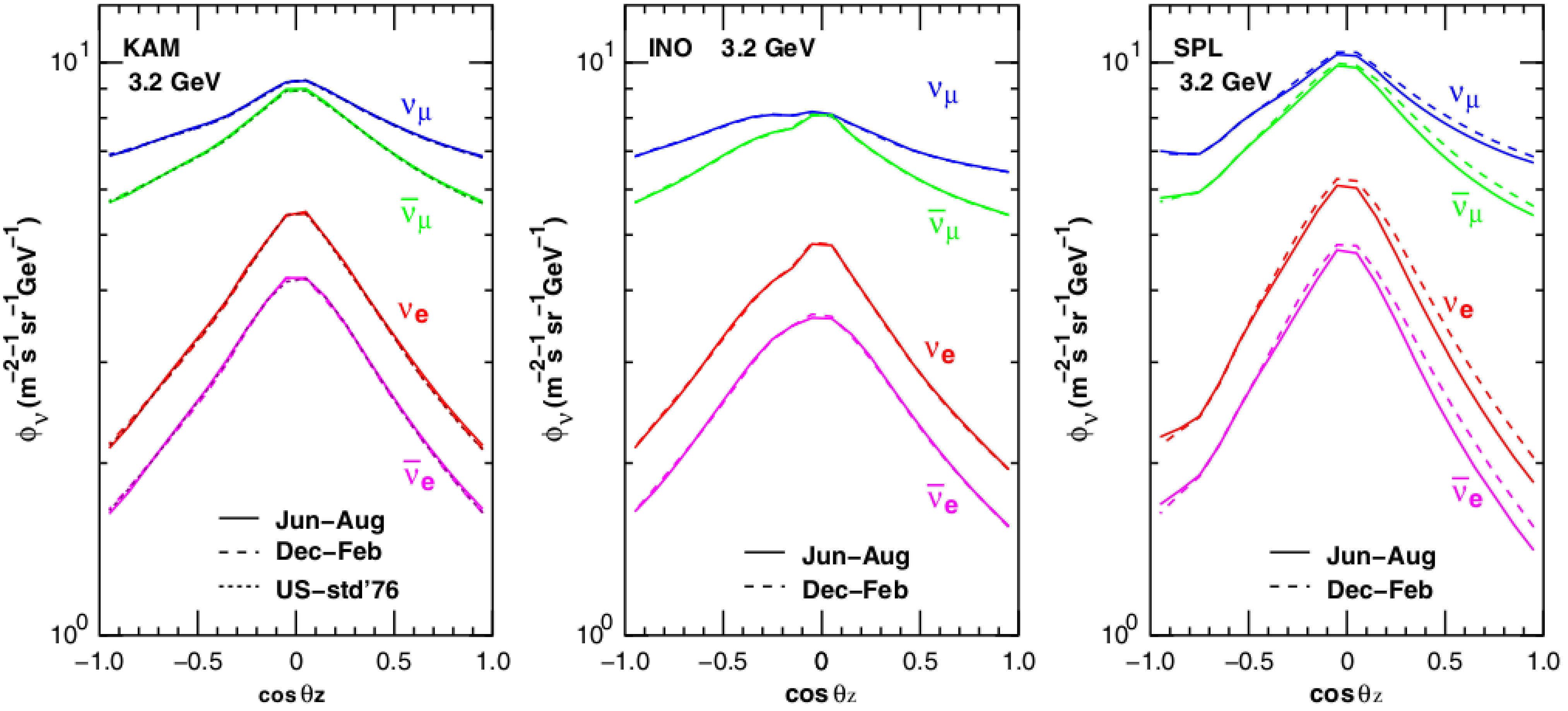


The variation of $\frac{\nu_\mu+\bar{\nu}_\mu}{\nu_e+\bar{\nu}_e}$ at South Pole and the difference from Kamioka are almost equal to the largest estimation of its uncertainty.

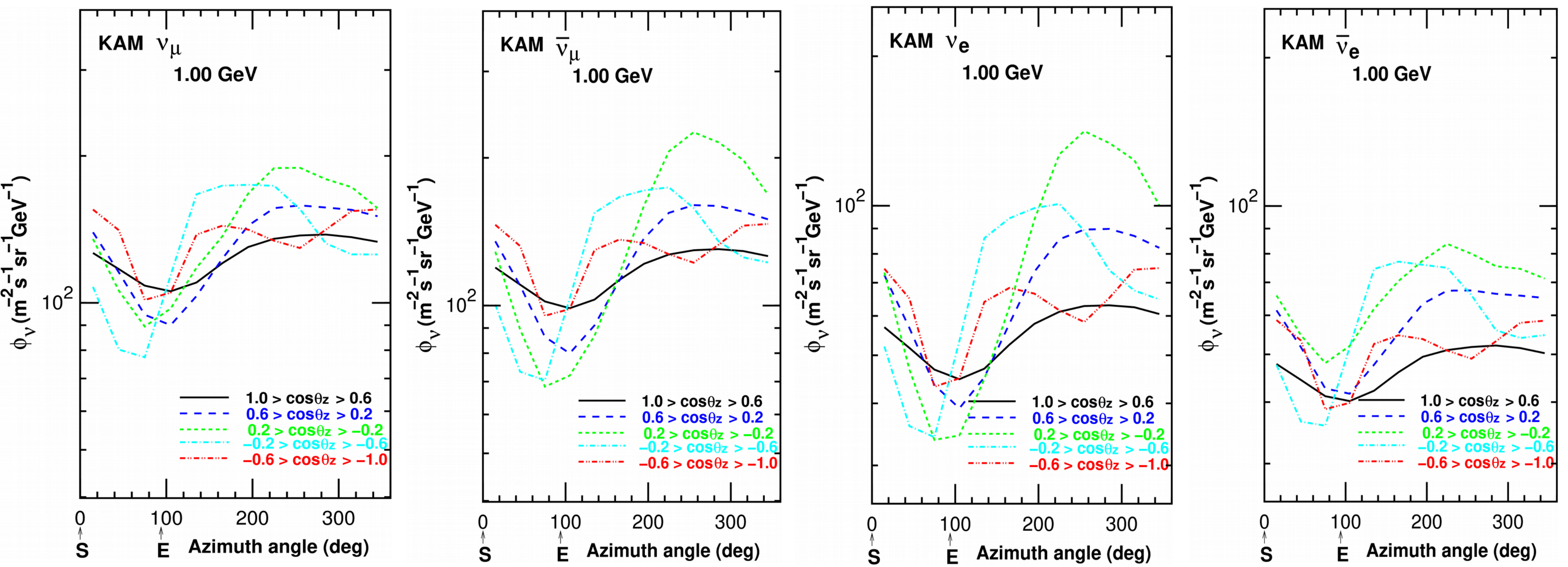
Zenith Angle Variation of Neutrino Fluxes at 1 GeV



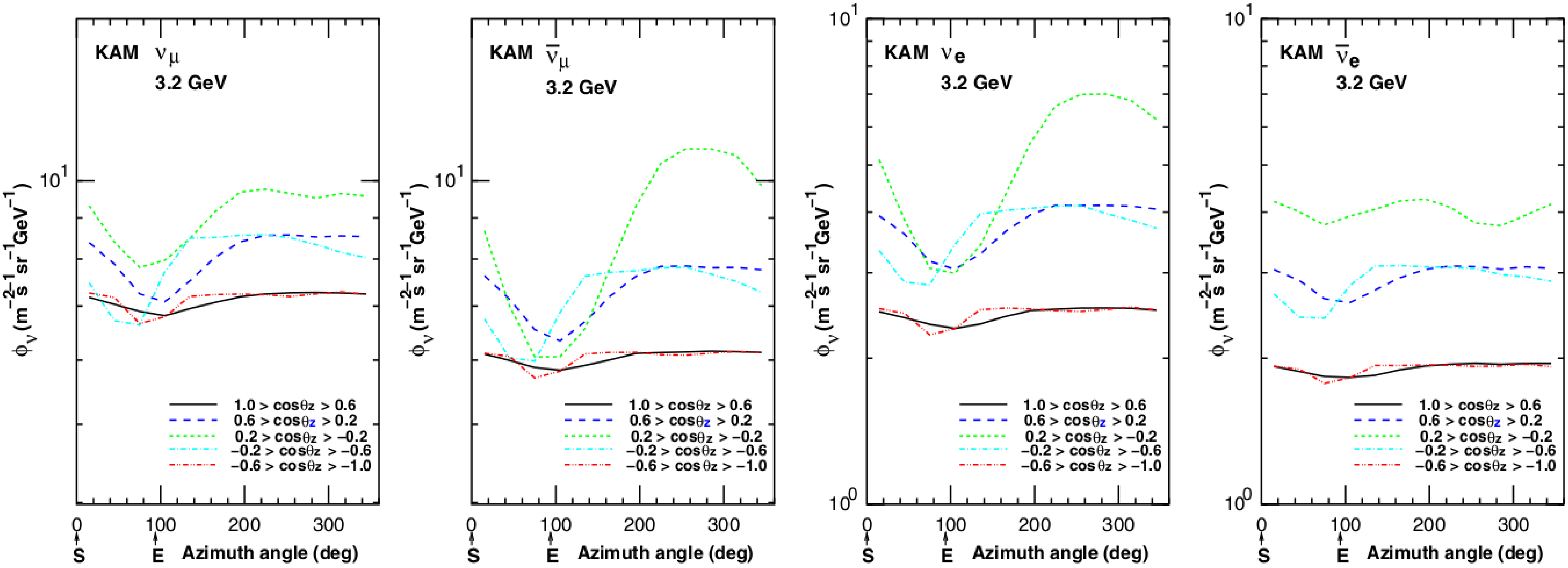
Zenith Angle Variation of Neutrino Fluxes at 3.2 GeV



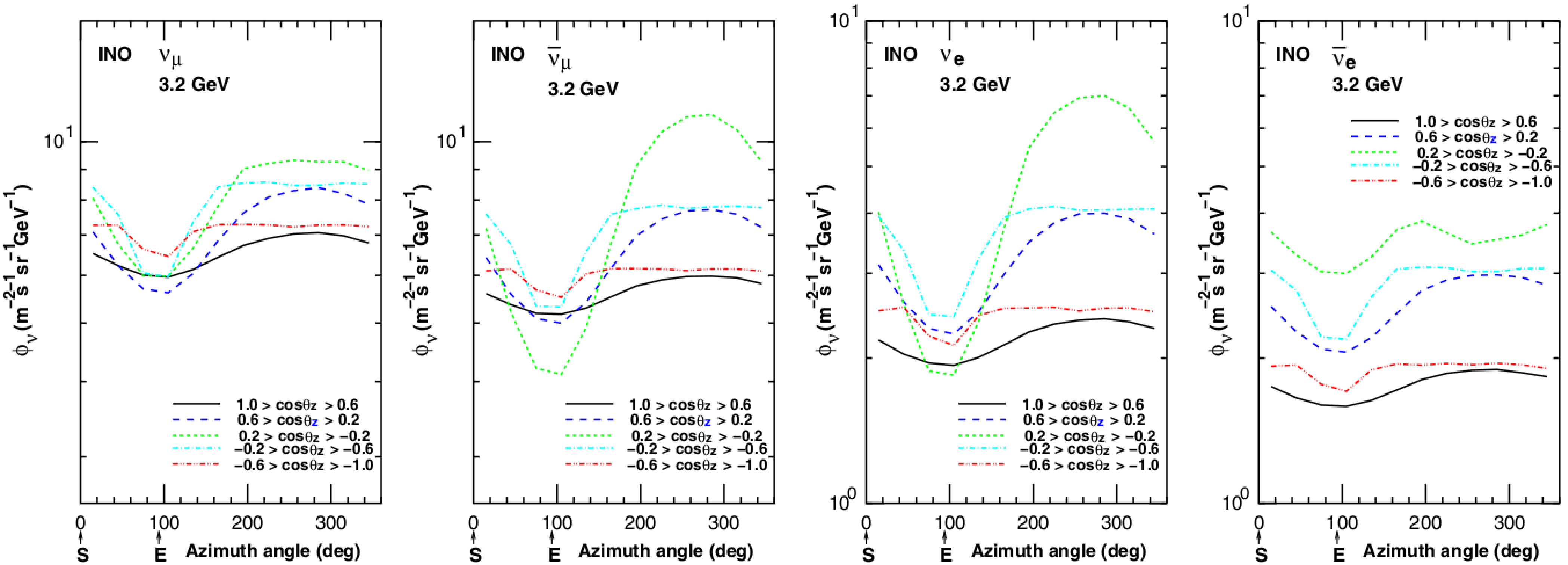
Azimuth Angle Variation of Neutrino Fluxes at 1 GeV at SK site



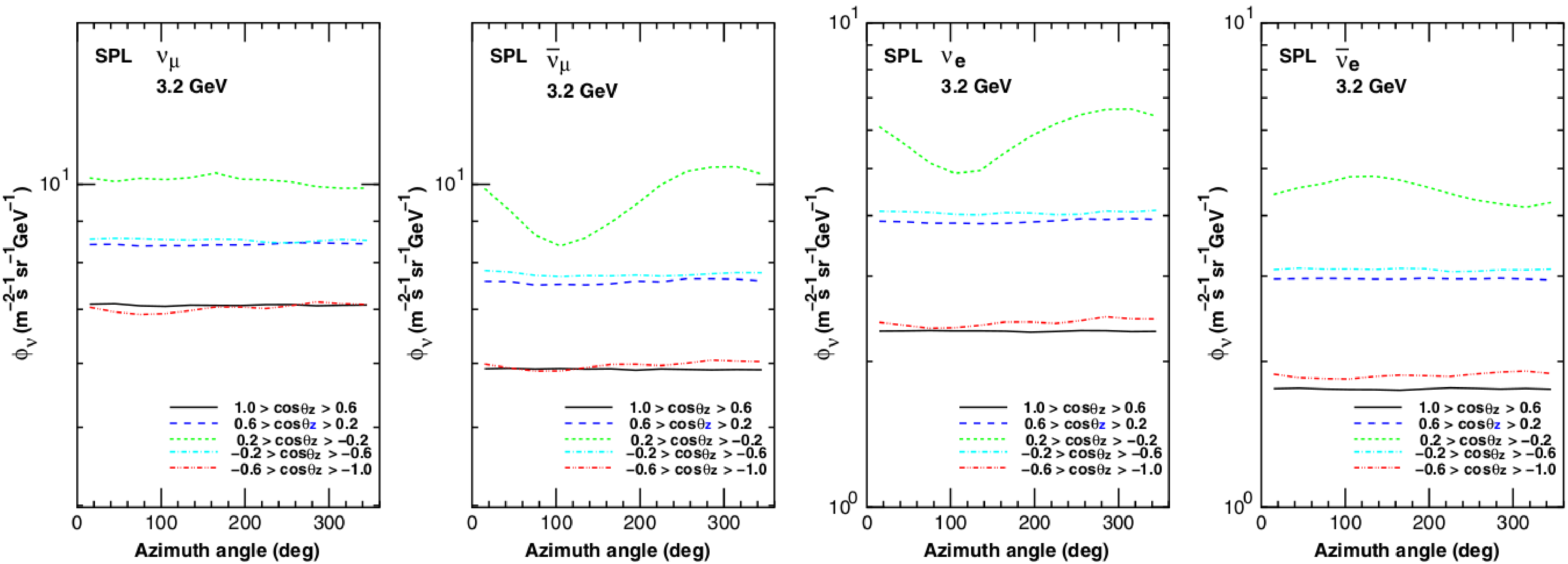
Azimuth Angle Variation of Neutrino Fluxes at 3.2 GeV at SK site



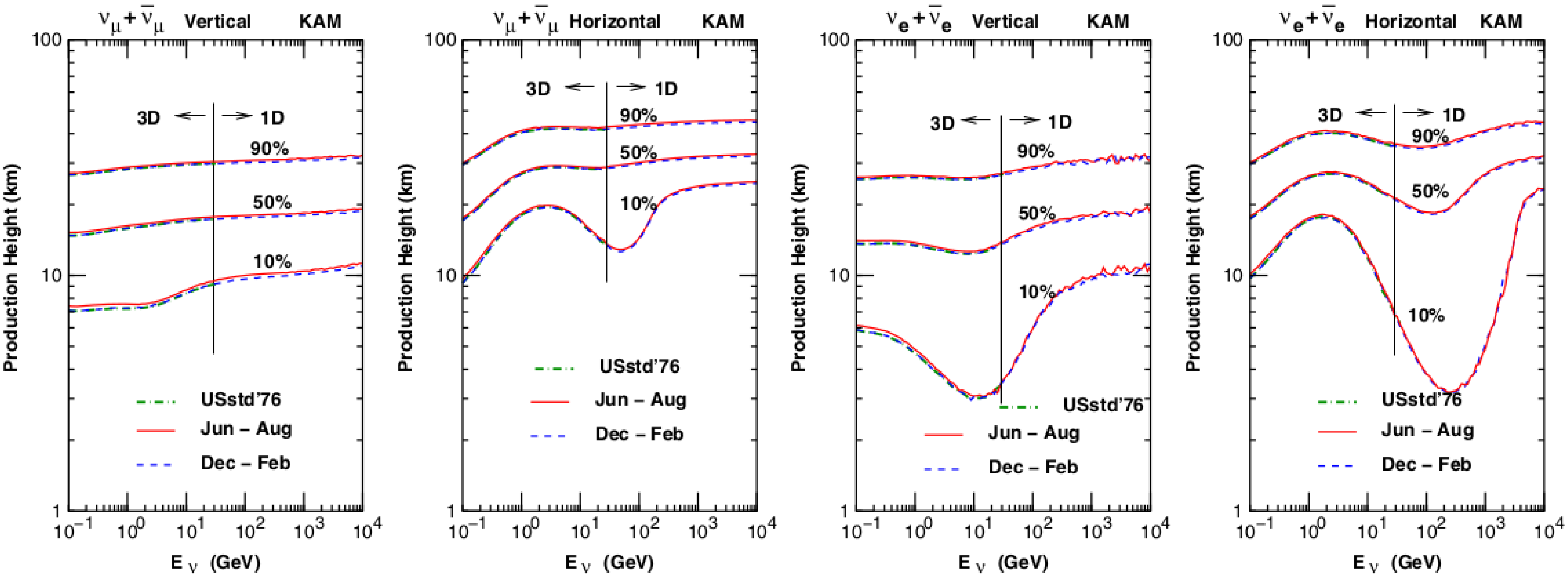
Azimuth Angle Variation of Neutrino Fluxes at 3.2 GeV at INO site



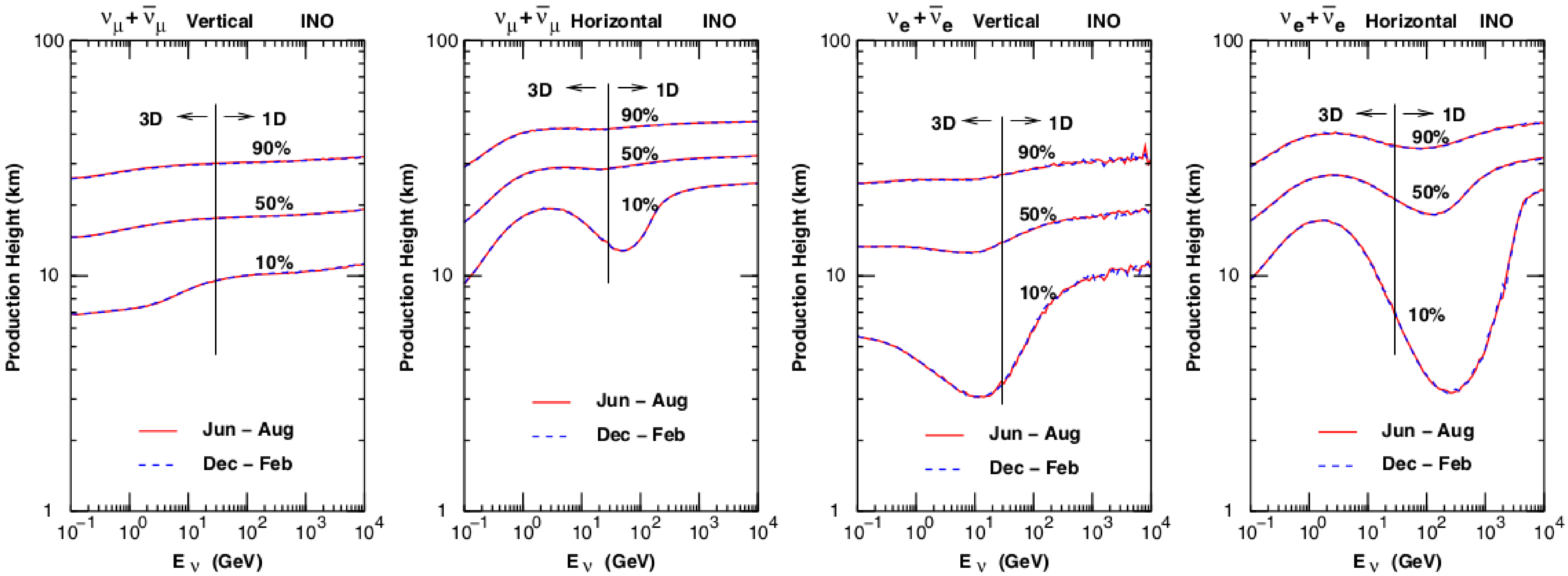
Azimuth Angle Variation of Neutrino Fluxes at 3.2 GeV at Suth Pole



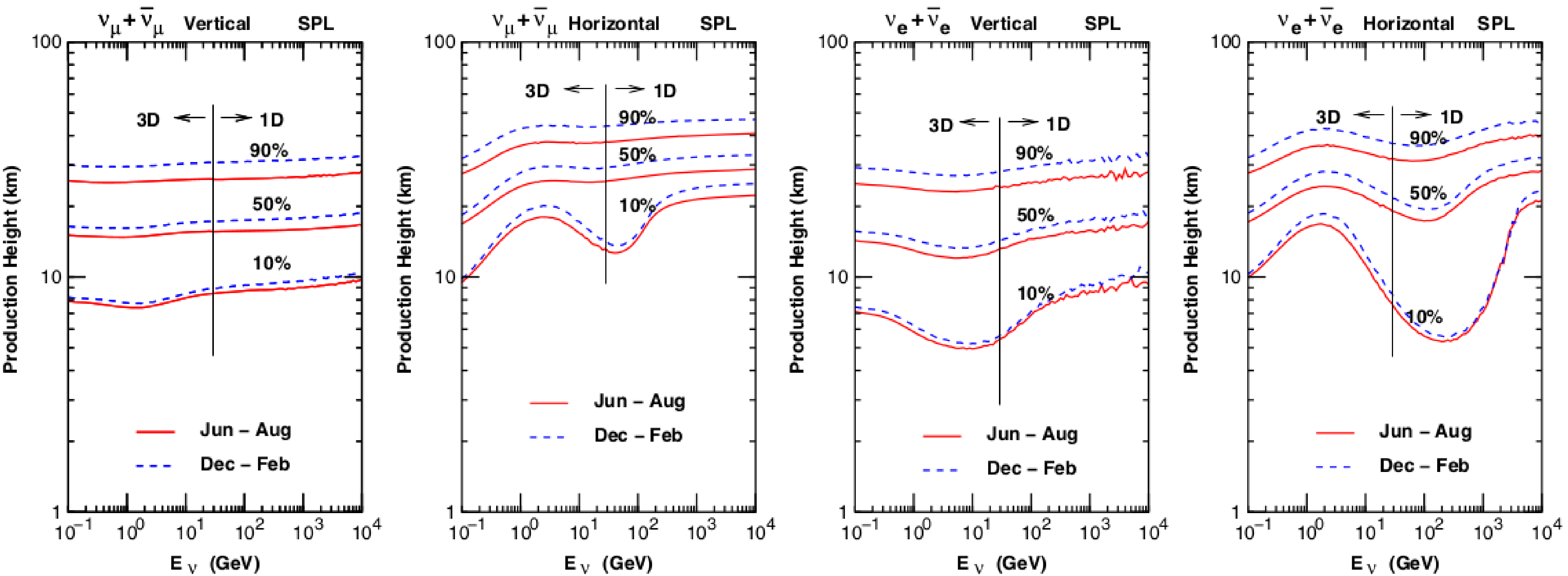
Cumulative Neutrino Production Height at SK site (Summed over all azimuth angles)



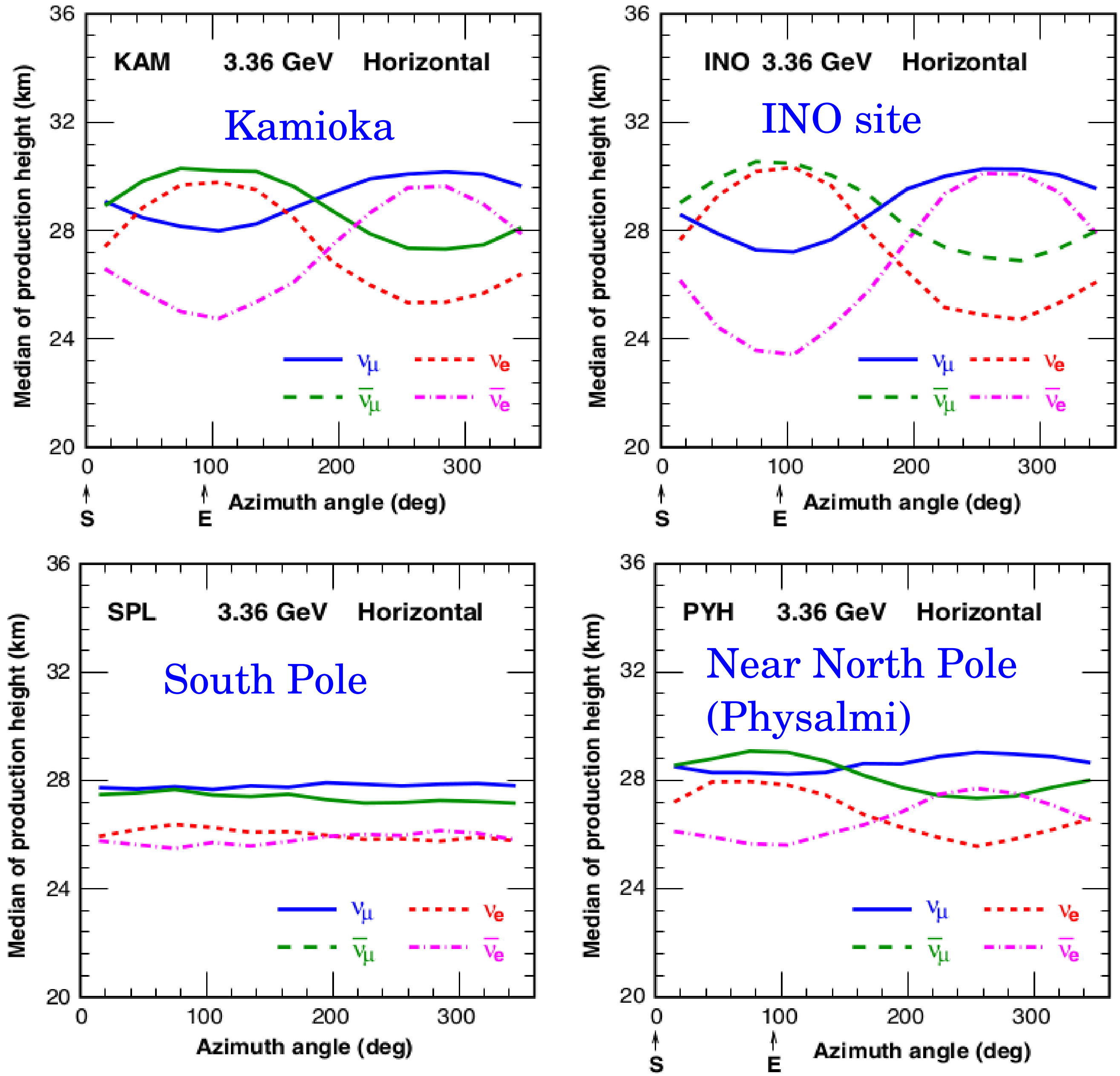
Cumulative Neutrino Production Height at INO site (Summed over all azimuth angles)



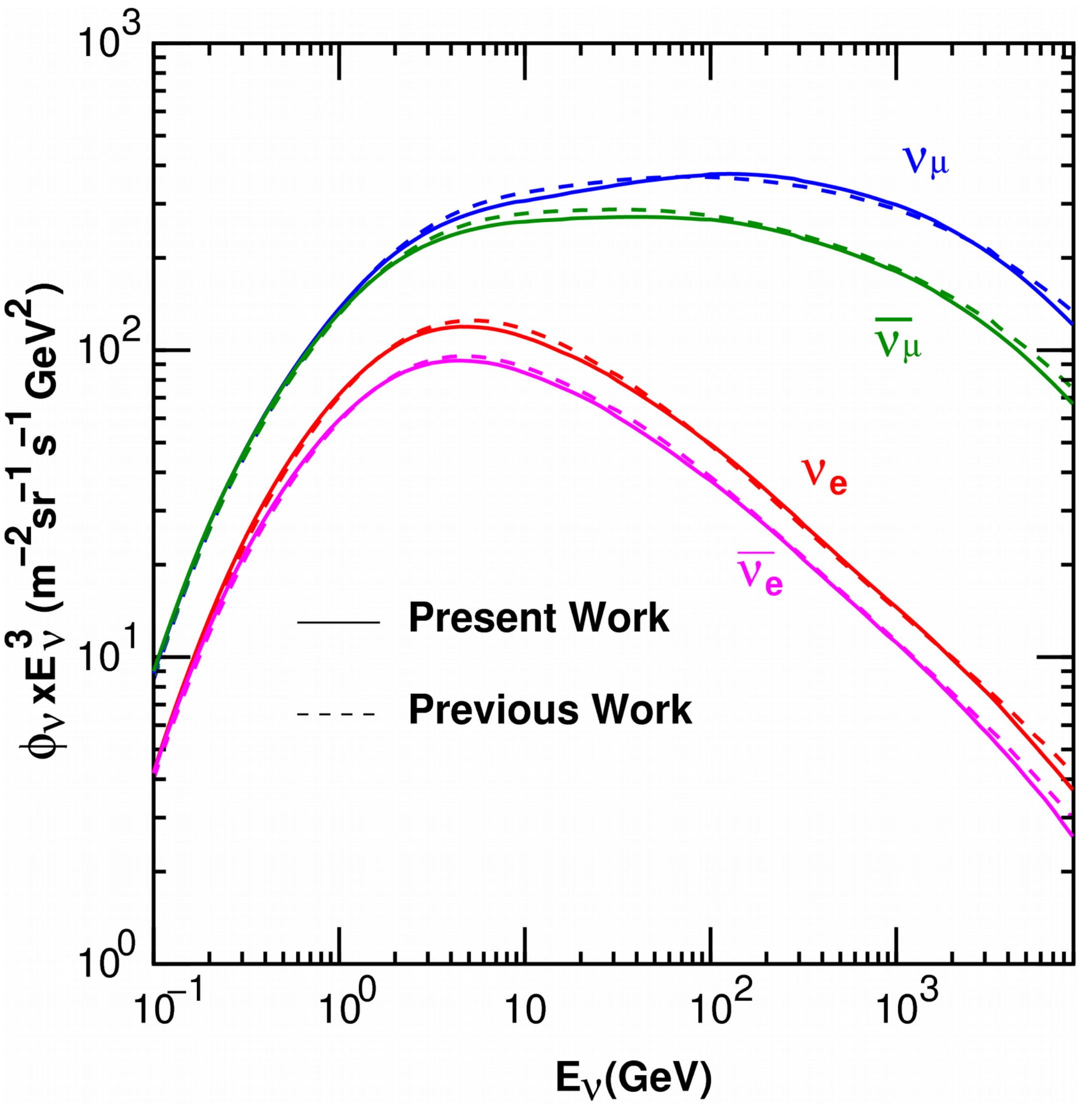
Cumulative Neutrino Production Height at South Pole (Summed over all azimuth angles)

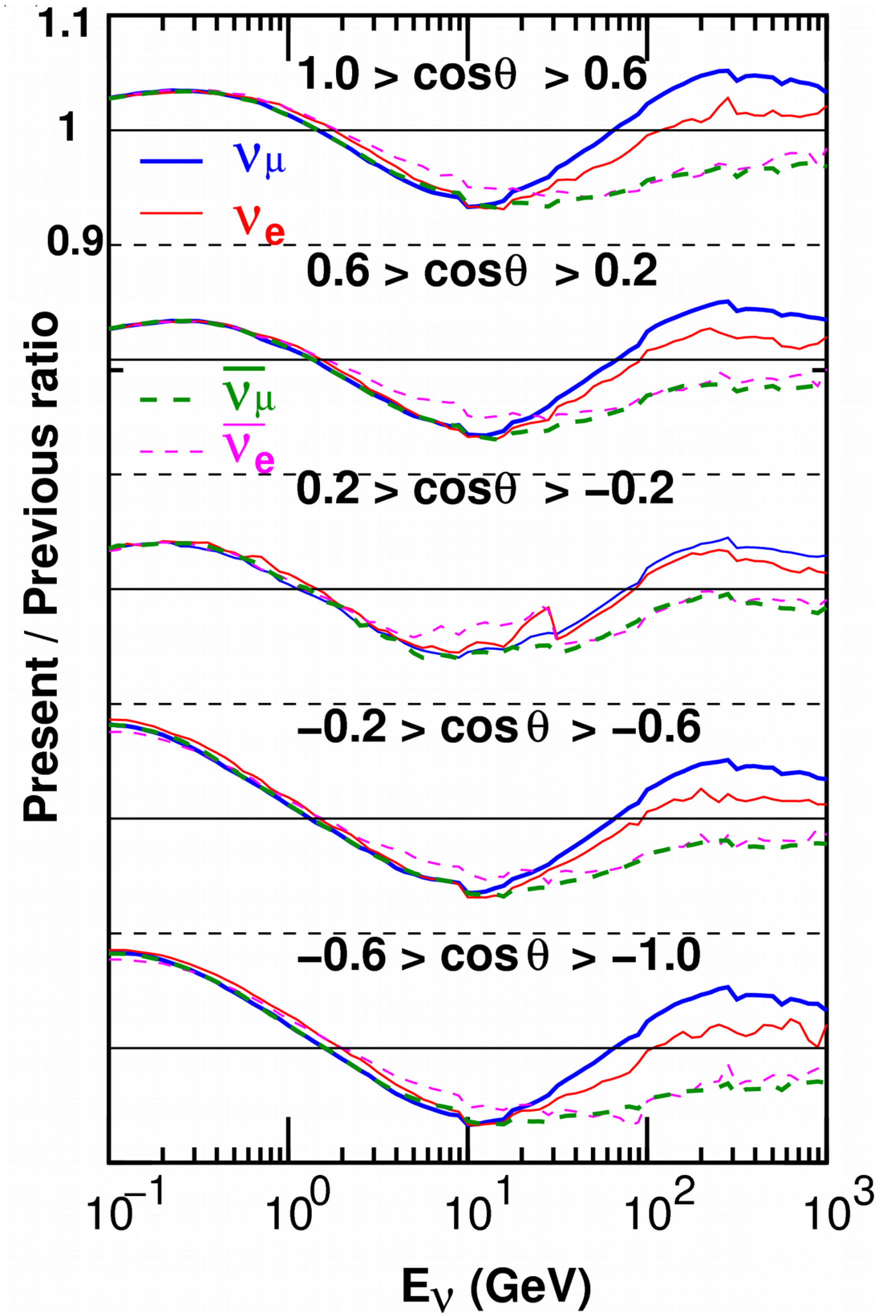


Azimuth Angle Variaiton of Neutrino Production Height

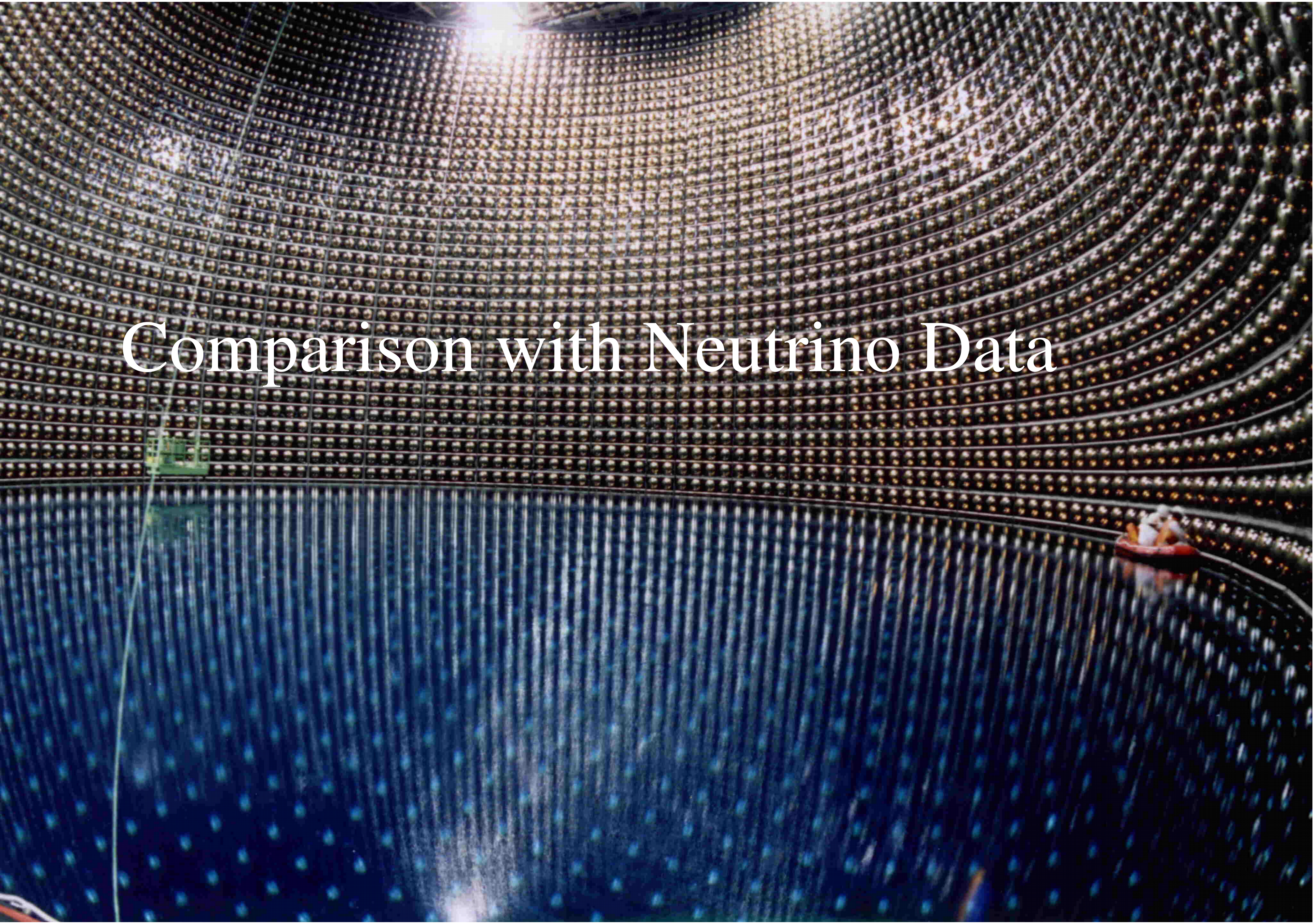


Based On AMS02 Obervation (Preliminary)





Comparison with Neutrino Data

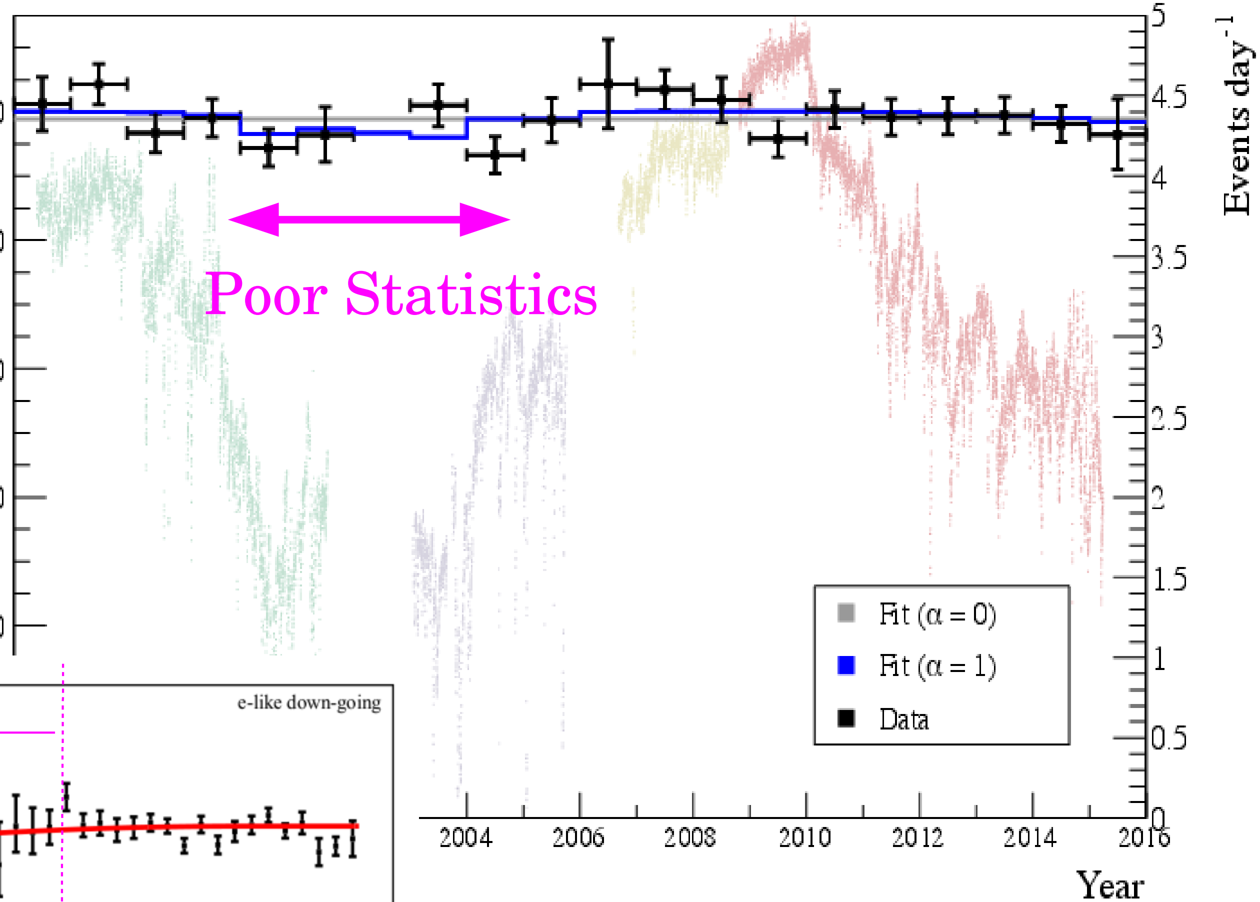


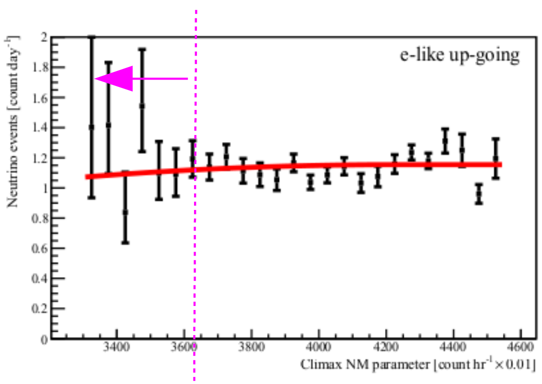
Atmospheric neutrino observed by SK

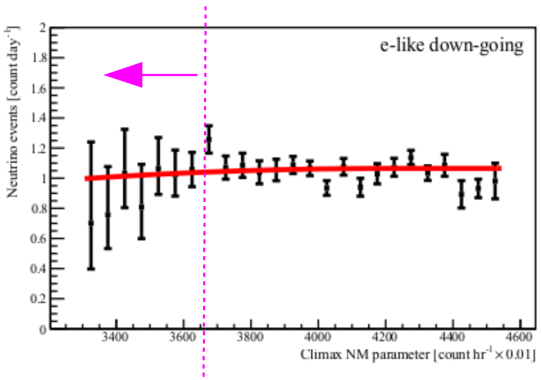
(Advertisement of the talk of Okumura-san)

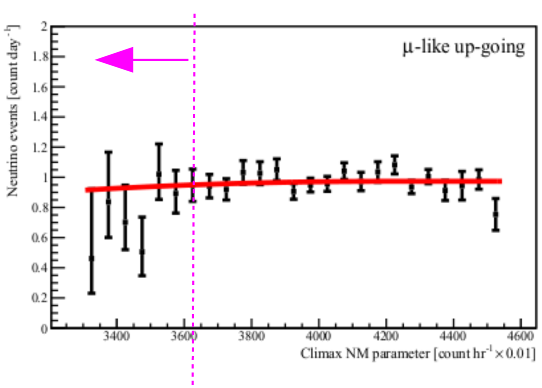
Solar Modulation of Atmospheric Neutrinos

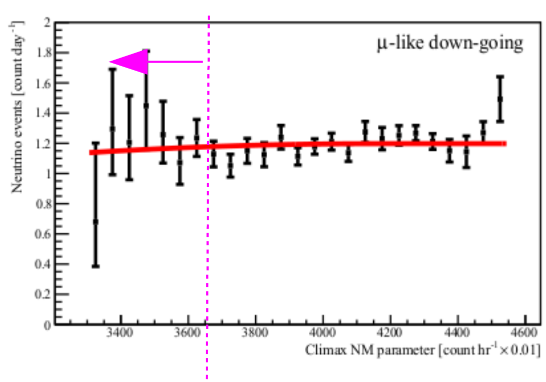
From PHD thesis of
E. Richard







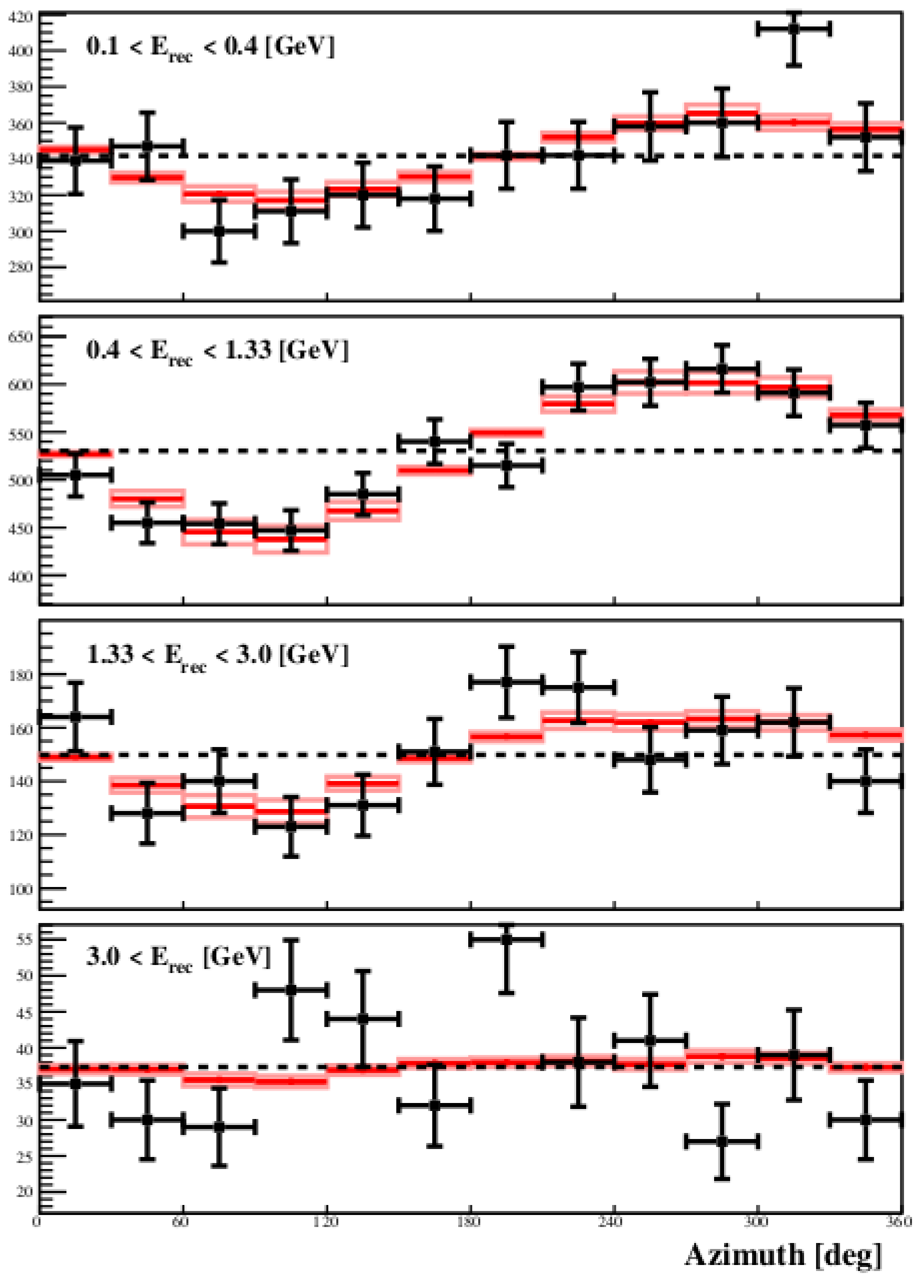




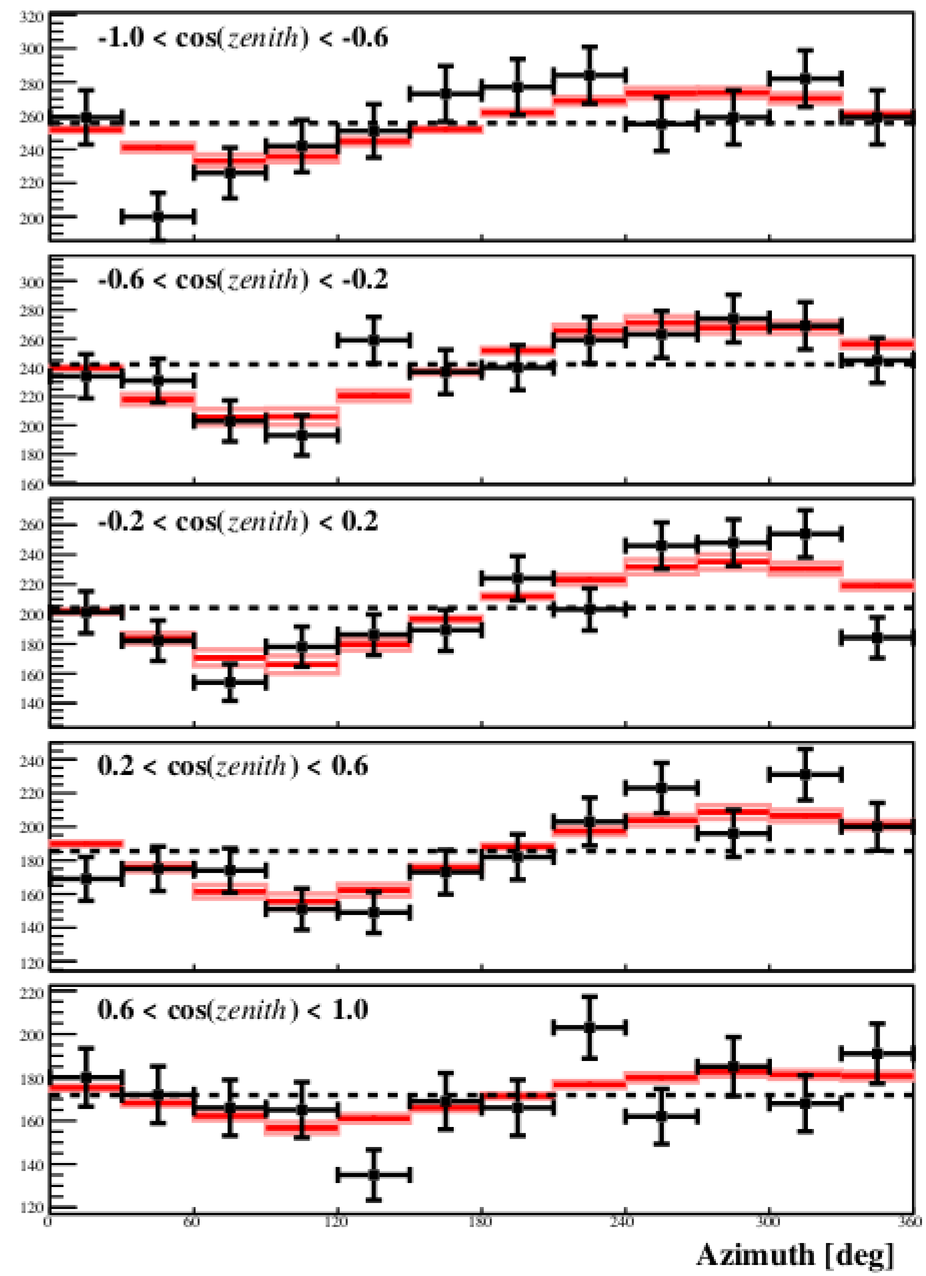
Best fit corresponds to 62 % of the predicted variations

Picked up mainly the forbush decrease ?

Observed Azimuthal Variation of ν_e flux (from PHD thesis of E.Richard)

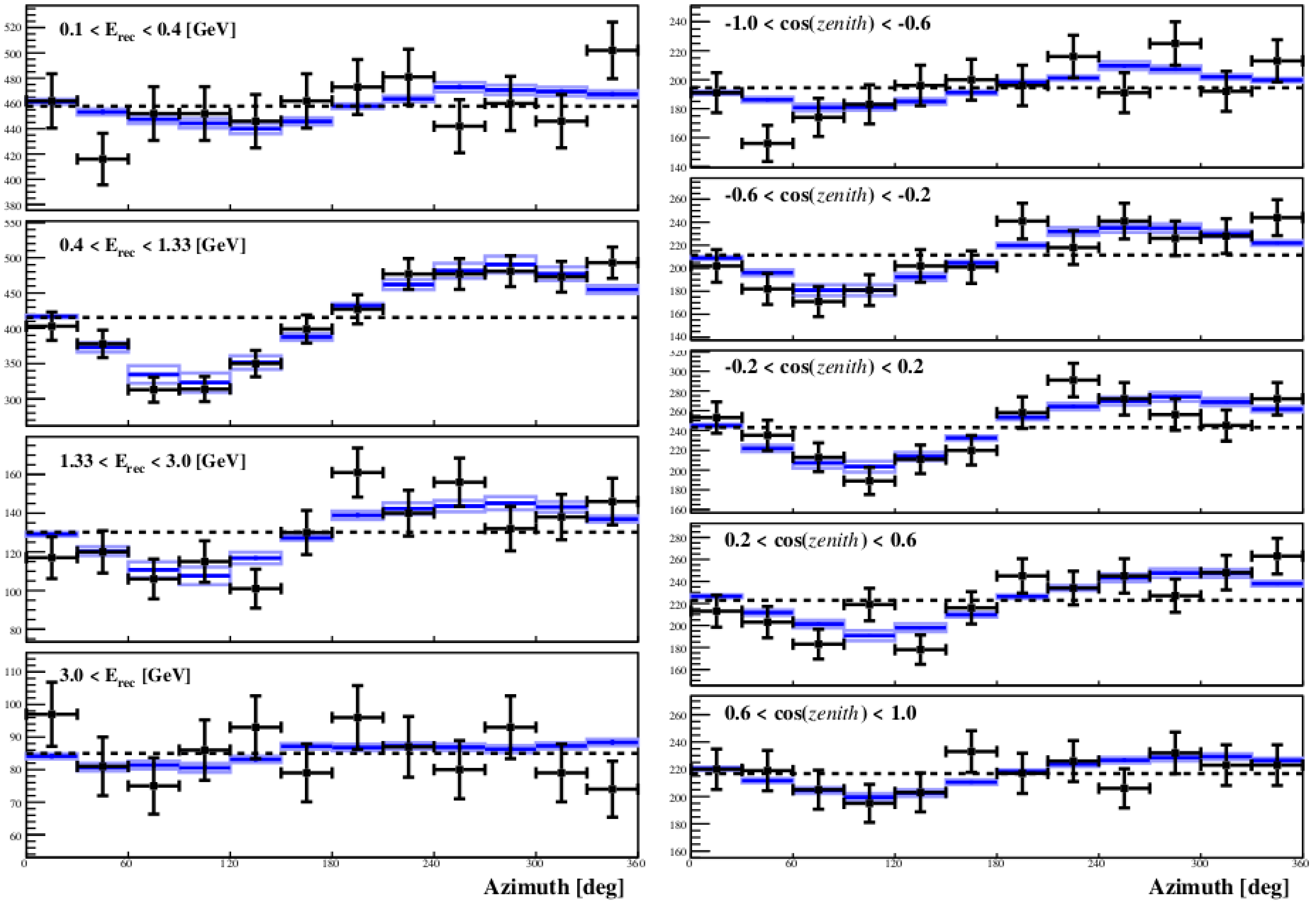


Energy Binned All Azimuth angles



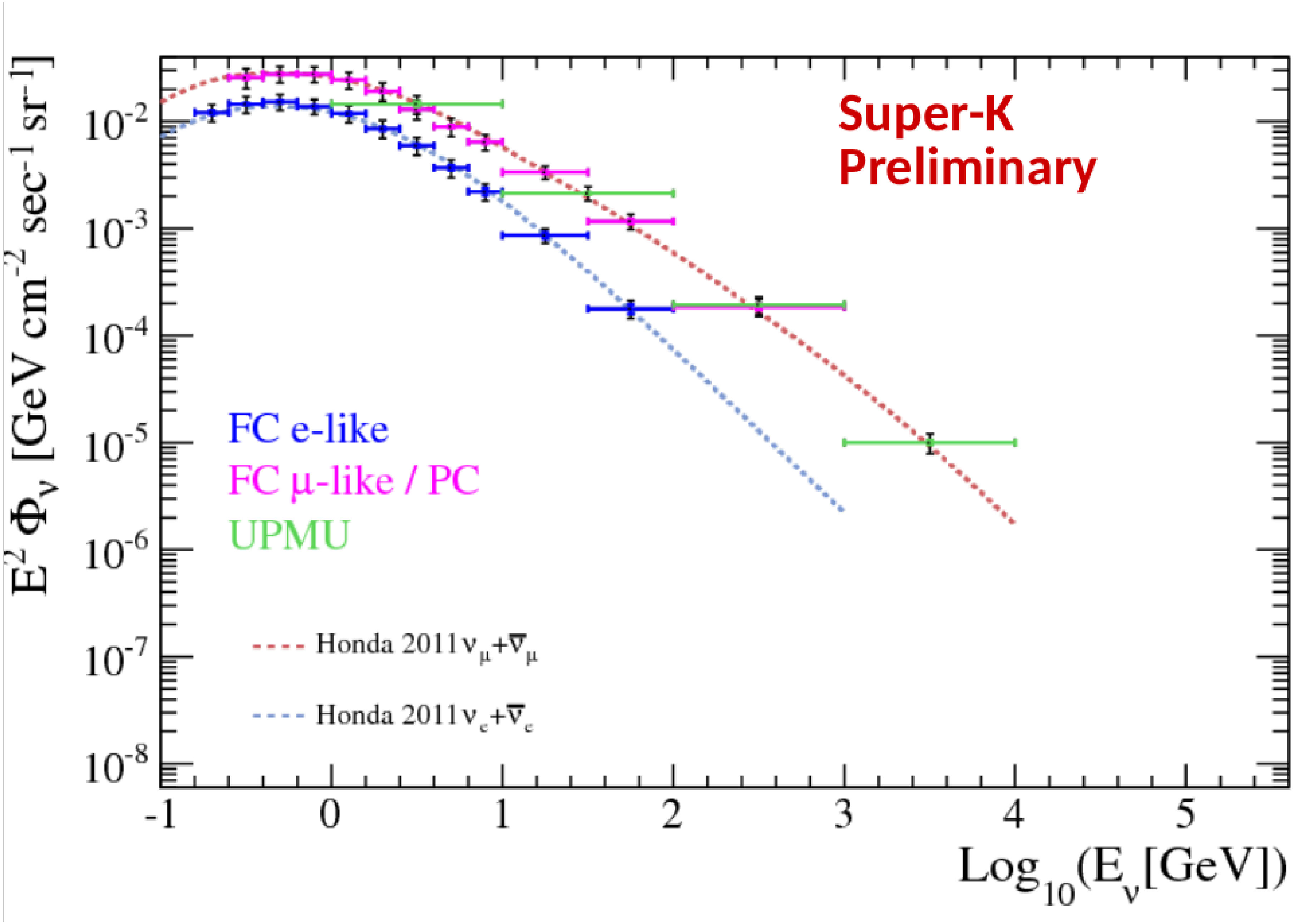
Zenith Angle Binned All Energies

Observed Azimuthal Variation of ν_μ flux (from PHD thesis of E.Richard)



Energy Binned All Azimuth angles

Zenith Angle Binned All Energies



From K.Okumura in ICRC2015

Summary

•We overviewed the calculation of atmospheric neutrino flux in HKKM.

•With NRLMSISE-00 atmosphere model, we find a large seasonal variation of neutrino flux at polar region. This also cause a variation in $\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e}$ ratio.

•We presented preliminary study based on AMS02 and BESS-polar. However, with the muon calibration, resulted atmospheric neutrino flux is very similar to the one with our (old) primary flux model.

•SK started to observe the predicted features of atmospheric neutrino flux.

•Advertisement: We are planning to record all the atmospheric neutrino on the earth. Then, we will be able to provide the atmospheric neutrino flux at any site on the Earth in a shorter period without re-calculation.

Back up

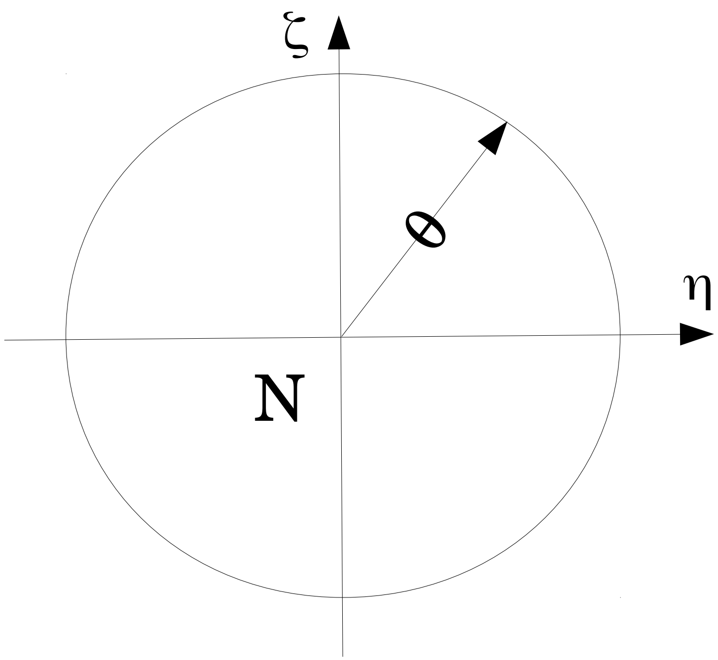
Assume the atmospheric neutrino flux is expanded as

$$\phi(\zeta,\eta)=\phi(0,0)+\frac{\partial\phi}{\partial\zeta}\zeta+\frac{\partial\phi}{\partial\eta}\eta+\frac{1}{2}\frac{\partial^2\phi}{\partial^2\zeta}\zeta^2+\frac{\partial^2\phi}{\partial\eta\partial\zeta}\zeta\eta+\frac{1}{2}\frac{\partial^2\phi}{\partial^2\eta}\eta^2+\ldots.$$

Average in a virtual detector with radius θ is given as

$$\phi_\theta\equiv\frac{1}{\pi\theta^2}\int_{\sqrt{\eta^2+\zeta^2}<\theta}\varphi(\eta,\zeta)\,d\eta\,d\zeta=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\eta^2}}^{+\sqrt{\theta^2-\eta^2}}\varphi(\eta,\zeta')\,d\zeta'\,d\eta$$

$$=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\zeta^2}}^{+\sqrt{\theta^2-\zeta^2}}\varphi(\eta',\zeta)\,d\eta'\,d\zeta$$



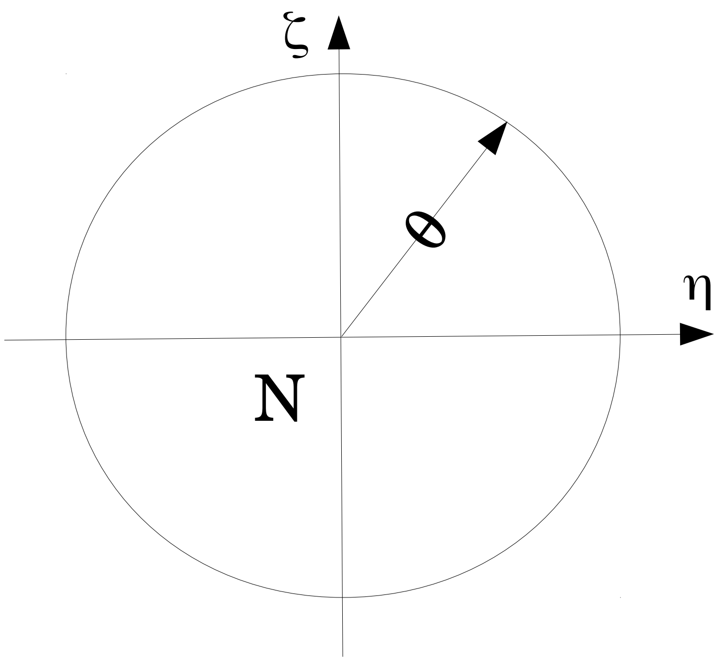
Assume the atmospheric neutrino flux is expanded as

$$\phi(\zeta,\eta)=\phi(0,0)+\frac{\partial\phi}{\partial\zeta}\zeta+\frac{\partial\phi}{\partial\eta}\eta+\frac{1}{2}\frac{\partial^2\phi}{\partial^2\zeta}\zeta^2+\frac{\partial^2\phi}{\partial\eta\partial\zeta}\zeta\eta+\frac{1}{2}\frac{\partial^2\phi}{\partial^2\eta}\eta^2+\ldots.$$

Average in a virtual detector with radius θ is given as

$$\phi_\theta\equiv\frac{1}{\pi\theta^2}\int_{\sqrt{\eta^2+\zeta^2}<\theta}\varphi(\eta,\zeta)d\eta d\zeta=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\eta^2}}^{+\sqrt{\theta^2-\eta^2}}\varphi(\eta,\zeta')d\zeta' d\eta$$

$$=\frac{1}{\pi\theta^2}\int_{-\theta}^{+\theta}\int_{-\sqrt{\theta^2-\zeta^2}}^{+\sqrt{\theta^2-\zeta^2}}\varphi(\eta',\zeta)d\eta' d\zeta$$



$$\int_{\sqrt{\eta^2+\zeta^2}<\theta}\eta d\eta d\zeta=\int_{\sqrt{\eta^2+\zeta^2}<\theta}\zeta d\eta d\zeta=0$$

$$\int_{\sqrt{\eta^2+\zeta^2}<\theta}\eta\zeta d\eta d\zeta=0$$

(continued)

$$\int_{\sqrt{\eta^2+\zeta^2}<\theta} \eta^2 \, d\eta \, d\zeta = \int_{\sqrt{\eta^2+\zeta^2}<\theta} \zeta^2 \, d\eta \, d\zeta = \int_{-\theta}^{+\theta} \int_{-\sqrt{\theta^2-\zeta^2}}^{+\sqrt{\theta^2-\zeta^2}} \eta'^2 \, d\eta' \, d\zeta$$

$$= \frac{2}{3} \int_{-\theta}^{+\theta} \sqrt{\theta^2-\zeta^2}^{\,3} \, d\zeta$$

$$= \frac{2}{3} \theta^4 \int_{-1}^{+1} \sqrt{1-t^2}^{\,3} \, dt$$

$$= \frac{1}{4} \pi \theta^4$$

Then we get

$$\phi_\theta \equiv \frac{1}{\pi \theta^2} \int_{\sqrt{\eta^2+\zeta^2}<\theta} \varphi(\eta, \zeta) \, d\eta \, d\zeta = \varphi(0,0) + \frac{1}{8} \left(\frac{\partial^2 \phi}{\partial^2 \zeta} + \frac{\partial^2 \phi}{\partial^2 \eta} \right) \theta^2 + \ldots.$$

Note, the factor before θ^2 would be a little different, due to the Jacobian for the integration on a sphere.

Gaisser Formula for illustration (by T.K.Gaisser at Takayama, 1998)

$$\Phi_\nu = \Phi_{primary} \otimes R_{cut} \otimes Y_\nu$$

$$\Phi_\mu = \Phi_{primary} \otimes R_{cut} \otimes Y_\mu$$

Where

$\Phi_{primary}$: Cosmic Ray Flux

$R_{cut} = R_{cut}(R_{cr}, latt., long., \theta, \phi)$: Geomagnetic field

$Y_\nu = Yield_\nu(h, \theta)$: Hadronic Interaction Model, Air Profile, and meson-muon decay

$Y_\mu = Yield_\mu(h, \theta)$: Hadronic Interaction Model, Air Profile, and meson decay

This formula illustrates 1D-calculation well

Cosmic rays in atmosphere

$$p_{CR} + [Air] \rightarrow \begin{pmatrix} n^{\pm} \cdot \pi^{\pm} \\ m \cdot \pi^{0} \end{pmatrix} + X(p, n, K, ...)$$

$$\pi^{0} \rightarrow 2\gamma$$

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$$\mu^{\pm} \rightarrow \nu_{e}(\bar{\nu}_{e}) + \bar{\nu}_{\mu}(\nu_{\mu}) + e^{\pm}$$

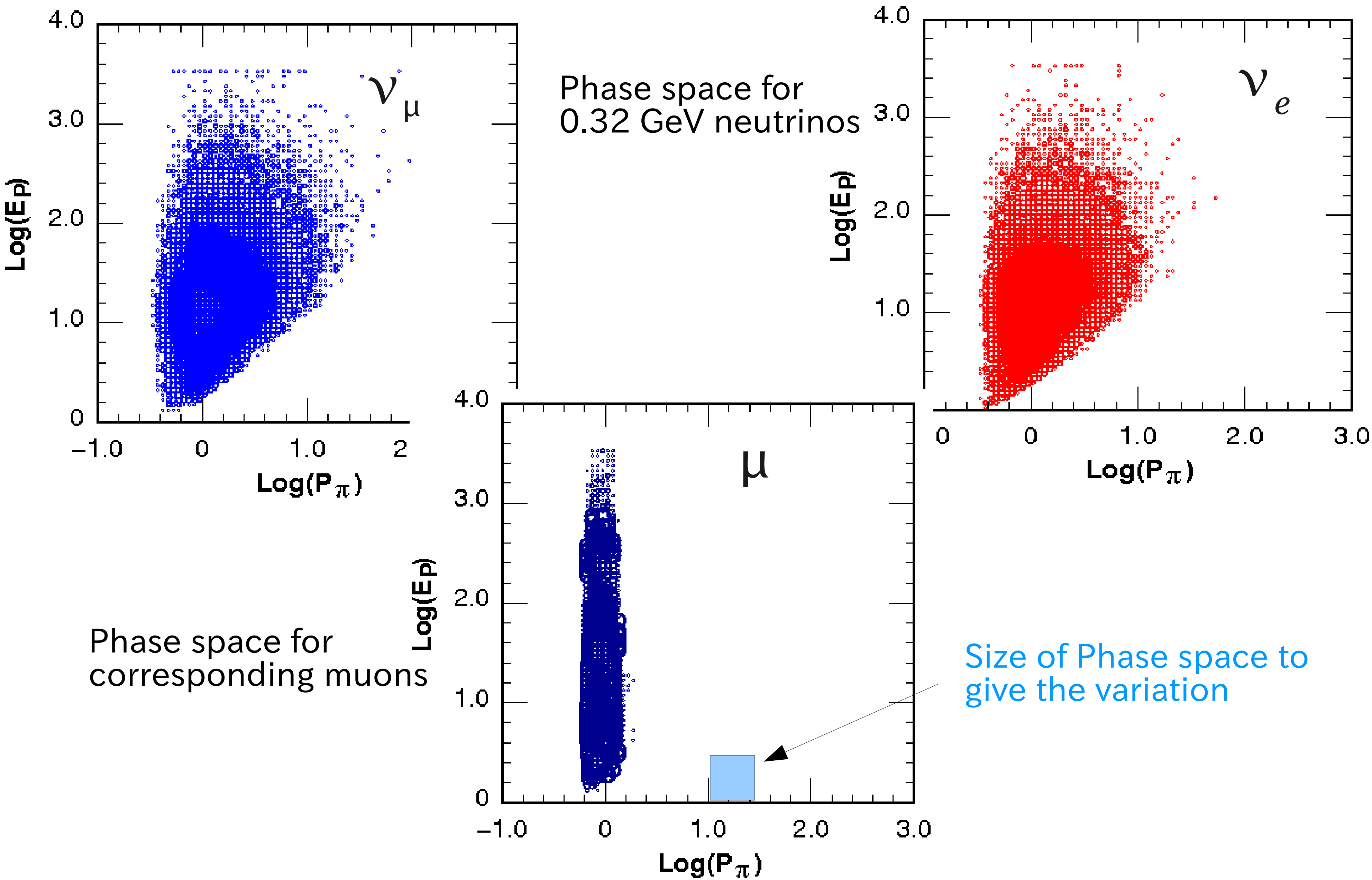
Atmospheric Neutrino

$$\nu_{\mu} : \nu_{e} \approx 2:1$$

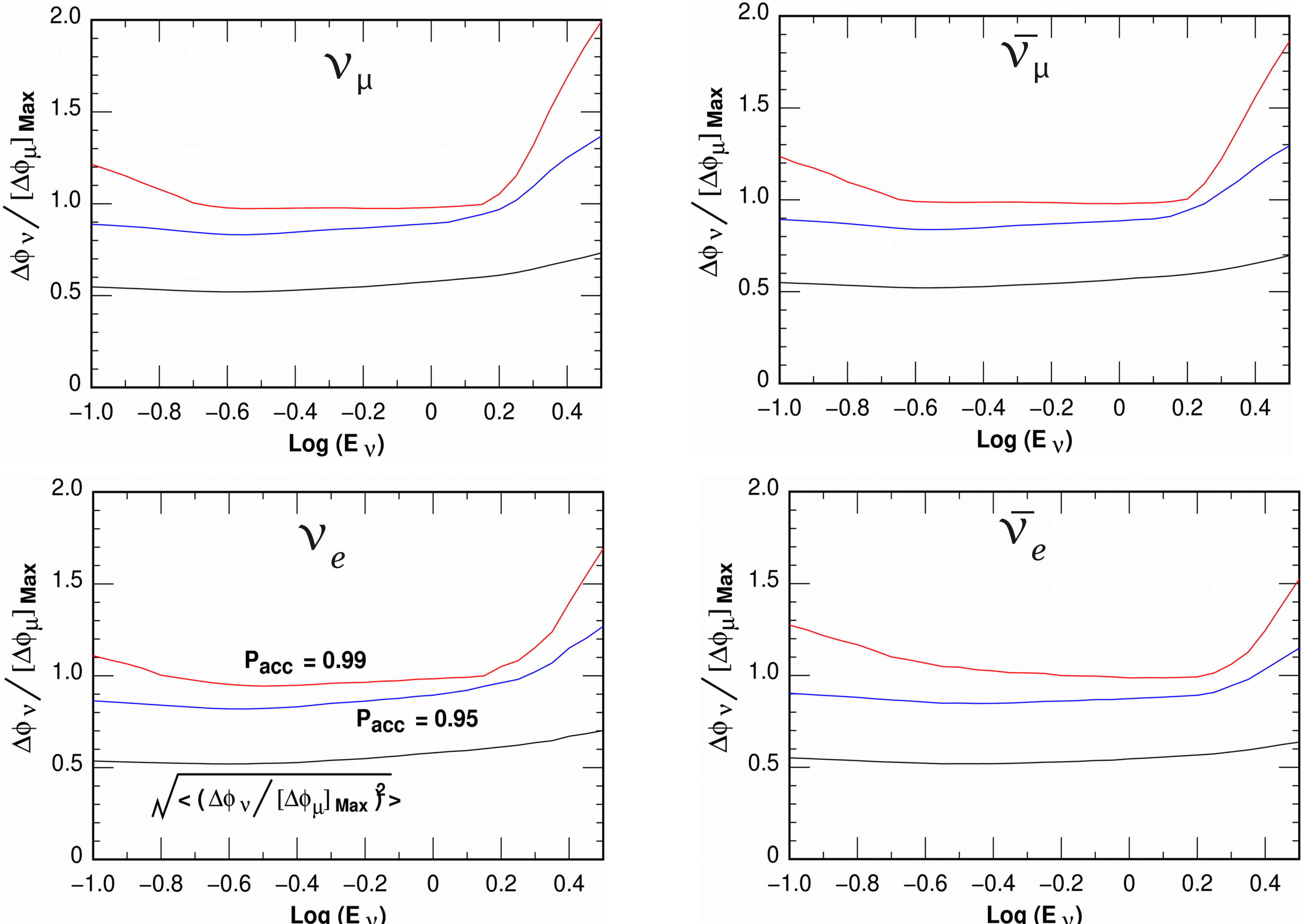
$\gamma, e^{\pm} \rightarrow$ EM-cascade → Air Shower

Other p's, n's, and sometimes π's repeat above interactions.

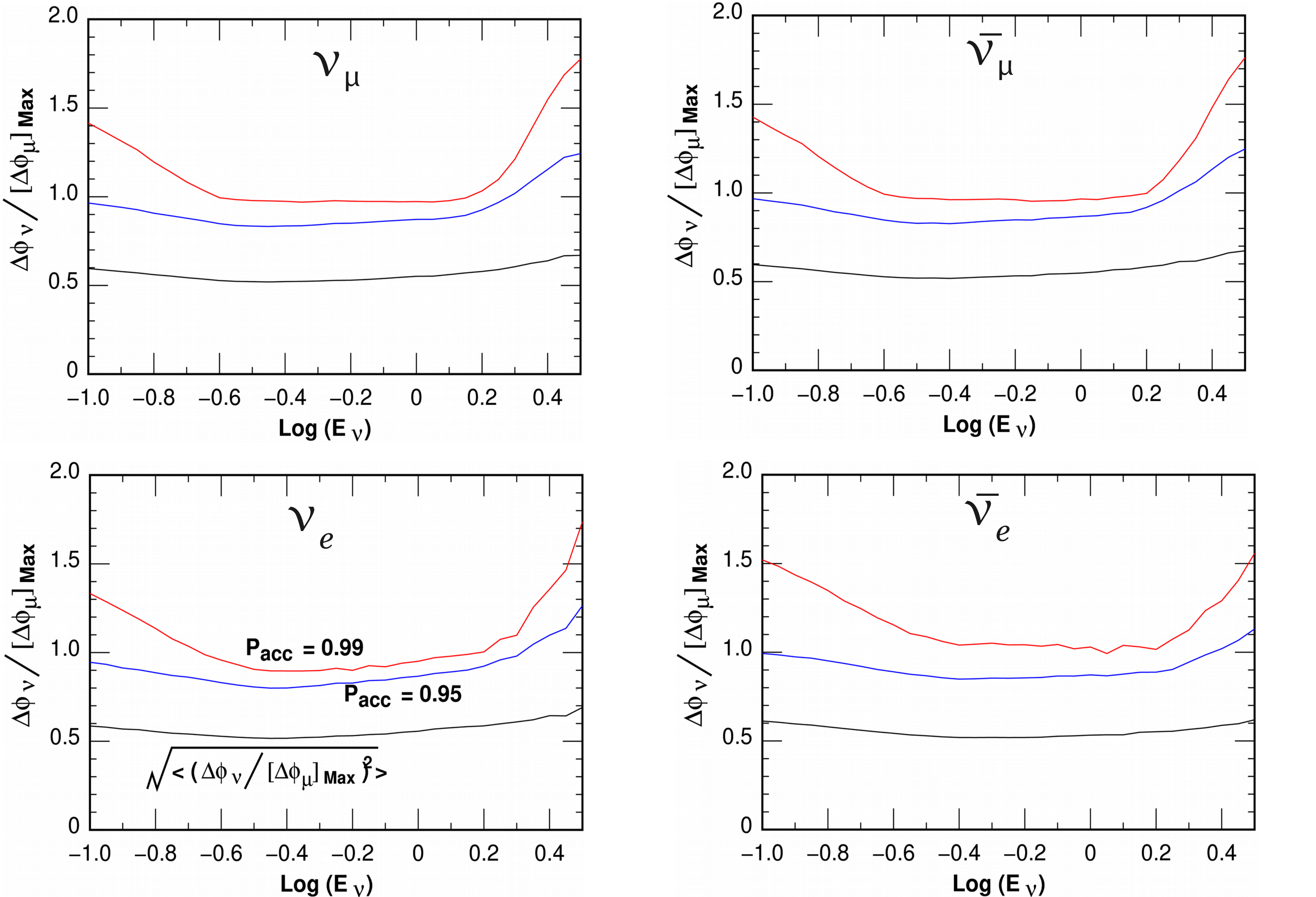
Analysis of calculation error:
Give Variations in the phase space and compare the variation of neutrino flux and the Maximum variation of muon flux in 0.5 ~ 2 GeV/c (μ+) and 0.5 ~ 4 GeV/c (μ-), where BESS Balloon observation was available.



Vertical neutrino flux



Horizontal neutrino flux

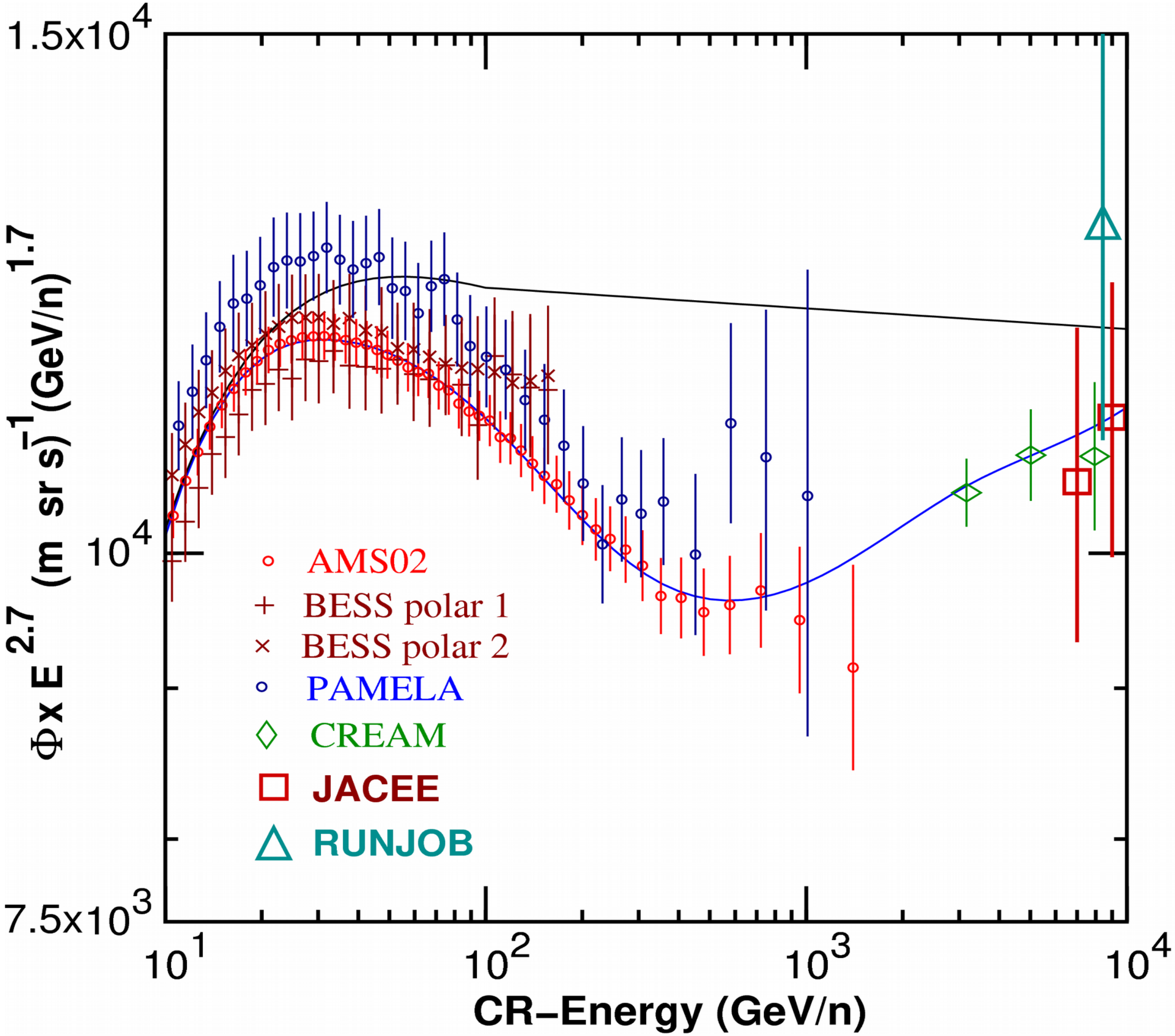


Impact of AMS02

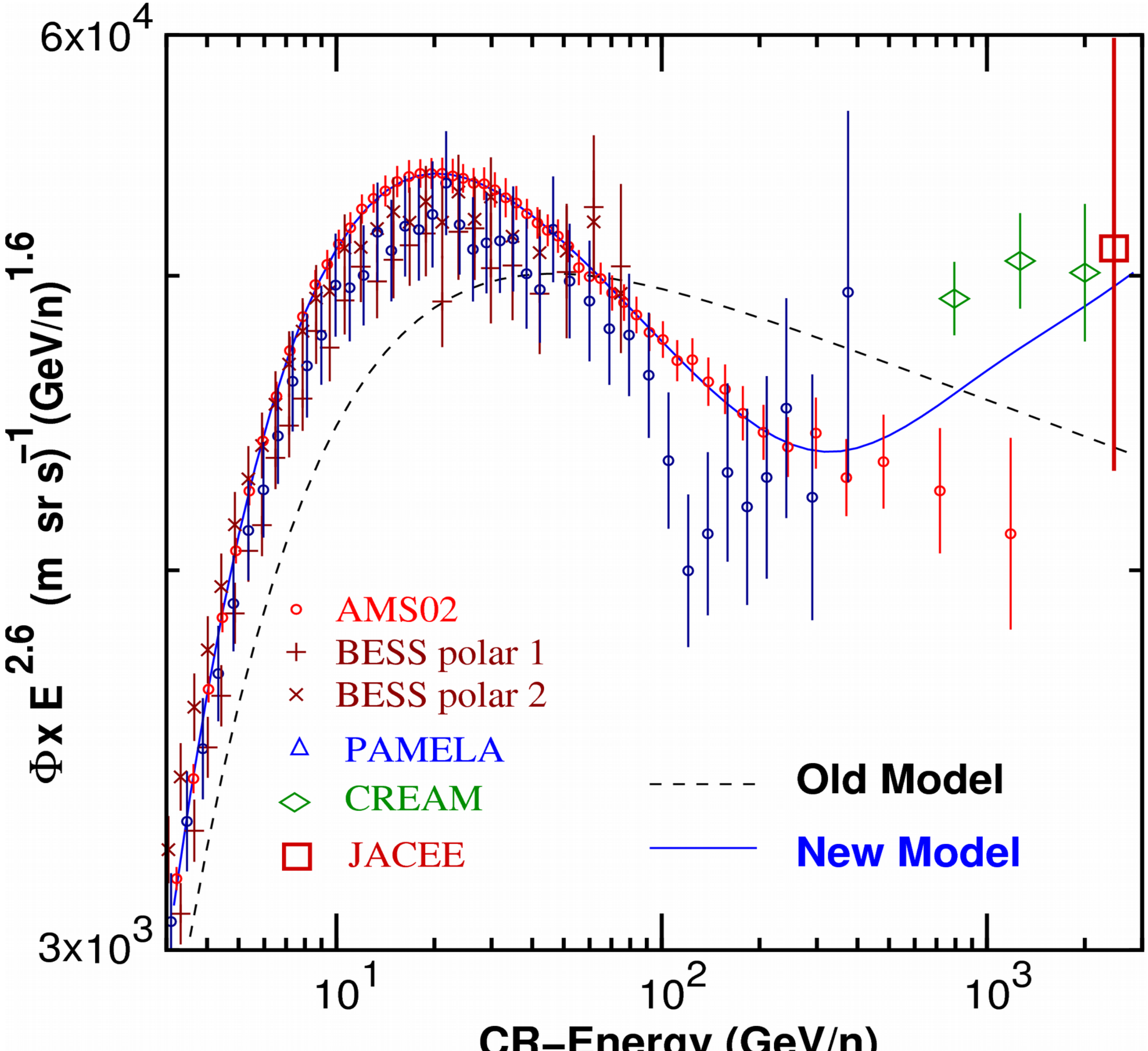
and

BESS-polar

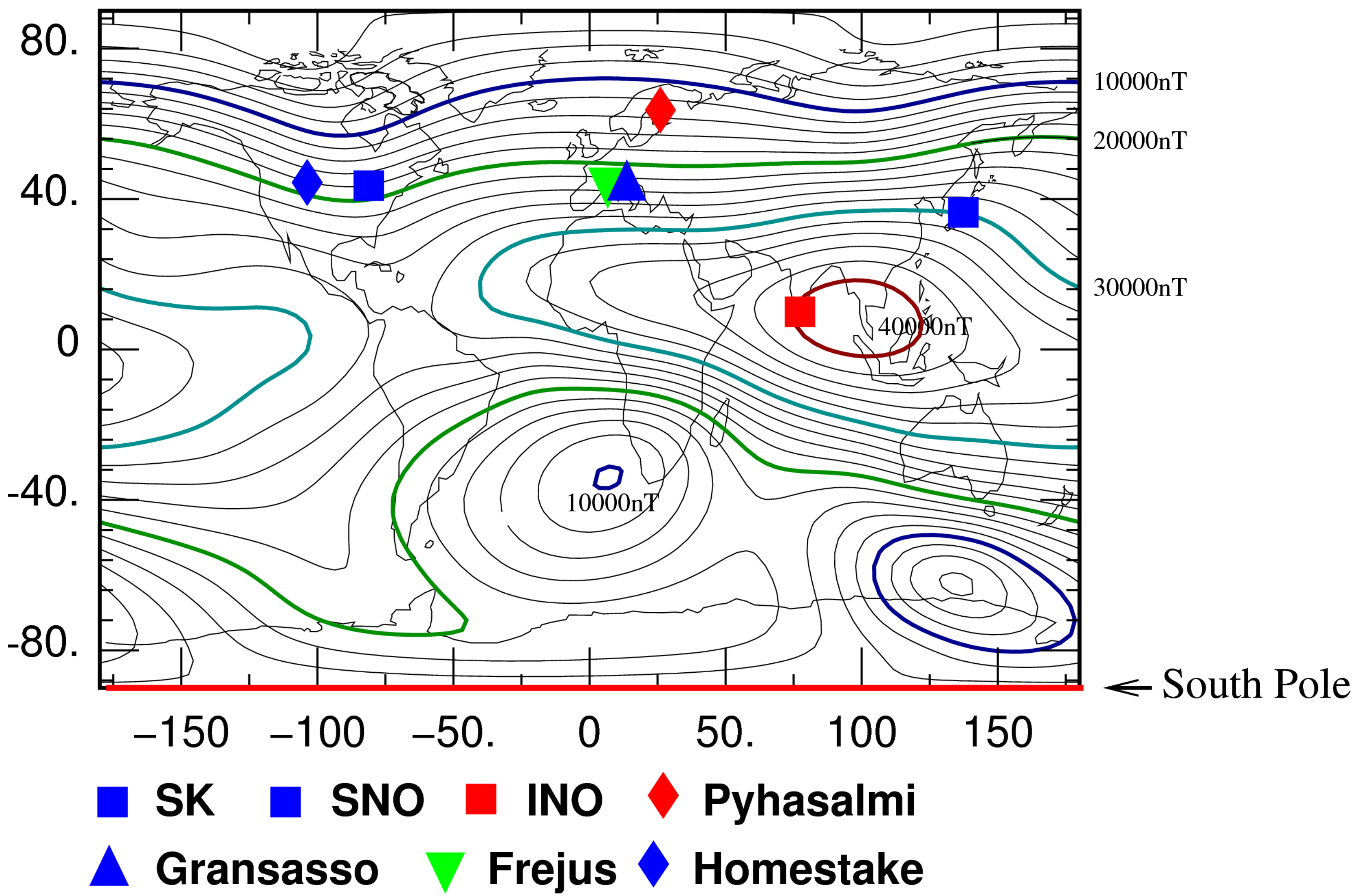
Proton closeup



Helium closeup



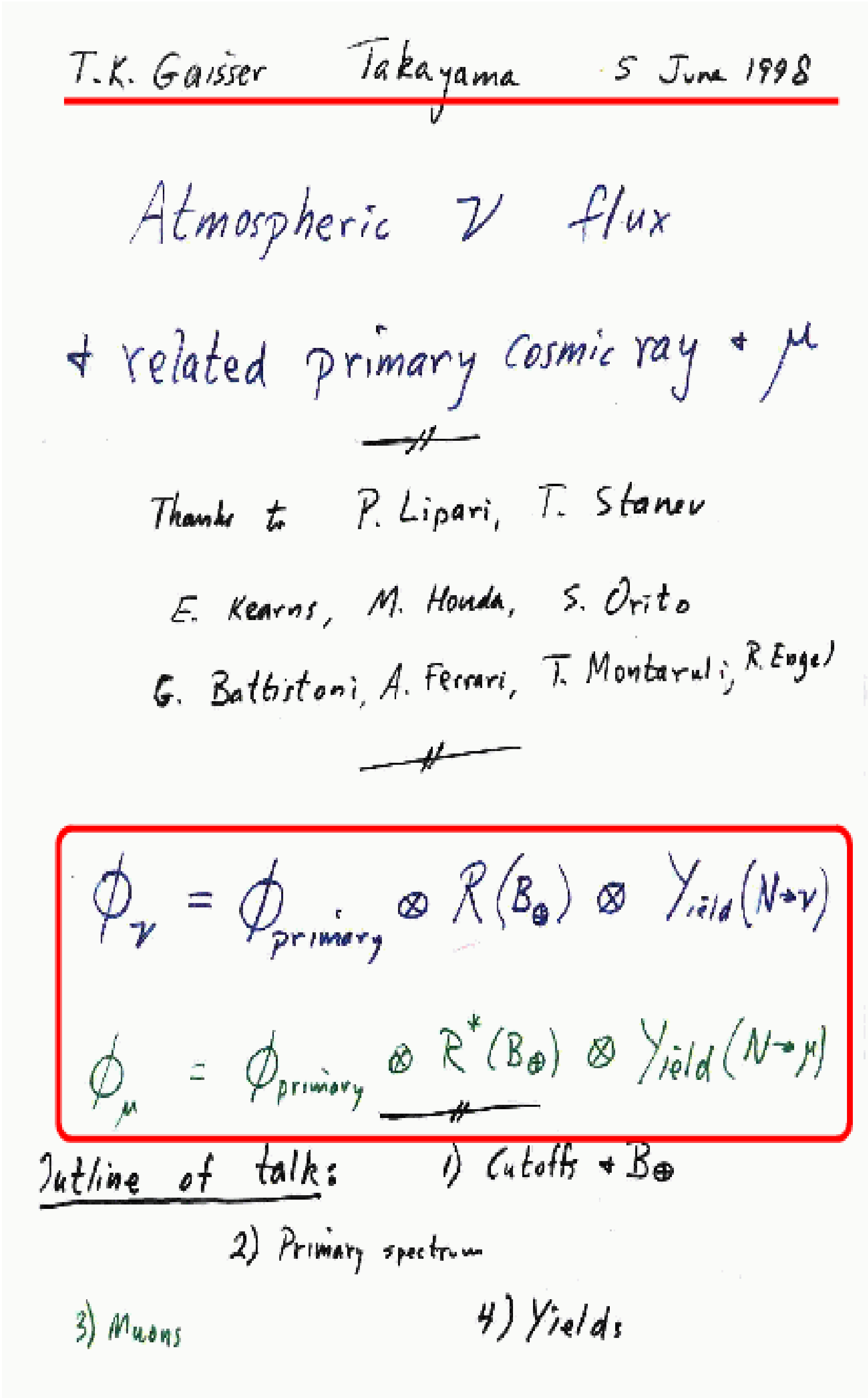
IGRF10 Geomagnetic Horizontal Field Strength



Overview

Primary cosmic ray flux
Interaction model

Calculation scheme
(include rigidity cutoff)
Atmosphere model


T.K. Gaisser Takayama 5 June 1998

Atmospheric ν flux

+ related primary cosmic ray + μ

Thanks to P. Lipari, T. Stanev

E. Kearns, M. Honda, S. Orito

G. Battistoni, A. Ferrari, T. Montaruli; R. Engel

$$\phi_\nu = \phi_{primary} \otimes R(B_\oplus) \otimes Yield(N \to \nu)$$

$$\phi_\mu = \phi_{primary} \otimes R^*(B_\oplus) \otimes Yield(N \to \mu)$$

Outline of talk: 1) Cutoffs + $B_\oplus$

2) Primary spectrum

3) Muons

4) Yields

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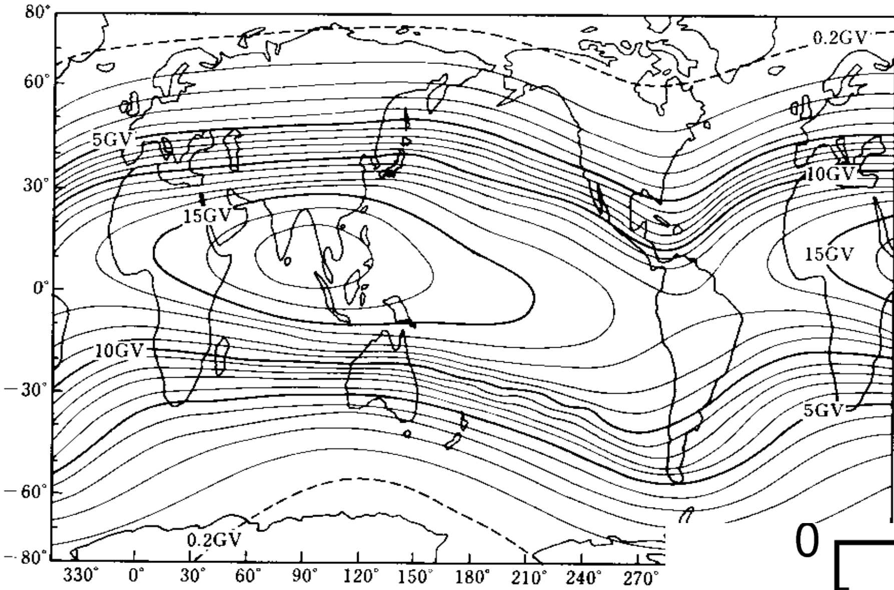
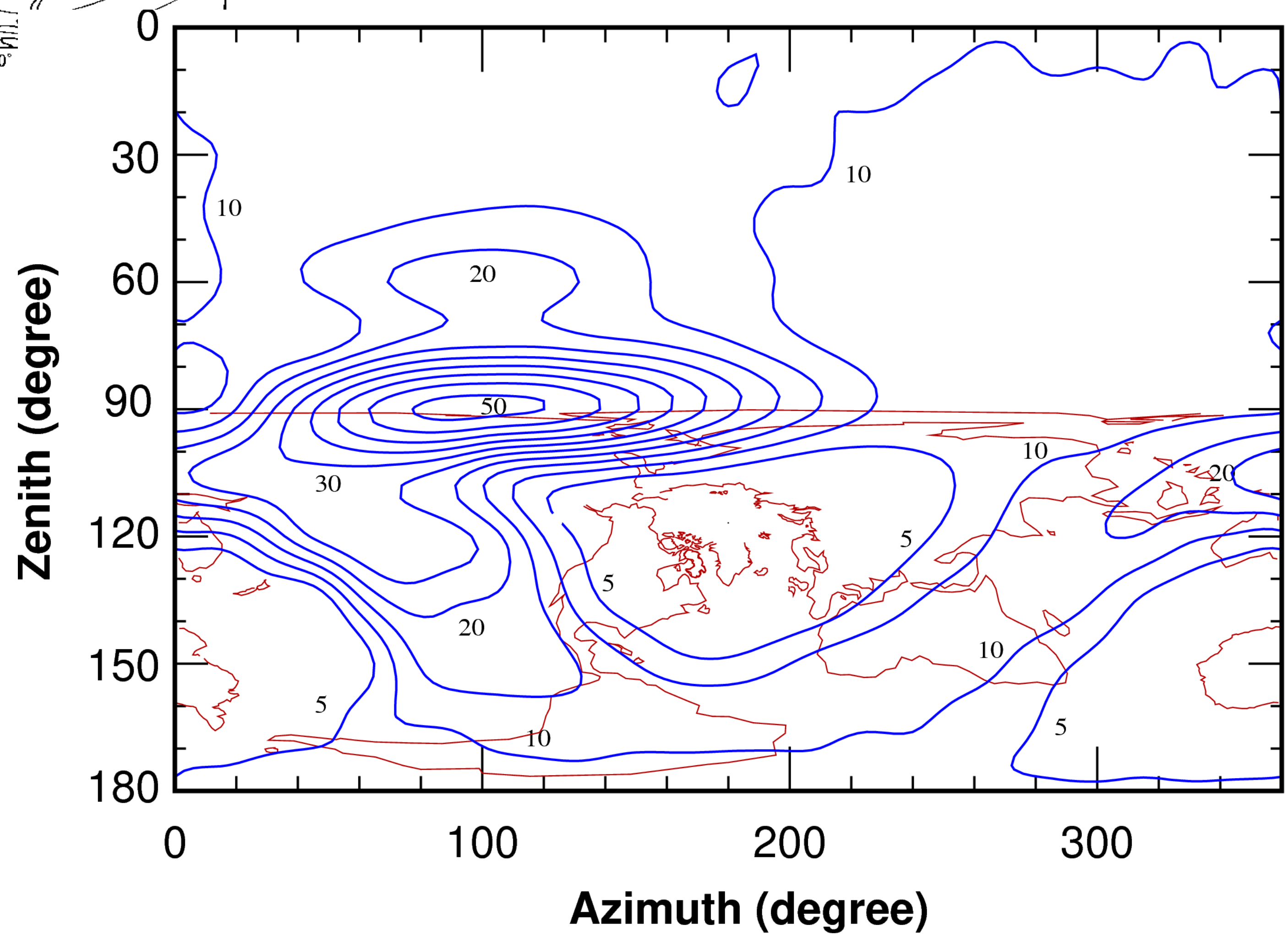


Rigidity Cutoff for
Vertical direction

Rigidity Cutoff
For SK direction



Rigity Cutoff and Geomagnetic Field

Horizontal component of geomagnetic field (IGRF2000?)