Neutrinoless double beta decay. What its observation would prove, and its nuclear matrix elements.

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From ``Reaching for the Horizon" the 2015 Long Range plan for the US Nuclear Science

RECOMMENDATION II

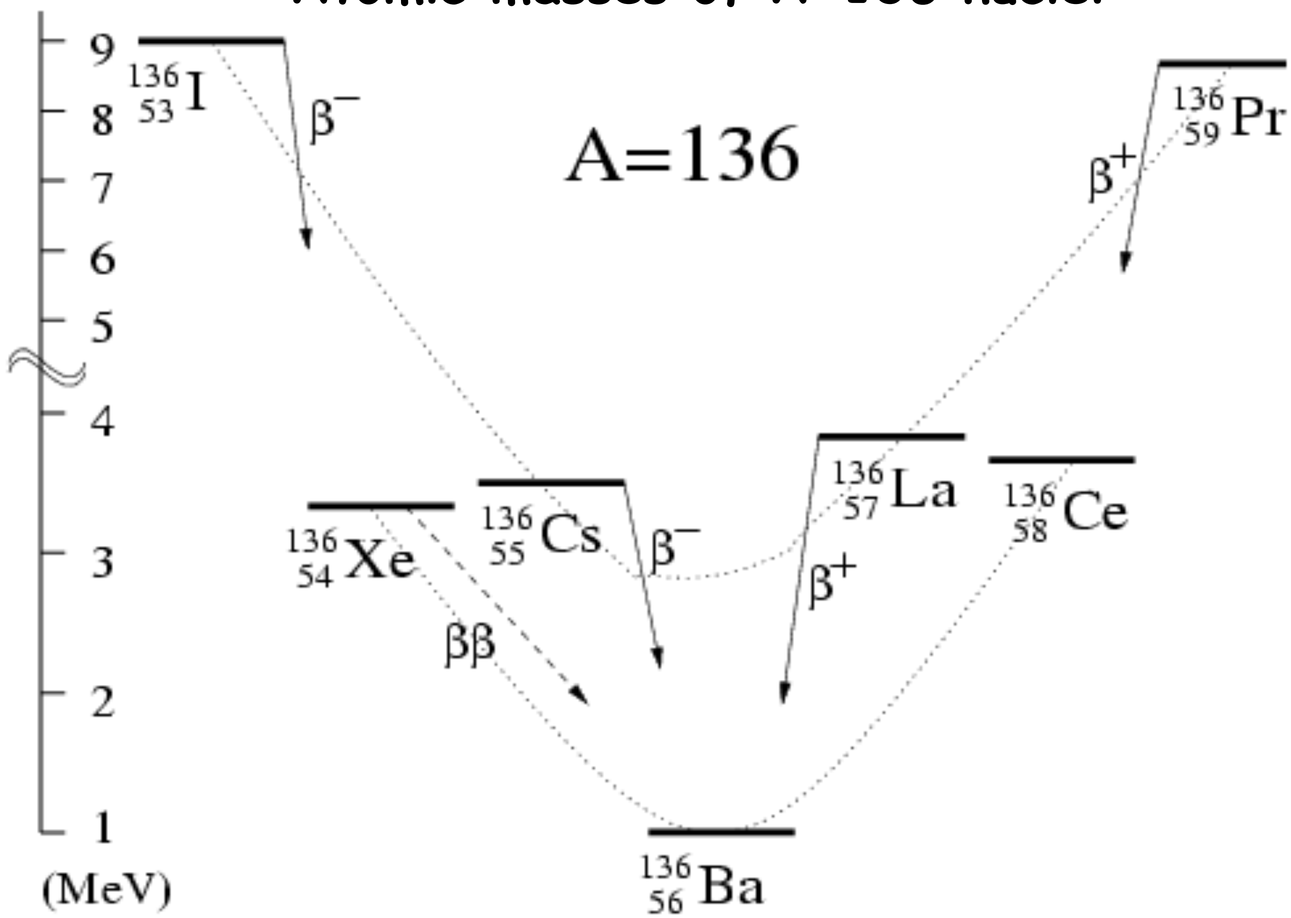
The excess of matter over antimatter in the universe is one of the most compelling mysteries in all of science. The observation of neutrinoless double beta decay in nuclei would immediately demonstrate that neutrinos are their own antiparticles and would have profound implications for our understanding of the matter-antimatter mystery.

We recommend the timely development and deployment of a U.S.-led ton-scale neutrinoless double beta decay experiment.

A ton-scale instrument designed to search for this as-yet unseen nuclear decay will provide the most powerful test of the particle-antiparticle nature of neutrinos ever performed. With recent experimental breakthroughs pioneered by U.S. physicists and the availability of deep underground laboratories, we are poised to make a major discovery.

Double ββ decay is observable because even-even nuclei are more bound than the odd-odd ones (due to the pairing interaction)

Atomic masses of A=136 nuclei



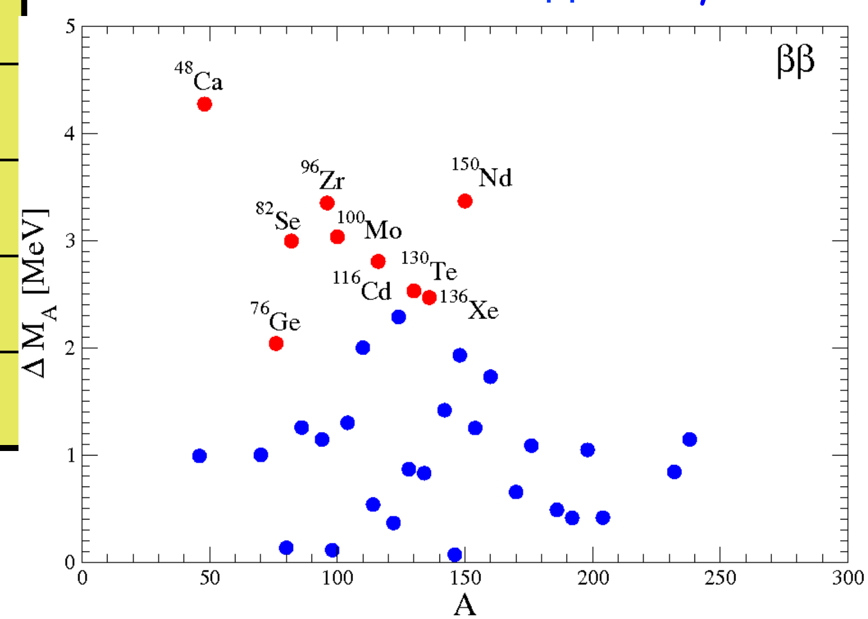
^{136}Xe and ^{136}Ce are stable against β decay (they exist in nature), but unstable against ββ decay ($\beta^-\beta^-$ for ^{136}Xe and $\beta^+\beta^+$ for ^{136}Ce)

Candidate nuclei for double beta decay with Q>2 MeV

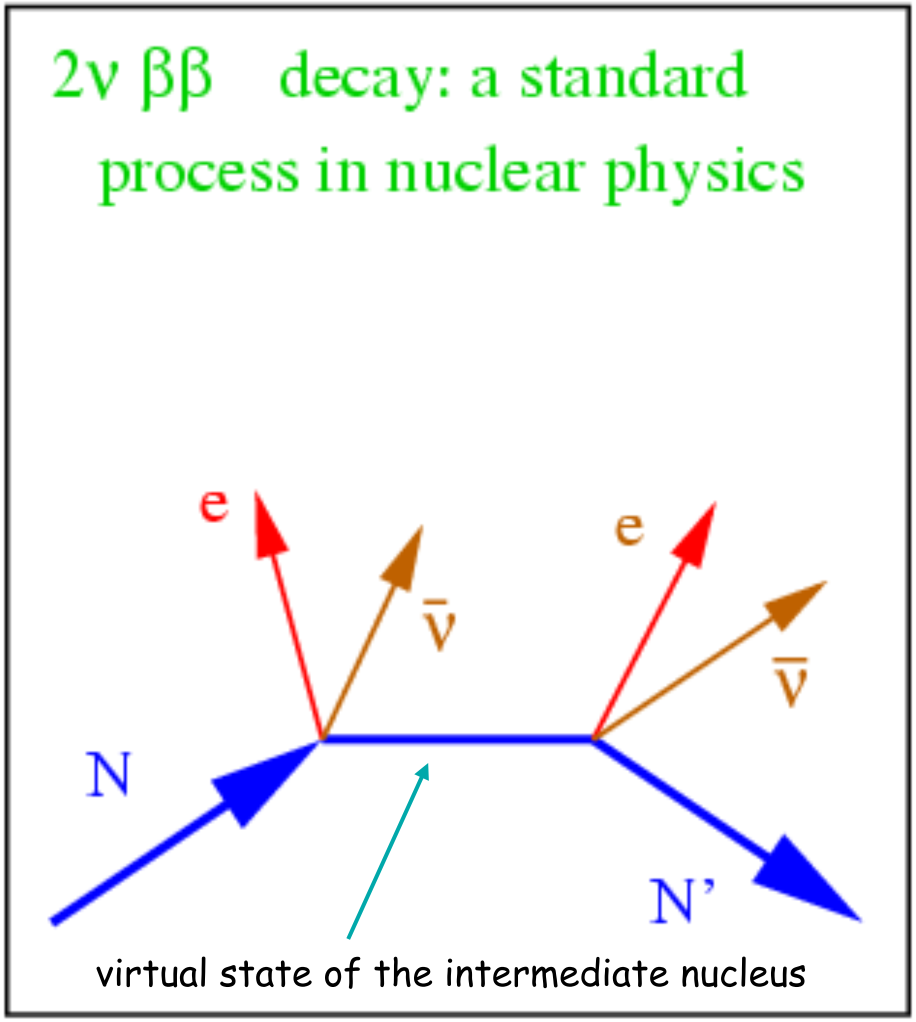
	Q (MeV)	Abund.(%)
$^{48}Ca \rightarrow ^{48}Ti$	4.271	0.187
$^{76}Ge \rightarrow ^{76}Se$	2.040	7.8
$^{82}Se \rightarrow ^{82}Kr$	2.995	9.2
$^{96}Zr \rightarrow ^{96}Mo$	3.350	2.8
$^{100}Mo \rightarrow ^{100}Ru$	3.034	9.6
$^{110}Pd \rightarrow ^{110}Cd$	2.013	11.8
$^{116}Cd \rightarrow ^{116}Sn$	2.802	7.5
$^{124}Sn \rightarrow ^{124}Te$	2.228	5.64
$^{130}Te \rightarrow ^{130}Xe$	2.533	34.5
$^{136}Xe \rightarrow ^{136}Ba$	2.479	8.9
$^{150}Nd \rightarrow ^{150}Sm$	3.367	5.6

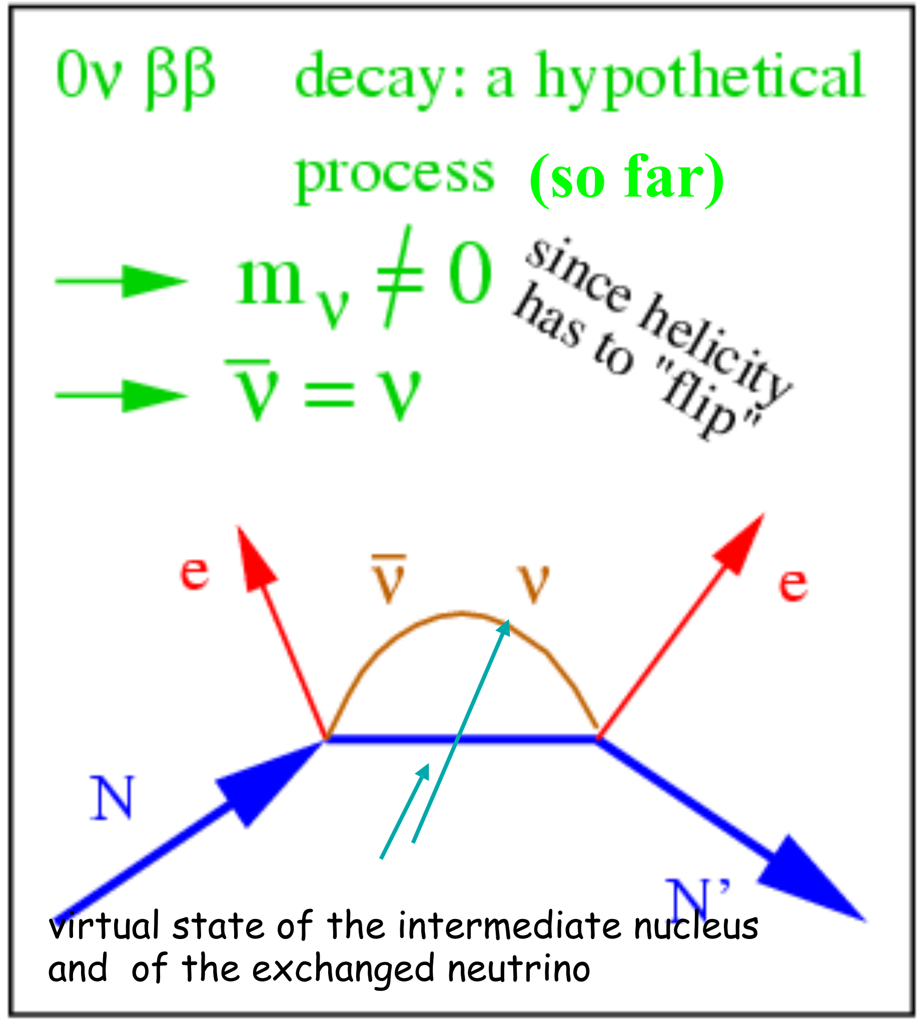
For most of the nuclei in this list the $2\nu\beta\beta$ decay has been observed. Typical half-life is 10^{20} years.

There are many other candidate nuclei with smaller Q, less suitable for observation of the $\beta\beta$ decay

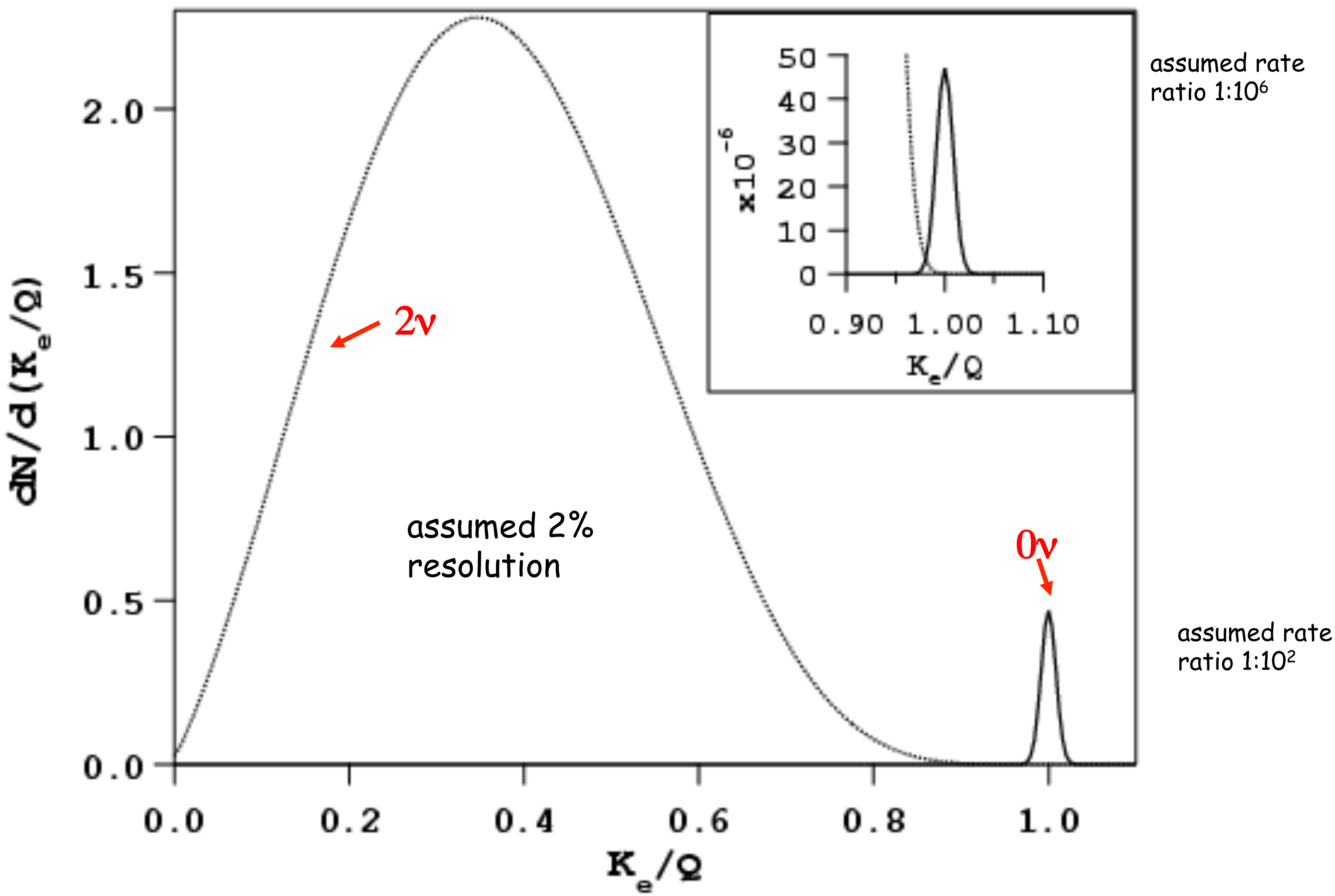


$\beta\beta$ decay can exist in two modes. The two-neutrino ($2\nu\beta\beta$) decay is an allowed but slow process, while the neutrinoless ($0\nu\beta\beta$) mode would violate the total lepton number conservation law and thus would be a sign of **new physics**





One can distinguish the two modes by measuring the sum electron energy. Ultimately, though, the 2ν decay is an unavoidable background to the 0νββ.



How can we tell whether the total lepton number is conserved?

A partial list of processes where the lepton number would be violated:

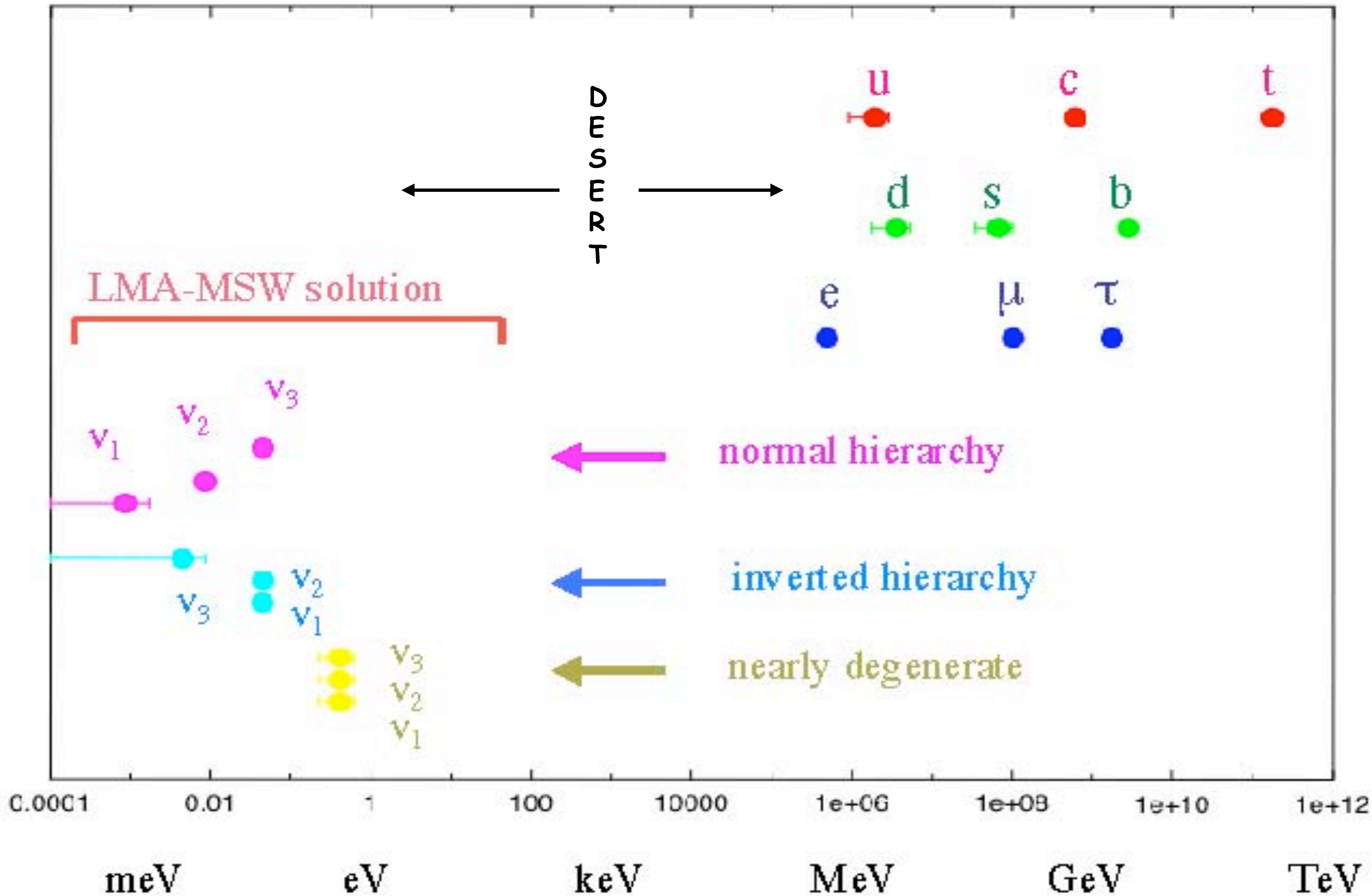
Neutrinoless $\beta\beta$ decay: $(Z,A) \rightarrow (Z\pm2,A) + 2e^{(\pm)}$, $T_{1/2} > \sim10^{26}$ y
Muon conversion: $\mu^- + (Z,A) \rightarrow e^+ + (Z-2,A)$, BR $< 10^{-12}$
Anomalous kaon decays: $K^+ \rightarrow \pi^-\mu^+\mu^+$, BR $< 10^{-9}$
Flux of $\bar{\nu}_e$ from the Sun: BR $< 10^{-4}$
Flux of ν_e from a nuclear reactor: BR < ?
Production at LHC of a pair of the same charge leptons, with no missing energy, through production of doubly charged scalar that decays that way?

Observing any of these processes would mean that the lepton number is not conserved, and that neutrinos are massive Majorana particles.

It turns out that the study of the $0\nu\beta\beta$ decay is by far the most sensitive test of the total lepton number conservation, so we restrict further discussion to this process.

Thanks to the fundamental discoveries of the last two decades we know that neutrino flavor is not conserved. From that it follows that neutrinos are massive and mixed The mass squared differences Δm^2_{solar}, and $\Delta m^2_{atmospheric}$ have been measured quite accurately, and the three mixing angles (θ_{12}, θ_{23}, θ_{13}) are known as well. However, we do not know the actual absolute neutrino mass, even though we do know that is is quite small, m_ν < few eV.

We know that ν masses are much much smaller than the masses of other fermions



Is that a "Hint of" a new mass-generating mechanism?

To solve the dilemma of `unnaturally' small neutrino mass we can give up on renormalizability and add operators of dimension d > 4 that are suppressed by inverse powers of some scale Λ, but are consistent with the SM symmetries.

Weinberg already in 1979 (PLR **43**, 1566) showed that there is **only one** dimension d=5 gauge-invariant operator given the particle content of the standard model:

$$L^{(5)} = C^{(5)}/\Lambda\ (\overline{L^c}\varepsilon H)(H^T\varepsilon L) + h.c.$$

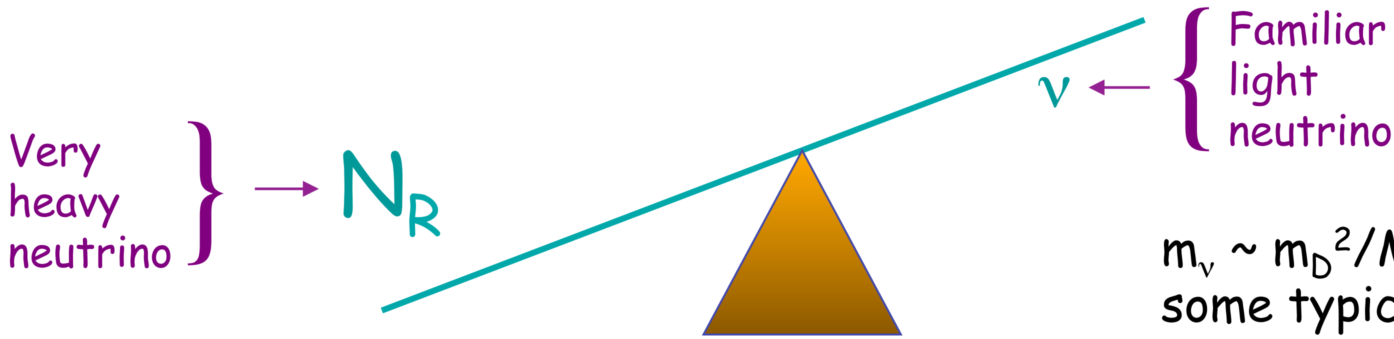
Here $L^c = L^T C$, where C is charge conjugation and $\varepsilon = -i\tau_2$. This operator clearly violates the lepton number by two units and represents neutrino Majorana mass

$$L^{(M)} = C^{(5)}/\Lambda\ v^2/2\ (\overline{\nu_L^{\,c}}\,\nu_L) + h.c.$$

If Λ is larger than v, the Higgs vacuum expectation value, the neutrinos will be `naturally' lighter than the charged fermions.

All other possible effective operators will be suppressed by higher powers of the energy scale Λ, i.e. Λ^{-n} with n > 2.

The See-Saw (type I) Mechanism was suggested already in ~1980 by Minkowski (1977), Gell-Mann, Ramond, and Slansky(1979), Yanagida(1979), Mohapatra and Senjanovic (1980). It is related to the finding of Weinberg (1979) that there is only one operator of dimension 5 (with only one power of the scale Λ_{LNV} in the denominator). It represents a neutrino Majorana mass realized in the see-saw model.



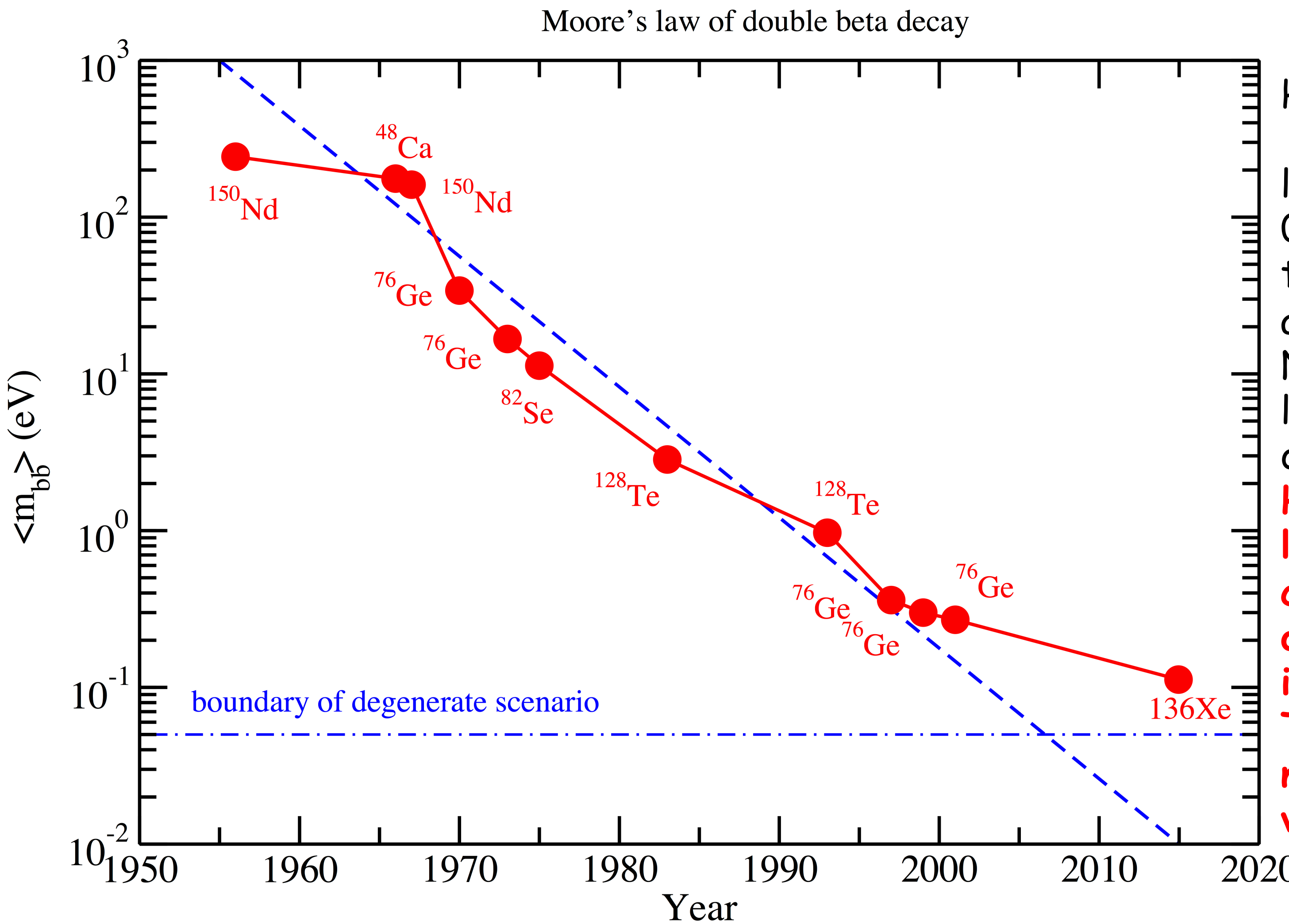
$m_\nu \sim m_D^2/M_R$ with m_D some typical Dirac fermion mass

In the light neutrino exchange, based on the above See-Saw type I, the decay rate is expressed as a product of three factors:

$$1/T_{1/2}^{0\nu} = G^{0\nu}(Q,Z)\ |M^{0\nu}|^2\ |\langle m_{\beta\beta}\rangle|^2, \quad \langle m_{\beta\beta}\rangle = |\Sigma_i U_{ei}^2\ m_i|\ ,$$

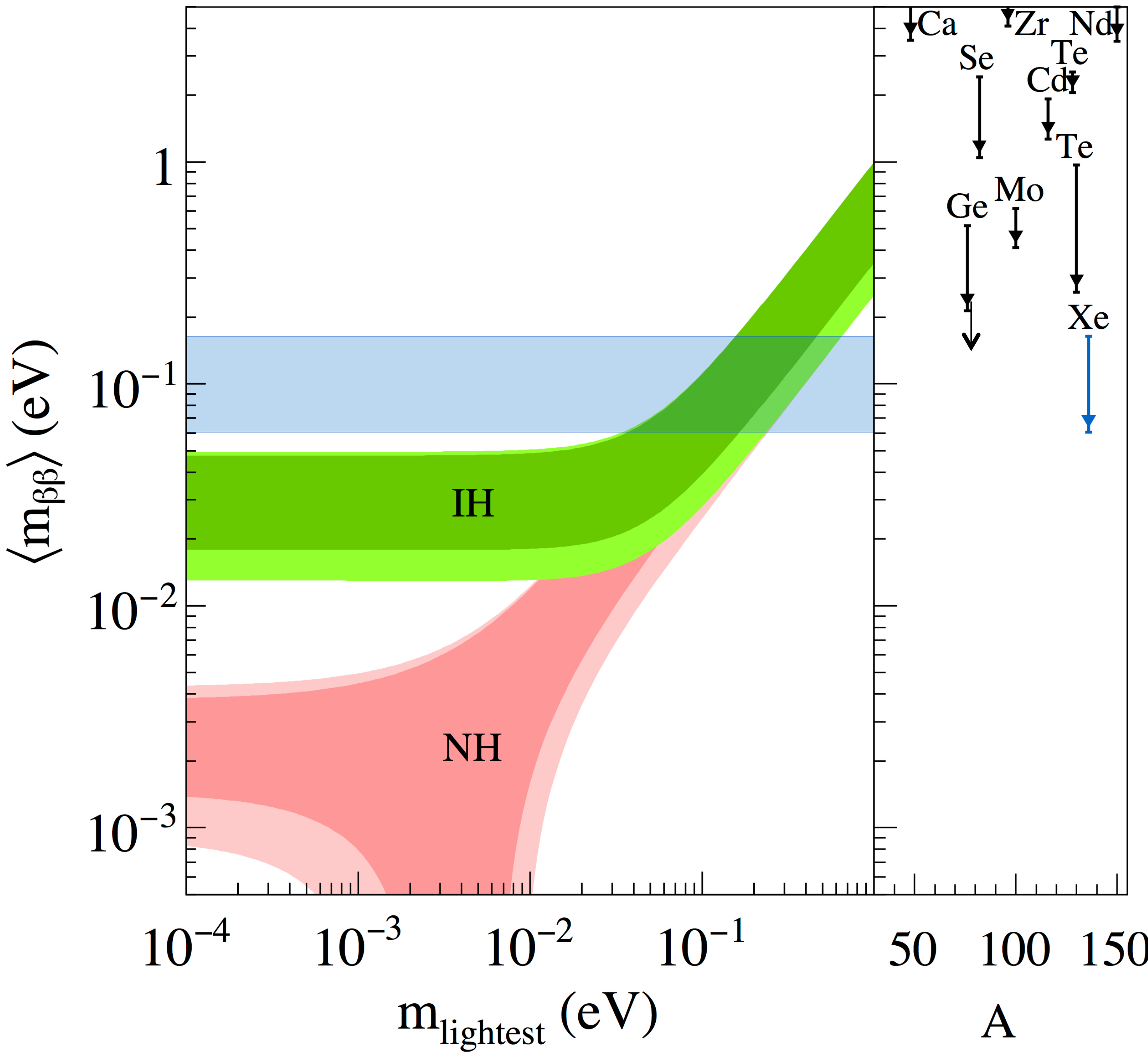
which represents a simple relation between the decay rate and the parameters of the neutrino mass matrix.

History of $0\nu\beta\beta$ decay



Historically, there are > 100 experimental limits on $T_{1/2}$ of the $0\nu\beta\beta$ decay. Here are the records expressed as limits on $\langle m_{\beta\beta}\rangle$. Note the approximate linear slope vs time on such semilog plot.

However, during the last decade the complexity and cost of such experiments increased dramatically The constant slope is no longer obviously visible.

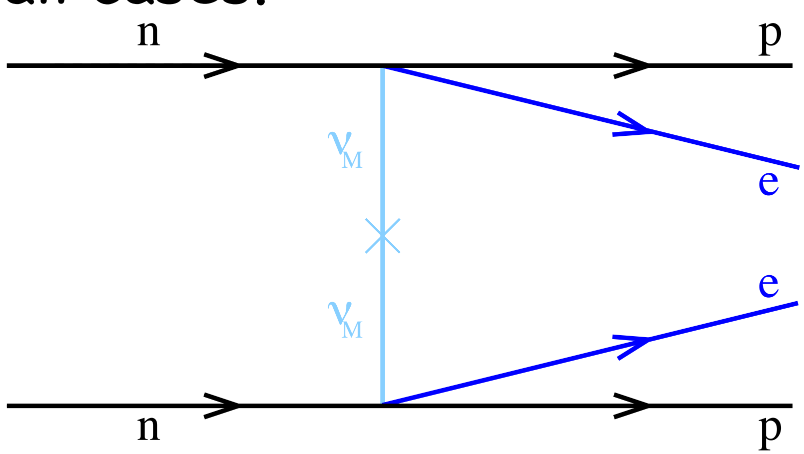


$\langle m_{\beta\beta} \rangle$ as a function of the mass of the lightest meutrino. Normal hierarchy in red, inverted hierarchy in green. The reach of the best experiments is indicated by the blue band. The sensitivity of the different tests is indicated in the right panel by the corresponding nuclei.

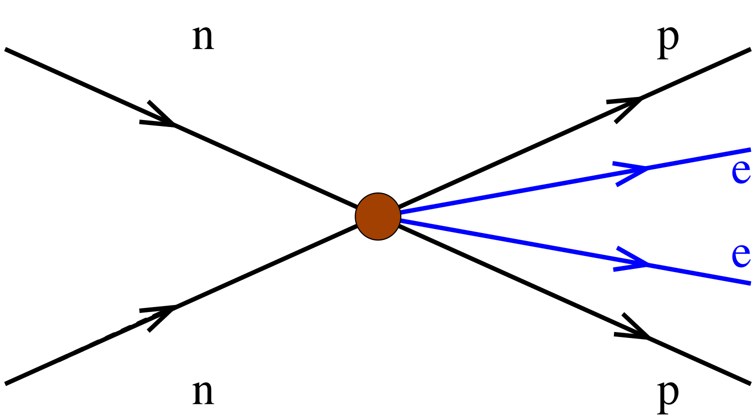
If (or when) the $0\nu\beta\beta$ decay is observed two problems must be still resolved:

a) What is the mechanism of the decay, i.e., what kind of virtual particle is exchanged between the affected nucleons (or quarks)?
b) How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding nuclear matrix elements?

There are two possible, and distinct in physics, but not in their signal signature, mechanisms of $0\nu\beta\beta$ decay. In the following I will concentrate on the simplest light Majorana neutrino exchange. Observation of the $0\nu\beta\beta$ decay will be a signal of ``**new physics**" beyond the standard model in all cases.



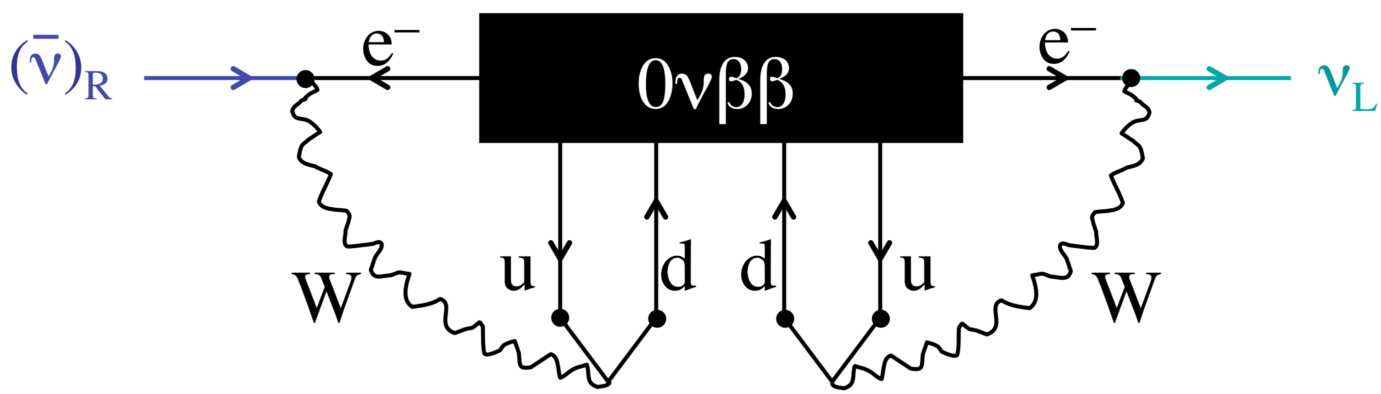
The long-range, an exchange of a light Majorana neutrino,
Neutrino mass is associated with the **See-saw type I** mechanism $m_\nu \sim v^2/M_N$, where M_N is the very heavy neutrino mass.



The short-range, an exchange of some heavy, often new, particle, it is therefore effectively a contact four nucleon vertex, represented by a dimension 9 operator.
The physics of this type of lepton number violation is present in the **see-saw type II** or **type III** models.

Whatever processes cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term and thus would represent ``New Physics'':

Schechter and Valle,82



By adding only Standard model interactions we obtain

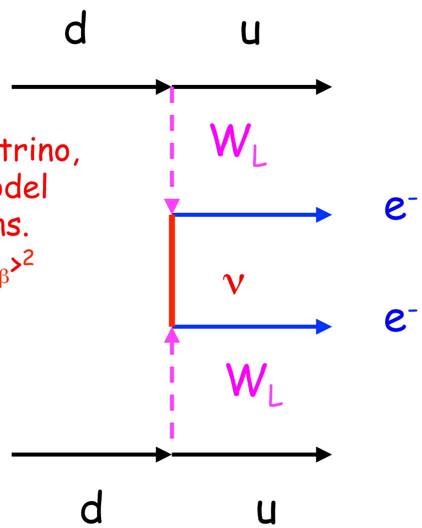
$(\bar{\nu})_R \rightarrow (\nu)_L$ ***Majorana mass term***

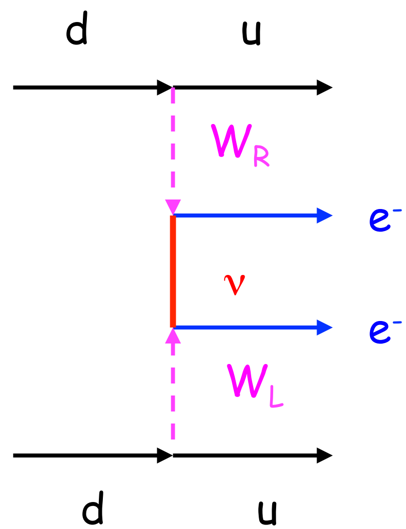
Hence observing the $0\nu\beta\beta$ decay guaranties that ν are massive Majorana particles. But the relation between the decay rate and neutrino mass might be complicated, not just as in the see-saw type I.

What is the nature of the `black box'? In other words, what kinds of effects can contribute to both categories of the $0\nu\beta\beta$ decay?

All these diagrams can in principle contribute to the $0\nu\beta\beta$ decay amplitude

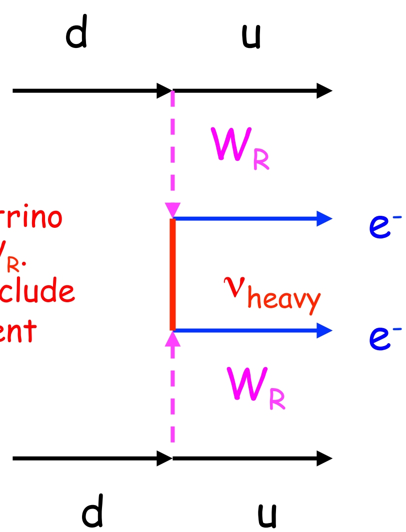
Light Majorana neutrino, only Standard Model weak interactions. Decay rate ~$\langle m_{\beta\beta}\rangle^2$

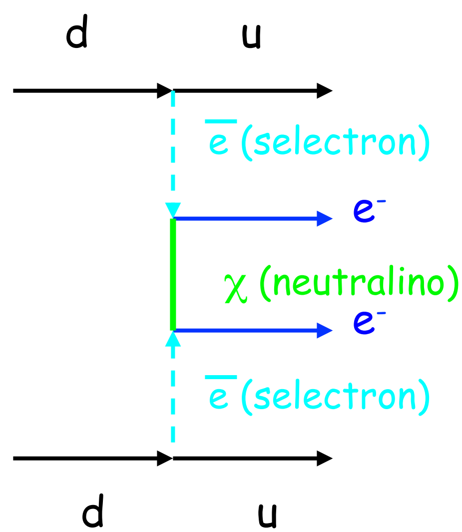




Light Majorana neutrinos. Model extended to include right-handed W_R. Mixing extended between the left and right-handed neutrinos. This is the mode where the rate ~λ^2 or η^2

Heavy Majorana neutrino interacting with W_R. Model extended to include right-handed current interactions.





Supersymmetry with R-parity violation. Many new particles invoked. Light Majorana neutrinos exist also.

It is well known that the amplitude for the light neutrino exchange scales as $\langle m_{\beta\beta} \rangle$. On the other hand, if heavy particles of scale Λ are involved the amplitude scales as $1/\Lambda^5$ (dimension 9 operator)

.

The relative size of the heavy (A_H) vs. light particle (A_L) exchange to the decay amplitude is (a crude estimate, due originaly to Mohapatra)

$A_L \sim G_F^2 \, m_{\beta\beta}/\langle q^2 \rangle$, $A_H \sim G_F^2 \, M_W^4/\Lambda^5$,

where Λ is the heavy scale and q ~ 100 MeV is the virtual neutrino momentum.

For Λ ~ 1 TeV and $m_{\beta\beta}$ ~ 0.1 – 0.5 eV $A_L/A_H \sim 1$, hence both mechanisms contribute equally. Thus, the existing $0\nu\beta\beta$ life-time limits constrain Λ_{LNV} to be larger than ~ TeV

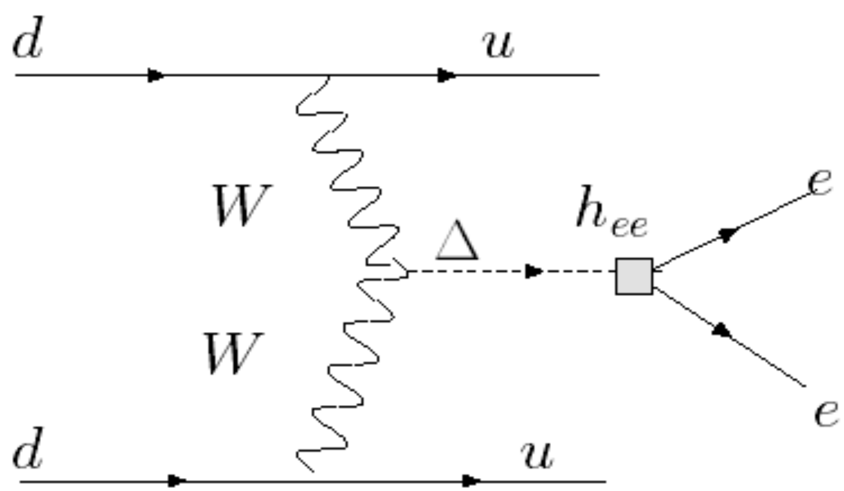
Lets consider briefly the particle physics models in which $0\nu\beta\beta$-decay of the short-range category might exist.
In them LNV violation is associated with low-scale (~TeV) physics, unlike see-saw with LNV at very high scale.

These models include e.g. Left-Right Symmetric Model (LRSM) and RPV SUSY [$R = (-1)^{3(B-L) + 2s}$]

Low scale LNV: Left-Right Symmetric Model (LRSM)

$$\mathcal{L}_{\delta^{\pm\pm}_{L,R}} = \frac{g}{2}\left[\delta^{++}_{L,R}\,\overline{l^c}\,(h_{L,R}\,P_{L,R})\,l + \delta^{--}_{L,R}\,\bar{l}\left(h^{\dagger}_{L,R}\,P_{R,L}\right)l^c\right]$$

The model includes a doubly charged Higgs that couples to leptons as shown



This is an example of 0νββ decay mediated by this coupling. The amplitude scales like

$$\frac{g_2^3\ \ h_{ee}}{M^3_{W_R}\,M^2_{\Delta}}$$

Another example is the exchange of heavy right-handed ν_R and two W_R that scales like

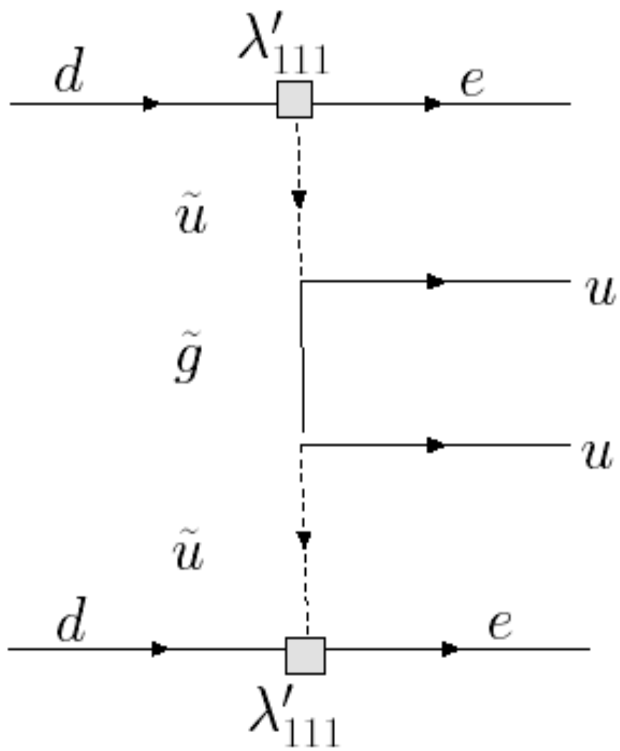
$$\frac{g_2^4}{M^4_{W_R}\,M_{\nu_R}}$$

In both cases the amplitude scales like $1/\Lambda^5$ with $\Lambda \sim M_{W(R)} \sim M_{\Delta} \sim M_{\nu\,(R)}$

Illustration II: RPV SUSY [R = (-1)$^{3(B-L) + 2s}$]

$$W_{RPV} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu'_i L_i H_u$$

0νββ



The 0νββ amplitude scales as

$$\frac{\pi\alpha_s \ \lambda'^2_{111}}{m_{\tilde{g}} \ m^4_{\tilde{f}}}$$

or in another example as

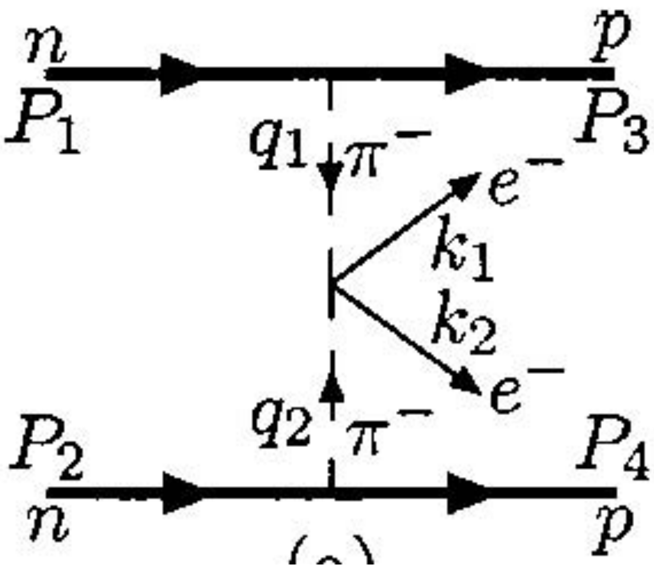
$$\frac{\pi\alpha_2 \ \lambda'^2_{111}}{m_{\chi} \ m^4_{\tilde{f}}}$$

Again with the characteristic $1/\Lambda^5$ scaling

Note in passing that less attention has been devoted in the past to the evaluation of the nuclear matrix elements for the case of heavy particle exchange (short-range contribution to 0νββ decay). Proper treatment of the nucleon-nucleon repulsion in that case is obviously crucial; it is traditionally treated crudely using nucleon form factors.

Including pion exchange avoids this problem and seems to lead to larger and more consistently evaluated matrix elements.

(Vergados 82, Faessler *et al.* 97, Prezeau *et al. 03*)



0νββ amplitude is contained in the ππee vertex

The study of lepton flavor violation (LFV) can help to decide what mechanism is responsible for the $0\nu\beta\beta$ decay if it is observed in a foreseeable future. The models that allow the existence of $0\nu\beta\beta$ decay at Λ_{LNV} ~ 1 TeV often include an enhancement of LFV as well.

This is based on “Lepton number violation without supersymmetry” Phys.Rev.D 70 (2004) 075007
V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.
and on “Neutrinoless double beta decay and lepton flavor violation” Phys. Rev. Lett. 93 (2004) 231802
V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.

Lepton flavor violation (LFV) involving charged leptons has not been observed as yet. The most sensitive limits are for the decay

$$B_{\mu\to e\gamma} = \Gamma(\mu\to e\gamma)/\Gamma(\mu\to e\nu_\mu\nu_e) < 1.2\times10^{-11}$$

New experiment, MEG at PSI, started data taking in 2008 and should reach sensitivity ~ 2 orders of magnitude better.

The "muon conversion" is constrained by

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))} < 8\times10^{-13}$$

Several proposals extending the sensitivity to $\sim10^{-17}$ have been proposed.

The fact that neutrinos have finite mass and that they mix will not make these LFV processes observable, they are suppressed by $(\Delta m^2/M_W^2)^2 \leq 10^{-50}$. Hence observation of them would imply "new physics" unrelated (or only indirectly related) to neutrino mass.

Summary so far:

1) Short-range contributions to the $0\nu\beta\beta$ decay with ~TeV mass scale can lead to the decay rate similar to that of light Majorana neutrino exchange with $\langle m_{\beta\beta} \rangle \sim 0.1 - 1$ eV.
2) In order to correctly interpret the experimental results and plan new experiments, it is important to determine the mechanism of the decay. Relation to LFV can help in that respect.
3) Next generation of experiments on LFV will extend the sensitivity considerably. In parallel, running of LHC will shed light on the existence of particles with ~TeV masses.

Lets restrict our further considerations to the simplest long-range mechanism involving the virtual exchange of a light Majorana neutrino.

As long as the mass eigenstates ν_i that are the components of the flavor neutrinos ν_e, ν_μ, and ν_τ are really Majorana neutrinos, the $0\nu\beta\beta$ decay **will occur,** with the rate

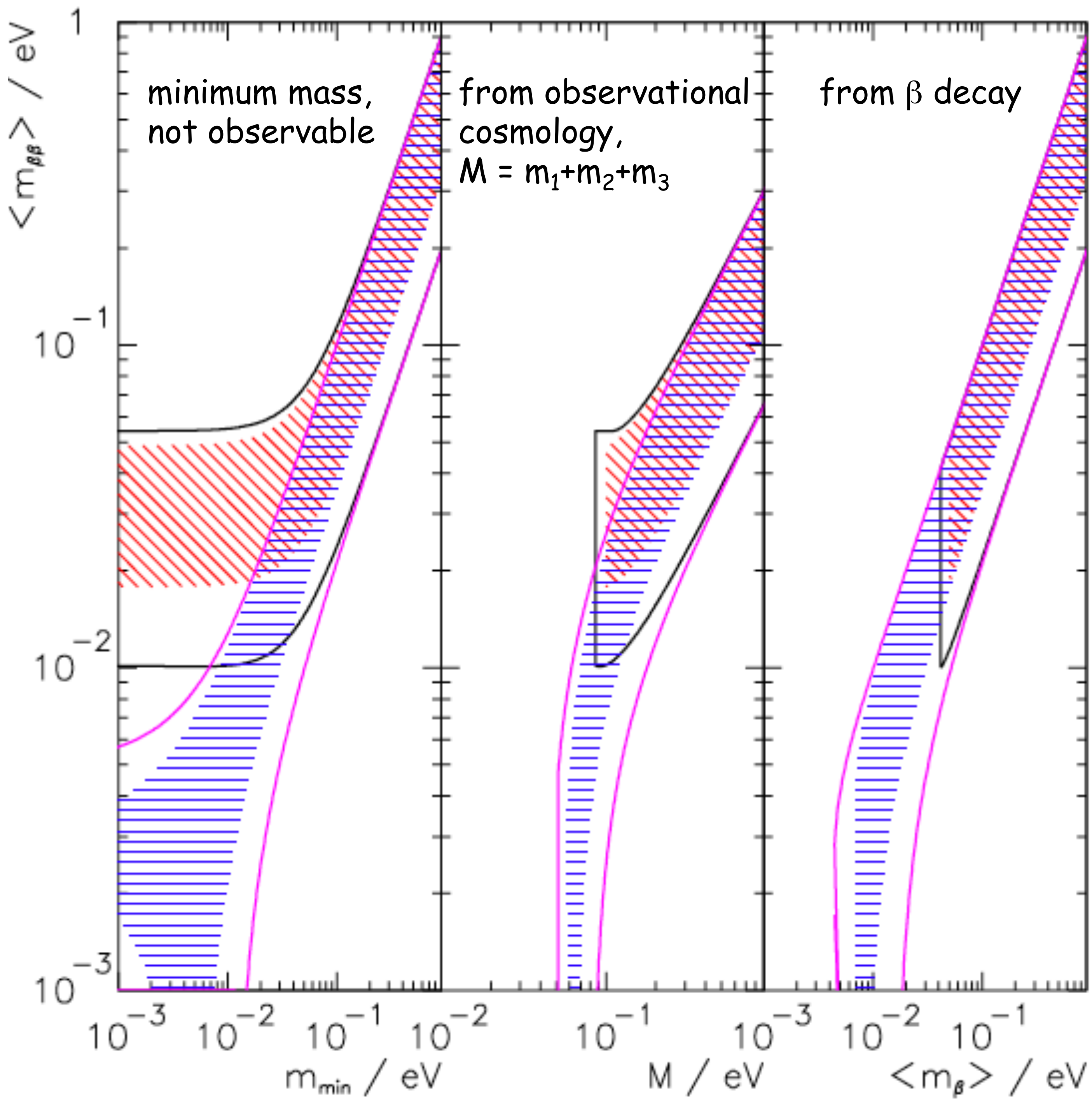
$$1/T_{1/2} = G(E_{tot}, Z)\,(M^{0\nu})^2 \langle m_{\beta\beta}\rangle^2,$$

where $G(E_{tot},Z)$ is easily calculable phase space factor, $M^{0\nu}$ is the nuclear matrix element, calculable with difficulties (and discussed later), and

$$\langle m_{\beta\beta}\rangle = |\ \Sigma_i\ |U_{ei}|^2 \exp(i\alpha_i)\ m_i\ |,$$

where α_i are unknown Majorana phases (only two of them are relevant).
Using the formula above we can relate $\langle m_{\beta\beta}\rangle$ to other observables related to the absolute neutrino mass.

$\langle m_{\beta\beta} \rangle$ vs. the absolute mass scales



blue shading: normal hierarchy, $\Delta m^2_{31} > 0$.

red shading: inverted hierarchy $\Delta m^2_{31} < 0$

shading: best fit parameters, lines 95% CL errors.

Thanks to A. Piepke

Note as a curiosity:
$\langle m_{\beta\beta} \rangle$ may vanish even though all m_i are nonvanishing
and all ν_i are Majorana neutrinos.

What can we do in that case?

In principle, although probably not in practice,
we can look for the lepton number violation
involving muons.

Numerical example: take $\theta_{13} = 0$, and Majorana phase $\alpha_2 - \alpha_1 = \pi$
(only for this choice of phases can $\langle m_{\beta\beta} \rangle$ vanish when $\theta_{13} = 0$).
$\langle m_{\beta\beta} \rangle = 0$ if $m_1/m_2 = \tan^2\theta_{12}$, with $m_2 = (m_1^2 + \Delta m_{sol}^2)^{1/2}$.
That happens for $m_1 = 4.58$ meV and $m_2 = 10$ meV
(this is, therefore, fine tuning).
But then $\langle m_{\mu e} \rangle = \sin 2\theta_{12} \cos\theta_{23}/2 \times (m_1 + m_2) = 4.78$ meV,
Which is, at least in principle, observable using
$\mu^- + (Z,A) \rightarrow e^+ + (Z-2,A)$.

Nuclear Matrix Elements:

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that again are bound in the ground state of the final nucleus.

It is therefore necessary to evaluate, with a sufficient accuracy, **the ground state wave functions of both nuclei**, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them.

This cannot be done exactly; some approximation and/or truncation is always needed. Moreover, unfortunately, there is no other analogous observable that can be used to judge the quality of the result.

Can one use the $2\nu\beta\beta$-decay matris elements for that?
What are the similarities and differences?

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states.
Both change two neutrons into two protons.

However, in $2\nu\beta\beta$ the momentum transfer $q <$ few MeV;
thus $e^{iqr} \sim 1$, **long wavelength approximation** is
valid, only the GT operator $\sigma\tau$ need to be considered.

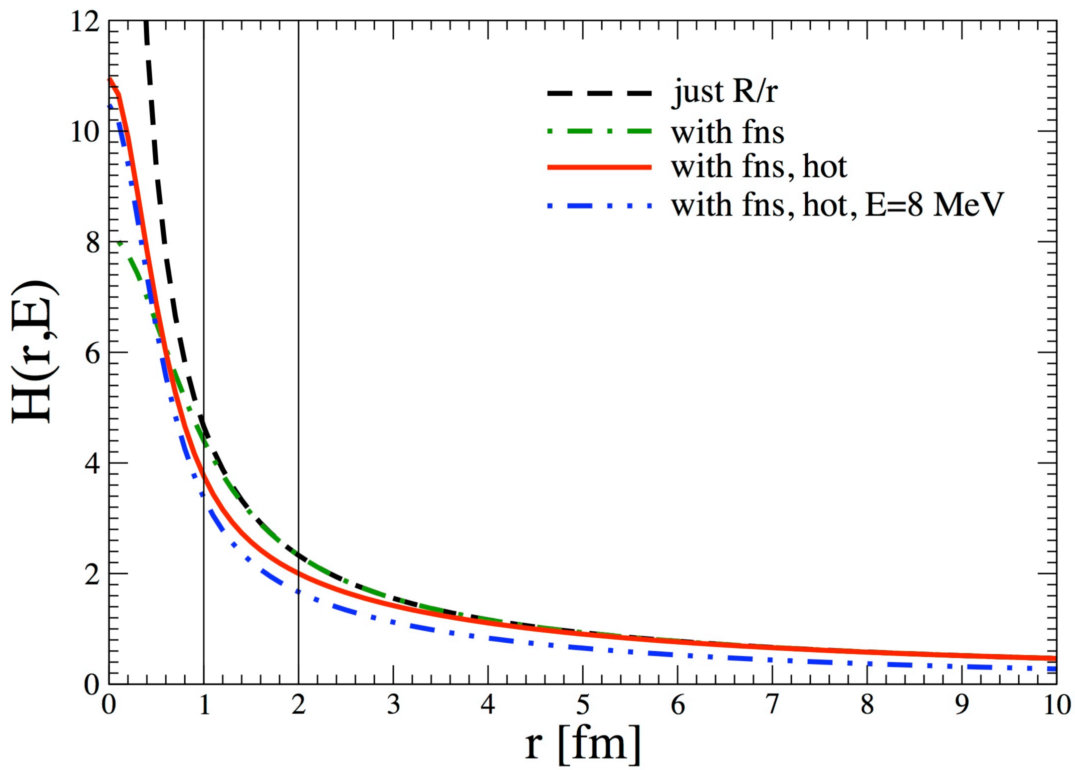
In $0\nu\beta\beta$ $q \sim 100\text{-}200$ MeV, $e^{iqr} = 1 +$ **many terms**, there
is no natural cutoff in that expansion.

Explaining $2\nu\beta\beta$-decay rate is necessary but not sufficient

**On the other hand since q is high in $0\nu\beta\beta$ the closure approximation
is adequate, while in the $2\nu\beta\beta$ we need to sum over all 1^+ intermediate
states in the odd-odd nucleus.**

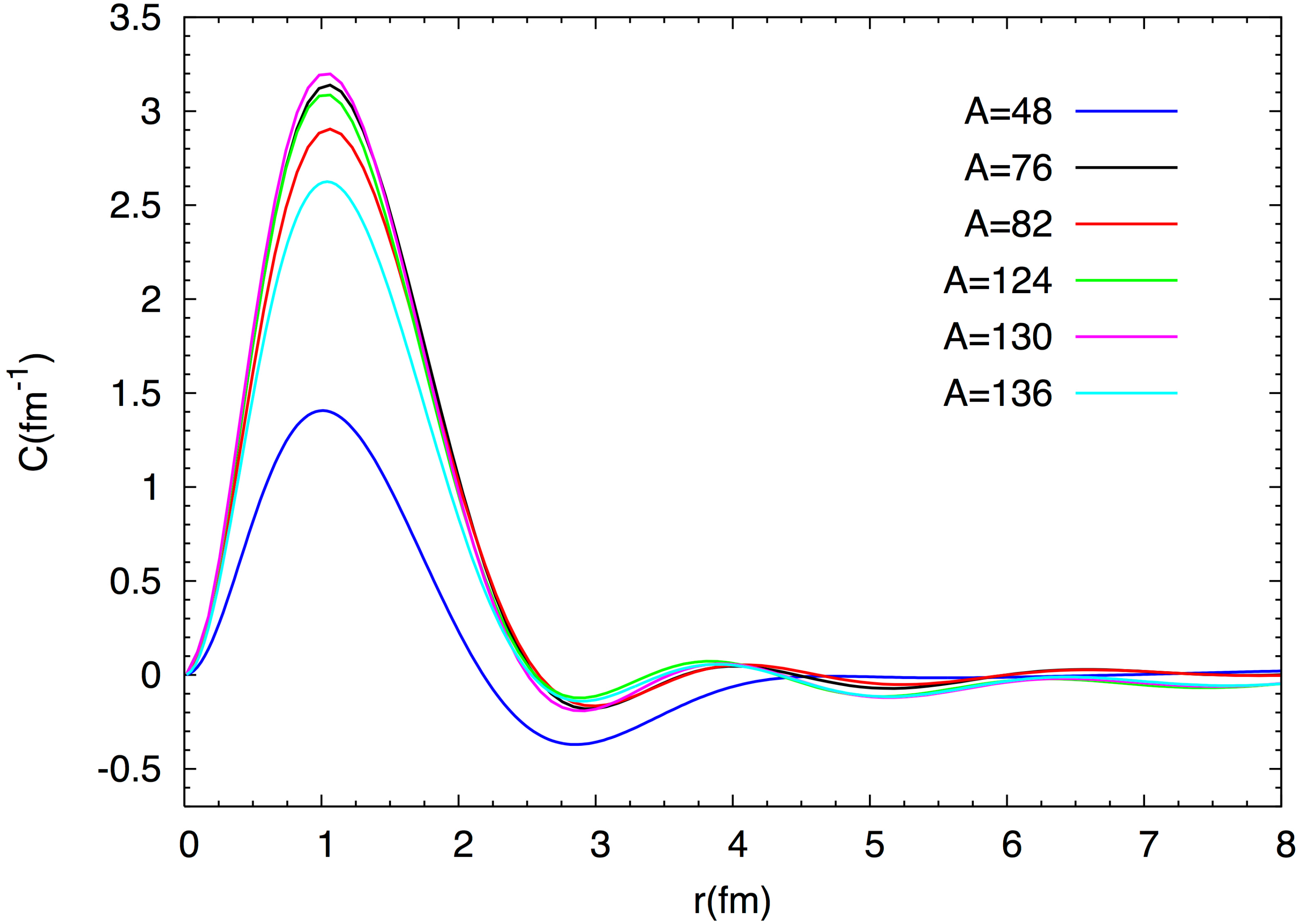
Transition operator contains $\tau^+_1\tau^+_2$ that change neutrons into protons plus in the GT part $\sigma_1\sigma_2$ and in the tensor part operator S_{12}. Each of these parts in multiplied by the `neutrino potential' (Fourier transform of the propagator) that introduces dependence on the radial distance between the nucleons.

$$H(r, E_m) = \frac{R}{2\pi^2} \int \frac{d\vec{q}}{\omega} \frac{1}{\omega + A_m} e^{i\vec{q}\cdot\vec{r}} = \frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + A_m)} = \frac{2R}{\pi} \int_0^\infty dq \frac{j_0(qr) q}{q + A_m} \ .$$



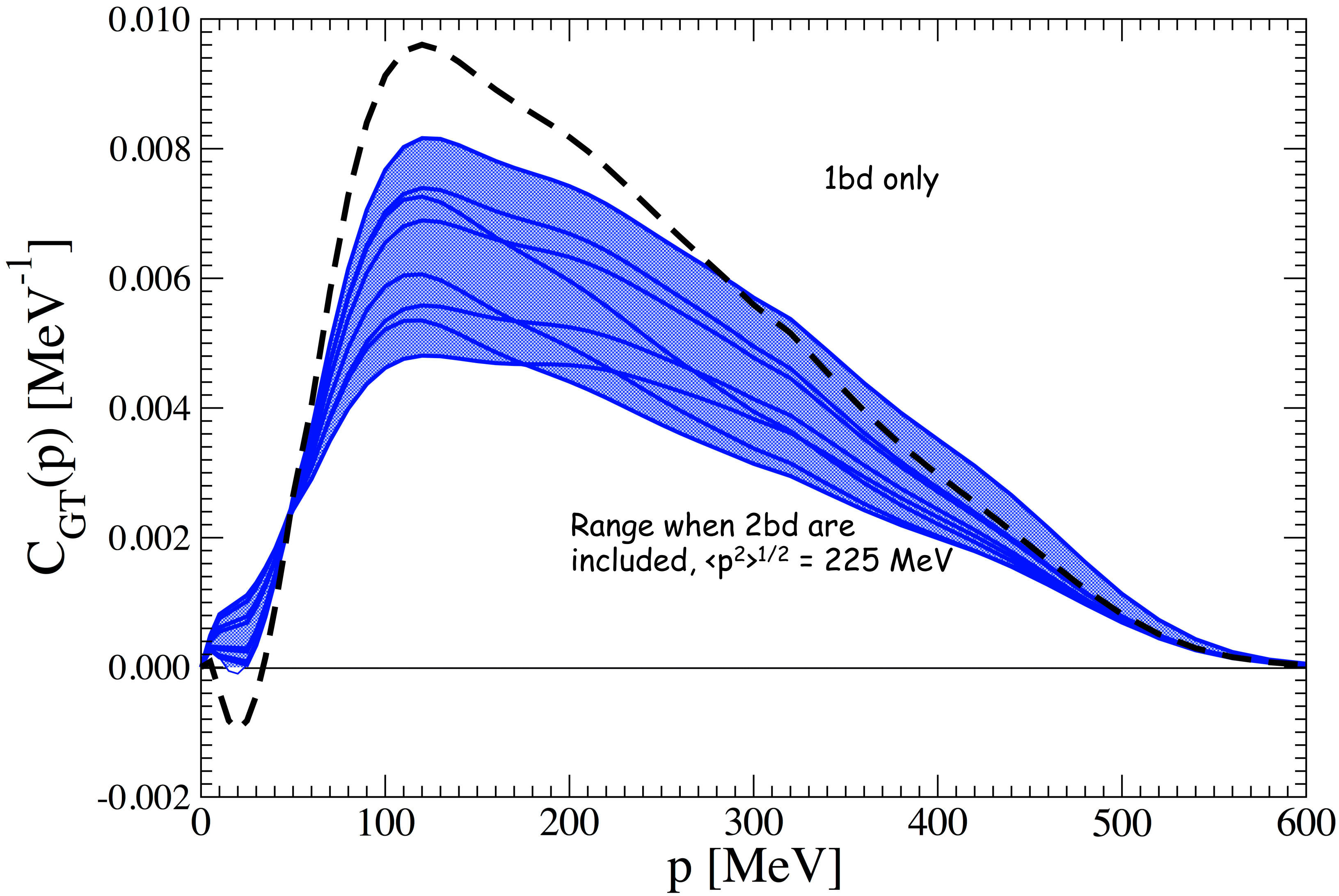
fns....nucleon finite size
hot...higher order terms in weak currents

The radial dependence of $M^{0\nu}$ ($M^{0\nu} = \int C(r)dr$) for the indicated nuclei.
Only distances r < 2-3 fm contribute, substantially than $R_{nucl.}$
(Identical result obtained in QRPA). Nuclear finite size and short range repulsion need to be included carefully.



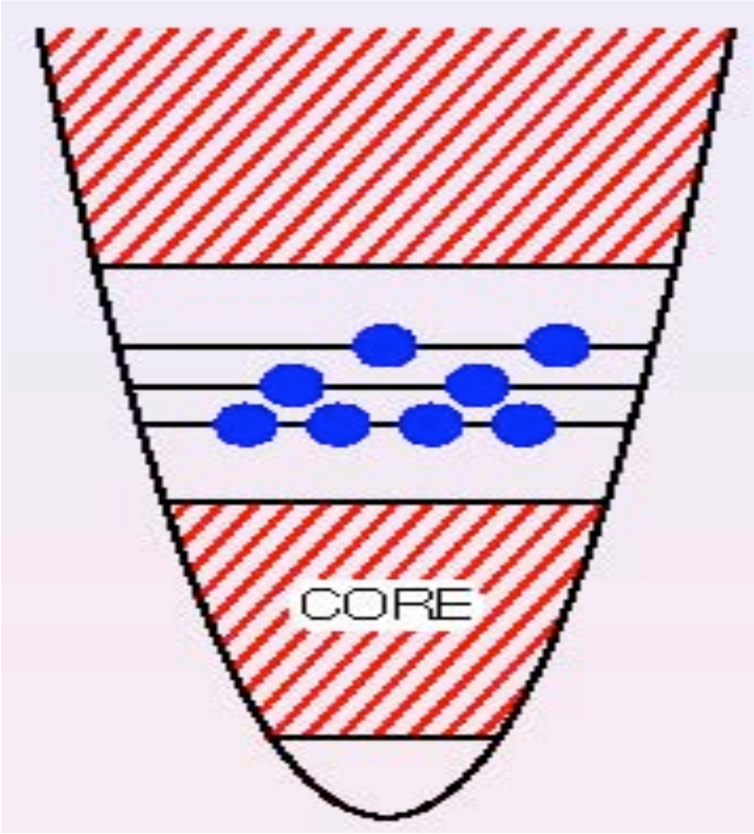
Shell model evaluation, Menendez et al., Nucl.Phys. **A818**, 139 (2009)

Momentum distribution of the contributions to $M^{0\nu}$.($M^{0\nu} = \int C(p)dp$). This example is for ^{136}Xe with QRPA. The $<p^2>^{1/2}$ is 15%-20% larger here than in the nuclear shell model.



From Engel, Simkovic and Vogel, PRC **89, 064308 (2014)**

Basic procedures: Treat the nucleus as a collection of protons and neutrons bound in a potential well, and interacting through an effective interaction. The procedure consists of several steps:



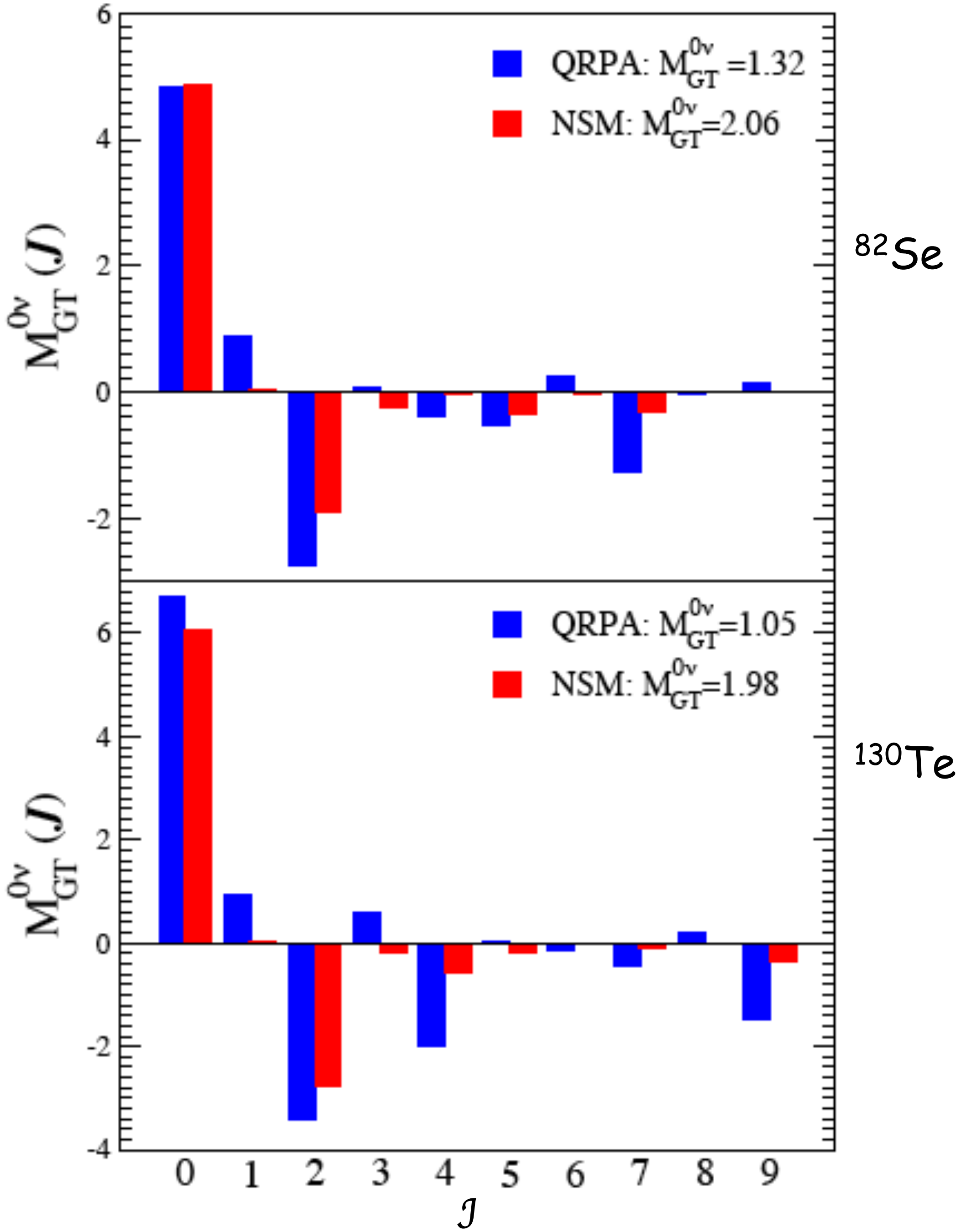
1) Define the valence space
2) Derive the effective hamiltonian H_{eff} using the nucleon-nucleon interaction plus some empirical nuclear data.
3) Solve the equations of motion to obtain the ground state wave functions

Two complementary procedures are commonly used:
a) Nuclear shell model (NSM)
b) Quasiparticle random phase approximation (QRPA)

In NSM a **limited** valence space is used but **all** configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$ calculations.

In QRPA a **large** valence space is used, but **only a class** of configurations is included. Describes collective states, but not details of dominantly few-particle states. Relatively simple, thus many $0\nu\beta\beta$ calculations.

Calculations of the $0\nu\beta\beta$ nuclear matrix elements were also performed in the Interacting Boson Model (IBM-2) as well as, more recently, using the Energy Density Functional method (or Generator Coordinate Method) that describes in particular quite well the effects related to the nuclear deformation and includes essentially unrestricted valence single particle space.

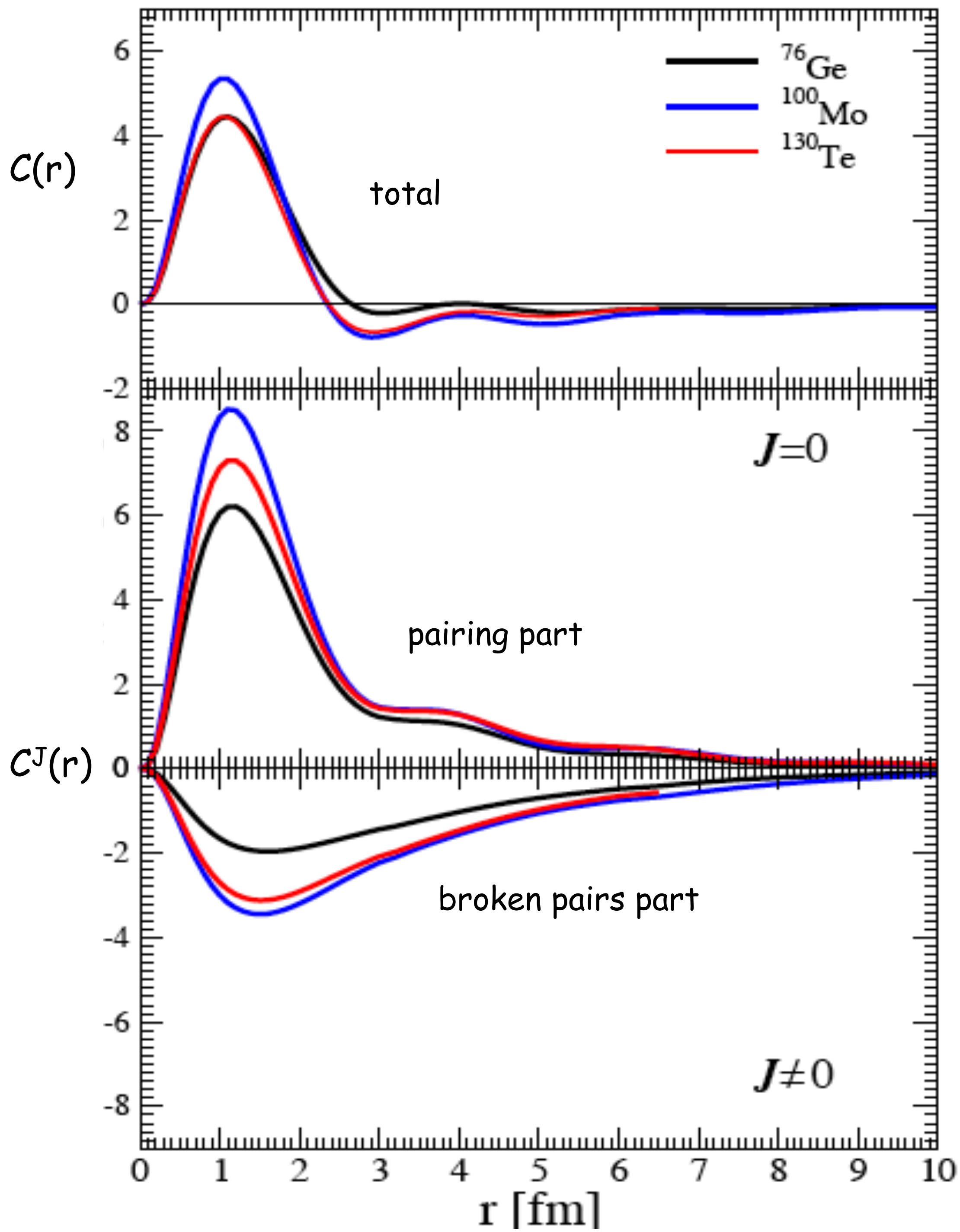


Why it is difficult to calculate the matrix elements accurately?

Contributions of different angular momenta J of the neutron pair that is transformed in the decay into the proton pair with the same J.

Note the opposite signs, and thus tendency to cancel, between the $J = 0$ (pairing) and the $J \neq 0$ (ground state correlations) parts.

The same restricted s.p. space is used for QRPA and NSM. There is a reasonable qualitative agreement between the two methods



The radial dependence of $M^{0\nu}$ for the three indicated nuclei. The contributions summed over all components ss shown in the upper panel. The `pairing' $J = 0$ and `broken pairs' $J \neq 0$ parts are shown separately below.

Note that these two parts essentially cancel each other for r > 2-3 fm. This is a generic behavior. Hence the treatment of small values of *r* and large values of *q* are quite important.

$$M^{0\nu} = \int C(r) dr$$

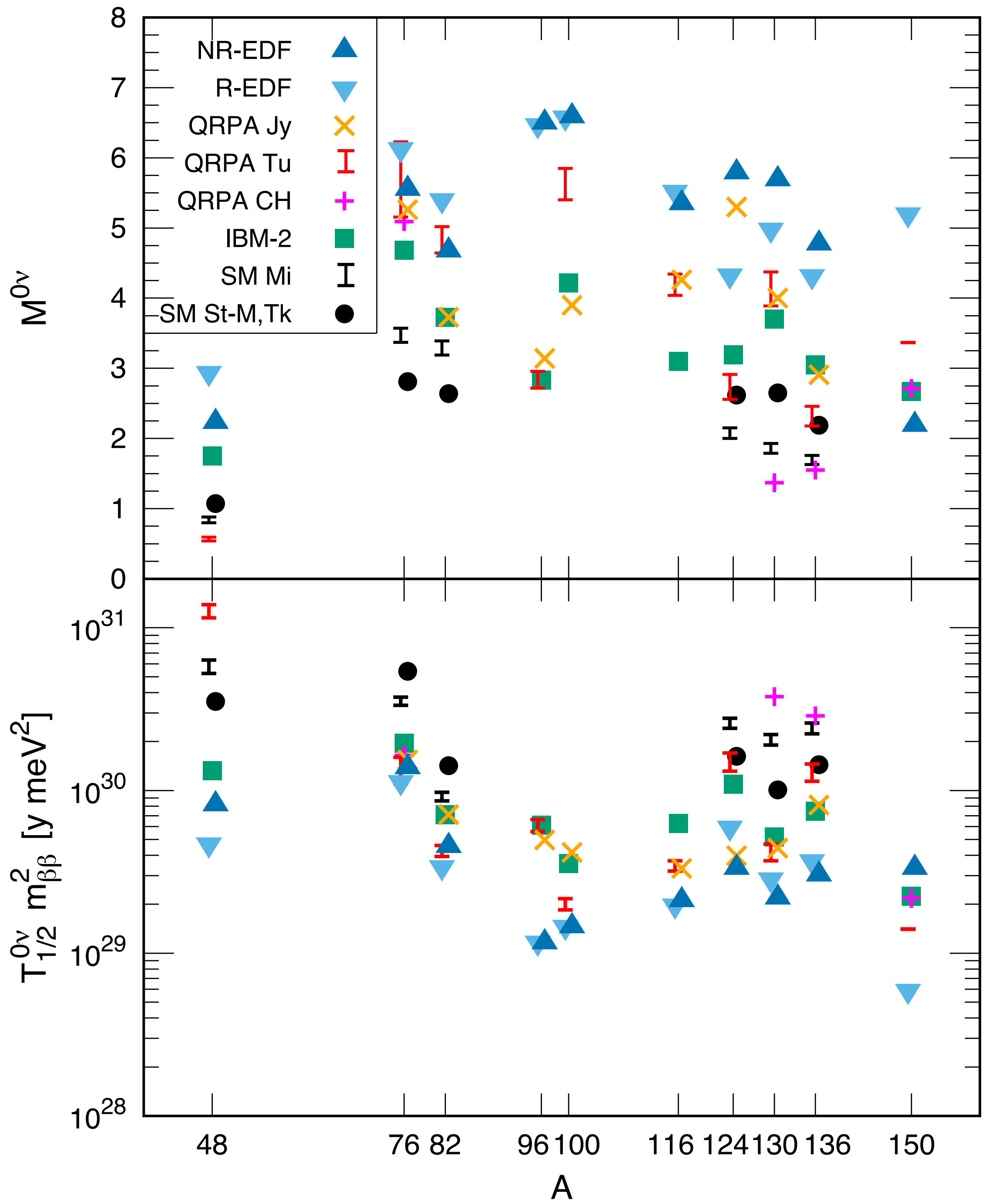
The finding that the relative distances r < 2- 3 fm, and correspondingly that the momentum transfer q > ~100 MeV means that one needs to consider a number of effects that typically play a minor role in the structure of nuclear ground states:

a) Short range repulsion
b) Nucleon finite size
c) Induced weak currents (Pseudoscalar and weak magnetism)

Each of these, with the present treatment, causes correction (or uncertainty) of ~20% in the $0\nu\beta\beta$ matrix element.

There is a consensus now that these effects must be included and even how they should be treated. Nevertheless, they obviously contribute a substantially uncertainty to the calculated values.

However, if the spread between the matrx element values evaluated using different nuclear models can be treated as a measure of uncertainty, it would dominate the final uncertainty by a big factor.



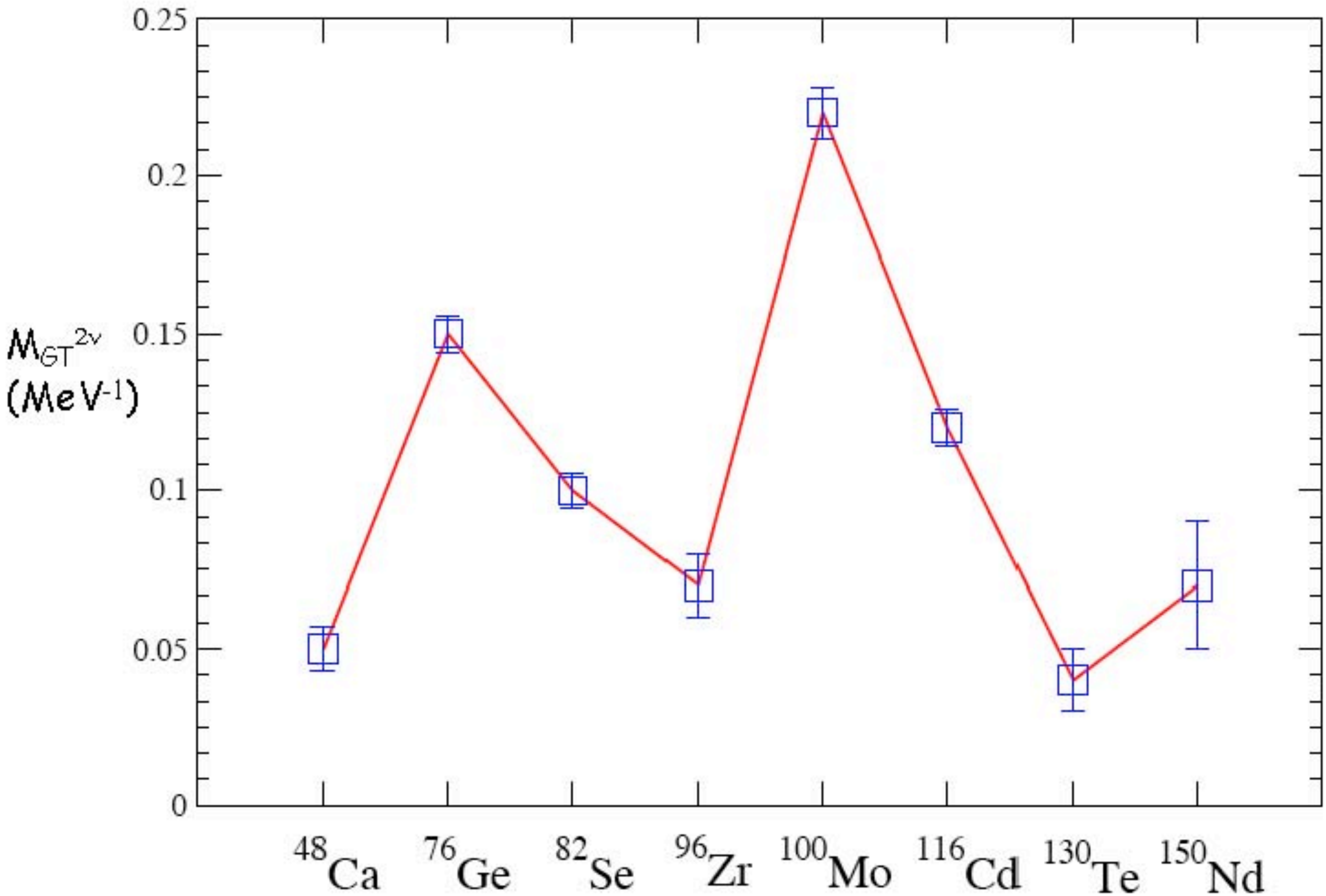
There are many evaluations of the matrix elements $M^{0\nu}$ using different methods and thus different approximations. It is difficult to conclude which of them is most realistic; each has its strong and weak points.

The spread of the $M^{0\nu}$ values for each nucleus is ~ 3. On the other hand, there is relatively little variation from one nucleus to the next.

In the lower panel the corresponding half-lives for $m_{\beta\beta}$ = 1 meV are shown. Obviously, the spread now is ~10, but again, there is no clear preference for any of the candidate nuclei.

Figure from review by Engel and Menendez

The 2ν matrix elements, unlike the 0ν ones, exhibit pronounced shell effects. They vary relatively fast as a function of Z or A.



Shell model (black dots) $M^{0\nu}$ are usually smallest, presumably for two reasons.

1) All configurations of valence nucleons are included. Complicated states with high seniority, that are absent in other approaches, further decrease the $M^{0\nu}$ values.
2) Only limited number of orbits can be included. In particular the spin-orbit partners are not included, except in ^{48}Ca. Perturbation tests suggest that including additional orbits would increase the $M^{0\nu}$ values.

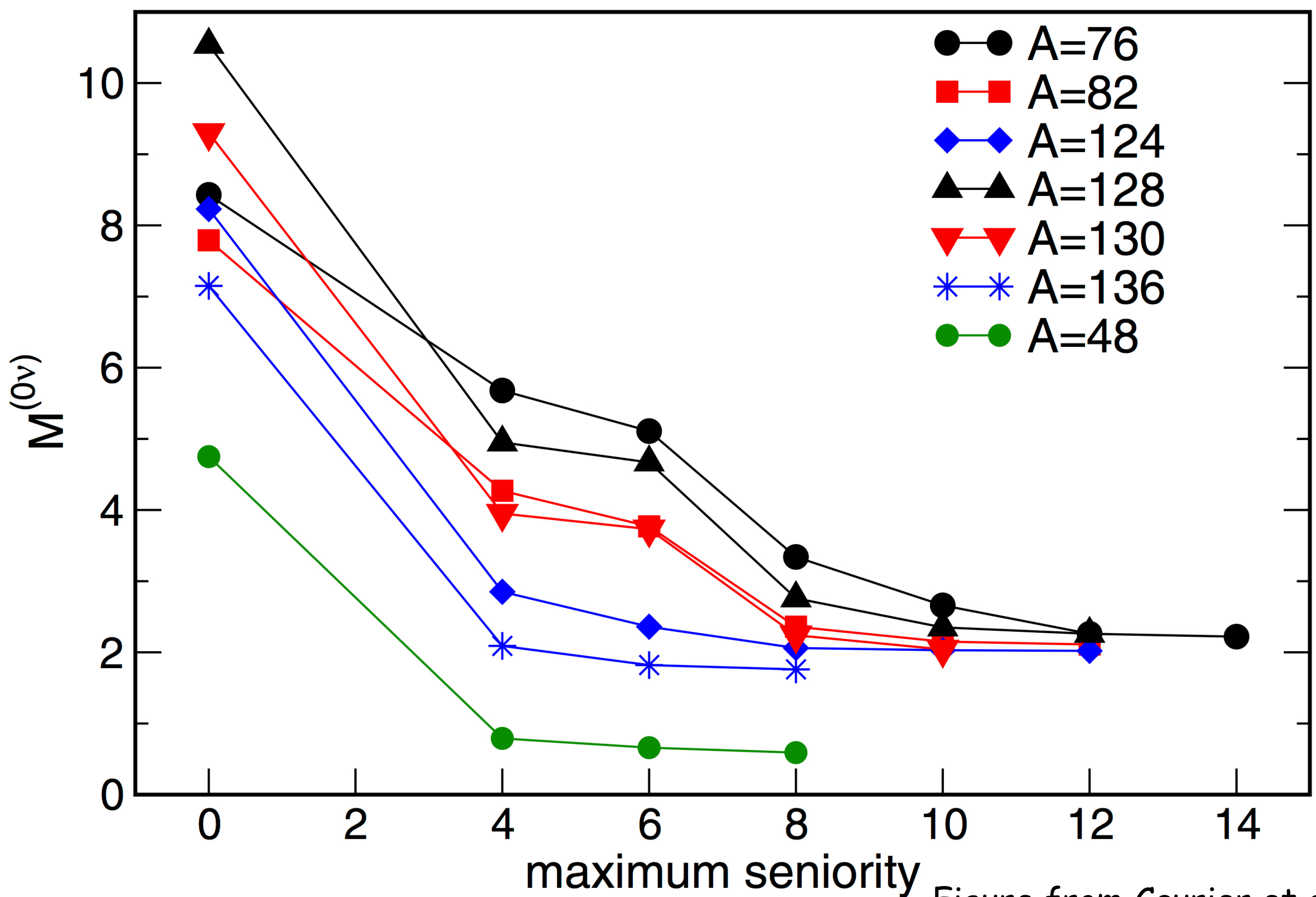


Figure from Caurier et al., PRL **100**, 052503 (2008)

The EDF/GCM (Energy Density Functional/Generator Coordinate Method) (blue triangles) gives typically largest $M^{0\nu}$ values. It has several advantages, large s.p. space, realistic treatment of deformation and standard pairing, projection on the correct angular momentum and particle number.

However, it does not include the isoscalar pairing(at least for now) , that is known to substantially reduce the $M^{0\nu}$ values.

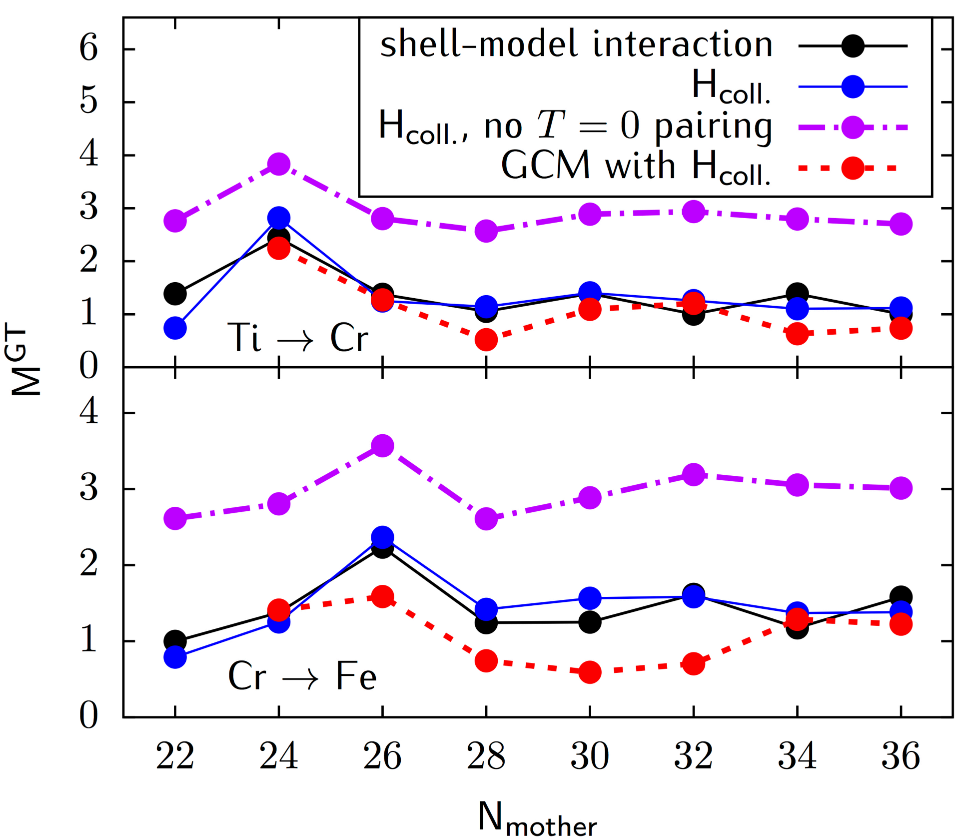


Illustration of the effect of isoscalar pairing for the GT (largest) part of $M^{0\nu}$. These isotope chains are not real $\beta\beta$ decay candidates, but the calculation shows that the effect is quite general.

From Menendez et al., PRC **93**,014305(2016)

Conclusions from the spread in calculated values:

1) It is difficult to decide which of the nuclear models used until now is most realistic.
2) We have reasons to believe that the NSM, which gives consistently the smallest values, is an underestimate.
3) In analogy, the EDF approach, which gives the largest values, is very likely an overestimate.
4) However, all model evaluations agree that there is no abrupt change in magnitude from one candidate nucleus to another one. Thus, at least from that point of view, there is no obvious advantage or disadvantage in using any of them.

However, we cannot exclude the possibility that all of the evaluations used so far leave out a common problem that may affect all of them. In fact, the quenching of the axial current matrix elements is an example of such an issue.

At this time the most widely discussed source of uncertainty is the ``effective" value of the axial current coupling constant g_A.

From the free neutron β decay the g_A = 1.27 is determined. However, it is possible, and there is some evidence, that in the nuclear many-body systems the effective values of g_A is different, and possibly smaller.

The decay rate of the $2\nu\beta\beta$ mode is proportional to $g_A{}^4$. For the $0\nu\beta\beta$ decay, the axial current part is still dominating. Thus any modification of g_A in heavy nuclei would affect the calculated half-life substantially.

The ISM predictions for the matrix element of several 2ν double beta decays (in MeV^{-1}). See text for the definitions of the valence spaces and interactions.

	$M^{2\nu}$(exp)	q	$M^{2\nu}$(th)	INT
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.047	kb3
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.048	kb3g
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.065	gxpf1
$^{76}Ge \rightarrow {}^{76}Se$	0.140 ± 0.005	0.60	0.116	gcn28:50
$^{76}Ge \rightarrow {}^{76}Se$	0.140 ± 0.005	0.60	0.120	jun45
$^{82}Se \rightarrow {}^{82}Kr$	0.098 ± 0.004	0.60	0.126	gcn28:50
$^{82}Se \rightarrow {}^{82}Kr$	0.098 ± 0.004	0.60	0.124	jun45
$^{128}Te \rightarrow {}^{128}Xe$	0.049 ± 0.006	0.57	0.059	gcn50:82
$^{130}Te \rightarrow {}^{130}Xe$	0.034 ± 0.003	0.57	0.043	gcn50:82
$^{136}Xe \rightarrow {}^{136}Ba$	0.019 ± 0.002	0.45	0.025	gcn50:82

From E. Caurier, F. Nowacki and A. Poves, Phys. Lett. **B711**, 62 (2012)

Since the $2\nu\beta\beta$ decay is simply two GT decays happening at once, the rate is proportional to g_A^2. Thus, by choosing $g_A^{eff} = q\, g_A$, with $q < 1$ so called quenching factor, it is possible to phenomenologically account for any discrepancy.

In M. Horoi and B.A.Brown, arXiv:1301.0256 more single-particle states were included, so that the Ikeda sum rule was obeyed. For ^{136}Xe 2ν matrix element the $M^{2\nu}$ = 0.020 MeV^{-1} was then obtained with quenching q=0.74. So, the inclusion of spin-orbit partners reduces the quenching value to more acceptable values.

In Corraggio et al. (1703.05087) the $2\nu\beta\beta$ decay in ^{130}Te and ^{136}Xe was treated in a realistic shell model, and $q \sim 0.65$ was required, less than above.

Quenching of GT matrix elements deduced from the β decay of the *sd* shell nuclei (A = 17-39). Comparison between the calculated and experimental strength. Typical reduction ~0.77. It is remarkable that one parameter is sufficient to bring the experiment and theory in agreement over a wide region of nuclei.

(from Brown & Wildenthal, Ann.Rev.Nucl. Part.Sci.**38**,(1988)29)

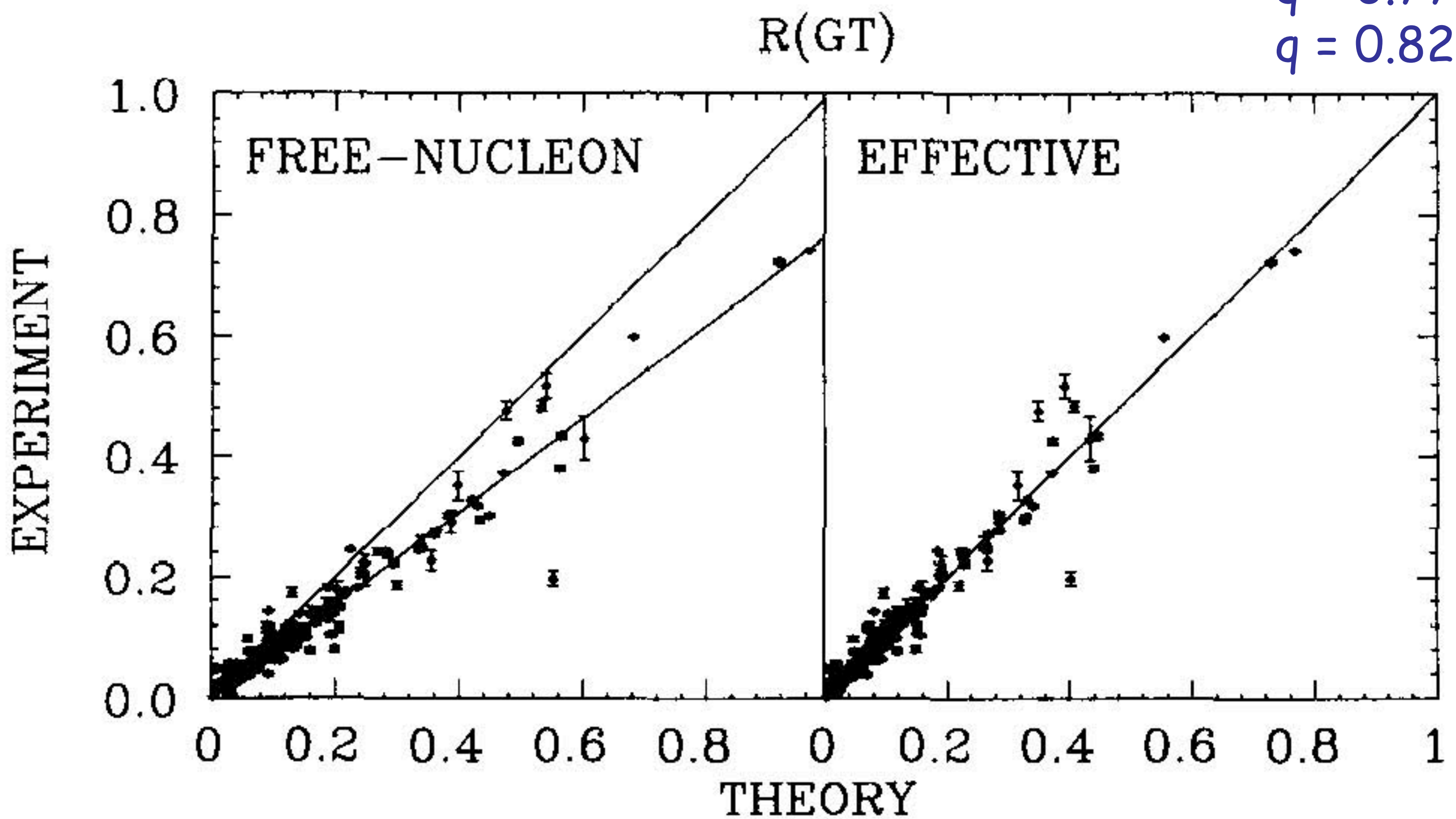
Other quenching factors

$q = g_A^{eff}/g_A$

$q = 0.744 \pm 0.015$ *pf*

$q = 0.77 \pm 0.02$ *sd*

$q = 0.82 \pm 0.02$ *p*



Quenching of the axial current operator

When the rate of ordinary β decay is calculated in the nuclear shell model, the corresponding theoretical Gamow-Teller matrix elements are typically larger than their experimental values. Their ratio, however, is nearly constant for a given group of nuclei, when the valence nucleons are in a specified shell.

To account for that effect the quenching factor $q < 1$ is introduced, that reduces the matrix elements of the GT operator $\sigma\tau$. A convenient way to handle that is to pretend that the coupling g_A is reduced to $q x g_A$.

Note that the total GT strength, the sum of squares of m.e. over all final states is constrained by the model independent **Ikeda sum rule**

$$S(\beta^-) - S(\beta^+) = 3(N-Z) \text{ , i.e. } S(\beta^-) > 3(N-Z)$$

That sum rule is fulfilled in theory if **all** single particle states of an oscillator shell are included in the calculation, including the spin-orbit partners.

The crucial question: Is the quenching needed in $0\nu\beta\beta$?
Are the quenching factors similar to those of $2\nu\beta\beta$?

Since the M_{GT} gives the largest contribution to $M^{0\nu}$, the $0\nu\beta\beta$ rate is approximately proportional to $g_A{}^4$.

Warning: If quenching of q=0.45 is needed, the $\langle m_{\beta\beta} \rangle$ sensitivity is reduced by q^2 = 0.2, i.e. 5 times.

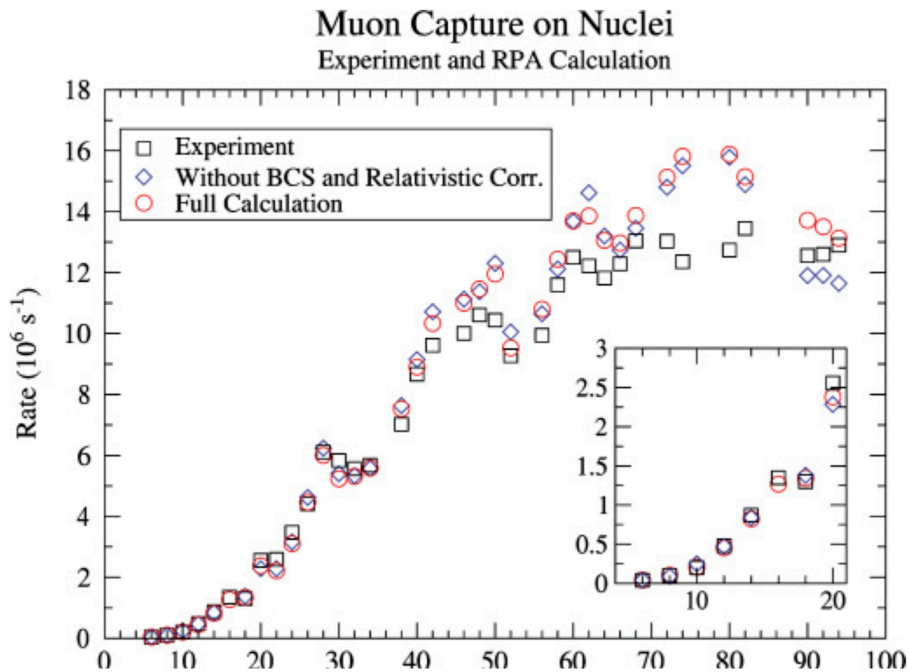
Remember, in $2\nu\beta\beta$ only intermediate 1^+ states participate and the momentum transfer q ~ few MeV.
In $0\nu\beta\beta$ many multipoles contribute and q ~ 100-200 MeV. So the answer to that question is not straightforward.

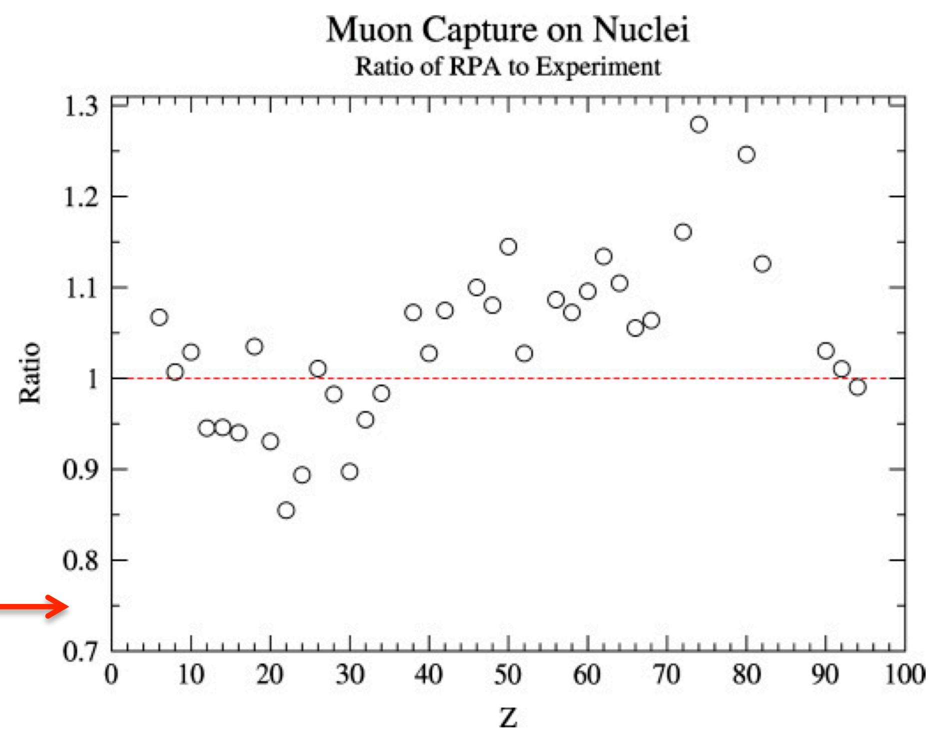
What about weak processes with other multipolarities, e.g. forbidden β decays or μ capture? Are they quenched?

Muon capture on nuclei,
$\mu^- + (Z,A) \rightarrow \nu_\mu + (Z-1, A)$.

Calculation using RPA
(see N. Zinner, K.Langanke and P. Vogel, Phys. Rev. C**74**, 024326(2006))

This process is dominated by dipole transitions.
No quenching is required.
In fact, using $g_A = 1.0$ would underestimate the rate by ~0.75.





Ratio for $g_A = 1.0$ →

Comparison of the calculated and experimental matrix elements for the full set of all known unique second forbidden β decays, |ΔI| = 3, Δπ = no (see G. Martinez-Pinedo and P. Vogel, Phys. Rev. Lett.**81**, 282(1998).

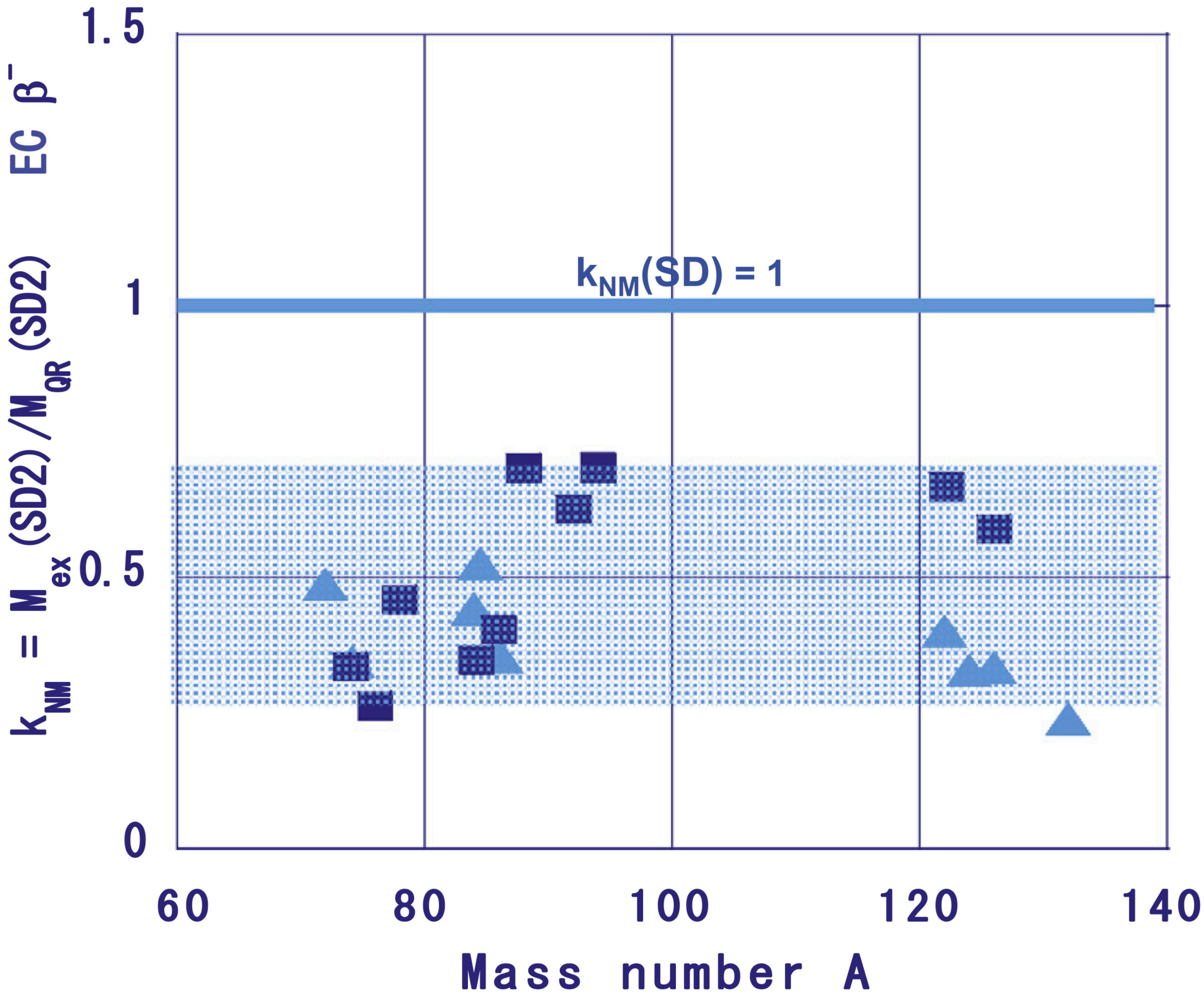
The average quenching factor is ~0.8; but ratios are not constant

TABLE I. Form factors and half-lives for the β^+ and β^- unique second-forbidden transitions.

	$R^{2\,A}F^0_{321}$ (fm^2)			Half-life (years)		
	HO	Woods-Saxon	Expt	HO	Woods-Saxon	Expt
^{54}Mn(β^+)^{54}Cr	7.82	7.76	7.1 ± 0.7	5.55×10^8	5.64×10^8	$(6.7 \pm 1.3) \times 10^8$
^{54}Mn(β^-)^{54}Fe	11.7	11.6		4.89×10^5	4.98×10^5	
^{22}Na(β^+)^{22}Ne	9.24	9.78	6.0 ± 0.8	2.04×10^3	1.87×10^3	$(4.8 \pm 1.3) \times 10^3$
^{26}Al(β^+)^{26}Mg [a]	2.44	2.78	2.38 ± 0.05	8.64×10^5	6.65×10^5	$(9.1 \pm 0.4) \times 10^5$
^{26}Al(EC)^{26}Mg [a]	2.44	2.78	2.39 ± 0.05	4.58×10^6	3.52×10^6	$(4.8 \pm 0.2) \times 10^6$
^{26}Al(EC)^{26}Mg [b]	12.6	13.8	8.8 ± 0.5	1.43×10^7	9.44×10^6	$(2.7 \pm 0.3) \times 10^7$
^{10}Be(β^-)^{10}B	23.1	23.3	20.4 ± 0.4	1.18×10^6	1.16×10^6	$(1.51 \pm 0.06) \times 10^6$

[a] The first-excited state at 1.809 MeV in ^{26}Mg.
[b] The second-excited state at 2.938 MeV in ^{26}Mg.

Matrix elements for unique first forbidden $2^- \to 0^+$ β decays in medium mass nuclei. The plotted ratio is M_{exp}/M_{QRPA} for both β^- and β^+ decays.



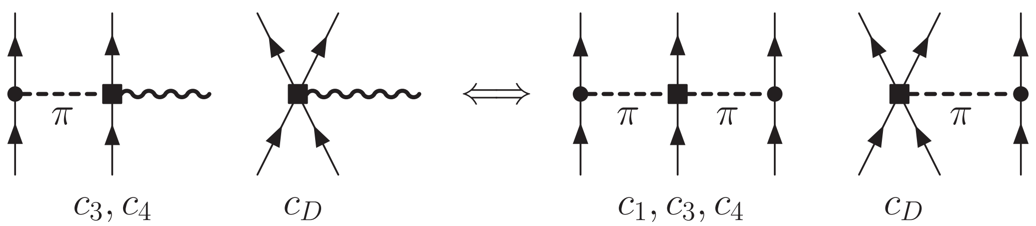
From Ejiri et al., Phys. Lett. B729, 27 (2014)

Determination of the quenching factor $q = g_A^{eff}/g_A$ from the $2\nu\beta\beta$ decay experimental matrix elements

1) In NSM $q \sim 0.74$ is obtained when the full oscillator shell is included
2) In QRPA there is no prediction. The isoscalar interaction constant g_{pp} is adjusted so that the $2\nu\beta\beta$ experimental half-life is obtained.
3) In EDF it is impossible, so far, to evaluate the spectrum of 1^+ states in the virtual intermediate odd-odd nucleus. Thus evaluation of $M^{2\nu}$, without closure, is impossible.
4) The same is true in IBM-2. However, the authors, Barea et al., Phys. Rev. *C* **87**, 014315 (2013) argue that the closure approximation might be acceptable and obtain quenching factors that are typically smaller than in ISM, e.g. for ^{136}Xe $q^{IBM\text{-}2} = 0.41$. Note, that this is obtained by assuming that the average energy denominator is $E = 1.12\ A^{1/2}$ MeV, roughly the energy of the giant GT resonance. The QRPA and/or NSM do not support this assumption.

One of the suggested explanations of quenching as due to the two-body currents (see Menendez, Gazit, and Schwenk, PRL **107**, 062501 (2011))
(This is related to the older ideas of couplings nucleons to the Δ isobars)

Using chiral effective field theory they derive expressions for a significant, and momentum dependent, modification of the axial weak current effective coupling. See also Klos, Menendez, Gazit, and Schwenk PRD **88**, 083516 for more developments of these ideas.

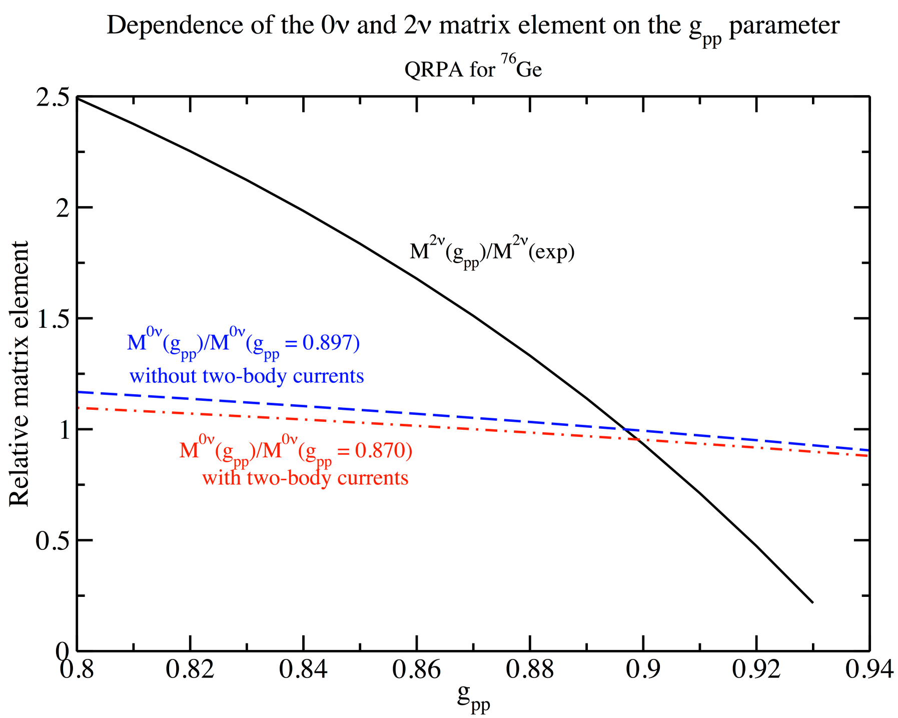


Chiral two-body currents and the 3N force depend on the same couplings. Their values are taken from the previous works.

For a variety of c_3 and c_4 values and c_D = 0 the quenching q = 0.66 – 0.85 is obtained for realistic nuclear densities.

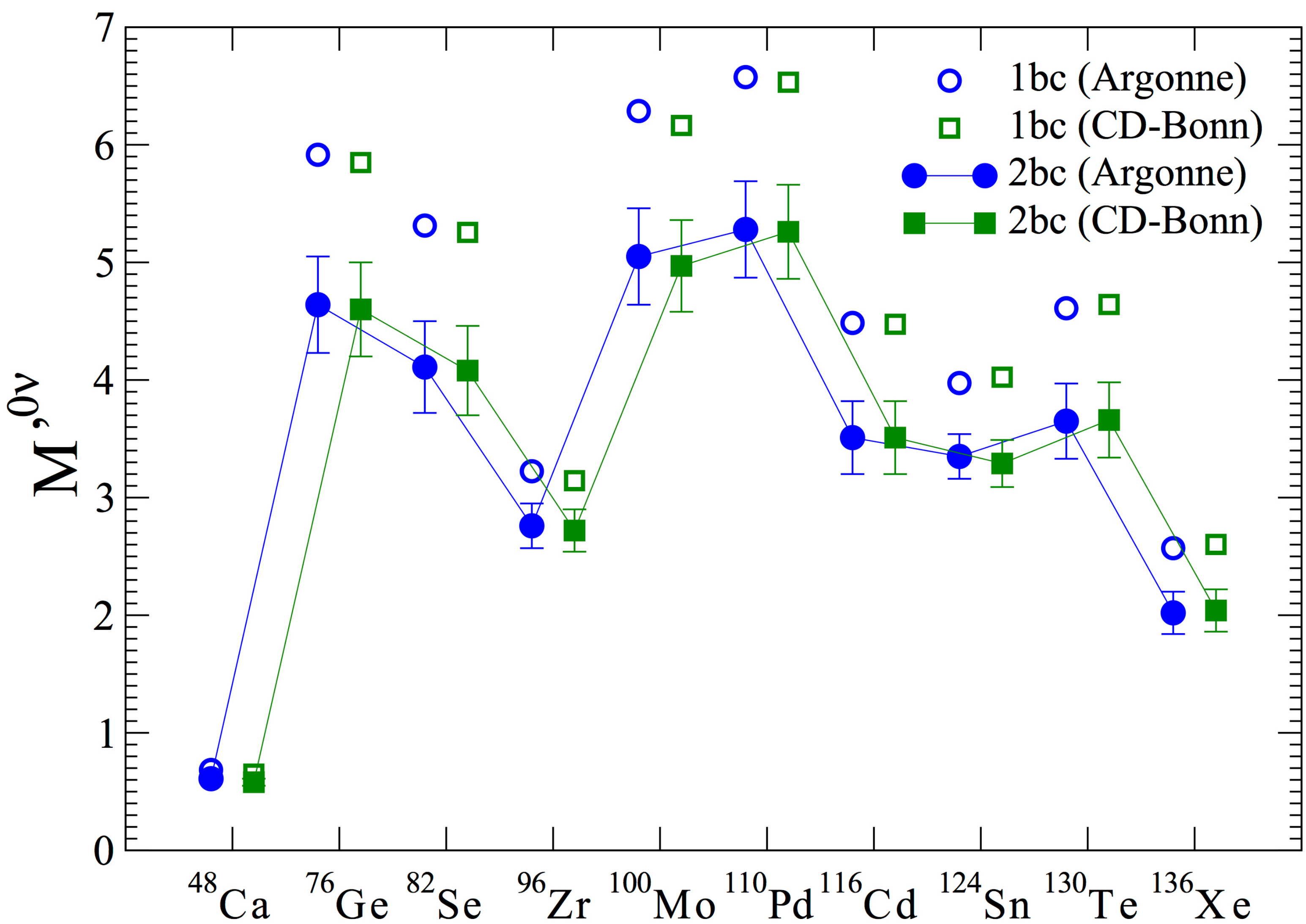
Application of chiral two body currents to the evaluation of $M^{0\nu\beta\beta}$ nuclear matrix elements within the QRPA.
(Simkovic, Engel, Vogel, Phys.Rev. C89 (2014), 064308)

The 2b currents are included, together with isospin restoration. The resulting $M^{0\nu\beta\beta}$ are reduced by **~20%**, while $M^{2\nu\beta\beta}$ are reduced by **66%** compared to the unquenched (i.e. one-body currents only) results. The reduction of $M^{0\nu\beta\beta}$ is somewhat less than in the shell model due to the usual g_{pp} adjustment, even though the effect is rather small.



Note the steep dependence of $M^{2\nu\beta\beta}$ on g_{pp} and the mild dependence of $M^{0\nu\beta\beta}$.
The values g_{pp} = 0.897 and 0.870 are the values that reproduce the experimental $M^{2\nu\beta\beta}$ without and with 2b currents.

Reduction by including the 2-body currents (full circles) compared to the 1-body currents only (empty circles).

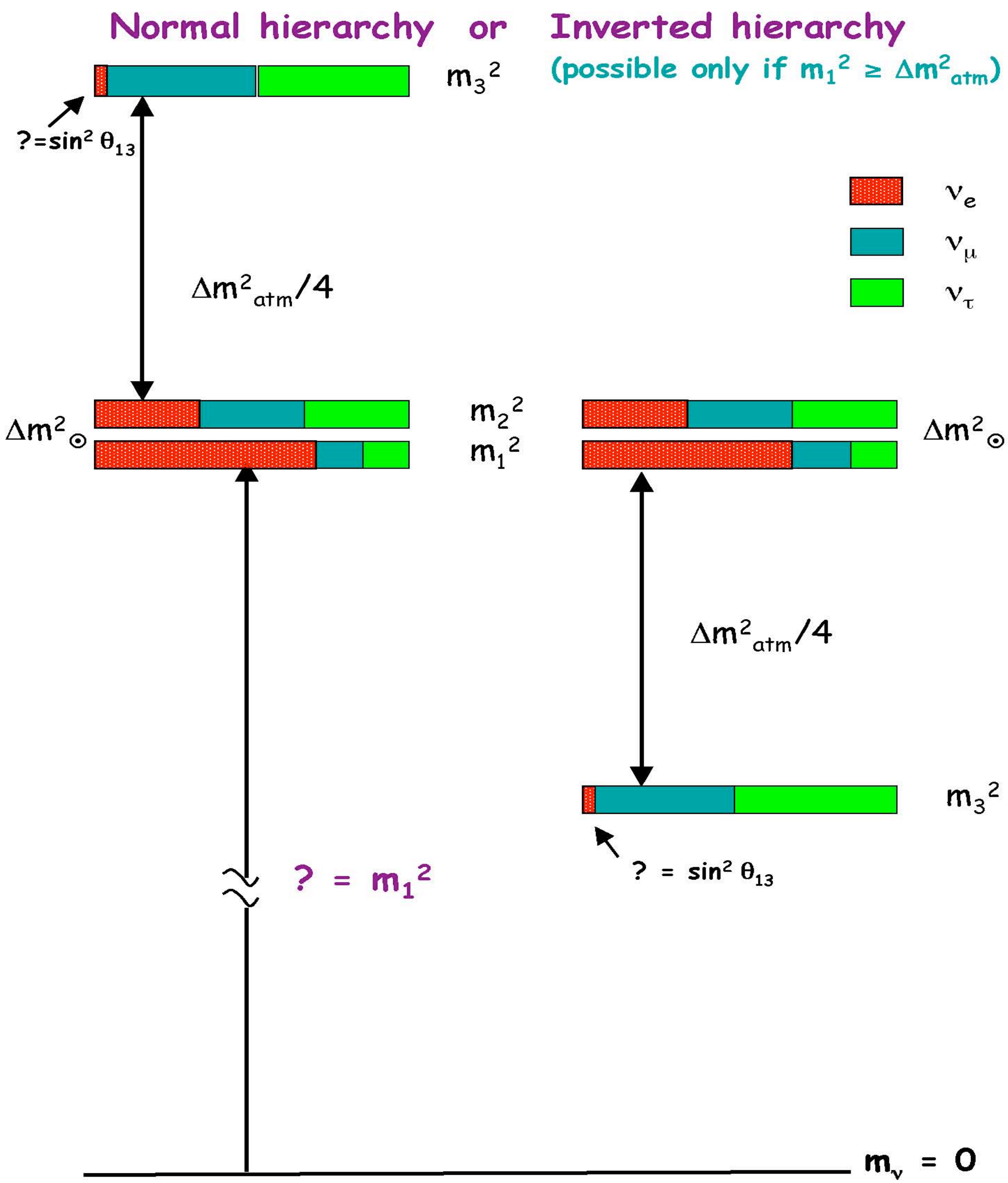


From Simkovic, Engel, Vogel, Phys.Rev. C89 (2014), 064308

Can we make any conclusions regarding g_A quenching?

1) Not really. However, all available evidence suggests that the $M^{0\nu}$ evaluated with g_A = 1.27, i.e. without quenching, is an overestimate.
2) The real issue is whether the matrix elements should be reduced by 20-30% or by a factor of of 3-5. The former would make experiments more difficult but still doable, the latter one would be a game changer.
3) Clearly, this is a crucial issue that needs a convincing solution.

spares



The following quantities are unknown at present:

a) The mass m_1

b) Whether the normal or inverted hierarchy is realized.

c) The CP phase δ

d) Most importantly, we do not know whether neutrinos are Dirac or Majorana, i.e. whether the total lepton number is conserved or not. Need to study $0\nu\beta\beta$ decay to decide this question.

SM extensions with low (~TeV) scale LNV contain additional operators that enhance $\mu \to e$ conversion

Left-right symmetric model,
R-parity violating SUSY, etc.
possibly $\Gamma_{0\nu\beta\beta}$ unrelated to $m_{\beta\beta}^2$

In them the double ratio,
(ratio of branching ratios)
$\mathcal{R} = B_{\mu \to e} / B_{\mu \to e\gamma} \gg 10^{-2}$

** ***In absence of fine-tuning or hierarchies in flavor couplings. Important caveat!***
See: V. Cirigliano et al., PRL**93**,231802(2004)

Thus the double ratio is a diagnostic tool: a link between LNV and LFV

Also, if LFV is not found in the next round of experiments, the TeV scale of LFV and LNV becomes even less probable.

Outline:

- Introduction: What is $\beta\beta$ decay, candidate nuclei, the allowed $2\nu\beta\beta$ mode, significance of the $0\nu\beta\beta$ mode, other tests of the lepton number conservation.

- Mechanism of the decay. Exchange of a light Majorana neutrino or a TeV scale physics?

- Effective Majorana mass $\langle m_{\beta\beta} \rangle$ and its relation to other ways of neutrino mass determination.

- Nuclear matrix elements. Issues related to nucleon structure. How important these things are and how to include them in the evaluation of nuclear matrix elements?

- Nuclear matrix elements and nuclear structure. Can we estimate the relevant uncertainty? What the comparison of different methods tells us?

Historical interlude:

$0\nu\beta\beta$-decay and right-handed currents (Doi *et al.* 1985)

$$H_W = G_F/\sqrt{2}\,[\, j_L J_L^\dagger + \eta j_R J_L^\dagger + \lambda j_R J_R^\dagger \,]\,,$$

Where $j_{L,R}$ is the lepton current, and $J_{L,R}$ hadron current.

From the $0\nu\beta\beta$ decay rate one can determine $\langle\lambda\rangle = \lambda\,\Sigma_j\, U_{ej}V_{ej}$, and $\langle\eta\rangle = \eta\,\Sigma_j\, U_{ej}V_{ej}$ (sum over light ν only, V_{ej} is the mixing matrix of the right-handed neutrinos)

The present limits are $\langle\lambda\rangle \leq \sim 10^{-6}$, $\langle\eta\rangle \leq \sim 10^{-8}$.

Can any of this realistically happen with the light neutrino exchange?

The parameters λ and η are obviously < 1.

In addition, if all neutrinos are light the sum $\Sigma_j U_{ej}V_{ej}$ vanishes due to unitarity of the mixing matrix.
To obtain a nonvanishing contribution we must then expand the neutrino propagator $1/(q^2 - m_\nu^2) \sim 1/q^2(1 + m_\nu^2/q^2)$ and have additional suppression $\Delta m_j^2/q^2$. This is small unless existence of O(keV) (or even O(100 keV)) very weakly coupled neutrinos is assumed.

Suppose that the right handed neutrinos are heavy. Then the mixing coefficient $V_{ej} \sim m_{light}/m_{heavy} \ll m_{light}/E_\nu \ll 1$.

Thus it is unlikely that the $0\nu\beta\beta$-decay mediated by λ and η, and mediated by a long range exchange, can compete with the one mediated by $\langle m_{\beta\beta}\rangle$ and is thus unobservable at present.