Beam loading

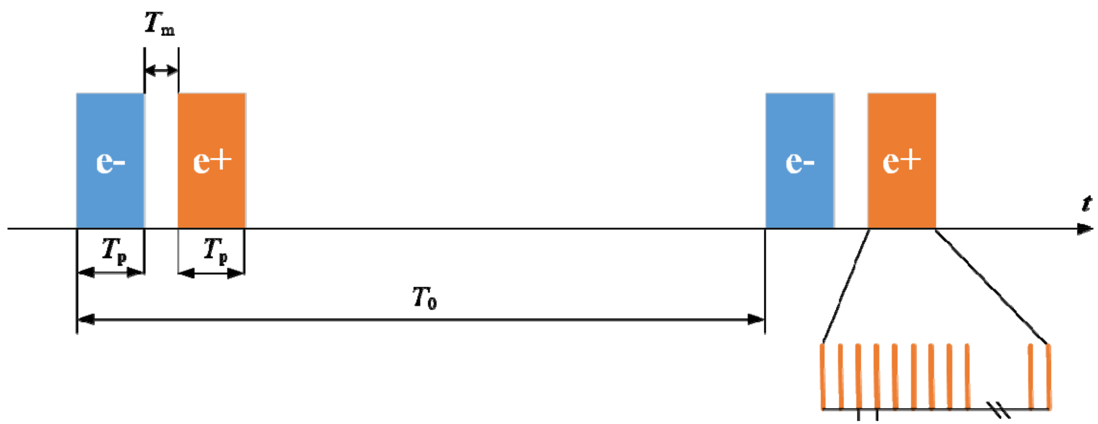
Sun Yuansehng,Zhang Yuan ,Zhai Jiyuan,Wang Na

2016.12.30

Outline

Beam-cavity interaction

Fill pattern



Voltage

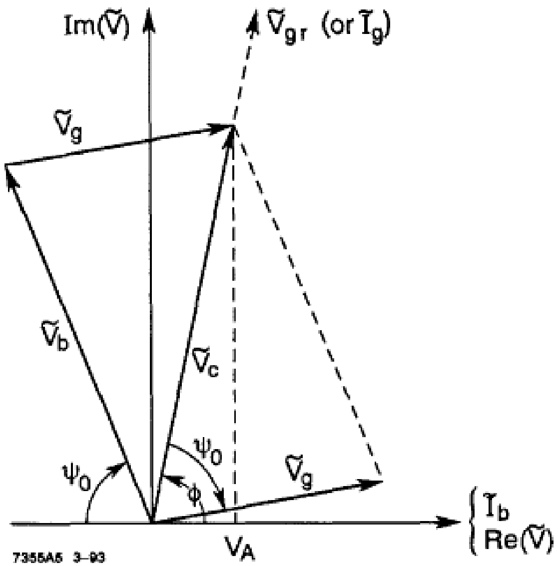


Figure 4.2.1. Phasor relationships for voltages in a PEP-II RF cavity at optimum tuning ($\tilde{I}_g$ colinear with $\tilde{V}_c$) and coupling (no reflected power).

$$\tilde{V}_c{}^n = \tilde{V}_b{}^n + \tilde{V}_g{}^n$$

The generator voltage is normally independent of n

ϕ : synchrotron phase

Beam-cavity interaction

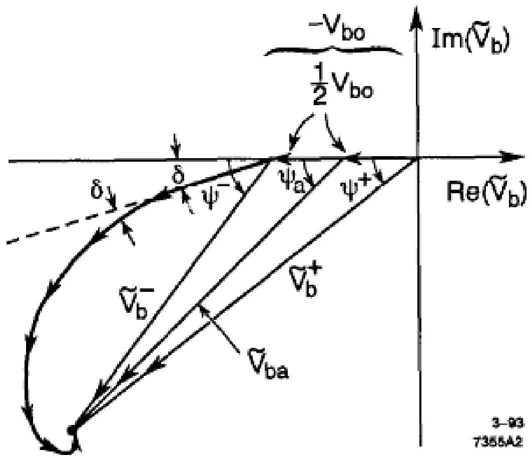


Figure 3.1.1. Diagram showing the buildup of the beam-induced voltage in a cavity by a train of equally-spaced bunches.

$\overline{V} = V_{b0} e^{(-\sigma + j\omega_R)t}$. $\delta = T_b(\omega_0 - \omega_R)$ $\sigma = 1/\tau$

$$k_\ell \equiv \frac{V^2}{4U} = \frac{\omega_0}{2}\left(\frac{R}{Q}\right) ; \qquad \frac{R}{Q} = \frac{V^2}{2\omega_0 U} ; \qquad \tau \equiv \frac{\Delta T_b}{T_f} ;$$

$$T_f = \frac{2Q_L}{\omega_0} = \frac{2Q}{\omega_0(1+\beta)} ; \qquad V_{b0} = 2k_\ell q = \omega_0 \left(\frac{R}{Q}\right) I_0 \Delta T_b = \frac{2 I_0 R}{1+\beta} \cdot \tau ,$$

$$\tilde{V}^+ = -V_{b0}\left(1 + e^{-\tau} e^{j\delta} + e^{-2\tau} e^{2j\delta} + \quad \right)$$

Beam-cavity interaction

Generator voltage is independent of *n*:

$$\boldsymbol{V}_g = \boldsymbol{V}_{c0} \exp(i\phi_0) - \boldsymbol{V}_{b0}$$

where nominal synchrotron phase:

$$\phi_0 = \cos^{-1}[(U_0 + U_{\mathrm{HOM}})/eV_{c0}]$$

and beam induced voltage of even distribution:

$$\boldsymbol{V}_{b0} = -\frac{2I_0 R}{1+\beta} \cos\psi \, e^{i\psi}$$

where the cavity detuning angle, detuning frequency and filling time:

$$\psi = \tan^{-1}(\Delta\omega T_{\mathrm{F}})$$

$$\Delta\omega = \omega_0 - \omega_{\mathrm{rf}}$$

$$T_{\mathrm{F}} = 2Q_{\mathrm{L}}/\omega_{\mathrm{rf}}$$

Beam-cavity interaction

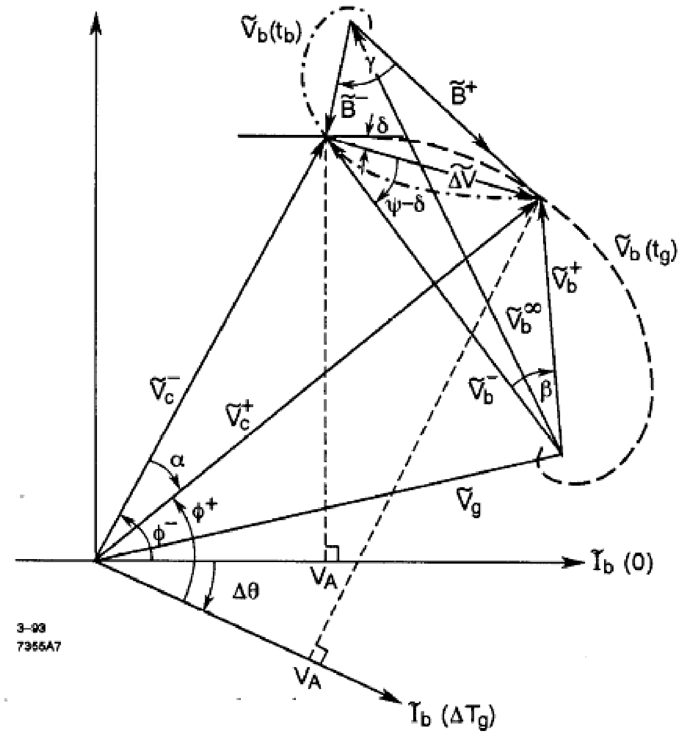


Figure 5.1.2. Phasor geometry for a beam with a gap with arbitrary τ_g and τ_b.

Beam-cavity interaction

$$\tilde{V}_b^+ = -V_{b0}\left(1 + e^{-\tau}\, e^{j\delta} e + e^{-2\tau}\, e^{2j\delta} + \ldots\right)$$

$$\delta = T_b(\omega_0 - \omega_{\mathrm{R}}); \qquad T_f = \frac{2Q_L}{w_0} = \frac{2Q}{\omega_0(1+\beta)}; \qquad \tau = \frac{\Delta T_b}{T_f};$$

$$\mathrm{V_{b0}} = 2\mathrm{k_l q}; = \omega_0 \left(\frac{R}{Q}\right) I_0 T_b$$

$$\mathrm{k}_l = \frac{V^2}{4U} = \frac{\omega_0}{2}\left(\frac{R}{Q}\right); \qquad \frac{R}{Q} = \frac{V^2}{2\omega_0 U};$$

$$\tilde{V_c}^n = \tilde{V_b}^n + \tilde{V_g}^n$$

$$\Delta\theta_{nn'} \approx \frac{\mathrm{Re}(\boldsymbol{V}_b^{n'} - \boldsymbol{V}_b^{n})}{V_{c0}\sin\phi_0}$$

Thanks