# Coupled-channel approach to spin partners of hadronic molecules

A.V. Nefediev

(MIPT, Moscow, Russia)

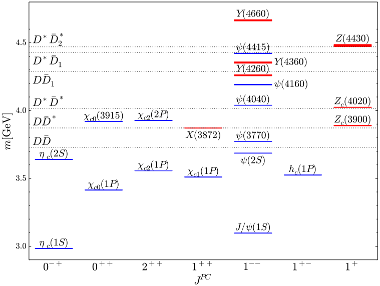
in collaboration with

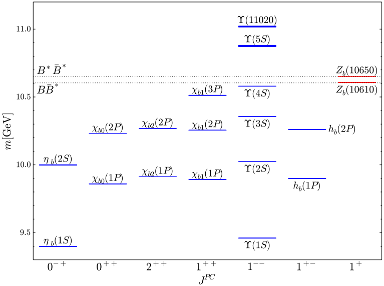
V. Baru, E. Epelbaum, A. A. Filin, F.-K. Guo, C. Hanhart, Yu. S. Kalashnikova, U.-G. Meißner, R. Mizuk, Q. Wang

Key refs: PRL115 (2015), 202001; PRD93 (2016), 074031; PLB763 (2016) 20; JHEP 1706 (2017) 158

QWG 2017

# Introduction




Many hadronic states are found in spectrum of heavy quarks which

- do not fit into the quark model scheme
- reside near $S$-wave open-flavour thresholds
- have large decay branchings to nearby channels

These are strong candidates to hadronic molecules

(to be distinguished from tetraquarks, Esposito et al (2014))

# Heavy-quark spin symmetry

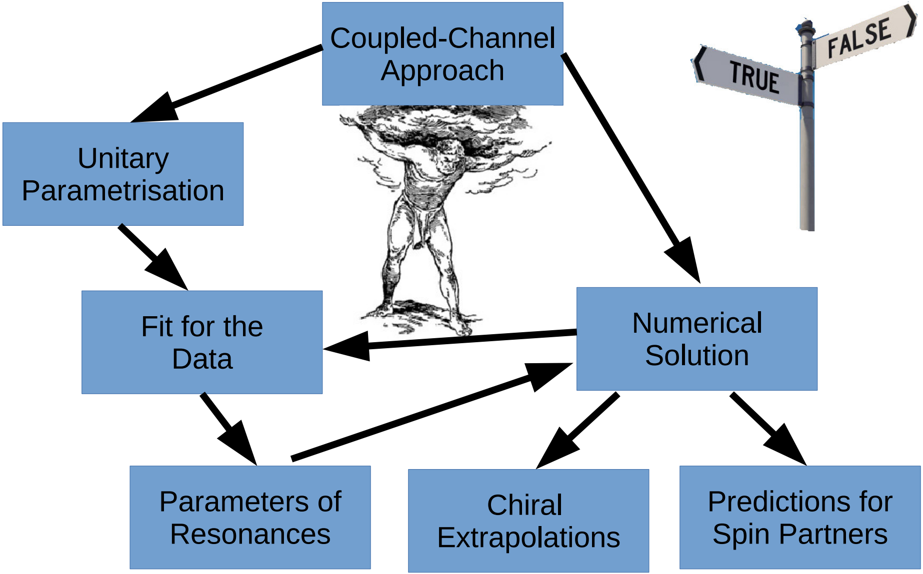
- Exotic $XYZ$ states contain heavy quarks (HQ)
- In the limit $m_Q \to \infty$ $(m_Q \gg \Lambda_{\rm QCD})$ spin of HQ decouples

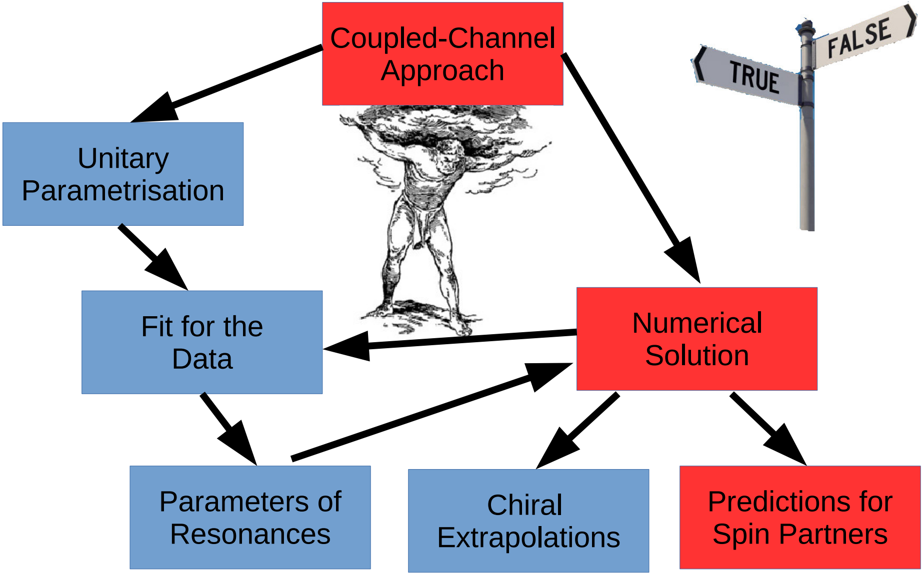
  $\Longrightarrow$ Heavy Quark Spin Symmetry (HQSS)
- For realistic $m_Q$'s HQSS is approximate but rather accurate symmetry of QCD
- HQSS is a tool to study properties of states with different HQ spin orientation

  $\Longrightarrow$ Spin partners
- Predictions of HQSS depend crucially on the nature of states under study

  (Cleven et al (2015))
- Disclaimer: In this talk, only molecular scenario is discussed
- Quarkonium component of the w.f. (if exists) may impact the predictions

  (Cincioglu et al. (2016))


Coupled-Channel Approach
Unitary Parametrisation
Fit for the Data
Numerical Solution
Parameters of Resonances
Chiral Extrapolations
Predictions for Spin Partners
TRUE
FALSE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# Parameters and Input

- Short-range elastic interactions $\Longrightarrow$ Low-Energy Constants
- Transition potential between channels $\Longrightarrow$ Coupling constants
- Overall normalisation constants
- Bare poles (not necessary for $Z_b$'s)

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HQ limit $\Longrightarrow$ Reduced # of parameters

- Ways to proceed
  - **Proper way:** combined coupled-channel fit for all measured channels
    Example of $Z_b(10610)$ and $Z_b(10650)$ (7 channels):

    $\Longrightarrow$ 7 parameters for profile of line shapes

    $\Longrightarrow$ 7 parameters for overall norms

    $\Longrightarrow$ CL$\simeq$50%
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Binding energies of $X(3872)$ and $Z_b(10610)/Z_b(10650)$ will be used as input

# Spin partners: Low-Energy Constants (LEC's)

- $J^{PC}$ states made of $P$ ($D$ or $B$) and $V$ ($D^*$ or $B^*$) meson:

$$\begin{aligned} 0^{++}: &\quad \{P\bar{P}(^1S_0), V\bar{V}(^1S_0)\} \\ 1^{+-}: &\quad \{P\bar{V}(^3S_1,-), V\bar{V}(^3S_1)\} \\ 1^{++}: &\quad \{P\bar{V}(^3S_1,+)\} \\ 2^{++}: &\quad \{V\bar{V}(^5S_2)\} \end{aligned}$$

- In HQ limit, short-range elastic potentials depend on two LEC's

(Grinstein et al. (1992), AlFiky et al. (2006), Nieves & Valderrama (2012))

$$V_{\rm LO}^{(0^{++})} = \frac{1}{4}\begin{pmatrix} 3C + C' & -\sqrt{3}(C - C') \\ -\sqrt{3}(C - C') & C + 3C' \end{pmatrix}$$

$$V_{\rm LO}^{(1^{+-})} = \frac{1}{2}\begin{pmatrix} C + C' & C - C' \\ C - C' & C + C' \end{pmatrix}$$

$$V_{\rm LO}^{(1^{++})} = V_{\rm LO}^{(2^{++})} = C$$

- No symmetry relates $C$ and $C'$ for different isospins
- HQ symmetry relates LEC's in $c$- and $b$-sector, but accuracy unclear

# Spin partners: Predictions of contact theory

- HQSS limit: $\delta = m_V - m_P \ll E_B \ll m$

  $\Longrightarrow$ two decoupled families

$$E^{(0)}_{2^{++}} = E^{(0)}_{1^{++}} = E^{(0)}_{1^{+-}} = E^{(0)}_{0^{++}} \qquad E^{(0)\prime}_{1^{+-}} = E^{(0)\prime}_{0^{++}}$$

(Hidalgo-Duque et al. (2013), our work (2016))

- In actuality: $E_B \ll \delta \ll m$

  $\Longrightarrow$ Expansion parameter $\sqrt{E_B/\delta}$

(Bondar et al. (2011), Voloshin (2011), Mehen & Powell (2011), our work (2016))

- $V^{(1^{++})}_{\rm LO} = V^{(2^{++})}_{\rm LO}$ (only one input needed)

  $\Longrightarrow$ $1^{++}$ bound state at $D\bar{D}^*$ threshold $(X(3872))$ implies

  $2^{++}$ bound state at $D^*\bar{D}^*$ threshold $(X_{c2})$

(Nieves & Valderama (2013))

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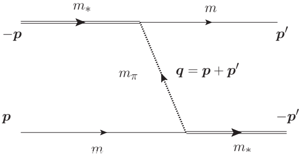
- $2^{++}$ state is uncoupled and has zero width

# One-pion exchange

- Pionic Lagrangian

$$\mathcal{L} = \frac{g_Q}{2f_\pi}\left(\boldsymbol{V}^\dagger \cdot \boldsymbol{\nabla}\pi^a \tau^a P + P^\dagger \tau^a \boldsymbol{\nabla}\pi^a \cdot \boldsymbol{V} + i[\boldsymbol{V}^\dagger \times \boldsymbol{V}] \cdot \boldsymbol{\nabla}\pi^a \tau^a\right)$$

- OPE potential




$$V^{ij}_{P\bar{V} \to \bar{P}V}(\boldsymbol{p}, \boldsymbol{p}') = -\frac{g_Q^2}{(4\pi f_\pi)^2}(\boldsymbol{\tau} \cdot \boldsymbol{\tau}^c)\frac{q_i q_j}{D_3(\boldsymbol{p}, \boldsymbol{p}')}$$

$$E_\pi = \sqrt{\boldsymbol{q}^2 + m_\pi^2}$$

$$D_3(\boldsymbol{p}, \boldsymbol{p}') = 2E_\pi\left[\left(m + \frac{\boldsymbol{p}^2}{2m} + m + \frac{\boldsymbol{p}'^2}{2m} + E_\pi\right) - (m_* + m + E)\right]$$

- When $m_* > m + m_\pi \implies$ Three-body cut
- When recoil terms neglected $\implies$ Static OPE
- When $q_i q_j \to \frac{1}{3}\boldsymbol{q}^2 \delta_{ij} \implies$ Central ($S$-wave) OPE

## Spin partners: OPE included on top of LEC's

- $P$-wave $V \to P(V)\pi$ vertices $\Longrightarrow$ Extended basis

$$
\begin{aligned}
0^{++}: &\quad \{P\bar{P}(^1S_0), V\bar{V}(^1S_0), V\bar{V}(^5D_0)\}\\
1^{+-}: &\quad \{P\bar{V}(^3S_1,-), P\bar{V}(^3D_1,-), V\bar{V}(^3S_1), V\bar{V}(^3D_1)\}\\
1^{++}: &\quad \{P\bar{V}(^3S_1,+), P\bar{V}(^3D_1,+), V\bar{V}(^5D_1)\}\\
2^{++}: &\quad \{P\bar{P}(^1D_2), P\bar{V}(^3D_2), V\bar{V}(^5S_2), V\bar{V}(^1D_2), V\bar{V}(^5D_2), V\bar{V}(^5G_2)\}
\end{aligned}
$$

Important and cannot be ignored:

- Coupled-channel dynamics
- High momenta ($q \sim 500$ MeV)
- $D$ waves ($q^2/m_\pi^2$ is large)
- Three-body dynamics ($c$-sector)

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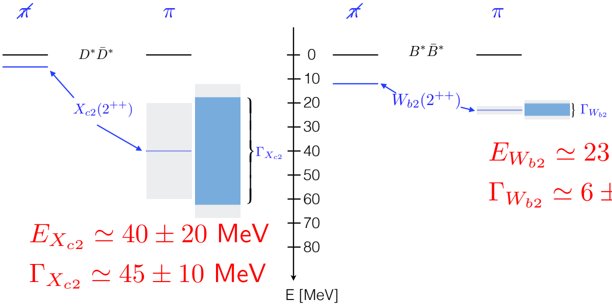
OPE couples $2^{++}$ channel to other channels $\Longrightarrow$ finite width

# Spin partners: Results for the $2^{++}$ state

**$c$-sector; $I = 0$** — (V.Baru et al. PLB763 (2016) 20)

**$b$-sector; $I = 1$** — (V.Baru et al. JHEP 1706 (2017) 158)




$E_{X_{c2}} \simeq 40 \pm 20$ MeV

$\Gamma_{X_{c2}} \simeq 45 \pm 10$ MeV

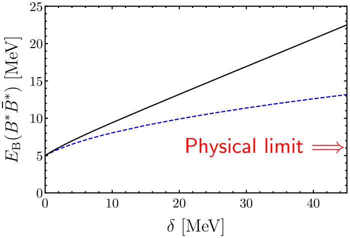
$E_{W_{b2}} \simeq 23 \pm 2$ MeV

$\Gamma_{W_{b2}} \simeq 6 \pm 1$ MeV

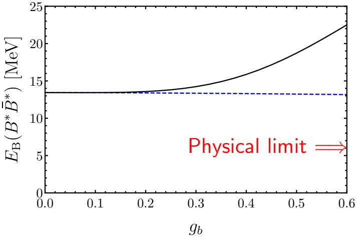
- Source of uncertainty: UV regulator ($c$-sector) and input ($b$-sector)
- Impact of HQSS violation:
  - Stronger in $c$-sector than in $b$-sector
  - Stronger than for perturbative pions (cf. Albaladejo et al. (2015))
- Role of three-body effects
  - Sizable in $c$-sector (reduce $E_{X_{c2}}$ and $\Gamma_{X_{c2}}$ by $\simeq 20\%$)
  - Marginal in $b$-sector

# $W_{b2}(2^{++})$: Dependence on input and parameters of interaction

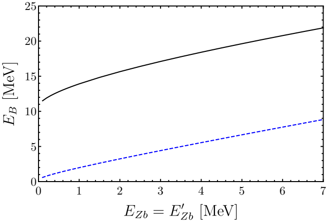
Dynamical OPE enhahces HQ symmetry breaking



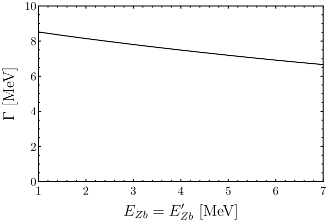

Dynamical OPE cannot be absorbed into LEC's




$E_B[W_{b2}]$ survives large input variations




$\Gamma[W_{b2}]$ is robust against input variations




Blue dashed line: contact+(static $S$-wave OPE)

Black Solid line: contact+(full dynamic OPE)

# Conclusions

- The proposed systematic approach to hadronic molecules respects all relevant symmetries (chiral symmetry, HQSS, unitarity, analyticity) and allows to
  - build a practical parametrisation for line shapes
  - extract parameters of resonances directly from data
  - build chiral extrapolations
  - investigate various molecular candidates in $c$- and $b$-sectors
- HQSS breaking and nonperturbative pions have significant impact on near-threshold states
- OPE is more operative in the $c$-sector than in the $b$-sector

# Conclusions

- $X_{c2}$ is broad and shifted away from the $D^*\bar{D}^*$ threshold (observable?)

- $W_{b2}$ is narrow and attracted to the $B^*\bar{B}^*$ threshold

  $\Longrightarrow$ Must be resolvable agaist the $B^*\bar{B}^*$ threshold

  $\Longrightarrow$ May be produced in $\Upsilon(5S) \to \gamma W_{b2}$ (Voloshin (2011))

  $\Longrightarrow$ May be seen in $B\bar{B}$, $B\bar{B}^*$, $\pi\chi_{b1}$, $\pi\chi_{b2}$, $\rho\Upsilon(1S)$ modes

- Predictions for the $Z_b$'s spin partners are rather accurate due to
  - Weak HQSS breaking in the $b$-sector $(\Lambda_{\rm QCD}/m_b \ll 1)$
  - No $\bar{b}b$ quarkonium admixture in w.f.'s (isovectors)
  - Small uncertainty caused by UV regulator (damped by large $m_b$)
  - Robust with respect to input variations (at least for the $2^{++}$ state)

# Backup

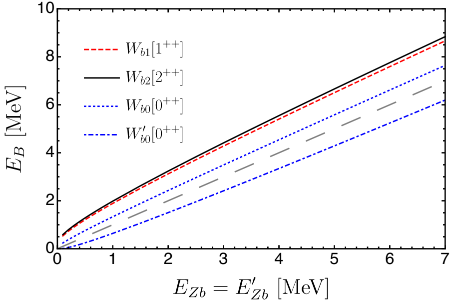
## Spin partners of $Z_b$'s: Results

| | $Z_{b1}(1^{+-})$ | $Z'_{b1}(1^{+-})$ | $W_{b0}(0^{++})$ | $W'_{b0}(0^{++})$ | $W_{b1}(1^{++})$ | $W_{b2}(2^{++})$ |
|---|---|---|---|---|---|---|
| $E_B$ | 5 (input) | 1 (input) | 5.3 $\pm$1.7 | — | 12.4$\pm$0.6 | 19.8$\pm$2.2 |
| $\Gamma_B$ | — | — | — | — | — | 4.6 $\pm$ 1.0 |
| $E_B$ | 1 (input) | 1 (input) | 0.7 $\pm$0.5 | — | 3.8$\pm$0.1 | 10.2$\pm$1.8 |
| $\Gamma_B$ | — | — | — | — | — | 6.2 $\pm$ 1.1 |

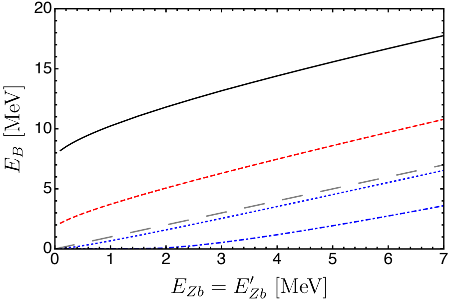
- Energies and widths are given in MeV
- Full calculation (contact+OPE+OEE)
- Uncertainty comes from UV regulator varied from 800 to 1500 MeV
- The $0^{++}$ partner at the $B^*\bar{B}^*$ threshold does not exist as a bound state

## Spin partners of $Z_b$'s: Dependence on input

Contact only theory

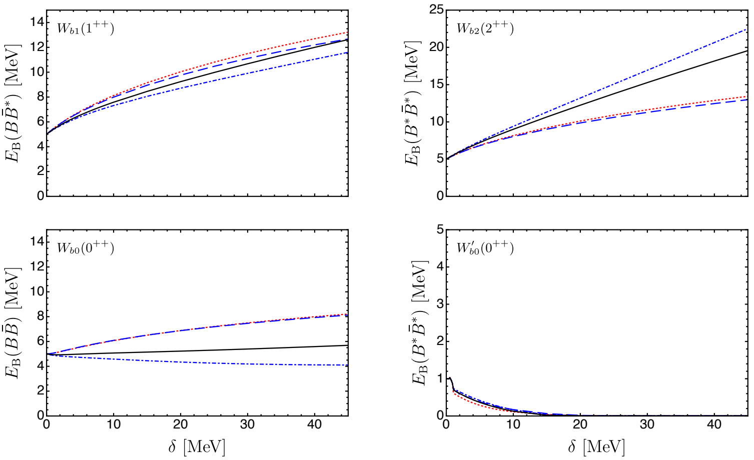



Full theory




- In the contact theory, all bound states turn to virtual states in unison
- In the full calculation, OPE preserves $2^{++}$ and (possibly) $1^{++}$ partners as bound states

# Spin partners of $Z_b$'s: HQSS breaking and SU(3) effects




Red dotted line: contact only

Blue dashed line: contact+central ($S$-wave) OPE

Blue dashed-dotted line: contact+full OPE

Black solid line: contact+full OPE+full OEE)

## Spin partners: HQSS breaking effects (contact theory)

- Relation for $2^{++}$ and $1^{++}$ partners ($\gamma = \sqrt{2\mu E_B}$)

$$\gamma_{X_2} = \left(1 - \frac{\delta}{2\bar{m}}\right)\gamma_X + \frac{\delta}{\pi\bar{m}}\Lambda + O\left(\frac{\gamma_X^2}{\Lambda}, \frac{\delta^2\Lambda}{\bar{m}^2}\right)$$

$$\delta = m_* - m \quad \bar{m} = \frac{1}{4}(3m_* + m)$$

- Relation for $1^{+-}$ partners

$$\gamma_{X_1'} = \left(1 - \frac{\delta}{2\bar{m}}\right)\gamma_{X_1} + \frac{\Lambda\delta}{\pi\bar{m}} - \frac{(\gamma_{X_1} - \gamma_X)^2}{\sqrt{\bar{m}\delta}} + i\frac{(\gamma_{X_1} - \gamma_X)^2}{\sqrt{\bar{m}\delta}} + \dots$$

- UV regulator-related uncertainties in $c$- and $b$-sectors

$$\left(\frac{\Lambda\delta}{\gamma\bar{m}}\right)_c \simeq 0.3 \qquad \left(\frac{\Lambda\delta}{\gamma\bar{m}}\right)_b \simeq 0.04$$