Fine and hyperfine heavy hybrid splittings

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Motivation

- Excited gluonic states have been long theorized as possible valence d.o.f of hadrons.
  - Glueballs: Purely gluonic states.
  - Hybrids: Quarks and gluonic excitations.
- However the unambiguous identification of such states has proven difficult.

* Glueballs: Expected to strongly mix with conventional mesons, exotic J^{PC} predicted at large masses (~ 4 GeV) by lattice.
* Heavy hybrids: $Q\bar{Q}$, $Q = c$, b and a gluonic excitation.
* Heavy Hybrids: Exotic J^{PC} already present for low lying heavy hybrid spin-multiplets: 1^{+-} (also low laying 0^{+-}, 2^{--}).

Motivation

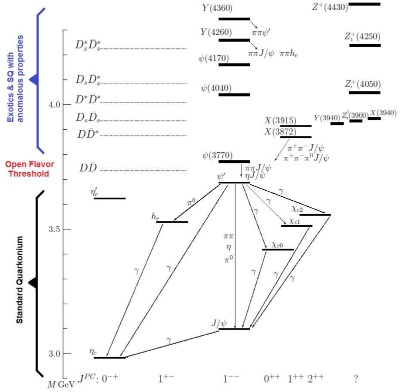
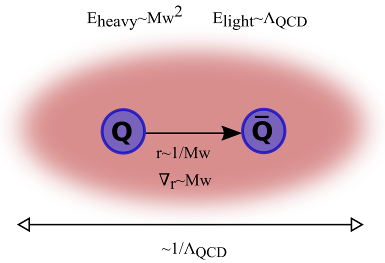


Figure:Voloshin 2008

- Heavy hybrid masses expected to be close or above open flavor thresholds.
- Many states discovered in the last decade in this region!
- The states that do not fit Quarkonium potential models are called Exotics and labeled Xs, Ys and Zs.
- Large experimental effort to study normal and Exotic quarkonium: BaBar, Belle2, BESIII, LHCb and Panda...

$\Rightarrow$ Understanding the spin-splittings in hybrid states will help to confirm or deny the experimental identification

Quarkonium Hybrid system scales



Characteristic Scales

- Heavy-quarks are non-relativistic $m_Q \gg \Lambda_{\mathrm{QCD}}$.
- Two components with very different dynamical time scales $\Lambda_{\mathrm{QCD}} \gg m_Q w^2$.
  - Excited gluonic state Λ_{QCD}.
  - Heavy-quark binding $m_Q w^2$ ($w \ll 1$ relative velocity).
  - Adiabatic expansion (Born-Oppenheimer approximation in atomic physics). Griffiths, Michael, Rakow 1983; Juge, Kuti, Morningstar 1998; Braaten, Langmack, Smith 2014; Meyer, Swanson 2015...

Quarkonium Hybrid system scales

Short distance regime

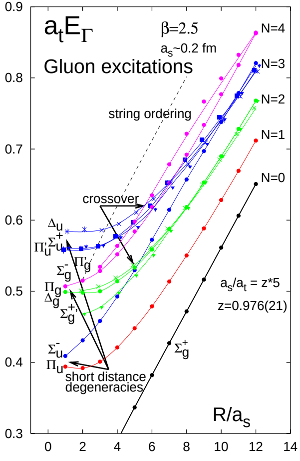
- Small Heavy-quark-antiquark distance $r \sim 1/(mw) < 1/\Lambda_{\rm QCD}$.
- Factorization of perturbative and nonperturbative physics.

Heavy Hybrids EFT

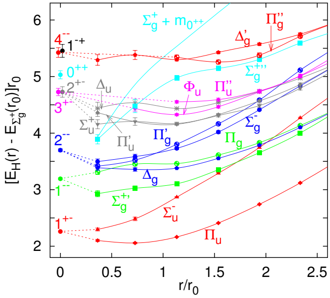
- Use the hierarchy of scales to describe the system.
  - Integrate out m_Q modes: NRQCD Caswell, Lepage 1986; Bodwin, Braaten and Lepage 1995
  - Integrate out $m_Q w \sim 1/r$ modes: (weakly-coupled) pNRQCD Pineda, Soto 1998; Brambilla, Pineda, Soto, Vairo 2000
  - Integrate out $\Lambda_{\rm QCD}$: Hybrid EFT Berwein, Brambilla, JTC, Vairo 2015; Brambilla, Krein, JTC, Vairo 2017; see also Oncala, Soto 2017

Characterization of Hybrid states at short distances

Heavy-quark-antiquark static energies in lattice NRQCD:



Juge, Kuti, Morningstar 2003



Juge, Kuti, Morningstar 2003, Foster, Michael 1999; Bali, Pineda 2004

- Static energies symmetry group $D_{\infty h}$ (like homonulcear diatomic molecules).
- At short distances the gluonic excited states are characterized by the Gluelump operators G_κ^{ia} ($\kappa = J^{PC}$). Brambilla, Pineda, Soto, Vairo, 1999

Gluelump Operators

- G_κ^{ia} are a basis of color-octet eigenstates of $h_0(\boldsymbol{R})$

$$h_0(\boldsymbol{R}) G_\kappa^{i\,a}(\boldsymbol{R})|0\rangle = \Lambda_\kappa G_\kappa^{i\,a}(\boldsymbol{R})|0\rangle$$

- The gluon Hamiltonian density leading order in the multipole expansion.

$$h_0(\boldsymbol{R}) = \frac{1}{2}\left(\boldsymbol{E}^a\boldsymbol{E}^a - \boldsymbol{B}^a\boldsymbol{B}^a\right)$$

- Λ_κ is the gluelump mass and it is a nonperturbative quantity.

Basis for hybrid states

- At the pNRQCD level a basis of hybrid states is defined as

$$|\kappa,\,\lambda\rangle = P^i_{\kappa\lambda} O^{a\,\dagger}(\boldsymbol{r},\boldsymbol{R})\, G_\kappa^{i\,a}(\boldsymbol{R})|0\rangle$$

- The hybrid EFT is formulated for the subspace spanned by

$$\int d^3r d^3R \sum_\kappa |\kappa,\,\lambda\rangle\, \Psi_{\kappa\lambda}(t,\,\boldsymbol{r},\,\boldsymbol{R})$$

Hybrid EFT and Spin-dependent operators

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \Big\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}_r^2}{M} P^{i}_{\kappa\lambda'} \Big\} \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \ldots$$

The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m$ and spin-dependent and independent parts

$$\begin{aligned}
V_{\kappa\lambda\lambda'}(r) &= V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m^2} + \ldots\,, \\
V^{(1)}_{\kappa\lambda\lambda'}(r) &= V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}(r)\,, \\
V^{(2)}_{\kappa\lambda\lambda'}(r) &= V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}(r)\,, \\
V^{(1)}_{1\lambda\lambda'\,SD}(r) &= V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{K}^{ij}_{1}P^{j}_{1\lambda'}\right)\cdot\boldsymbol{S}\,, \\
V^{(2)}_{1\lambda\lambda'\,SD}(r) &= V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^{i}_{1\lambda'}\right)\cdot\boldsymbol{S} + V_{1\,LSb}P^{i\dagger}_{1\lambda}(r)\left(\boldsymbol{L}^{i}_{Q\bar{Q}}\boldsymbol{S}^{j} + \boldsymbol{S}^{i}\boldsymbol{L}^{j}_{Q\bar{Q}}\right)P^{j}_{1\lambda'} \\
&\quad + V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^{j}_{1\lambda'}\left(\boldsymbol{S}^{i}_{1}\boldsymbol{S}^{j}_{2} + \boldsymbol{S}^{i}_{2}\boldsymbol{S}^{j}_{1}\right)
\end{aligned}$$

Hybrid EFT and Spin-dependent operators

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t, \boldsymbol{r}, \boldsymbol{R}) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}_r^2}{M} P^i_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t, \boldsymbol{r}, \boldsymbol{R}) \ldots$$

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$$V_{\kappa\lambda\lambda'}(r) = \boxed{V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'}} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r}{m}$$

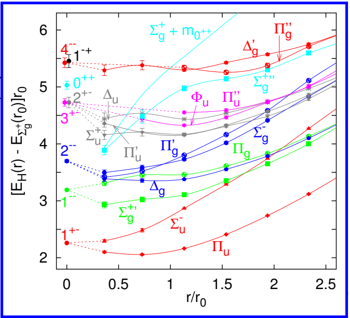
$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}$$

$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}$$

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{K}^{ij}_1 P^j_{1\lambda'}\right)$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^i_{1\lambda'} \ldots \bar{Q}\right) P^j_{1\lambda'}$$

$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^j_{1\lambda'}\left(\boldsymbol{S}^i_1\boldsymbol{S}^j_2 + \boldsymbol{S}^i_2\boldsymbol{S}^j_1\right)$$



The static potential can be matched to the lattice static energies.

The spectrum for $\kappa = 1^{+-}$ in this framework was obtained in Berwein, Brambilla, JTC, Vairo 2015

Hybrid EFT and Spin-dependent operators

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{M} P^{i}_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \ldots$$

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$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r) \left(P^{i\dagger}_{1\lambda} \boldsymbol{K}^{ij}_{1} P^{j}_{1\lambda'} \right) \cdot \boldsymbol{S}\,,$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r) \left(P^{i\dagger}_{1\lambda} \boldsymbol{L}_{Q\bar{Q}} P^{i}_{1\lambda'} \right) \cdot \boldsymbol{S} + V_{1\,LSb} P^{i\dagger}_{1\lambda}(r) \left(\boldsymbol{L}^{i}_{Q\bar{Q}} \boldsymbol{S}^{j} + \boldsymbol{S}^{i} \boldsymbol{L}^{j}_{Q\bar{Q}} \right) P^{j}_{1\lambda'}$$

$$+ V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)\boldsymbol{S}_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r) P^{i\dagger}_{1\lambda} P^{j}_{1\lambda'} \left(\boldsymbol{S}^{i}_{1}\boldsymbol{S}^{j}_{2} + \boldsymbol{S}^{i}_{2}\boldsymbol{S}^{j}_{1} \right)$$

Spin-dependent operators.

Hybrid EFT and Spin-dependent operators

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The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m$ and spin-dependent and independent parts

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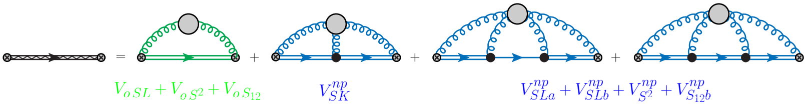
New operators not present in standard Quarkonium.

Matching of the Spin-dependent operators for $\kappa = 1^{+-}$

Matching diagrams in position space:



$$V_{1\,SK} = V^{np}_{SK}\,,$$
$$V_{1\,SLa} = V^{np}_{SLa} + V_{o\,SL}\,,$$
$$V_{1\,SLb} = V^{np}_{SLb}\,,$$
$$V_{1\,S^2} = V^{np}_{S^2} + V_{o\,S^2}\,,$$
$$V_{1\,S_{12}a} = V_{o\,S_{12}}\,,$$
$$V_{1\,S_{12}b} = V^{np}_{S_{12}b}\,.$$

Matching of the Spin-dependent operators for $\kappa = 1^{+-}$

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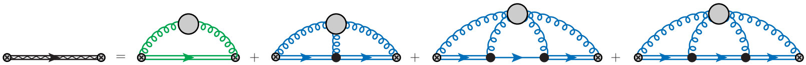


$$
\begin{aligned}
V_{1\,SK} &= V^{np}_{SK}\,, \\
V_{1\,SLa} &= V^{np}_{SLa} + V_{o\,SL}\,, \\
V_{1\,SLb} &= V^{np}_{SLb}\,, \\
V_{1\,S^2} &= V^{np}_{S^2} + V_{o\,S^2}\,, \\
V_{1\,S_{12}a} &= V_{o\,S_{12}}\,, \\
V_{1\,S_{12}b} &= V^{np}_{S_{12}b}\,.
\end{aligned}
$$

- The perturbative part is given by the octet quark-antiquark spin-dependent potential

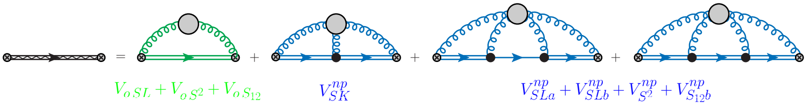
$$V_{o\,LS}(r) = \left(C_F - \frac{C_A}{2}\right)\left(\frac{c_s}{2} + c_F\right)\frac{\alpha_s(\nu)}{r^3}$$

$$V_{o\,S^2}(r) = \left[\frac{4\pi}{3}\left(C_F - \frac{C_A}{2}\right)c_F^2\alpha_s(\nu) + T_F\left(f_8(^1S_0) - f_8(^3S_1)\right)\right]\delta^3(\boldsymbol{r})$$

$$V_{o\,S_{12}}(r) = \left(C_F - \frac{C_A}{2}\right)\frac{\alpha_s(\nu)}{4r^3}$$

Matching of the Spin-dependent operators for $\kappa = 1^{+-}$

Matching diagrams in position space:



$$
\begin{aligned}
V_{1\,SK} &= V_{SK}^{np}\,, \\
V_{1\,SLa} &= V_{SLa}^{np} + V_{o\,SL}\,, \\
V_{1\,SLb} &= V_{SLb}^{np}\,, \\
V_{1\,S^2} &= V_{S^2}^{np} + V_{o\,S^2}\,, \\
V_{1\,S_{12}a} &= V_{o\,S_{12}}\,, \\
V_{1\,S_{12}b} &= V_{S_{12}b}^{np}\,.
\end{aligned}
$$

- The nonperturbative part is given in terms of gluon correlators $\widetilde{U}$

$$
\begin{aligned}
V_{SK}^{np} &= 2c_F \widetilde{U}_B^K \\
V_{SLa}^{np} &= -\frac{3c_F}{8}\widetilde{U}_{Ba}^o + c_s\left(\widetilde{U}_{Ea}^s + \frac{N_c^2-4}{8N_c^2}\widetilde{U}_{Ea}^o\right) \\
V_{SLb}^{np} &= -\frac{3c_F}{8}\widetilde{U}_{Bb}^o + c_s\left(\widetilde{U}_{Eb}^s + \frac{N_c^2-4}{8N_c^2}\widetilde{U}_{Eb}^o\right) \\
V_{S^2}^{np} &= -c_F^2\left(\widetilde{U}_{Ba}^s + \frac{N_c^2-1}{2N_c^2}\widetilde{U}_{Ba}^o\right) \\
V_{S_{12}b}^{np} &= -c_F^2\left(\widetilde{U}_{Bb}^s + \frac{N_c^2-1}{2N_c^2}\widetilde{U}_{Bb}^o\right)
\end{aligned}
$$

Nonperturvative Gluon correlators

$$\widetilde{U}_B^K = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \frac{1}{48 T_F} \int_{-T/2}^{T/2} dt \left[\langle 0 | \boldsymbol{G}^\dagger(T/2) \cdot (g\boldsymbol{B}_{\mathrm{adj}}(t) \times \boldsymbol{G}(-T/2)) | 0 \rangle \right] ,$$

$$\widetilde{U}_{Ba}^s + 4\widetilde{U}_{Bb}^s = \lim_{T\to\infty} \frac{e^{i\Lambda T}}{iT} \frac{N_c}{3 T_F} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0 | (\boldsymbol{G}^{a\dagger}(T/2) \cdot g\boldsymbol{B}^a(t))(g\boldsymbol{B}^a(t') \cdot \boldsymbol{G}^a(-T/2)) | 0 \rangle ,$$

$$3\widetilde{U}_{Ba}^s + 2\widetilde{U}_{Bb}^s = \lim_{T\to\infty} \frac{e^{i\Lambda T}}{iT} \frac{N_c}{3 T_F} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0 | \boldsymbol{G}^{a\dagger}(T/2) \cdot \Big((g\boldsymbol{B}^a(t) \cdot g\boldsymbol{B}^a(t'))\boldsymbol{G}^a(-T/2) \Big) | 0 \rangle ,$$

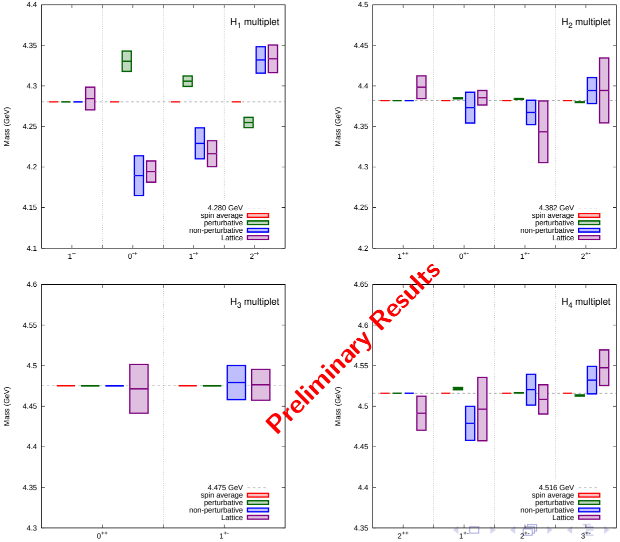
$$\widetilde{U}_{Ba}^o + 4\widetilde{U}_{Bb}^o = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \frac{1}{18 T_F^2} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0 | (\boldsymbol{G}^\dagger(T/2) \cdot g\boldsymbol{B}_{\mathrm{adj}}(t))(g\boldsymbol{B}_{\mathrm{adj}}(t') \cdot \boldsymbol{G}^a(-T/2)) | 0 \rangle ,$$

$$3\widetilde{U}_{Ba}^o + 2\widetilde{U}_{Bb}^o = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \frac{1}{18 T_F^2} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \langle 0 | \boldsymbol{G}^\dagger(T/2) \cdot \Big((g\boldsymbol{B}_{\mathrm{adj}}(t) \cdot g\boldsymbol{B}_{\mathrm{adj}}(t'))\boldsymbol{G}(-T/2) \Big) | 0 \rangle ,$$

- $\widetilde{U}_{Ea}^o$ and $\widetilde{U}_{Eb}^o$ are defined by replacing $\boldsymbol{B}$ for $\boldsymbol{E}$.
- The gluon correlators $\widetilde{U}$ are independent of r and the heavy quark flavor.

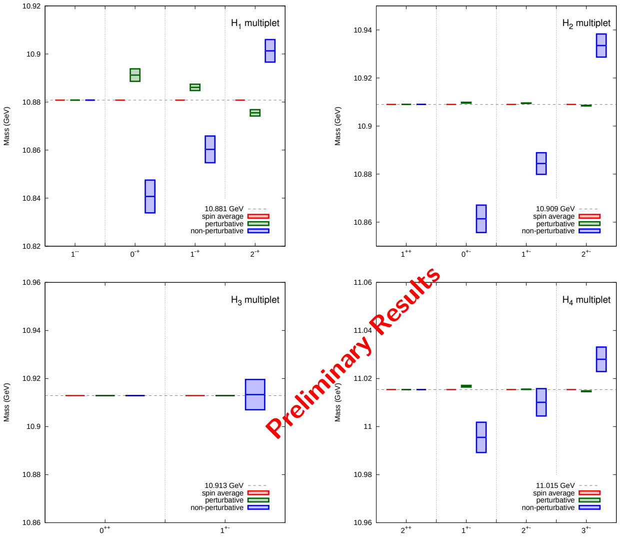
Charmonium hybrids

- The contributions of the spin-dependent operators are computed in standard QM perturbation theory.
- The value of the gluon correlators is fitted to reproduce the lattice spectrum of Liu et al 2012.



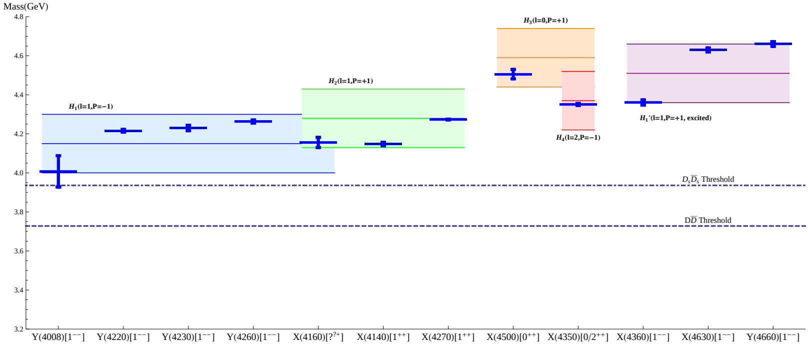
Bottomonium hybrids

- Extrapolation of the spin-splittings in the bottomonium sector.



Update: Identification with experimental states

- Neutral exotic charmonium states (Belle, CDF, BESIII, Babar, LHCb):



- The J^{PC} of all candidates correspond to spin-singlet hybrids except $X(4160)$ which is unknown.
- However, $M[Y(4220)] - M[X(4160)] = (60 \pm 28)$MeV is consistent with $M[H1(1^{--})] - M[H1(1^{-+})] \approx 50$MeV.
- $Y(4220)$ is one of the best candidates since it is not observed decaying into spin-triplet quarkonia. (However mixing with standard Quarkonia can allow for spin-flipping decays Oncala, Soto 2017)

Conclusions

- Quarkonium hybrids can be studied in a model independent way combining EFT with lattice inputs.
- In this framework we have obtained the $1/m$ and $1/m^2$ spin-dependent contributions to the spectrum.
- The matching coefficients have been characterized in terms of gluonic correlators.
- The hybrid EFT provides constrains to cross check lattice determinations of the charmonium hybrid spectrum.
- We have obtained the fine and hyperfine structure of the hybrid spectrum in the bottom sector, where direct lattice predictions are still sparse.

Thank you for your attention