B_c decays from highly improved staggered quarks and NRQCD

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B_c decays @ LHCb

- B_c mesons are being produced in sufficient quantity for detailed measurements at LHCb, yet there are no lattice simulations of B_c semileptonic decays needed for extracting $|V_{cb}|$.
- Intriguing persistent anomalies in 'R-ratios' for $b \to c$ semileptonic transitions, also just observed in $B_c \to J/\psi\, l\nu$ decays at LHCb.
- Today's talk will focus on progress in simulating $B_c \to \eta_c\, l\nu$ and $B_c \to J/\psi\, l\nu$ semileptonic decays.
  - Technical progress in simulation of heavy quarks.
  - Results and discussion on impact for phenomenology.

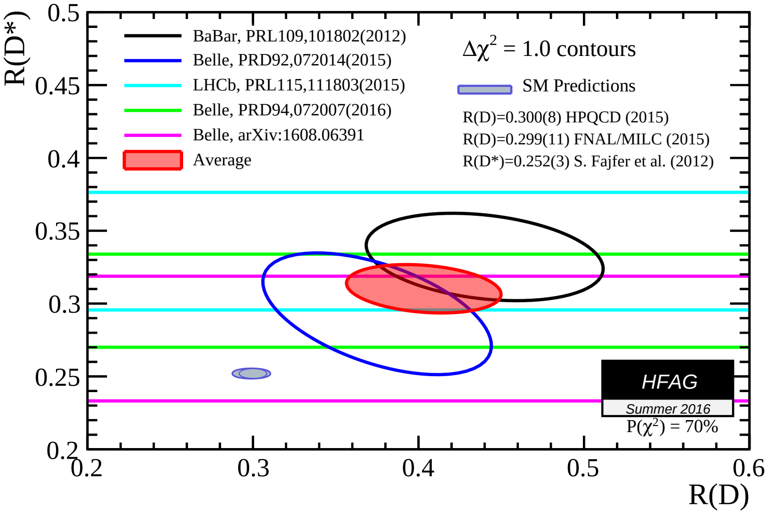
R-ratios

R-ratio for $B_c \to J/\psi$ semileptonic decay defined as

$$R(B_c \to J/\psi) = \frac{\mathcal{B}(B_c \to J/\psi\, \tau\nu)}{\mathcal{B}(B_c \to J/\psi\, l\nu)}\,, \quad l = \mu,\, e\,,$$

- Test lepton flavour universality.
- There are persistent few-sigma anomalies in the ratios $R(B \to D^*)$ and $R(B \to D)$ involving the same $b \to c$ transition.
- The current work will provide reliable SM determination for $R(B_c \to J/\psi)$, to be compared with recent measurement by LHCb.
  lhcb-public.web.cern.ch/lhcb-public/

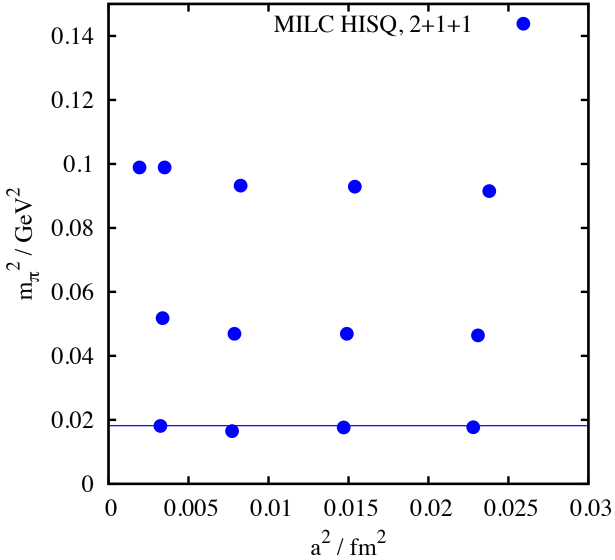
$R(D^{(*)})$ – Expt'l status



c and b quarks on the lattice

- Treatment of c and especially b quarks challenging in lattice simulations due to lattice artifacts which grow as $(am_q)^n$.
- HPQCD treats the charm quark relativistically using a highly improved staggered quark (HISQ) action.
- Generally one uses an effective theory framework to handle the b quark, here focus on NRQCD.
- HPQCD use two complementary approaches for b quarks:
  - Improved non-relativistic formalism (NRQCD) at m_b.
  - Highly improved relativistic action at small a, extrapolate $m_h \to m_b$.

MILC ensemble parameters [1004.0342,1212.4768]



DiRAC II computing

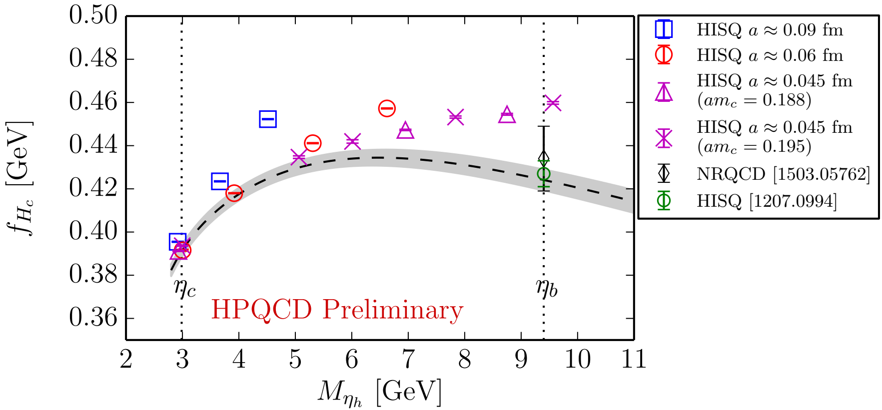
Quark propagator inversions carried out on the Darwin cluster at Cambridge.

Includes:

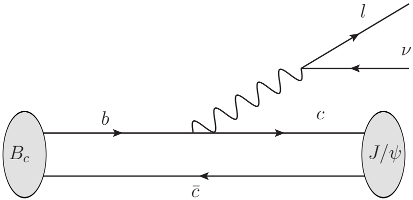
- 9600 Intel Sandy Bridge cores
- 2.6 GHz, 4 GB RAM/core
- 2 PB storage

Part of STFC's HPC facility for theoretical particle physics and astronomy.

Spanning c to b with HISQ



B_c semileptonic decays



Semileptonic decays

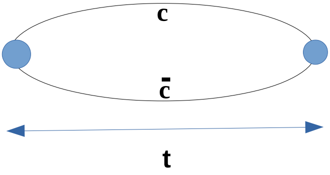
- Study of $B_c \to \eta_c$, $B_c \to J/\psi$ decay matrix elements.
- We work in the frame where the B_c is at rest.
- The form factors which parametrise the matrix elements are functions of q^2, where q is the four-momentum transferred to the leptons.
  - $q^2_{\mathrm{max}} = (M - m)^2$, zero recoil of decay hadron.
  - $q^2 = 0$, maximum recoil of decay hadron.
- Matrix elements are determined by simultaneous fitting of three-point and two-point functions.

$B_c \to \eta_c$ and J/ψ

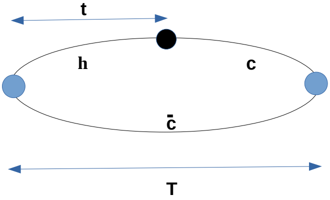
- All quarks are heavy (m_c or heavier) $\to$ good signal, can control calculation across full q^2 range, both in NRQCD and full relativistic calculation.
- NRQCD $b \to c$ transition currents of the form: $(1 + \alpha_s z + \cdots)(J_\mu^{(0)} + J_\mu^{(1)} + \cdots)$
  - Syst. error from the matching uncertainty appears in all $b \to c$ semileptonic transition calculations $B_{(s)} \to D_{(s)}^{(*)}$.
  - Can cross-check NRQCD systematics using the fully relativistic formulation.
- $B_c \to \eta_c$ (pseudoscalar): $f_0(q^2)$, $f_+(q^2)$
  $B_c \to J/\psi$ (vector): $A_{0,1,2,3}(q^2)$ (three independent), $V(q^2)$

Semileptonic decays – meson correlators

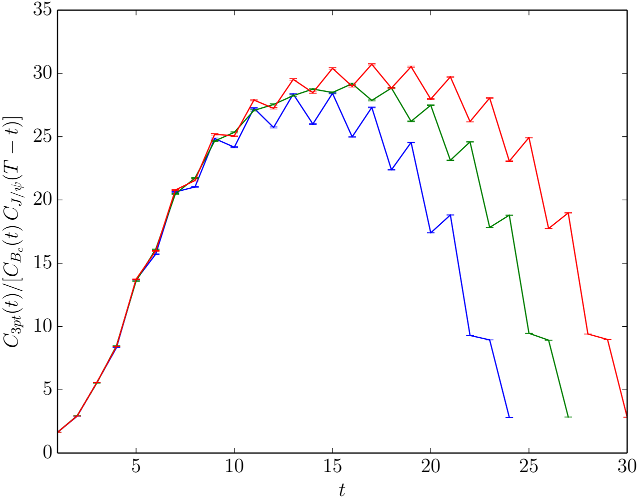
Two-point functions:



Three-point functions:



Semileptonic decays



$B_c \to \eta_c$ form factors

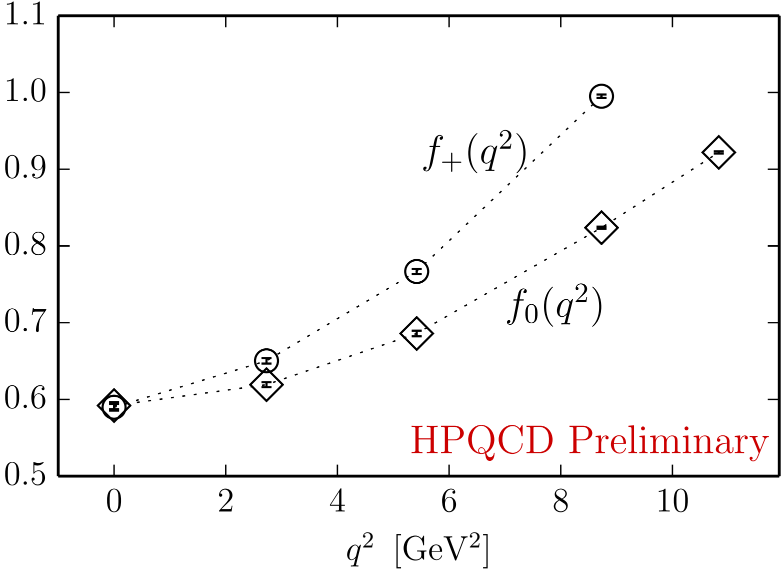
$$Z\langle\eta_c(p)|V^\mu|B_c(P)\rangle = f_+(q^2)\left[P^\mu + p^\mu - \frac{M^2-m^2}{q^2}q^\mu\right] + f_0(q^2)\,\frac{M^2-m^2}{q^2}\,q^\mu\,,$$

From PCVC,

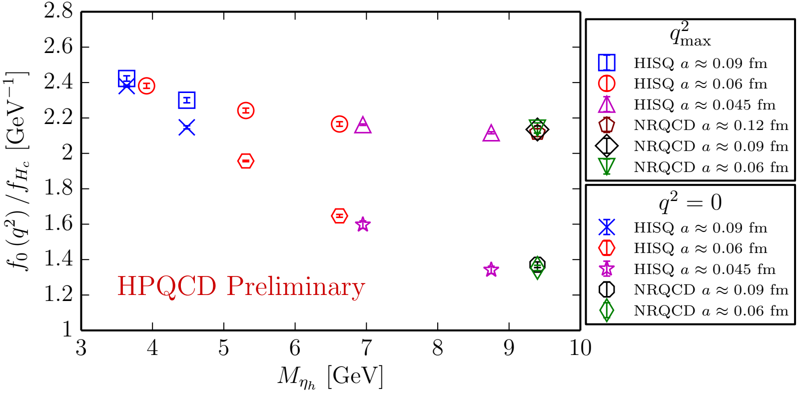
$$\langle\eta_c(p)|S|B_c(P)\rangle = \frac{M^2-m^2}{m_{b0}-m_{c0}}f_0(q^2)$$

Find Z by calculating both matrix elements at q^2_{max}.

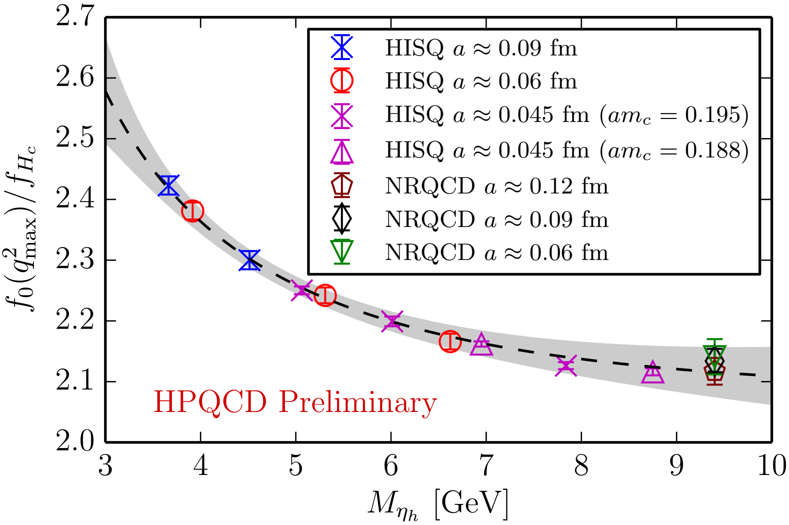
NRQCD $B_c \to \eta_c$ form factors



$f_0(q^2 = 0, \text{max})/f_{H_c}$ compared to NRQCD



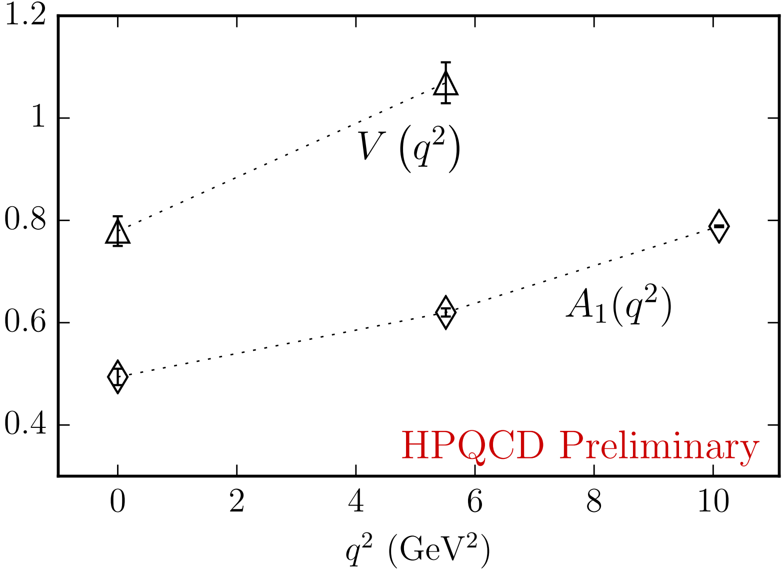
$f_0(q^2_{\mathsf{max}})/f_{H_c}$



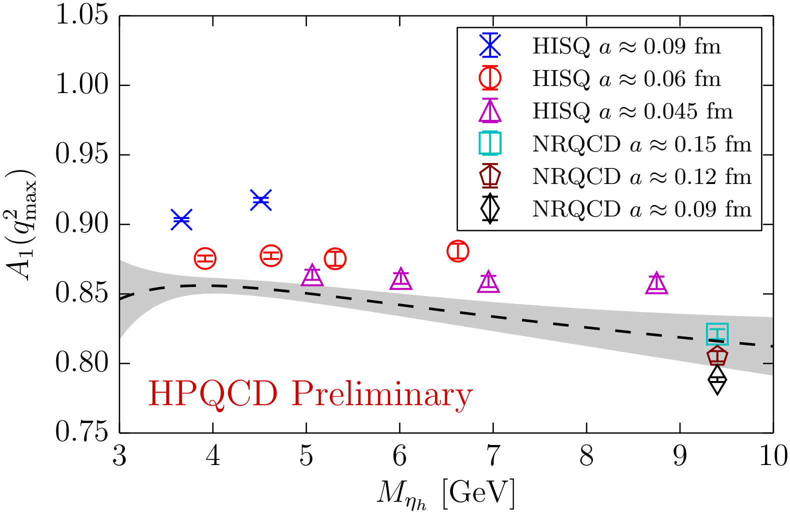
$B_c \to J/\psi$ form factors

$$\langle J/\psi(p,\varepsilon)|V^\mu - A^\mu|B_c(P)\rangle = \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m}\varepsilon^*_\nu p_\rho P_\sigma \, V(q^2) - (M+m)\varepsilon^{*\mu}\, A_1(q^2) + \frac{\varepsilon^*\cdot q}{M+m}\,(p+P)^\mu \; A_2(q^2) + 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu \, A_3(q^2) - 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu \, A_0(q^2)$$

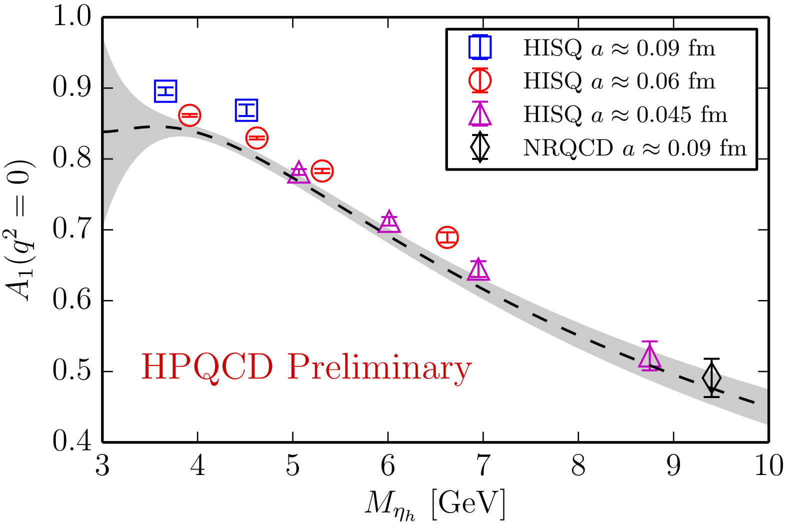
NRQCD $B_c \to J/\psi$ form factors



$A_1(q^2_{\mathrm{max}})$

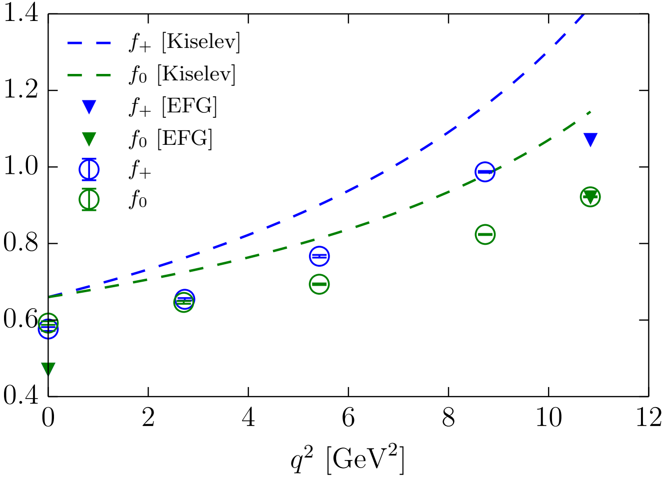


$A_1(q^2 = 0)$

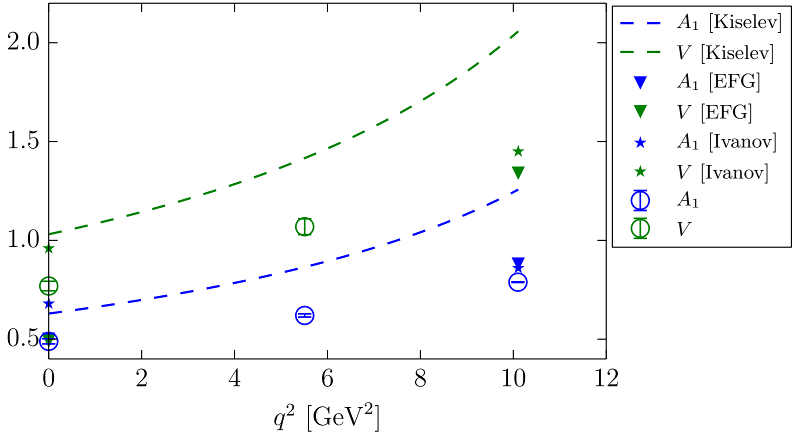


HISQ $a \approx 0.09$ fm
HISQ $a \approx 0.06$ fm
HISQ $a \approx 0.045$ fm
NRQCD $a \approx 0.09$ fm
HPQCD Preliminary
$A_1(q^2 = 0)$
M_{η_h} [GeV]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
3
4
5
6
7
8
9
10

Comparisons $B_c \to \eta_c$ [hep-ph/0211021,0306306]

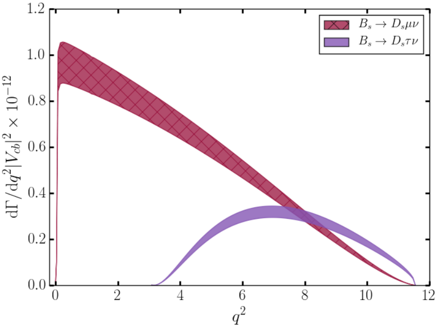


Comparisons $B_c \to J/\psi$ [hep-ph/0007169,0211021,0306306]



$B_s \to D_s$ [1703.09728]

Recent result from HPQCD used NRQCD + improved staggered (asqtad) quarks to obtain $\frac{d\Gamma}{dq^2}$ over full q^2 range.



- $R(D_s) = 0.314(6)$
- Rel. q model result $R(D_s) = 0.274(20)$ [A. Bhol (2014)]

See also ETMC [1310.5238], FNAL/MILC [1202.6346], and HPQCD (HISQ) in progress [E. McLean]

$B_{(s)} \to D^*_{(s)}$

Results at zero recoil from NRQCD.

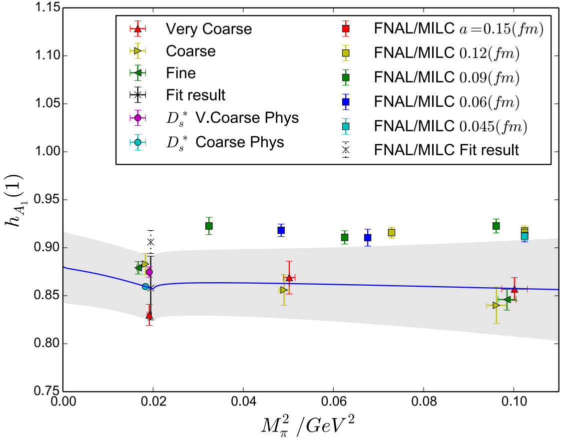


Figure courtesy Judd Harrison [1612.06716]

Summary & Future work

- A promising approach to study of $b \to c$ transitions:
  - Lattice NRQCD with HISQ quarks, plus
  - Fully relativistic formulation, extrapolate m_h to m_b.
- Proof-of-principle demonstrated for B_c semileptonic decay.
  - Controlled calculation over full q^2 range.
  - Good agreement seen with NRQCD results.
- Outputs:
  - B_c to $J/\Psi \to$ new possible determination of $|V_{cb}|$.
  - Reliable SM prediction for $R(B_c \to J/\psi)$.
  - Improved understanding of NRQCD currents feeds into additional calculations (B to D, B to D^*, ...).
  - Expand relativistic calculations, e.g. to $B_s \to D_s^*$ at zero recoil.

Thank you!

Form factor extrapolation

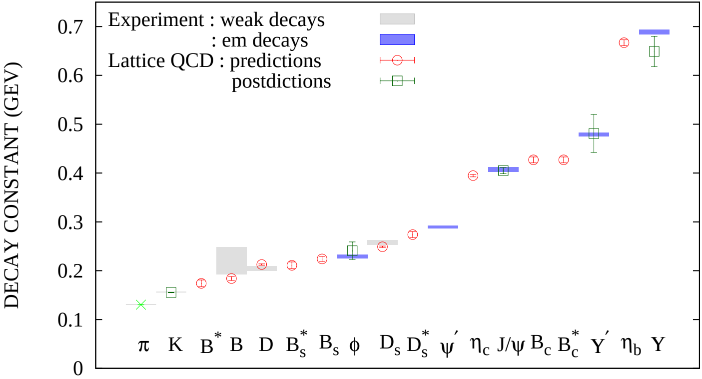
Generic HQET-inspired fit form given by

$$F(q^2, M_{\eta_h}, a^2) = A(q^2)\left(\frac{M_{\eta_h}}{M_0}\right)^b \times$$
$$\left[\sum_{ijkl} c_{ijkl}(q^2)\left(\frac{M_0}{M_{\eta_h}}\right)^i \left(\frac{am_c}{\pi}\right)^{2j}\left(\frac{am_h}{\pi}\right)^{2k}\left(\frac{a\Lambda_{\mathrm{QCD}}}{\pi}\right)^{2l}\right].$$

Continuum result evaluated at $m_h = m_b$:

$$F(q^2, M_{\eta_b}, 0) = A(q^2)\left(\frac{M_{\eta_b}}{M_0}\right)^b \sum_i c_{i000}(q^2)\left(\frac{M_0}{M_{\eta_h}}\right)^i$$

Decay constants – summary plot.



Tuning to the physical point

Bare quark masses are input parameters to lattice simulations. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud0} \rightarrow m_\pi^2$
- $m_{s0} \rightarrow m_K^2$
- $m_{c0} \rightarrow m_{\eta_c}$

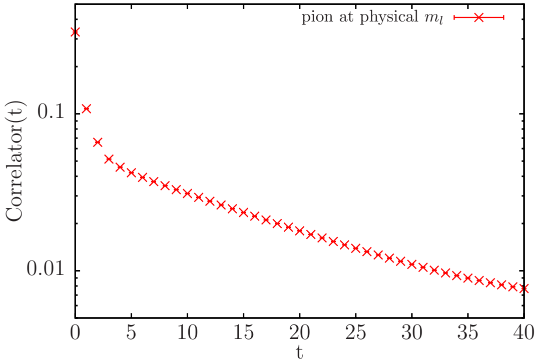
Tuning performed at multiple lattice spacings, defining a continuum trajectory for which $a^2 \rightarrow 0$ limit can be taken.

- Rest of physics is then prediction of QCD.
- Parameters can be varied away from physical values.. understand effect of quark mass, quantify systematics, etc.

Lattice QCD simulations - III

Energies and matrix elements are determined by fitting (sums of) exponentials.

$$\langle \pi(t)\ \pi^\dagger(0) \rangle \underset{\text{large } t}{\longrightarrow} \frac{|\langle 0|\pi|\pi\rangle|^2}{2m_\pi} e^{-m_\pi t} \propto f_\pi^2\, e^{-m_\pi t}$$



Simulating charm

Heavy quarks are challenging to simulate.

- Requires $am_0 < 1$ to keep discretization effects under control.
- Need large enough box to minimize finite-volume effects $\rightarrow N_{\text{site}}$ large.

These conditions can be satisfied by using a highly improved action (e.g. HISQ).

Staggered Quarks

Staggered quark action is defined in terms of one-component Grassman fields.

$$\mathcal{L}_{\text{naive}} = \bar{\chi}(x)\left[\eta^\mu(x)\Delta_\mu(U(x)) + m\right]\chi(x)$$

where

$$\eta^\mu(x) = (-1)^{\sum_{\nu<\mu} x^\nu} .$$

- $2a$ translation symmetry
- Represents four identical “tastes” of quark in the continuum.
- Reduce to one dynamical quark via “rooting” procedure.

HISQ Action

[hep-lat/0610092]

$$\Delta_\mu^{\mathrm{HISQ}} = \Delta_\mu(W) - \frac{a^2}{6}(1+\epsilon)\Delta_\mu^3(X)$$

- Removes all $\mathcal{O}(a^2)$ discretisation effects.
- Highly suppressed one-loop taste exchange errors.
- No tree-level $\mathcal{O}((am)^4)$ errors to leading order in v/c.