Determination of heavy quark masses from heavy-light meson masses

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Outline

Introduction

- Six of the fundamental parameters of the Standard Model are quark masses
  - because of confinement they cannot be measured directly
  - must be extracted indirectly from hadron masses
- For observable particles such as the electron
  - the position of the pole in the propagator is the definition of its mass
  - the pole mass is the rest mass of an isolated particle
- The masses of quarks can be defined as theoretical parameters
  - renormalized, *e.g.*, in the $\overline{\text{MS}}$ scheme at a given scale μ
  - PDG values for the $\overline{\text{MS}}$ masses renormalized at scale $\mu{=}2$ GeV
    $m_u(\mu){=}2.15 \pm 0.15$, $m_d(\mu){=}4.7 \pm 0.2$, $m_s(\mu){=}93.5 \pm 2$ in MeV
    and for the charm and bottom quarks
    $\overline{m}_c = m_c(m_c){=}1.28 \pm 0.025$ GeV and $\overline{m}_b = m_b(m_b) = 4.18 \pm 0.3$ GeV
- Precise values of m_b and m_c are needed for precise calculations in SM and BSM
  - Goal: to calculate $\overline{\text{MS}}$ masses of bottom and charm quarks
  - How: from heavy-light (or heavy-heavy) meson masses calculated on lattice

Meson mass $\leftrightarrow$ quark pole mass $\leftrightarrow$ quark $\overline{\text{MS}}$ mass

Pole mass of a quark

- The pole mass cannot be measured by experimentalist
  - * Obstacle: confinement
- The pole mass cannot be defined by theorists in an unambiguous way
  - * Obstacle: divergence of perturbation theory
    The perturbative relation between the pole mass and the $\overline{\text{MS}}$ mass is divergent due to renormalons
- However, the pole mass appears as an intermediate quantity (a bridge) relating the $\overline{\text{MS}}$ mass of the heavy quark and meson masses; HQET:

$$M_H = m_Q + \overline{\Lambda} + \frac{\mu_\pi^2}{2m_Q} - \frac{\mu_G^2(m_Q)}{2m_Q} + \cdots$$

  - $\overline{\Lambda}$: energy of quarks and gluons inside the system
  - $\mu_\pi^2/2m_Q$: kinetic energy of the heavy quark inside the system
  - $\mu_G^2(m_Q)/2m_Q$: hyperfine energy due to heavy quark's spin (μ_G^2 runs)

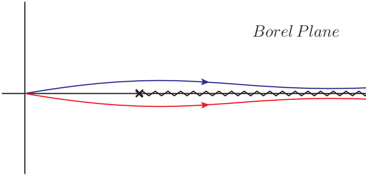
Renormalons in Pole Mass

- With $\overline{m}$ being the $\overline{\text{MS}}$ mass of a heavy quark at scale $\mu = \overline{m}$

$$m_{\text{pole}} = \overline{m}\left(1 + \sum_{n=0}^{\infty} r_n\, \alpha_s^{n+1}(\overline{m})\right), \qquad r_n \propto (2\beta_0)^n \Gamma(n+b+1) \text{ as } n \to \infty$$

- The divergent expression can be interpreted using the Borel transform

involves an integral of form $\displaystyle\int_0^\infty dz\, \frac{e^{-z/(2\beta_0\alpha_s)}}{(1-z)^{1+b}}$



- We use *minimal-renormalon-subtracted (MRS) mass*
  (a modified version of the RS mass [A. Pineda hep-ph/0105008])
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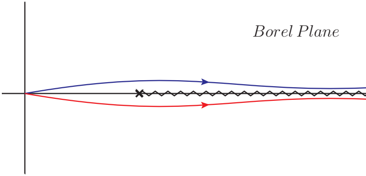

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  well-behaved expression between $\overline{\text{MS}}$ and MRS masses of the heavy quark

  $m_{\text{pole}} \to m_{\text{MRS}} + \mathcal{O}(\Lambda_{QCD})$

- We exploit four-loop relation between the pole and $\overline{\text{MS}}$ masses [hep-ph/1606.06754], and define

$$m_{h,\mathrm{MRS}} = \overline{m}_h \left(1 + \sum_{n=0}^{3} \left[r_n - R_n \right] \alpha_s^{n+1}(\overline{m}_h) + \mathcal{O}(\alpha_s^5) \right) + \mathcal{J}_{\mathrm{MRS}}(\overline{m}_h) + \Delta m_{(c)}$$

$\mathcal{J}_{\mathrm{MRS}}(\overline{m}_h)$:	contribution from the leading renormalon (see backup slides)
$-R_n$:	subtracting the leading renormalon from the perturbative series
$\Delta m_{(c)}$:	for contribution from the charm quark [arXiv:1407.2128]

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- The relation between the MRS and $\overline{\mathrm{MS}}$ masses for a theory with $n_l = 3$ active quarks, and $R_0 = 0.535$:

$$r_n = (0.4244,\ 1.0351,\ 3.6932,\ 17.4358, \ldots)$$
$$R_n = (0.5350,\ 1.0691,\ 3.5966,\ 17.4195, \ldots)$$

for $n = 0, 1, 2, 3, \ldots$. And their differences

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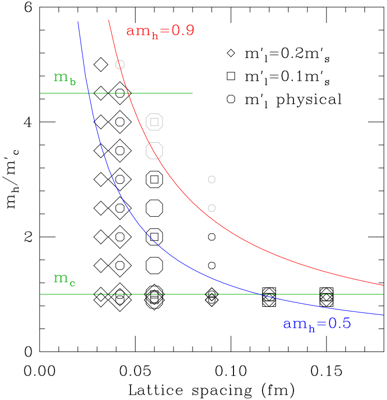
We use the MRS mass to analyze pseudoscalar meson masses calculated in lattice-QCD simulations with heavy-quarks masses ranging from charm to bottom

Heavy-light mesons with HISQ action

We use HISQ ensembles (generated by MILC) with (2+1+1)-flavor of dynamical quarks



- We have 24 Ensembles:
  - 6 lattice spacings
  - several sea masses
- We calculate masses of pseudoscalar mesons for various light and heavy quarks with masses:
  - heavy valence: $m_c \lesssim m_h \lesssim m_b$
- We use only $am_h < 0.9$ to avoid large discretization errors

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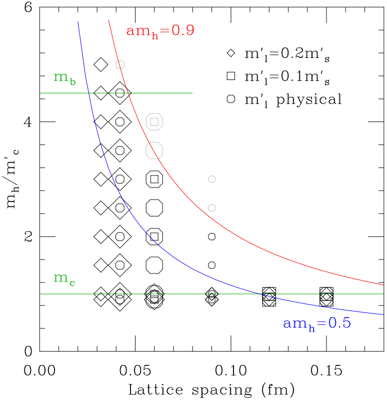

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EFT description of heavy-light meson masses

We employ HQET and heavy-meson (staggered) ChPT to describe the dependence of meson masses on both heavy and light quark masses and incorporate (taste-breaking) lattice artifacts

Mapping bare lattice masses to the $\overline{\text{MS}}$ and MRS masses

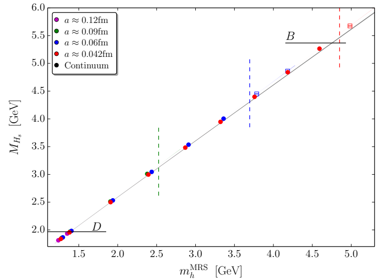
- In lattice simulations with a heavy quark h, we have access to its bare mass in lattice units ($am_{h,0}$)
- To map the bare mass of the h quark, $am_{h,0}$, to its $\overline{\text{MS}}$ mass:
  1) Introduce a "reference mass" and consider the ratio

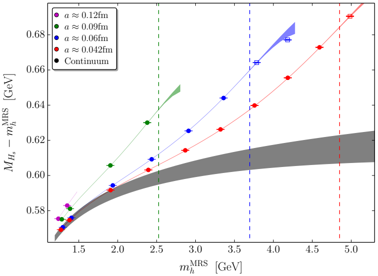
$$\frac{m_{h,\overline{\text{MS}}}(\mu)}{m_{r,\overline{\text{MS}}}(\mu)} = \frac{am_{h,0}}{am_{r,0}} + \mathcal{O}(a^2),$$

     where the LHS holds with any mass-independent renormalization scheme, and the RHS relies on the remnant chiral symmetry of staggered fermions
  2) Set the reference mass to $0.4am_{s,0}$ and treat $m_{r,\overline{\text{MS}}}(\mu)$ as a fit parameter
  3) Incorporate lattice artifacts by parameterizing the a^2 corrections
  4) Calculate $m_{h,\overline{\text{MS}}}(\mu)$
  5) Use continuum-limit relations to map $m_{h,\overline{\text{MS}}}(\mu)$ to the MRS mass $m_{h,\text{MRS}}$
  6) Finally, plug $m_{h,\text{MRS}}$ in EFT description of masses of heavy-light mesons

A sample EFT fit to pseudoscalar-meson masses

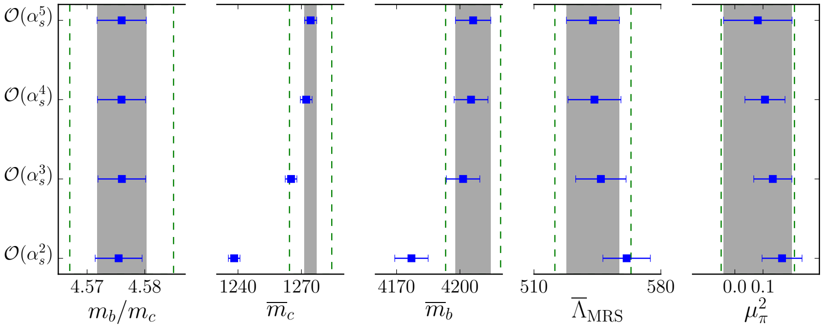
- We use 384 lattice data point and 72 parameters in our EFT fit function
- We use constrained fitting procedure [hep-lat/0110175]
- We impose constraints, *e.g.*, $\mu_G^2(m_b) = 0.35(7)\ \mathrm{GeV}^2$ [arXiv:1307.4551] and $R_0 = 0.535(10)$ for the overall normalization of the leading renormalon as prior values to the corresponding fit parameters.
- We perform a combined-correlated fit (χ^2/d.o.f $= 329/312$)
- After extrapolation to continuum, we determine HQET matrix elements
- Fixing the meson mass to M_{D_s} and M_{B_s} we determine the masses of the charm and bottom quarks





Stability of results under variation in number of loops

- We use
  - four-loop relation between the pole and $\overline{\text{MS}}$ mass
  - five-loop results for the quark mass anomalous dimension
  - five-loop results for beta function
- The plot shows the dependence of our final results on number of loops



Preliminary results

- The strange, charm and bottom quark masses in a theory with 4 dynamical quarks

$$
\begin{aligned}
m_{s,\overline{\mathsf{MS}}}(\mu) &= 92.66(28)_{\mathsf{stat}}(40)_{\mathsf{sys}}(48)_{\alpha_s}(11)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV} \\
\overline{m}_c &= 1274(3)_{\mathsf{stat}}(3)_{\mathsf{syst}}(9)_{\alpha_s}(0)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV} \\
\overline{m}_b &= 4206(8)_{\mathsf{stat}}(8)_{\mathsf{syst}}(6)_{\alpha_s}(1)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV}
\end{aligned}
$$

  where $\overline{m}_h = m_{h,\overline{\mathsf{MS}}}(m_{h,\overline{\mathsf{MS}}})$ and $\mu = 2$ GeV.

- In a theory with 5 dynamical quarks, we have

$$
\overline{m}_b^{(n_f=5)} = 4200(8)_{\mathsf{stat}}(8)_{\mathsf{sys}}(6)_{\alpha_s}(1)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV}
$$

- Uncertainties:

"stat") Statistics and EFT fit

"syst") Various systematic uncertainties in inputs: FV, EM, topological charge freezing, contamination from higher order states...

α_s) Uncertainty in the strong coupling constant $\alpha_{s,\overline{\mathsf{MS}}}(5\,\mathrm{GeV}; n_f=4) = 0.2128(25)$ [HPQCD, arXiv:1408.4169]

$f_{\pi,\mathsf{PDG}}$) Uncertainty in the PDG value of $f_{\pi^\pm} = 130.50(13)$ MeV, which is used for scale setting

Preliminary results for HQET parameters

- For HQET parameters we have

$$\overline{\Lambda}_{\mathrm{MRS}} = 543(15)_{\mathsf{stat}}(9)_{\mathsf{syst}}(13)_{\alpha_s}(1)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV}$$
$$\mu_\pi^2 = 0.08(12)_{\mathsf{stat}}(03)_{\mathsf{syst}}(05)_{\alpha_s}(00)_{f_\pi,\mathsf{PDG}}\,\mathrm{GeV}^2$$
$$\mu_G^2(m_b) = 0.31(02)_{\mathsf{stat}}(05)_{\mathsf{syst}}(01)_{\alpha_s}(00)_{f_\pi,\mathsf{PDG}}\,\mathrm{GeV}^2$$

(recall that the prior value of $\mu_G^2(m_b)$ is set to $0.35(7)\,\mathrm{GeV}^2$ [arXiv:1307.4551])

- In the RS scheme at scale $\nu_f = 1$ GeV, we find

$$\overline{\Lambda}_{\mathsf{RS}}(1\,\mathrm{GeV}) = 627(15)_{\mathsf{stat}}(9)_{\mathsf{syst}}(21)_{\alpha_s}(1)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV}$$

- The MRS mass for the charm and bottom quarks (in a theory with 3 massless quarks + 1 charm quark)

$$m_{c,\mathrm{MRS}} = 1392(20)_{\mathsf{stat}}(3)_{\mathsf{sys}}(5)_{\alpha_s}(0)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV}$$
$$m_{b,\mathrm{MRS}} = 4754(9)_{\mathsf{stat}}(8)_{\mathsf{sys}}(10)_{\alpha_s}(1)_{f_\pi,\mathsf{PDG}}\,\mathrm{MeV}$$

Preliminary results for the up and down quark masses

- To calculate the light quark masses we combine our determination of $m_{s,\overline{\mathrm{MS}}}(2\mathrm{GeV})$ and separate determination of mass ratios m_s/m_l and m_d/m_u

$$\begin{aligned} m_{d,\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= 4.70(3)_{\mathrm{stat}}(4)_{\mathrm{sys}}(2)_{\alpha_s}(1)_{f_{\pi,\mathrm{PDG}}}\,\mathrm{MeV} \\ m_{u,\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= 2.12(2)_{\mathrm{stat}}(3)_{\mathrm{sys}}(1)_{\alpha_s}(0)_{f_{\pi,\mathrm{PDG}}}\,\mathrm{MeV} \end{aligned}$$

- m_u and m_d values depend on separate calculation of EM effects on light-light mesons (by MILC collaboration); that calculation is being finalized

Conclusion

- We developed a method based on HQET to extract quark masses from heavy-light meson masses
- We employed heavy-meson (staggered) ChPT to describe the dependence of heavy-light mesons on masses of light valence and sea quarks, and we performed a combined correlated, multidimensional fit to 384 data at multiple lattice spacings
- We presented preliminary results for strange-, charm- and bottom-quark masses
- We presented preliminary results for HQET parameters: $\overline{\Lambda}$ (in MRS and RS schemes), μ_π^2 and and $\mu_G^2(m_b)$
- Combined with a separate determination of quark mass ratios m_s/m_l and m_d/m_u, and our preliminary result for strange-quark mass, we presented preliminary results for the up- and down-quark masses

Thanks for your attention!

back-up slides

- $\mathcal{J}_{\mathrm{MRS}}(\mu)$ is defined as

$$\mathcal{J}_{\mathrm{MRS}}(\mu) = \frac{R_0}{2\beta_0}\mu e^{-1/[2\beta_0\alpha_{\mathrm{g}}(\mu)]}\sum_{n=0}^{\infty}\frac{1}{n!(n-b)}\left(\frac{1}{2\beta_0\alpha_{\mathrm{g}}(\mu)}\right)^n$$

  where $b = \beta_1/(2\beta_0^2)$, R_0 is the overall normalization of the leading renormalon in the pole mass, and $\alpha_{\mathrm{g}}(\mu)$ is the coupling constant in the scheme with

$$\beta\left(\alpha_{\mathrm{g}}(\mu)\right) = -\frac{\beta_0\alpha_{\mathrm{g}}^2(\mu)}{1-(\beta_1/\beta_0)\alpha_{\mathrm{g}}(\mu)}$$

- For the relations between the RS and MRS schemes:

$$m_{\mathsf{RS}}(\nu_f) = m_{\mathrm{MRS}} - \mathcal{J}_{\mathrm{MRS}}(\nu_f)$$
$$\overline{\Lambda}_{\mathsf{RS}}(\nu_f) = \overline{\Lambda}_{\mathrm{MRS}} + \mathcal{J}_{\mathrm{MRS}}(\nu_f)$$