$\psi(2S)$ and $\Upsilon(3S)$ hadroproduction in the Parton Reggeization Approach: yield, polarization, and the role of fragmentation

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QWG-2017,
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Motivation.

The talk is based on: [B. A. Kniehl, M. A. Nefedov, V. A. Saleev, Phys. Rev. D **94**, 054007 (2016)]

Two main pillars of physics of heavy quarkonium production at hadron colliders:

- NRQCD-factorization ($c\bar{c} \to (J/\psi,\ \chi_{cJ},\ \psi(2S),\ldots) + X$)
- QCD-factorization ($p + p \to c\bar{c} + X$): Collinear Parton Model + radiative corrections

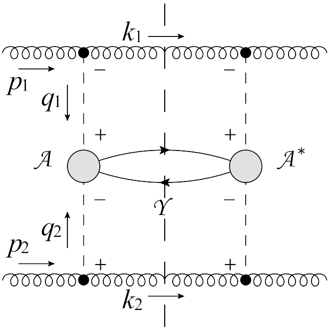
Main observable – p_T-spectrum (+ polarization!), 3 kinematic regions:

- Small $p_T \ll M$ (assuming $M \gg \Lambda_{QCD}$): *Sudakov region*, large logarithms – $\log^2(p_T/M)$.
- "Moderate" $p_T \sim M$: NLO corrections in CPM are significant ($>$ factor-2), $\log 1/x$-effect? ($x \sim M/\sqrt{S} \ll 1$)
- High $p_T \gg M$: *Fragmentaion region*, large logarithms $\log(p_T/M)$.

We need to fit all regions simultaneously, to understand HQ production! Unified description of regions 1 and 2 + p_T**-spectrum for** $2 \to 1$ **production** $\Rightarrow$ k_T**-factorization.**

Parton Reggeization Approach

Traditionally, k_T**-factorization** is motivated starting from the BFKL evolution equation ($\log 1/x$-resummation). We derive the factorization formula of **PRA** starting from the Collinear Parton Model (see Sec. II of [A. Karpishkov, M. Nefedov, V. Saleev, hep-ph/1707.04068] for the details).



Auxiliary CPM subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2).$$

Modified Multi-Regge-Kinematics approximation (mMRK-approximation) for $\overline{|\mathcal{M}|^2}$ $\quad (z_1 = q_1^+/p_1^+,\ z_2 = q_2^-/p_2^-)$:

$$\overline{|\mathcal{M}|^2}_{\text{mMRK}} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{gg}(z_1) P_{gg}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $\overline{|\mathcal{A}_{PRA}|^2}$ – gauge-invariant PRA amplitude with **off-shell** (**Reggeized**) initial-state partons:

$$\boxed{q_{1,2}^{\mu} = x_{1,2} P_{1,2}^{\mu} + q_{T1,2}^{\mu},\ \ q_{1,2}^2 = -t_{1,2} < 0}.$$

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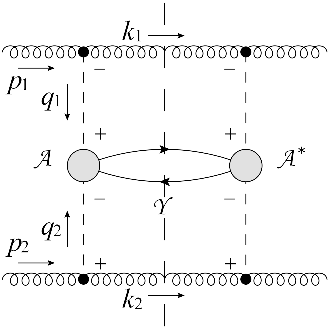


Regions of validity of mMRK approximation ($z_1 = q_1^+/p_1^+$, $z_2 = q_2^-/p_2^-$):

- Collinear region: $t_{1,2} \ll \mu^2,\ 0 \leq z_{1,2} \leq 1$
- Multi-Regge region: $t_{1,2} \sim \mu^2,\ z_{1,2} \ll 1.$

Multi-Regge Kinematics = **large Rapidity Gaps**:

$$y(k_1) - y(P_{\mathcal{A}}) \sim \log \frac{1}{z_1},\ y(P_{\mathcal{A}}) - y(k_2) \sim \log \frac{1}{z_2}.$$

Parton Reggeization Approach

Substituting the mMRK approximation for $\overline{|\mathcal{M}|^2}$ to the CPM factorization fromula, we obtain the PRA factorization formula:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \tilde{\Phi}_g(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \tilde{\Phi}_g(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\rm PRA},$$

where $\tilde{\Phi}$ are the tree-level "uninitegrated PDFs" (next slide) and

$$d\hat{\sigma}_{\rm PRA} = \frac{\overline{|\mathcal{A}_{PRA}|^2}}{2Sx_1x_2} \cdot (2\pi)^4 \delta\left(q_1 + q_2 - P_{\mathcal{A}}\right) d\Phi_{\mathcal{A}}.$$

Note: The flux-factor of CPM $I = 2Sx_1x_2$ for the **off-shell** initial-state partons. Other approaches in the literature are **NOT** consistent with Multi-Regge limit!

Unintegrated PDFs

The tree-level "uninitegrated PDFs":

$$\tilde{\Phi}_g(x,t,\mu^2) = \frac{1}{t}\frac{\alpha_s}{2\pi}\int\limits_x^1 dz\ P_{gg}(z)\cdot\frac{x}{z}f_g\left(\frac{x}{z},\mu^2\right),$$

contain singularities for $t_{1,2}\to 0$ and $z_{1,2}\to 0$. Introduction of the **Sudakov formfactor** ($T_i(t,\mu^2)$ $\Rightarrow$ resummation of $\log^2(t/\mu^2)$-corrections in LLA) and **rapidity-ordering condition** for $z_{1,2}$, converts them into well-known (in k_T-factorization community) Kimber-Martin-Ryskin unPDFs [KMR, 2001]:

$$\Phi_i(x,t,\mu^2) = \frac{T_i(t,\mu^2)}{t}\frac{\alpha_s(t)}{2\pi}\sum_{j=q,\bar{q},g}\int\limits_x^1 dz\ P_{ij}(z)\cdot\frac{x}{z}f_j\left(\frac{x}{z},t\right)$$
$$\times\ \theta\left(1-\Delta_{KMR}(t,\mu^2)-z\right),$$

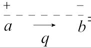
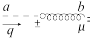
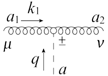
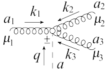
normalized such as:

$$\int\limits_0^{\mu^2} dt\ \Phi_i(x,t,\mu^2) = xf_i(x,\mu^2).$$

PRA amplitudes

In PRA, the **gauge-invariant** matrix elements with **off-shell** initial-state partons are obtained in the framework of Effective Field Theory for Multi-Regge processes in QCD, introduced by L. N. Lipatov [Lipatov, 1995].

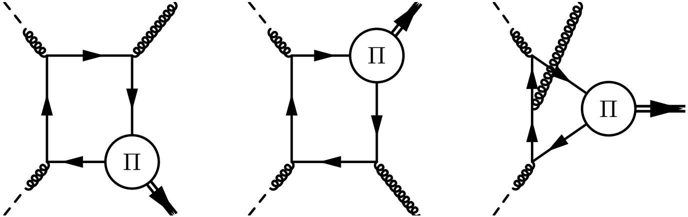
Part of the Feynman rules:

 $= \frac{-i\delta_{ab}}{2q^2}$	 $= (-iq^2) n^{\mp}_{\mu} \delta_{ab}$
	$g_s f_{aa_1a_2} \left(n^{\mp}_{\mu} n^{\mp}_{\nu}\right) \frac{q^2}{k_1^{\mp}}$
	$i g_s^2 \left(n^{\mp}_{\mu_1} n^{\mp}_{\mu_2} n^{\mp}_{\mu_3}\right) \frac{q^2}{k_3^{\mp}} \left[\frac{f_{aba_1} f_{ba_2a_3}}{k_1^{\mp}} + \frac{f_{aba_2} f_{ba_1a_3}}{k_2^{\mp}}\right]$

where $n^{\mu}_{\pm} = 2P^{\mu}_{2,1}/\sqrt{S}$, $n_+^2 = n_-^2 = 0$, $n^+ n^- = 2$.

Induced vertices of interaction of **Reggeized gluon (R)** with n gluons are $O(g_s^n)$, due to the Wilson lines in the Lagrangian of EFT.

$R + R \to Q\bar{Q}\left[{}^3S_1^{(1)}\right] + g$ amplitude.



Collinear limit:

$$\int_0^{2\pi} \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2}\to 0} \overline{|\mathcal{A}_{PRA}|^2} = \overline{|\mathcal{A}_{CPM}|^2}$$

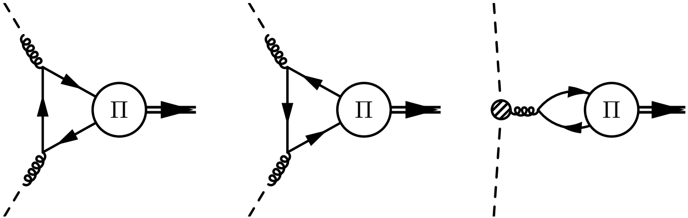
Squared amplitude **coincides** with the squared amplitude, obtained in the "old k_T-factorization" prescription:

$$\varepsilon^\mu(q_{1,2}) \to \frac{q^\mu_{T1,2}}{\sqrt{t_{1,2}}},$$

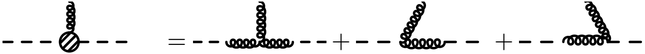
due to the Slavnov-Taylor identities.

$2 \to 1$ amplitudes.

Processes $R + R \to Q\bar{Q}\left[{}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_J^{(1,8)}\right]$



Lipatov vertex:



Squared PRA amplidude **coincides** with the amplitudes obtained earlier in the “old k_T-factorization” [Kniehl, Saleev, Vasin, 2006].
For **single** heavy quarkonium production, PRA = “old k_T-factorization” with KMR unPDF and CPM flux factor for **all subprocesses.**
For **pair** production, the amplitudes are different, see the talk by Zhiguo He.

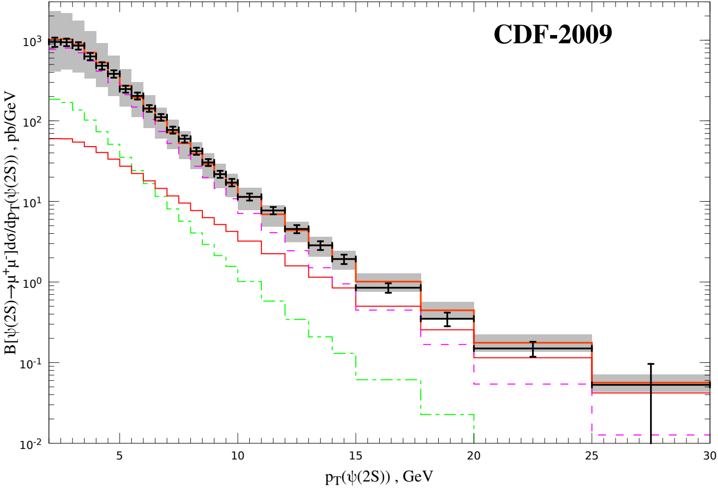
Heavy quarkonium production in PRA

The fits of Color-Octet LDMEs for J/ψ, $\psi(2S)$ and χ_{cJ}-production in the LO of PRA has been performed since [Kniehl, Vasin, Saleev, 2006], and in [Nefedov, Saleev, Shipilova, 2012] the LHC data has been included.
The fit of CO LDMEs for $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ production, based on LHC data, has been performed in [Nefedov, Saleev, Shipilova, 2013].
Strategy of the present work:

- Concentrate on the most "clean" states: $\psi(2S)$ and $\Upsilon(3S)$ which are minimally affected by feed-down decays. For $\psi(2S)$ there is no excited states with $M_{\psi(2S)} < M < 2M_D$. For $\Upsilon(3S)$ there is only $\chi_b(3P)$, with unknown branchings.
- Include Tevatron ($p_T < 30$ GeV) and **latest** LHC (p_T up to 100 GeV) data into the fit.
- Include **fragmentation mechanism**, to describe high-p_T data and obtain consistent description for all values of $p_T \Rightarrow$ reliable LDMEs.
- Obtain the predictions for **polarization** of $\psi(2S)$ and $\Upsilon(3S)$ and compare them with the **data**.

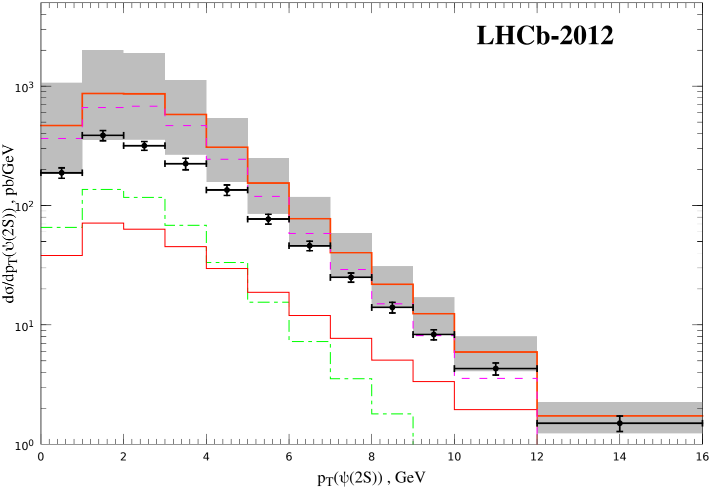
In such a way we obtain the clear and unambiguous manifestation of *polarization puzzle.*

Prompt $\psi(2S)$-production. Fit of the CDF-2009 data. $\sqrt{S} = 1.96$ TeV.



Contributions: ${}^3S_1^{(1)}$, ${}^3S_1^{(8)}$, ${}^1S_0^{(8)}$.

Prompt $\psi(2S)$-production. Description of the LHCb-2012 data. $\sqrt{S} = 7$ TeV, $2 < y < 4.5$.



Contributions: ${}^3S_1^{(1)}$, ${}^3S_1^{(8)}$, ${}^1S_0^{(8)}$.

Fit results

$$M_R^{\mathcal{H}} = \left\langle \mathcal{O}^{\mathcal{H}}\left[{}^1S_0^{(8)}\right]\right\rangle + \frac{R_{\mathcal{H}}}{M_{\mathcal{H}}^2}\left\langle \mathcal{O}^{\mathcal{H}}\left[{}^3P_0^{(8)}\right]\right\rangle$$

LDME	Fusion	Fragmentation	NLO CPM [1,2]	NLO CPM [3]
$\left\langle \mathcal{O}^{\psi(2S)}\left[{}^3S_1^{(1)}\right]\right\rangle$ /GeV^3	0.65 ± 0.06	0.65 ± 0.06	0.76	0.76
$\left\langle \mathcal{O}^{\psi(2S)}\left[{}^3S_1^{(8)}\right]\right\rangle$ /GeV^3 $\times 10^3$	1.84 ± 0.23	2.57 ± 0.09	1.2 ± 0.3	2.80 ± 0.49
$M_R^{\psi(2S)}$/GeV^3 $\times 10^2$	3.11 ± 0.14	2.70 ± 0.11	2.0 ± 0.6	0.37 ± 4.85
$R_{\psi(2S)}$	23.0 ± 1.0	23.0 ± 1.0	23.5	23.0
χ^2/d.o.f.	0.6	1.1	0.56	2.84
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(1)}\right]\right\rangle$ /GeV^3	3.54	–	3.54	–
$\left\langle \mathcal{O}^{\Upsilon(3S)}\left[{}^3S_1^{(8)}\right]\right\rangle$ /GeV^3 $\times 10^2$	2.73 ± 0.15	–	2.71 ± 0.13	–
$M_R^{\Upsilon(3S)}$/GeV^3 $\times 10^2$	0.00 ± 0.18	–	1.083 ± 1.66	–
$R_{\Upsilon(3S)}$	22.1 ± 0.7	–	22.1	–
χ^2/d.o.f.	9.7	–	3.16	–

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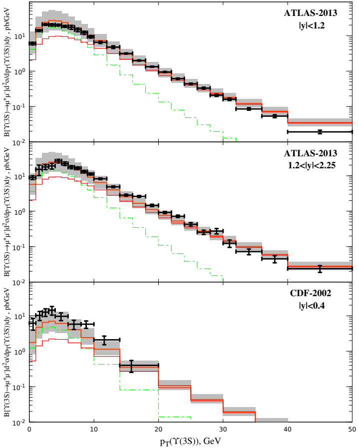
$\Upsilon(3S)$-production. Fit of the ATLAS and CDF data



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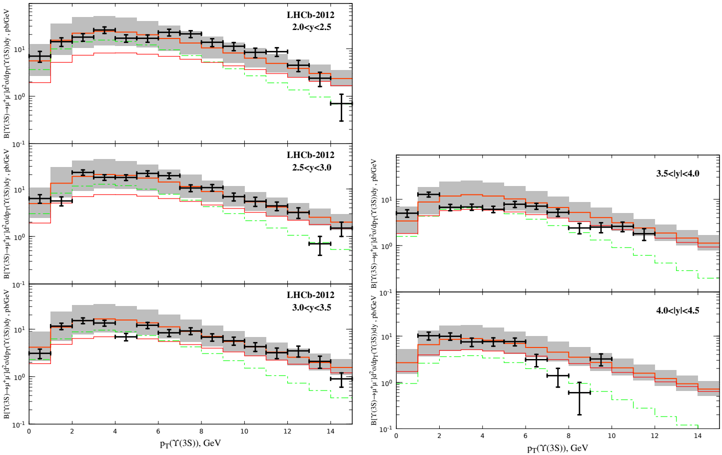
$\Upsilon(3S)$-production. Description of the LHCb data.



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Fragmentation mechanism

In the LO + Leading Logarithmic Approximation, only the production of ${}^3S_1^{(8)}$-state recieves the corrections $\sim \log(p_T/M)$. They can be taken into account by the introduction of fragmentation function $D_{g\to\mathcal{H}}(z,\mu_F^2)$:

$$\frac{d\sigma}{dp_T^{\mathcal{H}}dy_{\mathcal{H}}}(pp\to\mathcal{H}+X)=\int\limits_0^1 dz\frac{d\sigma}{dp_T^g dy_g}(pp\to g+X)\cdot D_{g\to\mathcal{H}\left[{}^3S_1^{(8)}\right]}(z,\mu_F^2),$$

which evolves with the scale μ_F^2 according to the DGLAP equations, with the following initial condition at the starting scale $\mu_{F0}^2 = M^2$:

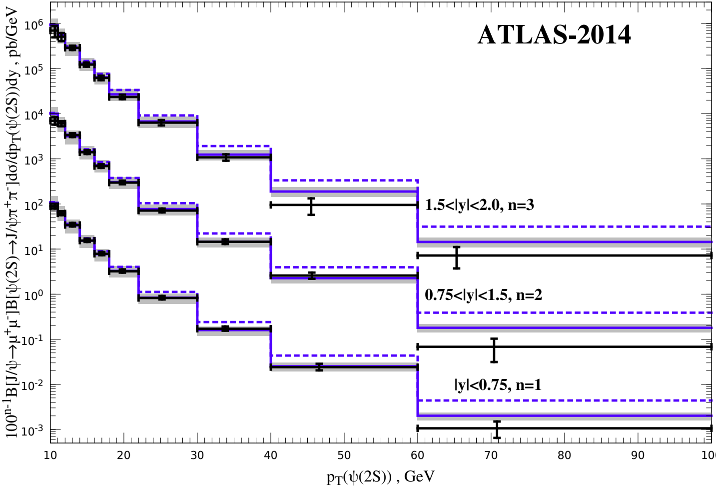
$$D_{g\to\mathcal{H}\left[{}^3S_1^{(8)}\right]}(z,\mu_{F0}^2)=\frac{\pi\alpha_s(\mu_{F0}^2)}{6M_{\mathcal{H}}^3}\left\langle\mathcal{O}^{\mathcal{H}}\left[{}^3S_1^{(8)}\right]\right\rangle\delta(1-z).$$

The gluon production cross-section in PRA is:

$$\frac{d\sigma}{dp_T^g dy_g}=\frac{1}{(p_T^g)^3}\int\limits_0^\infty dt_1\int\limits_0^{2\pi}d\phi_1\Phi_g(x_1,t_1,\mu_F^2)\Phi_g(x_2,t_2,\mu_F^2)\overline{|\mathcal{M}(RR\to g)|^2},$$

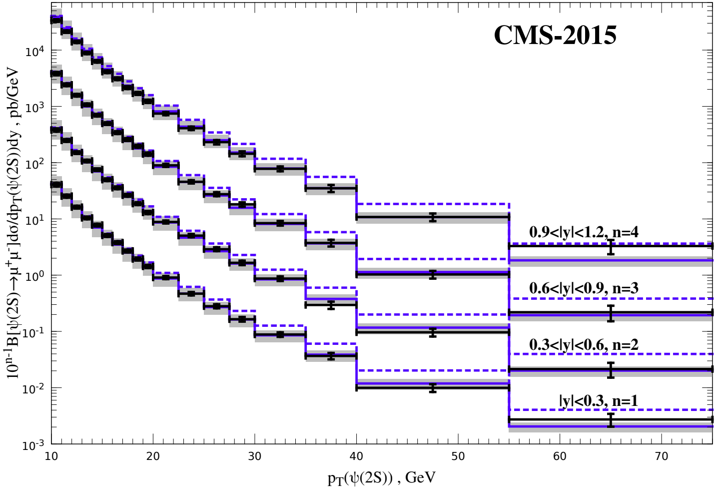
where $\overline{|\mathcal{M}(RR\to g)|^2}=(3/2)\pi\alpha_s(\mu_R^2)(p_T^g)^2$ is the square of Lipatov vertex.

Fit of the ATLAS data ($\sqrt{S} = 7$ TeV).



Dashed histogram – no fragmentation, **solid histogram** – fragmentation included.

Fit of the CMS data ($\sqrt{S} = 7$ TeV).



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Polarization observables

The angular distribution of decay muons in the rest frame of the heavy quarkonium can be parametrized as:

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos(2\varphi) + \lambda_{\theta\varphi} \sin(2\theta)\cos\varphi.$$

The polarization parameter λ_θ can be calculated as:

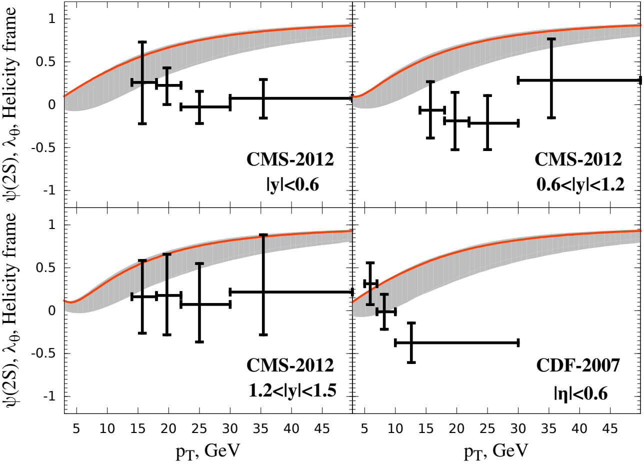
$$\lambda_\theta = \frac{\sigma^{\mathcal{H}} - 3\sigma_L^{\mathcal{H}}}{\sigma^{\mathcal{H}} + \sigma_L^{\mathcal{H}}},$$

where σ_L was estimated using a simple model expression [Cho, Leibovich, 1996]

$$\begin{aligned} \sigma_L^{\mathcal{H}} &= \sigma_0^{\mathcal{H}}\left[{}^3S_1^{(1)}\right] + \sigma_0^{\mathcal{H}}\left[{}^3S_1^{(8)}\right] + \frac{1}{3}\left(\sigma^{\mathcal{H}}\left[{}^1S_0^{(8)}\right] + \sigma^{\mathcal{H}}\left[{}^3P_0^{(8)}\right]\right) \\ &\quad + \frac{1}{2}\left(\sigma_1^{\mathcal{H}}\left[{}^3P_1^{(8)}\right] + \sigma_1^{\mathcal{H}}\left[{}^3P_2^{(8)}\right]\right) + \frac{2}{3}\sigma_0^{\mathcal{H}}\left[{}^3P_2^{(8)}\right]. \end{aligned}$$

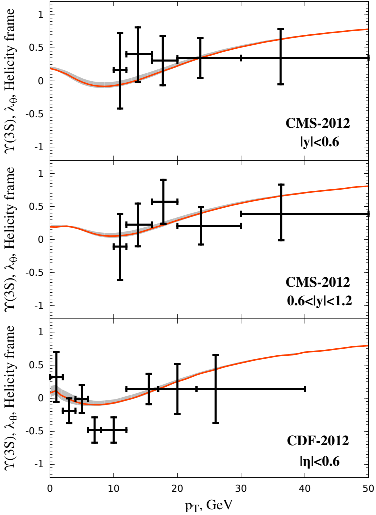
The matrix elements corresponding to $\sigma_{|J_z|}^{\mathcal{H}}[{}^{2S+1}L_J]$ are derived in the paper for the **$\hat{s}$-channel helicity frame**.

Polarization of $\psi(2S)$



"Polarization puzzle" is reproduced. In the model, polarization at high p_T is mostly transverse, due to $g^\star \to c\bar{c}\left[{}^3S_1^{(8)}\right]$-transition.

Polarization of $\Upsilon(3S)$



There is essentially no tension with data for polarization of $\Upsilon(3S)$, due to the smaller fraction of ${}^3S_1^{(8)}$ state at high p_T.

Conclusions

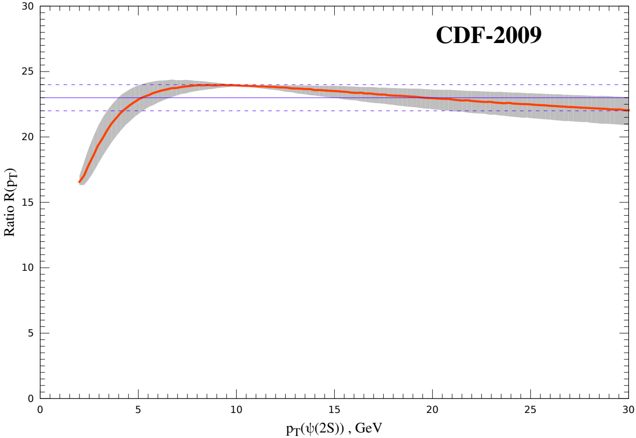
- Description of the production of heavy quarkonia in the LO of PRA is **the same** as in the NLO of CPM. Both hierarchy of contributions and values of LDMEs are similar. The CO contributions are necessary, contrary to some statements in the literature [Baranov, Lipatov, Zotov, 2012]. *The discrepancy has been caused by the use of* $2\hat{s} = 2M^2$ *instead of* $2Sx_1x_2 = M^2 + \mathbf{p}_T^2$ *flux-factor for the* $2 \to 1$ *subprocess in this paper.*
- The fragmentation mechanism is **important** at high p_T in PRA. In CPM it can be less significant, because FSR of 1 gluon is explicitly taken into account.
- The polarization problem for $\psi(2S)$ is reproduced. It is **very robust** and can be solved only by some kind of depolarizing final-state interaction.

Thank you for your attention!

Backup slides

Plot of $R_{\mathcal{H}}(p_T)$

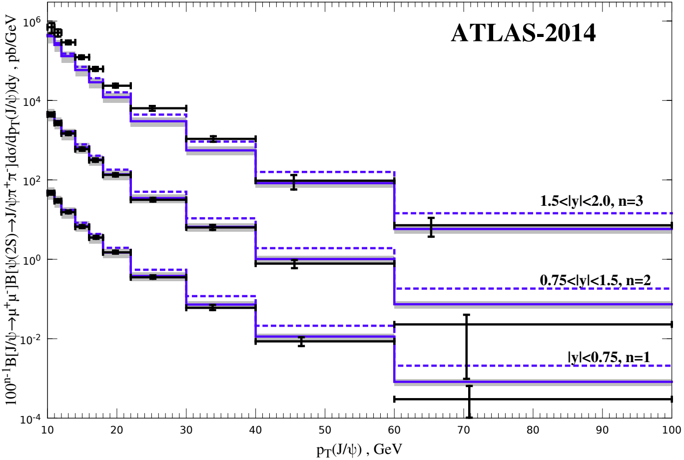
$$R_{\mathcal{H}}(p_T) = \frac{M_{\mathcal{H}}^2 \sum\limits_{J=0}^{2} (2J+1) d\sigma/dp_T \left[{}^3P_J^{(8)}\right]}{d\sigma/dp_T \left[{}^1S_0^{(8)}\right]}$$



Description of $p_T(J/\psi)$-spectrum from $\psi(2S) \to J/\psi + \pi^+\pi^-$

The approximate formula for p_T-shift in the decay $\mathcal{H}_1 \to \mathcal{H}_2 + X$ is used:

$$\langle p_T^{\mathcal{H}_2} \rangle = \frac{M_{\mathcal{H}_2}}{M_{\mathcal{H}_1}} p_T^{\mathcal{H}_1} + \mathcal{O}\left(\frac{(\Delta M)^2}{M^2}, \frac{M}{p_T} \right)$$



Polarization transfer model

$$\sigma_L^{\mathcal{H}} = \sigma_0^{\mathcal{H}}\left[{}^3S_1^{(1)}\right] + \sigma_0^{\mathcal{H}}\left[{}^3S_1^{(8)}\right] + \frac{1}{3}\left(\sigma^{\mathcal{H}}\left[{}^1S_0^{(8)}\right] + \sigma^{\mathcal{H}}\left[{}^3P_0^{(8)}\right]\right) + \frac{1}{2}\left(\sigma_1^{\mathcal{H}}\left[{}^3P_1^{(8)}\right] + \sigma_1^{\mathcal{H}}\left[{}^3P_2^{(8)}\right]\right) + \frac{2}{3}\sigma_0^{\mathcal{H}}\left[{}^3P_2^{(8)}\right].$$

- ${}^3S_1^{(8)}$ – emission of (at least) two **very soft** gluons, which can not flip the spin of heavy quark (HQSS)
- $\sum_{J_{2z}=0,\pm1} |\langle J_1 = 1, J_{1z} = 0; J_2 = 1, J_{2z} \, | J = 0, J_z = 0\rangle|^2 = \frac{1}{3},$
- $\sum_{J_{2z}=0,\pm1} |\langle J_1 = 1, J_{1z} = 0; J_2 = 1, J_{2z} \, | J = 1, J_z = 1\rangle|^2 = \frac{1}{2},$
  $\sum_{J_{2z}=0,\pm1} |\langle J_1 = 1, J_{1z} = 0; J_2 = 1, J_{2z} \, | J = 2, J_z = 1\rangle|^2 = \frac{1}{2},$
- $\sum_{J_{2z}=0,\pm1} |\langle J_1 = 1, J_{1z} = 0; J_2 = 1, J_{2z} \, | J = 2, J_z = 0\rangle|^2 = \frac{2}{3}.$