Determination of the strong coupling constant from lattice QCD

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Determination of the strong coupling constant from the moments of pseudo-scalar charmonium correlators

Maezawa, PP, Phys. Rev. D 94 (2016) 034507

$$\alpha_s(M_Z) = 0.11622(84)$$

Determination of the strong coupling constant from the energy of static quark anti-quark pair

Bazavov, Brambilla, Gacia i Tormo, PP, Soto, Vairo, Phys. Rev. D 90 (2014) 074038

$$\alpha_s(M_Z) = 0.1166^{+0.0012}_{-0.0008}$$

Some lattice details

Highly improved Staggered Quark (HISQ) action and tree-level improved gauge action

HotQCD gauge configurations : 2+1 flavor QCD
physical m_s , $m_l = m_s/20$: $m_K = 504$ MeV, $m_\pi = 161$ MeV
Bazavov et al, PRD90 (2014) 094503

Lattice spacing set by the r_1 scale

$$\left(r^2 \frac{dE_0(r)}{dr} \right)_{r=r_1} = 1$$

$r_1 = 0.3106(14)(8)(4)$ fm (pion decay constant)

Temperature is varied by the lattice spacing a

$T = (1/N_\tau a)$

Many lattice spacings available, $a_{min} = 0.041$fm

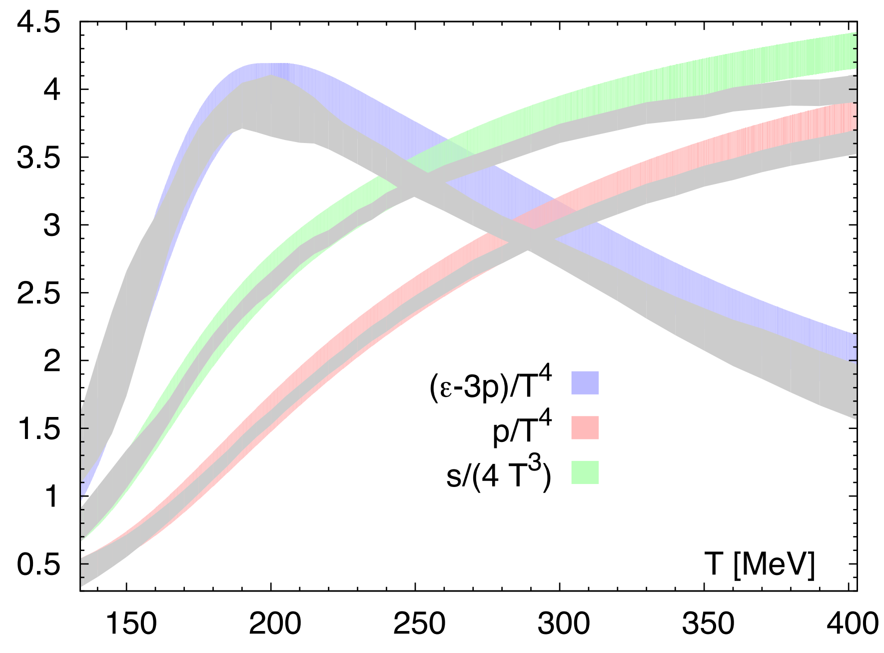
Additional gauge configurations for $m_l = m_s/5$ on 64^4 lattices with $a = 0.035$ fm, 0.029 fm and 0.025 fm to obtain the static quark potential at shorter distances

Bazavov, PP, Weber, arXiv:1710:05024

statistics for the $T=0$ runs:

24^3x32:	4-8K TU
32^4, 32^3x64:	7-40K TU
48^4:	8-16K TU
48^3x64:	8-9K TU
64^4 :	9K TU

in molecular dynamic time units (TU)



Moments of charm current correlators

We use moments method pioneered by HPQCD and Karlsruhe group:

$$G(t) = a^6 m_{c0}^2 \sum_{\mathbf{x}} \langle j_5(\mathbf{x}, t) j_5(0,0) \rangle, \; j_5 = \bar{\psi}_c \gamma_5 \psi_c$$

$$G_n = \sum_t (t/a)^n \, G(t)$$

Calculated continuum perturbation theory to order α_s^3

$$G_n = \frac{g_n(\alpha_s(\mu), m_c(\mu))}{m_c^{n-4}(\mu)}, \quad g_n = \sum_j g_{nj}(m_c, \mu) \alpha_s^j(\mu)$$

To cancel lattice effects consider the reduced moments $R_n = \left(\frac{G_n}{G_n^0} \right)^{1/(n-4)}$

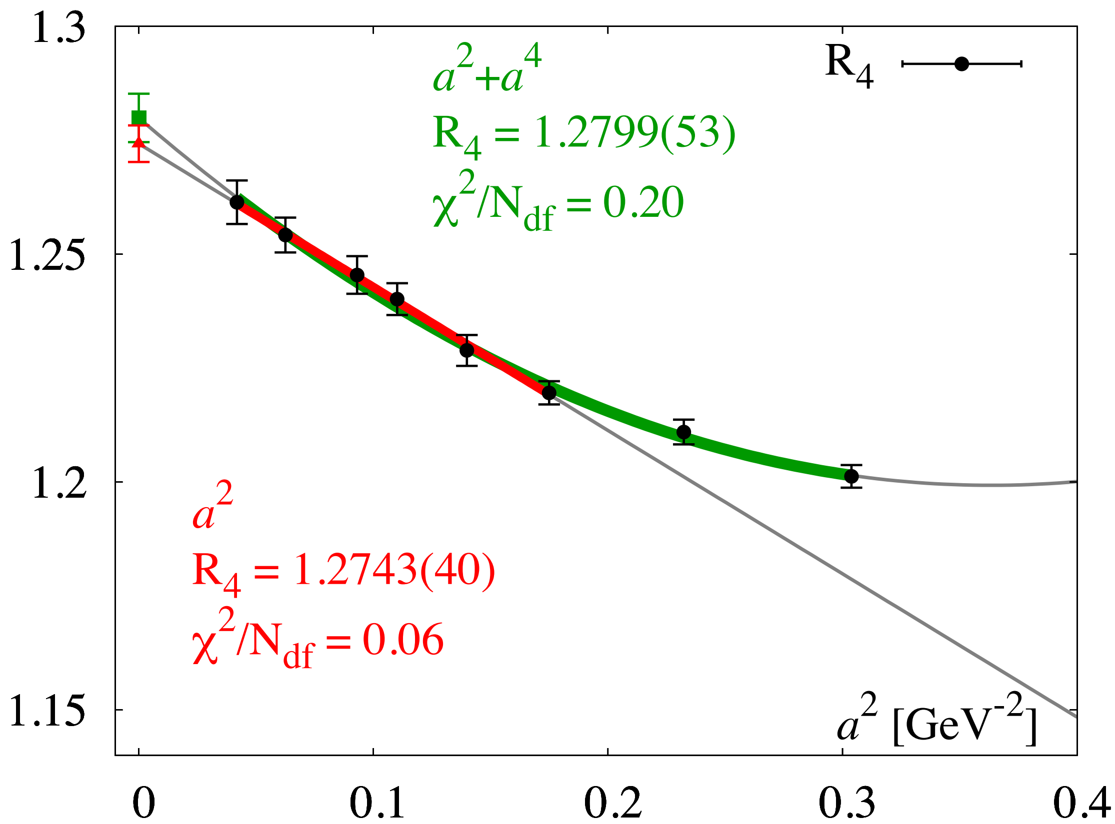
and similarly on the weak coupling side:

$$r_n = \sum_j r_{nj}(m_c, \mu) \alpha_s^j(\mu)$$

Effects of charm loop are 0.7% for R_4 and 0.1% for R_6 in perturbation theory
This information can be used to correct for charm loops in for in 2+1 flavor simulations

Allison et al, PRD78 (2008) 054513

Moment of charm current correlators and coupling constant



In the continuum limit the results for the 4th moment agree with HPQCD; our central value is slightly smaller

HPQCD 2008: Allison et al, PRD78 (2008) 054513

HPQCD 2014: Chakraborty et al, PRD 91 (2015) 054508

Determination at the lowest scale !

$\mu = m_c$ ➡ No large logs in r_4 ➡

Stat. cont. pert.

$$\alpha_s(m_c) = 0.3697\,(54)\,(64)\,(15)$$

Moments:

$$\alpha_s(1.5\text{GeV}, n_f = 3) = 0.3316(69)$$

$$\alpha_s(M_Z, n_f = 5) = 0.11622(84)$$

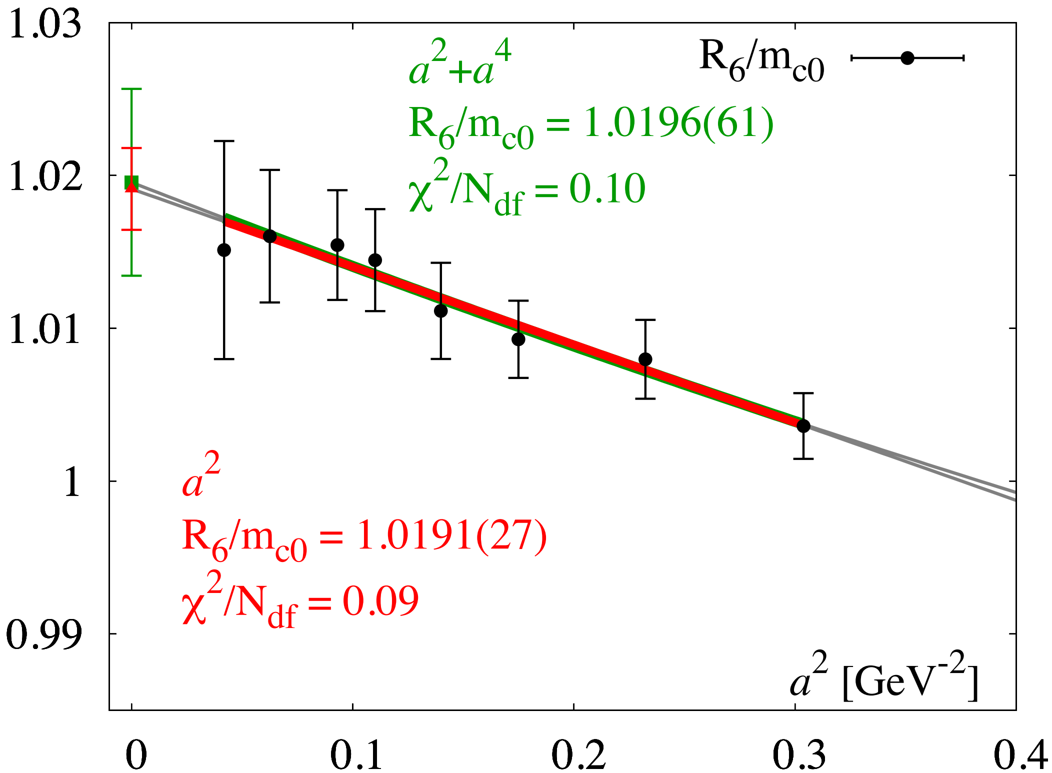
Static energy, Brambilla et al, PRD90 ('14) 074038

$$\alpha_s(1.5\text{GeV}, n_f = 3) = 0.336(+12)(-08)$$

$$\alpha_s(M_Z, n_f = 5) = 0.1166(+12)(-08)$$

also agrees with 2008 HPQCD value: $\alpha_s(M_Z, n_f = 5) = 0.1174(12)$

Moment of charm current correlators and charm mass



Use 6[th] moment to determine the charm quark mass

$$R_6/m_{c0} = 1.0191(27)$$

$$\mu = m_c$$

stat. pert. α_s scale

$$m_c(\mu = m_c, n_f = 3) = 1.2668(33)(34)(79)(73)\ \text{GeV}$$

HPQCD 2014: Chakraborty et al, PRD 91 (2015) 054508 $m_c(m_c) = 1.2733(76)\text{GeV}$

Static quark anti-quark energy in perturbation theory

Potential from pNRQCD, ultrasoft logs, renormalon

$$E_0(r) = V_s(r,\nu,\mu) + \delta_{US}(r,\nu,\mu) + RS(\rho)$$

Problem : Either we have a large log(*vr) or r*-dependent renormalon term (large uncertainty*)*

Solution: First take the derivative in *r* (RS term is gone) the re-sum the large logarithm *v=1/r*
⇒ calculate the force

$$F(r,\frac{1}{r}) = \frac{C_F}{r^2}\alpha_s(1/r)[1 + \frac{\alpha_s(1/r)}{4\pi}\left(\tilde{a}_1 - 2\beta_0\right) + \frac{\alpha_s^2(1/r)}{(4\pi)^2}\left(\tilde{a}_2 - 4\tilde{a}_1\beta_0 - 2\beta_1\right)$$

$$+\frac{\alpha_s^3(1/r)}{(4\pi)^3}\left(\tilde{a}_3 - 6\tilde{a}_2\beta_0 - 4\tilde{a}_1\beta_1 - 2\beta_2\right)$$

$$-\frac{\alpha_s^2(1/r)}{(4\pi)^2}\frac{a_3^L}{2\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(1/r)} - \frac{\alpha_s^2(1/r)}{(4\pi)^2}\frac{4C_A^3\pi^2\eta_0}{3}\left(\alpha_s(\mu) - \alpha_s(1/r)\right)$$

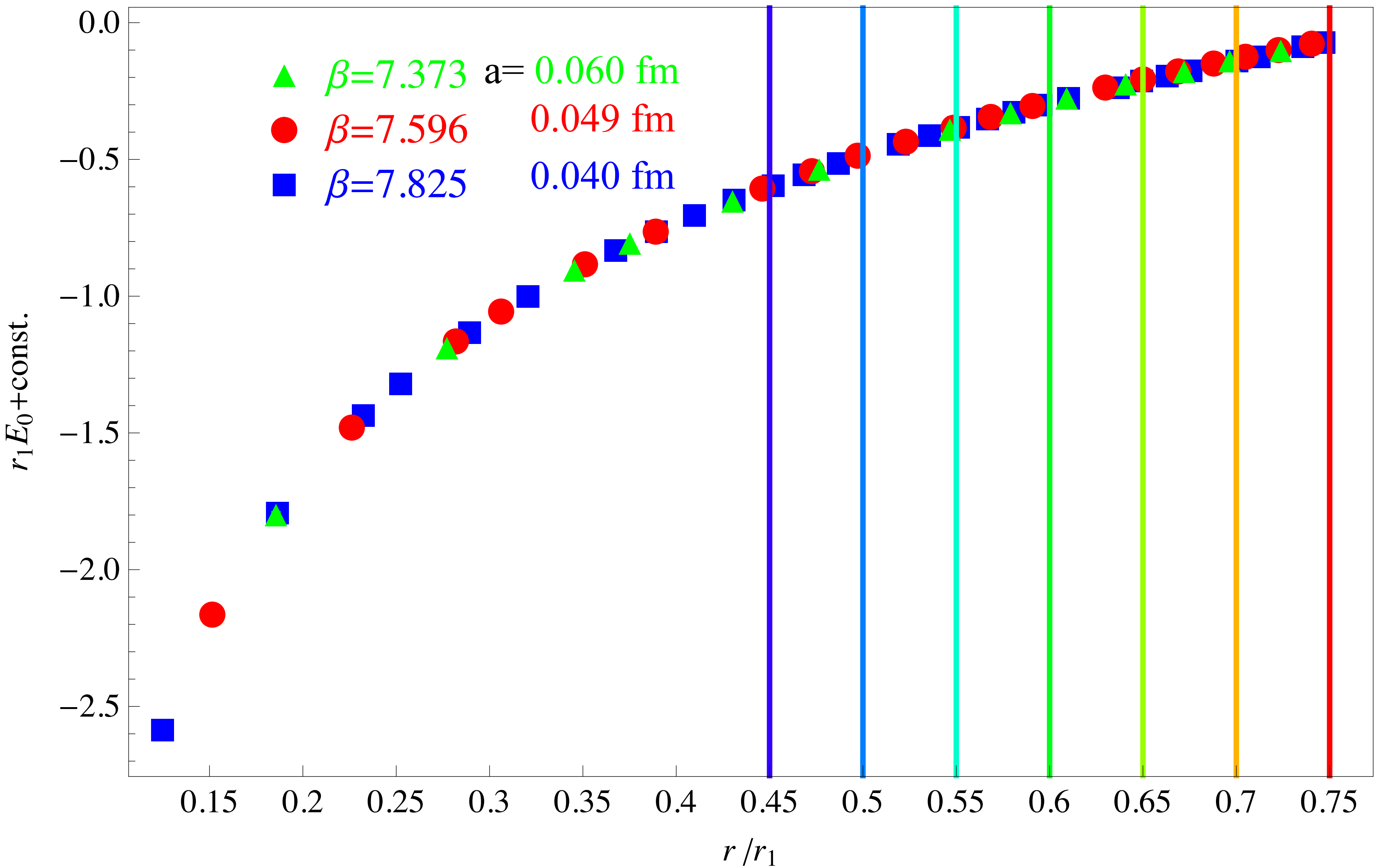
$$+\frac{\alpha_s^3(1/r)}{(4\pi)^3}8C_A^3\pi^2\left(2 - \frac{\tilde{a}_1}{\beta_0}\right)\ln\frac{\alpha_s(\mu)}{\alpha_s(1/r)} + \frac{\alpha_s^2(1/r)\alpha_s(\mu)}{(4\pi)^3}a_3^L\ln\frac{C_A\alpha_s(1/r)}{2r\mu}$$

$$+\mathcal{O}(\alpha_s^4, \alpha_s^5\ln\alpha_s)\Big]$$

see Garcia i Tormo, MPLA 28 133028 for a review

$$E_0(r) = \int_{r^*}^{r} dr' F(r') + const$$

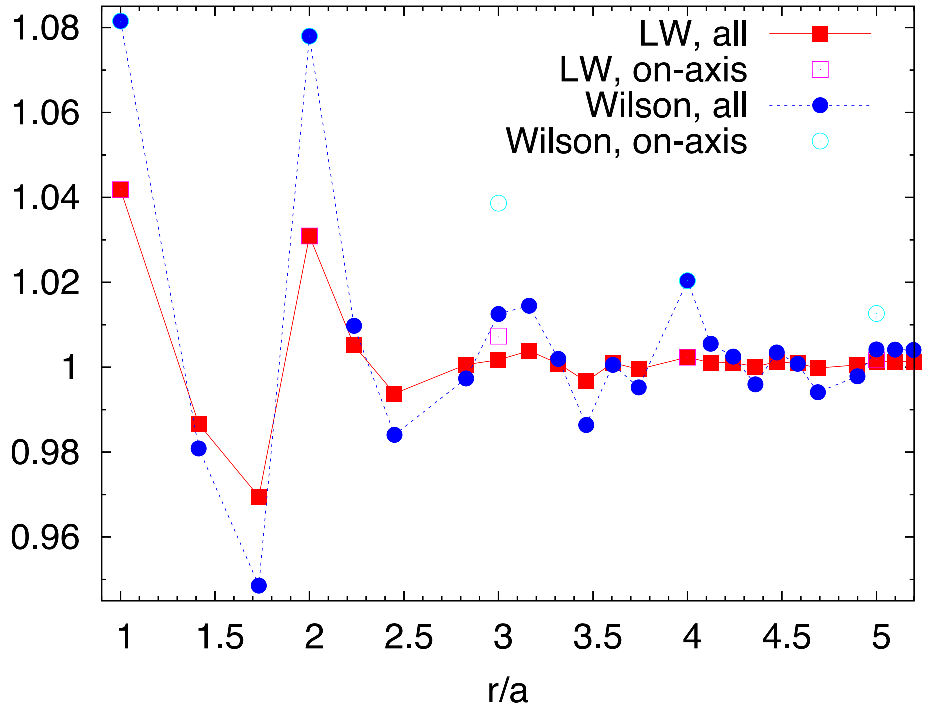
Static quark anti-quark energy on the lattice



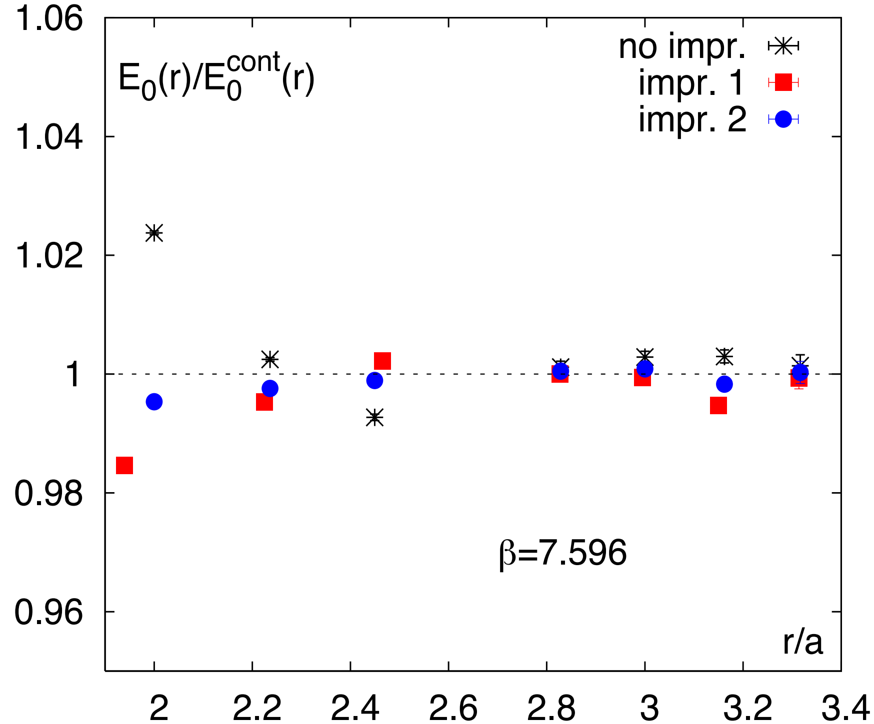
Cutoff effects in the potential are small even at short distances

Static quark anti-quark energy on the lattice (cont'd)

$E_0(r)/E_0^{cont}(r)$ at tree level (free theory)



$\beta = 7.596$



Use the lattice data at the highest beta and for $r/a>2.6$ to provide a continuum estimate for the static energy $E_0^{cont}(r)$

The cutoff effects in the free theory and QCD are quite similar

No cutoff effects visible in the data for $r/a>2.6$ within statistical errors

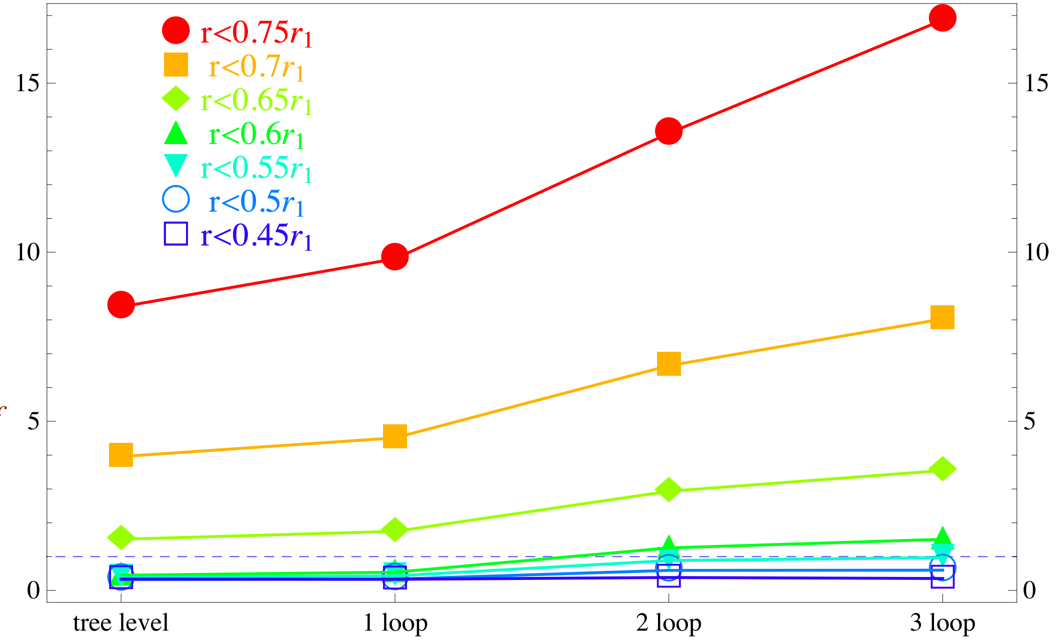
Very little cutoff dependence even for $r/a<2.6$ if tree level improvement is used $r \rightarrow r_I = C_{latt}^{-1}$

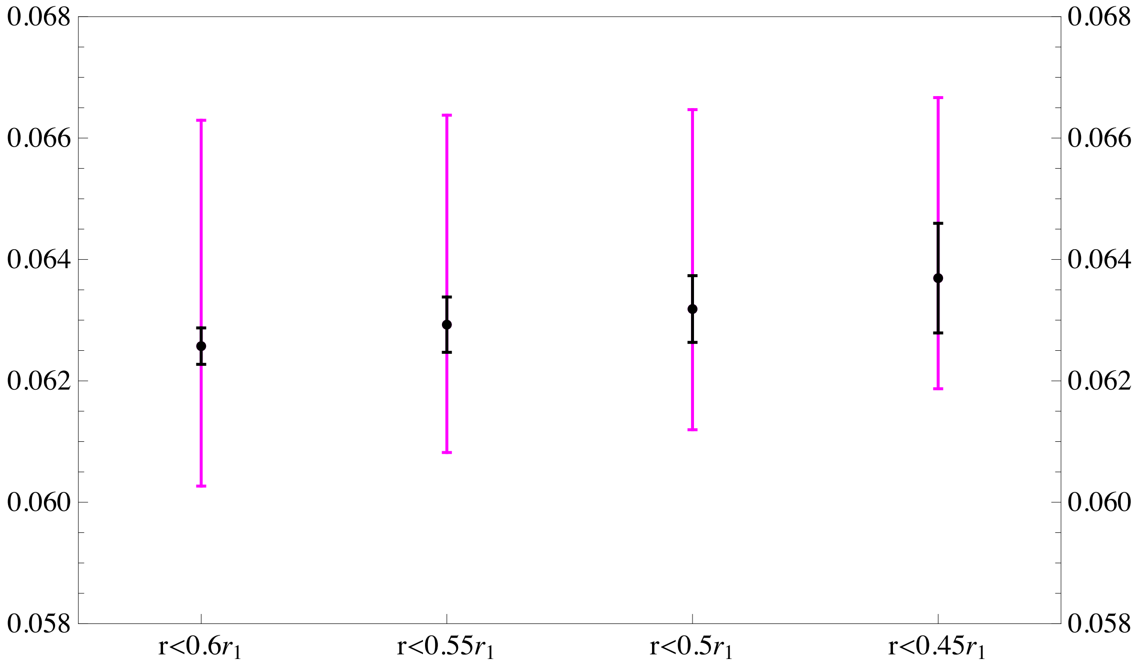
Use the values of $E_0(r)/E_0^{cont}(r)$ to correct for the residual cutoff effects

Fitting the lattice results on the static energy

Example: $\beta = 7.825$

One can fit the data with tree-level 1-loop, 2-loop or 3-loop expressions for $r<0.5\ r_1$ up to an additive constant specified by matching the perturnative potential to the lattice one at $(r/a)^2 = N_{ref}$





Obtain $a\Lambda_{\overline{MS}}$ or $r_1\Lambda_{\overline{MS}}$ from the fit

Perturbative errors (from scale variations $\nu = [1/(\sqrt{2}r), \sqrt{2}/r]$ or higher orders $\pm\alpha_s^4/r^2$) dominate:

3-loop: $r_1\Lambda_{\overline{MS}} = 0.486^{+0.028}_{-0.018}$

Fitting the lattice results on the static energy (cont'd)

For the final result we also included the leading ultra-soft log with $\mu=1.26/r_1 \sim 0.8$ GeV
Results at different lattice spacings (β) are similar :

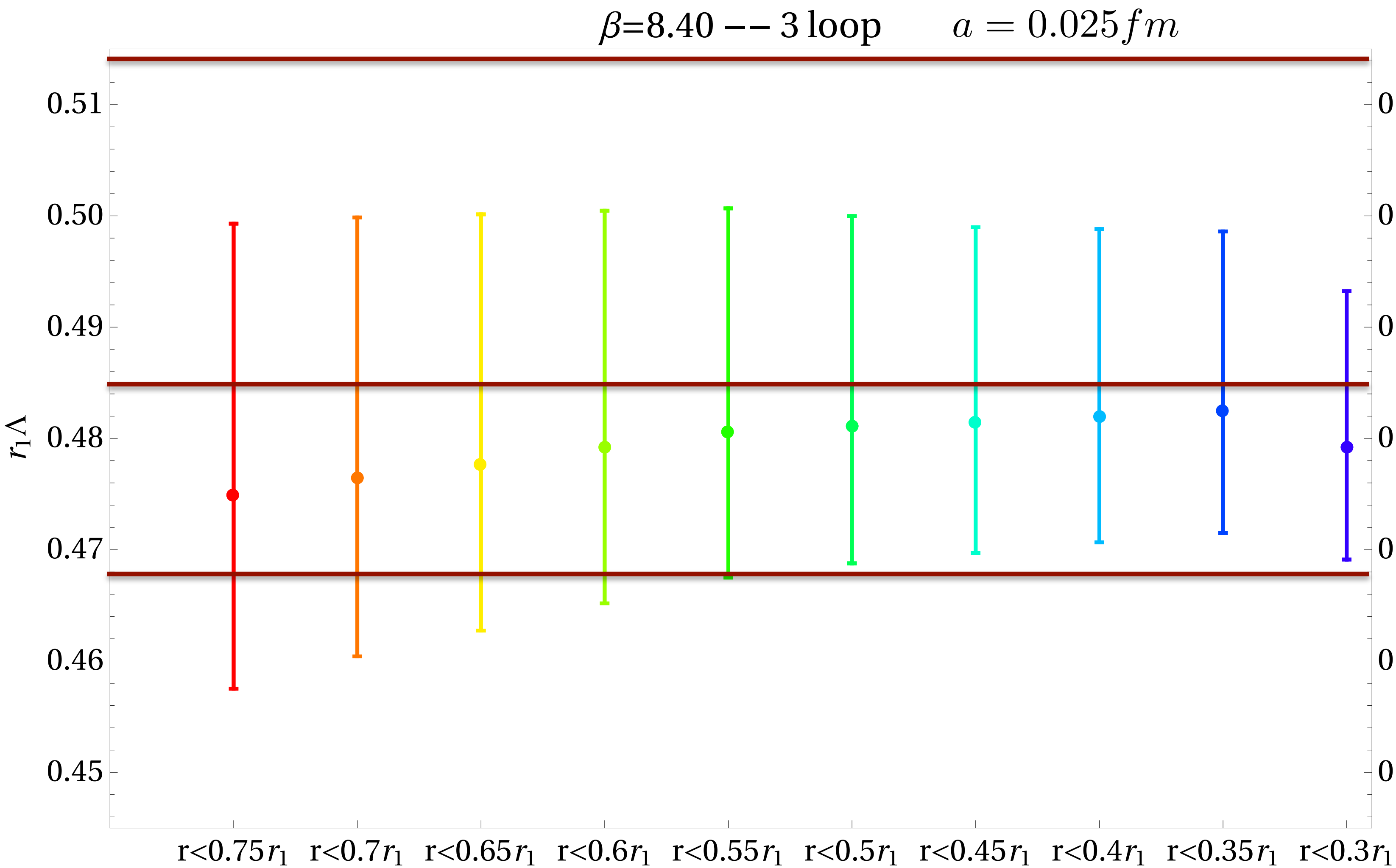
	$a\Lambda_{\overline{MS}}$; $N_{\rm ref}=7$	$a\Lambda_{\overline{MS}}$; $N_{\rm ref}=9$	$a\Lambda_{\overline{MS}}$; range	$r_1\Lambda_{\overline{MS}}$; range
$\beta=7.373$	$0.0957^{+0.0046}_{-0.0028}$ ± 0.0017	$0.0957^{+0.0046}_{-0.0028}$ ± 0.0017	$0.0957^{+0.0046}_{-0.0028}$ ± 0.0017	$0.4949^{+0.0240}_{-0.0144} \pm 0.0086 \pm 0.0025$ $= 0.4949^{+0.0256}_{-0.0170}$
$\beta=7.596$	$0.0781^{+0.0046}_{-0.0029}$ ± 0.0007	$0.0785^{+0.0046}_{-0.0029}$ ± 0.0007	$0.0783^{+0.0048}_{-0.0031}$ ± 0.0010	$0.4961^{+0.0303}_{-0.0197}{}^{+0.0066}_{-0.0061} \pm 0.0044$ $= 0.4961^{+0.0313}_{-0.0211}$
$\beta=7.825$	$0.0644^{+0.0032}_{-0.0019}$ ± 0.0006	$0.0643^{+0.0032}_{-0.0020}$ ± 0.0008	$0.0643^{+0.0033}_{-0.0021}$ ± 0.0008	$0.4944^{+0.0256}_{-0.0159} \pm 0.0065 \pm 0.0037$ $= 0.4944^{+0.0267}_{-0.0175}$
Average				$\mathbf{r_1\Lambda_{\overline{MS}} = 0.495^{+0.028}_{-0.018}}$

$$\Lambda_{\overline{MS}} = 314.5^{+17.6}_{-11.7} \pm 1.7 \text{ MeV} = 315^{+18}_{-12} \text{ MeV}$$

$$\alpha_s(1.5 \text{ GeV}, n_f=3) = 0.336^{+0.012}_{-0.008}$$

$$\alpha_s(M_Z, n_f=5) = 0.1166^{+0.0012}_{-0.0008}$$

Cross-checks using the new very fine lattices



The new very fine lattices give results for that are comparable with the 2014 results but with significantly reduced uncertainty.

Summary

The strong coupling constant has been determined using 2+1 flavor HISQ lattices provided by HotQCD Collaboration from:

Moments of the pseudo-scalar charmonium correlators

$$\alpha_s(M_Z) = 0.11622(84)$$

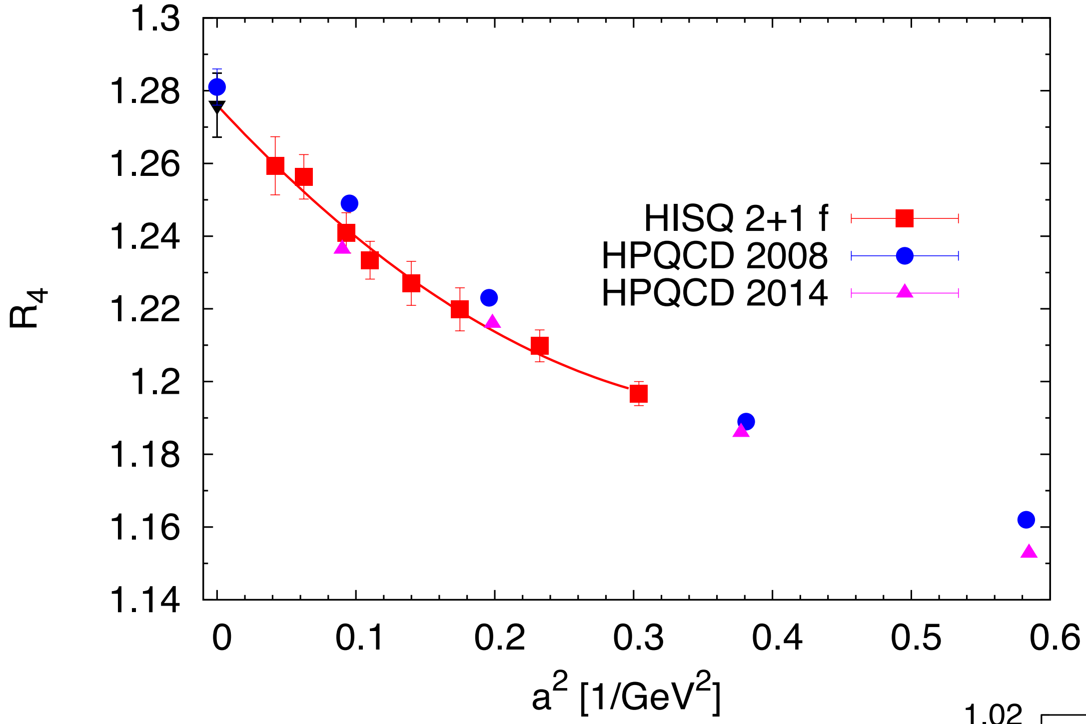
Static quark anti-quark energy

$$\alpha_s(M_Z) = 0.1166^{+0.0012}_{-0.0008}$$

The two results are consistent but lower than many lattice determinations and the FLAG average

New results on the static quark anti-quark energy on a=0.035, 0.029 and 0.025 fm lattices confirm the previous results and the preliminary analysis indicates that the errors are down by factor 1.5

Back-up slides:



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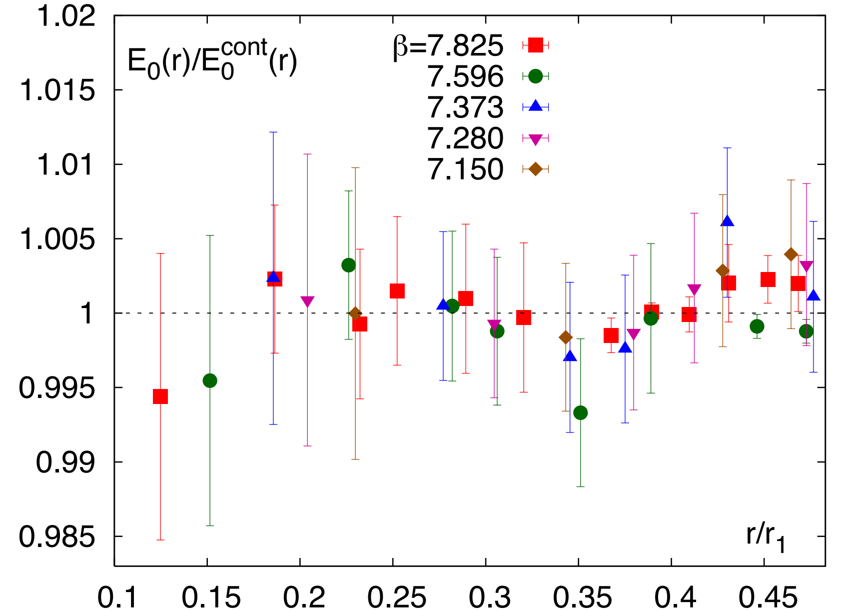
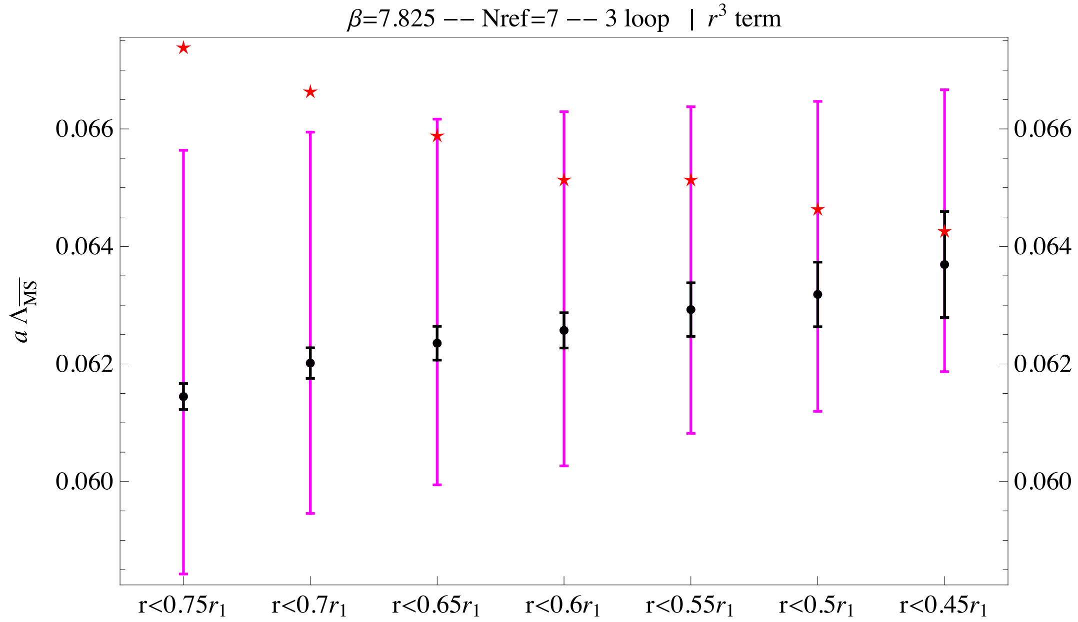
HPQCD 2008: Allison et al, PRD78 (2008) 054513

HPQCD 2014: Chakraborty et al, PRD 91 (2015) 054508

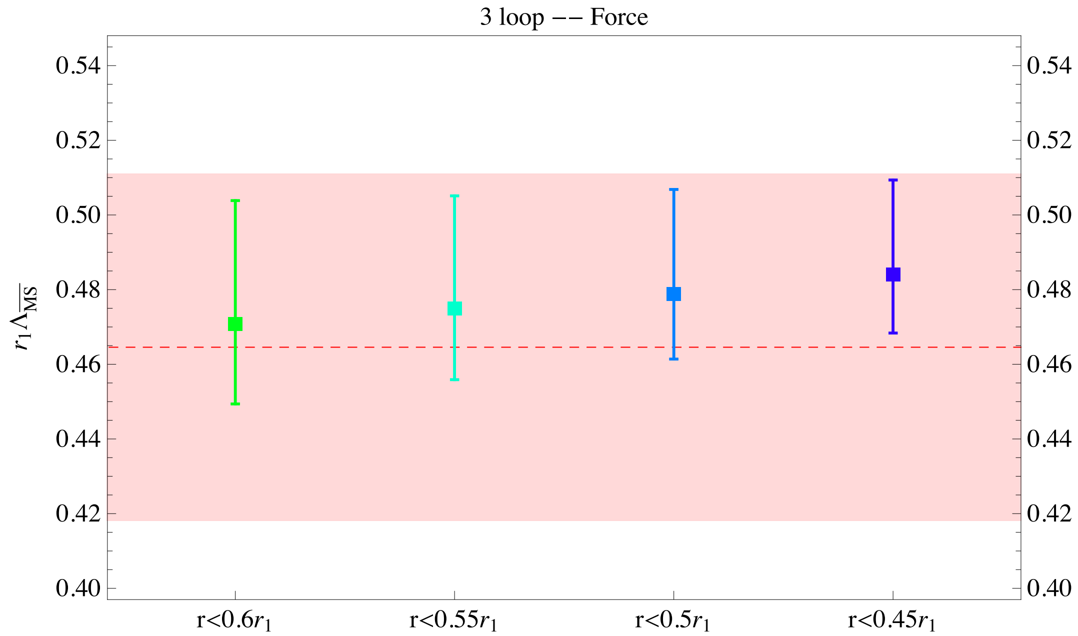
Data after corrections



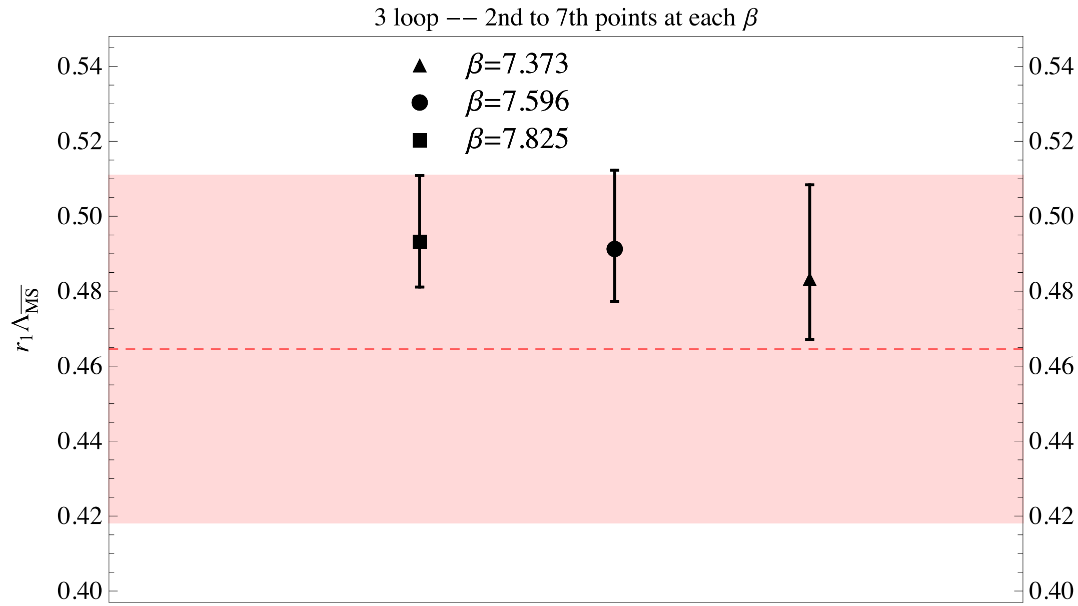
Correction from condensate:



Using the force:



Using only $1<(r/a)^2<7$:



Using only $(r/a)^2>6$: