Perturbative Matching Breakdown and Bottomonium HFS in Lattice NRQCD

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Topics discussed

- NRQCD theory of heavy quarkonium
  - *main concept*
  - *lattice NRQCD*
  - *matching*
- Coulomb dynamics on the lattice
  - *four-quark operators and Coulomb artifacts*
  - *analytical solution*
  - *breakdown of perturbative matching*
- HFS in bottomonium
  - *consistent treatment of Coulomb artifacts*
  - *QCD vs Lattice vs Experiment*

Based on

A.Penin, A. Rayyan, arXiv:1710.03244 [hep-lat]

T. Liu, A.Penin, A. Rayyan, JHEP **1702**, *084 (2017)*.

M.Baker, A.Penin, D.Seidel, N.Zerf, Phys. Rev. D **92**, *054502 (2015)*

B.Kniehl, A.Penin, A.Pineda, V.Smirnov, M.Steinhauser,

Phys. Rev. Lett. **92**, *242001 (2004)*

QCD $\to$ NRQCD

Caswell, Lepage; Bodwin, Braaten, Lepage

$$\overline{q}\,(i\gamma^\mu D_\mu - m_q)\,q$$

$\Downarrow$

hard scale m_q *integrated out*

$$\psi^\dagger\left(iD_0 + \frac{\boldsymbol{D}^2}{2m_q}\right)\psi - \frac{c_F\,g_s}{2m_q}\,\psi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{B}\psi + \ldots$$

Lattice NRQCD

- Main concept:
  - *take* $vm_b \ll 1/a \ll m_b$ *as the NRQCD factorization scale*
  - *compute NRQCD Wilson coefficients in perturbative QCD*
  - ↳ *"matching"*
  - *simulate the spectrum at finite lattice spacing*

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  - *take $vm_b \ll 1/a \ll m_b$ as the NRQCD factorization scale*
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  - ➥ *“matching”*
  - *simulate the spectrum at finite lattice spacing*

- Why not lattice QCD?
  - *to suppress finite size effects* ⇨ $L \gg \Lambda_{QCD}$
  - *to accommodate relativistic b-quark* ⇨ $a \ll 1/m_b$
  - ➥ *too demanding for hardware, only extrapolation to physical m_b*

Matching

- Basic idea
  - *in a given order in* α_s *and* v

$$\boxed{\text{QCD amplitude} = \text{NRQCD amplitude}}$$

➥ *equation for the Wilson coefficients*

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- Well understood concept?

Matching

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$$\boxed{\text{QCD amplitude} = \text{NRQCD amplitude}}$$

↪ *equation for the Wilson coefficients*

- Well understood concept?

- *Do not understand me too quickly!*

(a French saying)

Four-quark operators

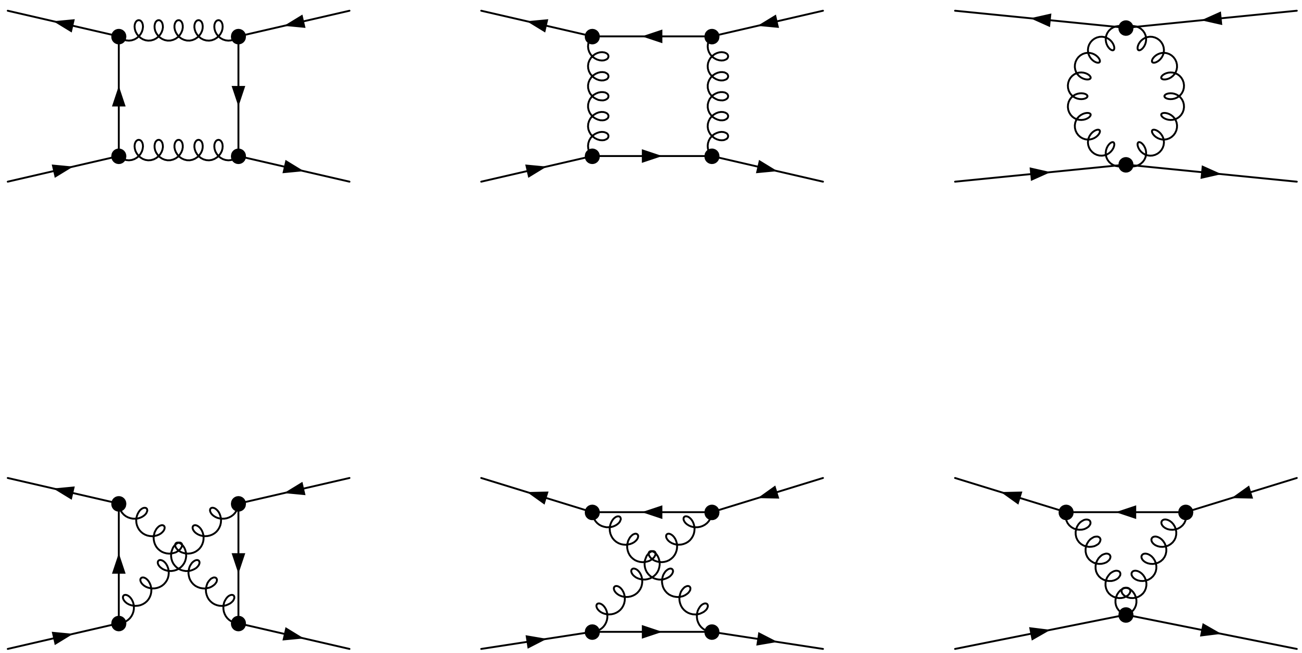
- One-particle irreducible amplitudes

  ➥ *contact interaction*

$$\delta\mathcal{L}_{NRQCD} = \sum_i \frac{C_F\alpha_s}{m_q^2} C_i O_i \,,$$

$$O_i = \psi^\dagger \mathbf{\Gamma}_i \psi \chi_c^\dagger \mathbf{\Gamma}_i \chi_c$$

1PI amplitudes



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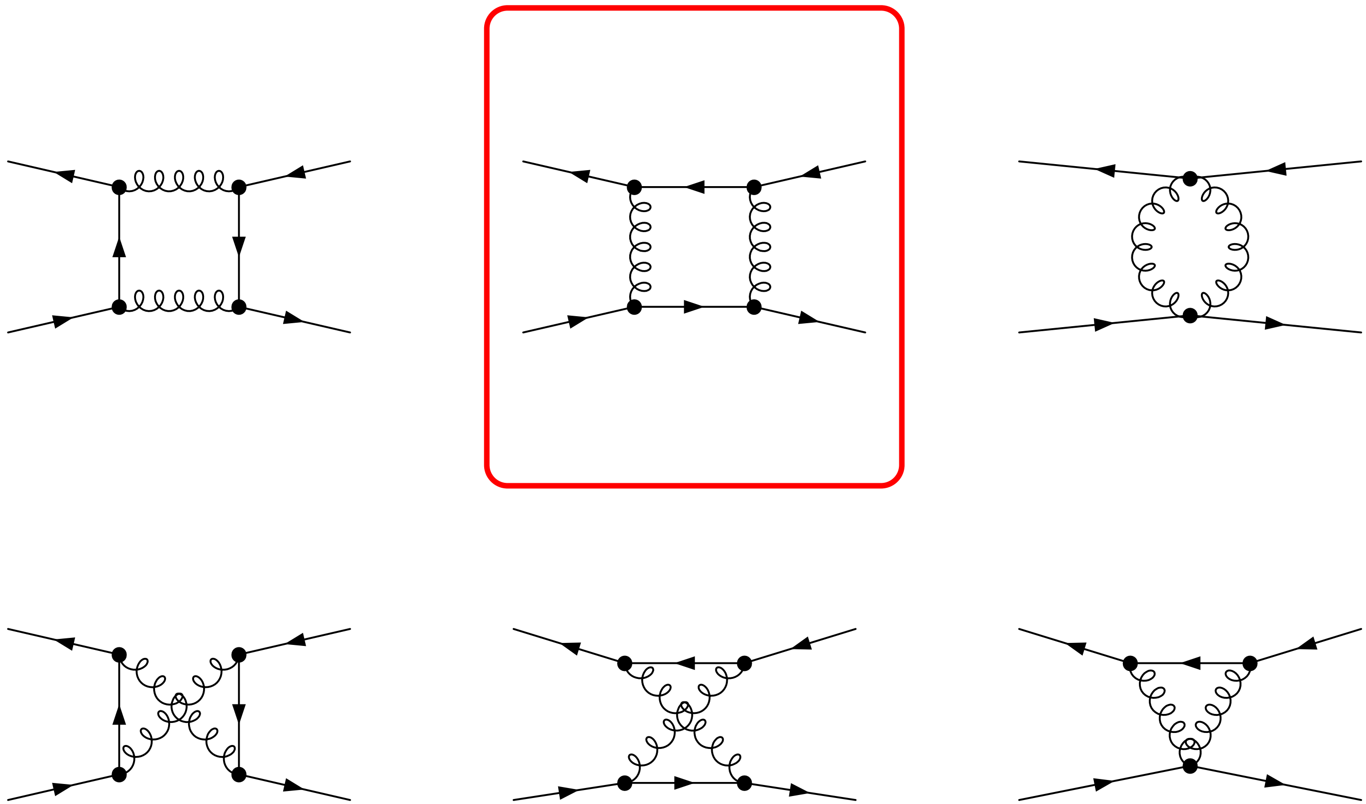
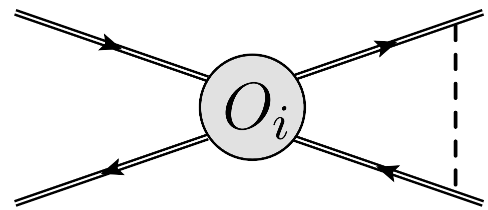


Coulomb artifacts

- Diagram with a Coulomb pinch



- Static Coulomb interaction of nonrelativistic quarks

$$\delta C_i O_i \propto \mathcal{M}^{\rm cont}_{\rm NRQCD} - \mathcal{M}^{\rm lat}_{\rm NRQCD}$$

  - *n-loop Coulomb artifacts*

$$\delta C_i \propto (a/r_{\rm Bohr})^n \sim (\alpha_s a m_q)^n, \qquad n = 1, 2, \ldots$$

➥ *can be absorbed into the Coulomb matrix element*

Matching of Coulomb artifacts

- Perturbative matching

$$\delta E \propto \delta C_i \langle O_i \rangle$$

- Schrödinger matching

$$\delta E \propto \left(1 - \frac{|\psi_{\rm lat}(0)|^2}{|\psi_{\rm cont}(0)|^2}\right) \langle O_i \rangle$$

  - *all orders in* α_s

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  - *all orders in* α_s

- ***No agreement already in one loop!***
  - *no linear artifacts in Schrödinger matching*
  - *breakdown of perturbative matching*

Coulomb problem on the lattice

Penin, Rayyan, arXiv:1710.03244

- Solution of the finite-difference equation $r = an$

$$\psi_l(n) = \frac{1}{\pi^{1/2}(1+a^2)^{1/4}} e^{-n \sinh^{-1} a} = \psi_c(r) + \mathcal{O}(a^2)\,,$$

  - *continuum limit* $\psi_c(r) = e^{-r}/(\sqrt{\pi})$

- Fourier transform

$$\tilde{\psi}_l(p) = 4\pi a^3 \sum_{n=0}^{\infty} n^2 \psi_l(n) \frac{\sin(nap)}{nap}$$

$$\tilde{\psi}_l(p) = \frac{8\sqrt{\pi}}{(1+a^2)^{1/4}} \frac{(a/2)^4}{\left(\sin^2(pa/2) - a^2 E_l(a)/2\right)^2} \frac{\sin(ap)}{ap} = \tilde{\psi}_c(p) + \mathcal{O}(a^2)\,,$$

  - *continuum limit* $\tilde{\psi}_c(p) = \frac{8\sqrt{\pi}}{(p^2+1)^2}$

Coulomb problem on the lattice

- Inverse Fourier transform
  - *valid only for* $n \neq 0$

$$\psi_l(n) = \frac{1}{2\pi^2}\int_0^{\frac{\pi}{a}} \mathrm{d}p\, p^2\, \tilde{\psi}_l(p)\, \frac{\sin(nap)}{nap}$$

- Wave function at the origin
  - *actual solution*

$$\psi_l(0) = \frac{1}{\sqrt{\pi}(1+a^2)^{1/4}} = \psi_c(0)\left(1 - \frac{a^2}{4} + \mathcal{O}(a^4)\right),$$

  - *momentum space value (agrees with perturbative matching)*

$$\psi_p(0) = \frac{1}{\sqrt{\pi}}\left(1 - \frac{\sqrt{1+a^2}-1}{a}\right) = \psi_c(0)\left(1 - \frac{a}{2} + \mathcal{O}(a^2)\right)$$

Coulomb problem on the lattice

- Radial derivative at the origin

  - *Actual solution*

$$\psi_l'(0) = \frac{\psi_l(1) - \psi_l(0)}{a} = -\psi_c(0) + \mathcal{O}(a)\,,$$

➥ *correct continuum limit* $\psi_c'(0) = -\psi_c(0)$

  - *Momentum space result*

$$\psi_p'(0) = \frac{\psi_l(1) - \psi_p(0)}{a} = -\frac{1}{2}\psi_c(0) + \mathcal{O}(a)\,,$$

➥ *wrong continuum limit, pathological function*

The reason of the breakdown

- Coulomb Hamiltonian at $r \to 0$

$$H \sim \frac{1}{r}\frac{d}{dr} + \frac{1}{r},$$

- Balance of *kinetic* and *potential* energy

➥ $\psi_c'(0) = -\psi_c(0)$

- Expansion in α_s cannot be used at $r = 0$!

➥ *i.e. perturbative matching breaks down*

Bottomonium hyperfine splitting

$$E_{\rm hfs} = M(\Upsilon_{1S}) - M(\eta_b)$$

A zoo of contradictory results

NLL NRQCD (Kniehl *et al*, 2004) $E_{\rm hfs}^{\rm th} = 41 \pm 11({\rm th})^{+9}_{-8}(\delta\alpha_s)\ \ {\rm MeV}$

$\Upsilon(3S)$ *decays* (Babar, 2008) $E_{\rm hfs}^{\rm exp} = 71.4 \pm 2.7\,({\rm syst})\,^{+2.3}_{-3.1}\,({\rm stat})\ {\rm MeV}$

NLO $\mathcal{O}(v^4)$ *lattice NRQCD* (HPQCD, 2012) $E_{\rm hfs}^{\rm th} = 70 \pm 9\ \ {\rm MeV}$

$h_b(1P)$ *decays* (Belle, 2012) $E_{\rm hfs}^{\rm exp} = 57.9 \pm 2.3\ \ {\rm MeV}$

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lattice QCD (extrapolation) (HPQCD, 2013) $E_{\rm hfs}^{\rm th} = 53 \pm 5\ \ {\rm MeV}$

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HFS in lattice NRQCD

$$\mathcal{L}_\sigma = \frac{c_F}{2m_q}\psi^\dagger \boldsymbol{B\sigma}\psi + (\psi \to \chi_c) + d_\sigma \frac{C_F\alpha_s}{m_q^2}\psi^\dagger \boldsymbol{\sigma}\psi\chi_c^\dagger \boldsymbol{\sigma}\chi_c,$$

- c_F ➪ chromomagnetic moment
- d_σ ➪ local four-quark interaction
  - *contradictory results*

Spurious linear Coulomb artifacts

Hammant, Hart, Hippel, Horgan, Monahan, Phys.Rev.Lett. **107**, 112002 (2011)

Coulomb artifacts removed by asymptotic expansion

M.Baker, A.Penin, D.Seidel, N.Zerf, Phys. Rev. D **92**, 054502 (2015)

Bottomonium spectrum in lattice NRQCD

- Heavy quark-antiquark propagator asymptotic behavior
  - *bound state parameters*
- Actual simulations $a \sim 1/(vm_b)$
  - *ultraviolet* $1/(am_b)$ *terms are suppressed*
  - *Coulomb artifacts become crucial* $\alpha_s am_b \sim 1$
- Extrapolation $a \to 0$
  - *fit the lattice data by a polynomial in* a *with vanishing linear term*
  - *justified since* $1/(am_b)$ *terms are numerically small*
  - $(\alpha_s am_b)^n$, $(a\Lambda_{QCD})^n$ *artifacts are removed*

Bottomonium spectrum in lattice NRQCD

- Problem of perturbative matching
  - *introduces spurious linear Coulomb artifacts*
  - ↪ *systematic error of the fit with no linear term*

Bottomonium spectrum in lattice NRQCD

- Problem of perturbative matching
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- The fix
  - *remove linear artifact from Wilson coefficient by*

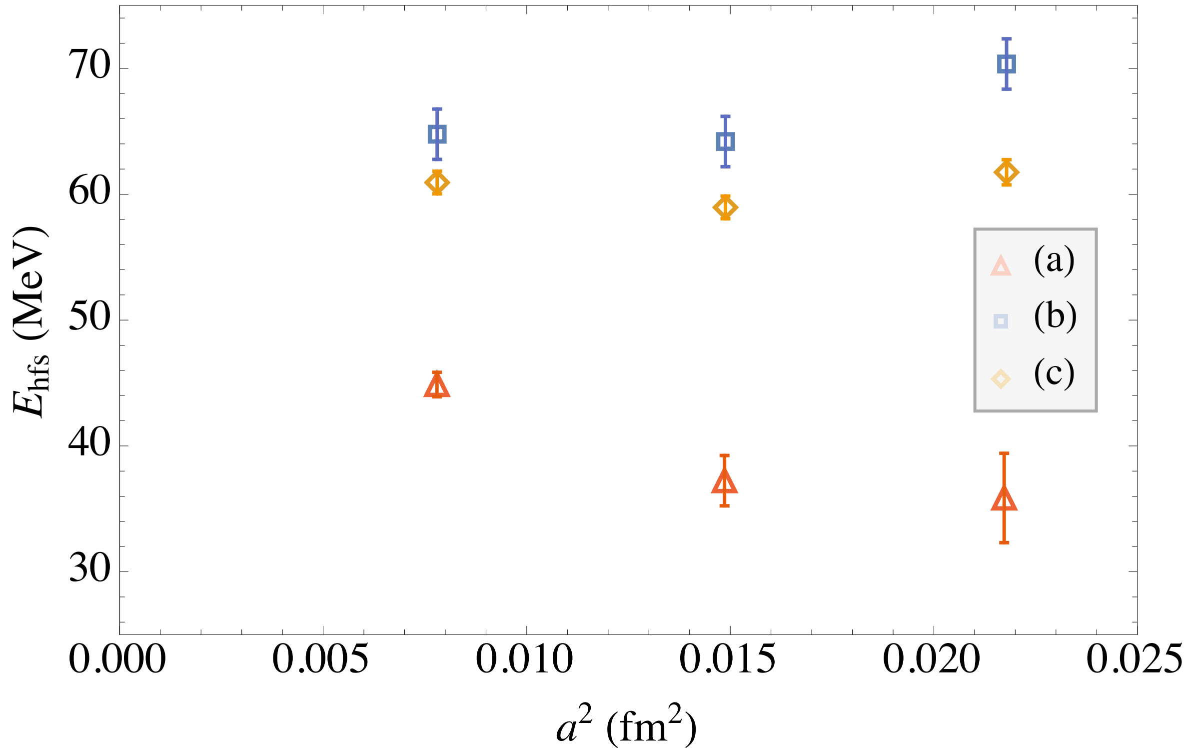
    asymptotic expansion in a (M.Baker *et al*, Phys.Rev. **D92**, 054502 (2015))

    or numerical fit (T. Liu, A.Penin, A. Rayyan, JHEP **1702**, 084 (2017))
  - *treat Coulomb artifacts by*

    Schrödinger matching

    and/or $a \to 0$ *extrapolation*

NLO $\mathcal{O}(v^6)$ lattice QCD data



(a) *linear artifact subtracted (slope agrees with a^2 Coulomb artifact)*

(b) *naive perturbative matching*

(c) *no four-quark operators*

Status of bottomonium HFS

Experiment	
$h_b(1P)$ decays (Belle, 2012)	57.9 ± 2.3

Theory	
NRQCD, NLL (Kniehl et al., 2004)	41 ± 14
Lattice QCD (HPQCD, 2013)	53 ± 5
Lattice NRQCD (Baker et al., 2015)	52.9 ± 5.5

Summary

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- Matching of NRQCD on the lattice is more fun than it was appreciated
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- Systematic matching procedure for Coulomb artifacts:
  - *subtract linear Coulomb artifact from Wilson coefficients*
  - *use Schrödinger matching or extrapolation for higher order artifacts*

Summary

- Matching of NRQCD on the lattice is more fun than it was appreciated
  - *perturbative matching breaks down*
- Systematic matching procedure for Coulomb artifacts:
  - *subtract linear Coulomb artifact from Wilson coefficients*
  - *use Schrödinger matching or extrapolation for higher order artifacts*
- Continuum QCD, lattice QCD and Belle now agree

➥ η_b *mass puzzle is solved!*