# Heavy Quark-Diquark Symmetry: Doubly Heavy Baryons and Tetraquarks

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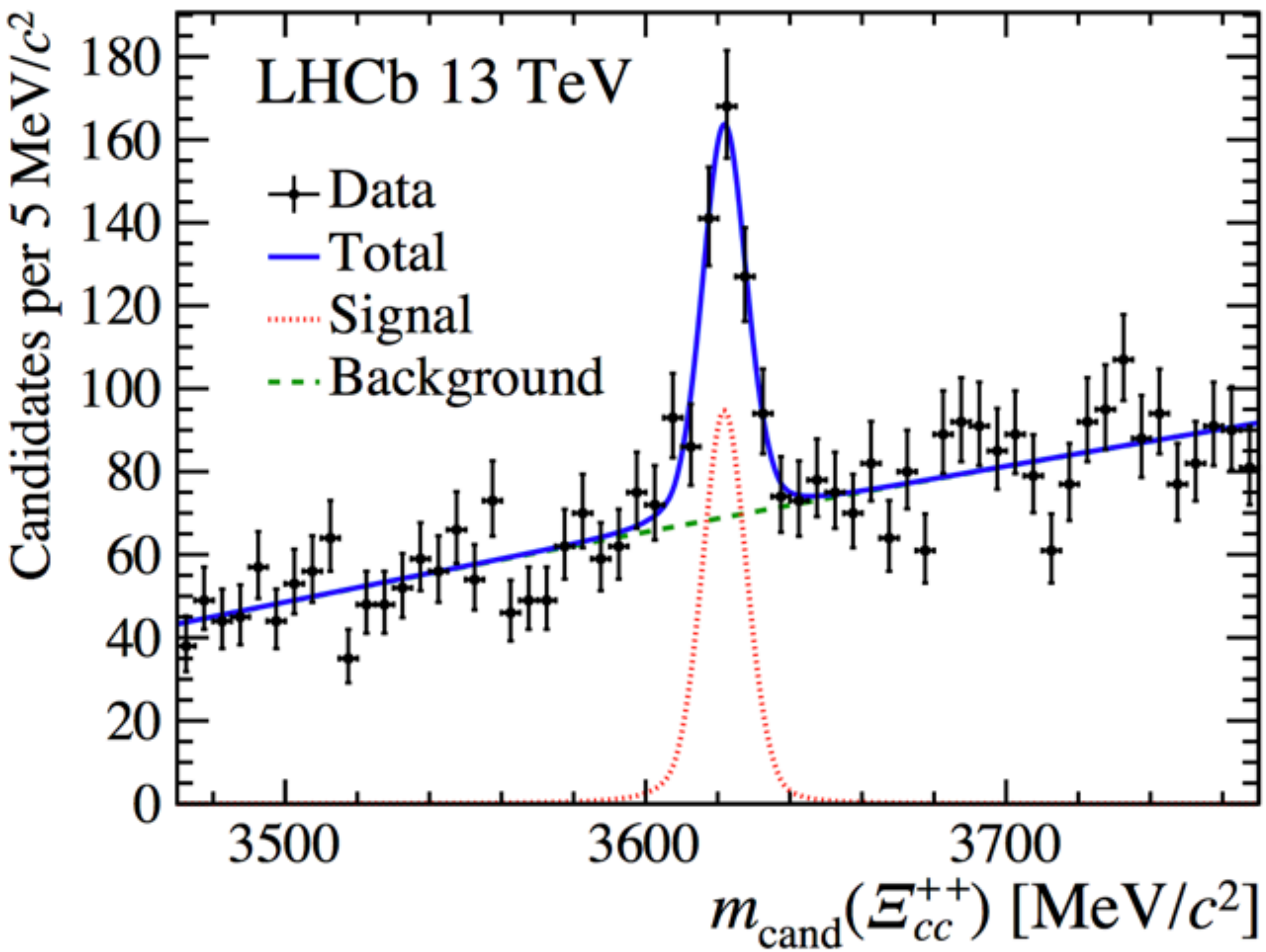
# LHCb discovery of $\Xi_{cc}^{++}$

R. Aaij, et. al. (LHCb) , PRL **119** 112001 (2017)




$\Lambda_c^+ K^- \pi^+ \pi^+$

$$m_{\Xi_{cc}^{++}} = 3621.4 \pm 0.72 \pm 0.27 \pm 0.14\,\mathrm{MeV}$$

## Confirming Doubly Charm Interpretation of $\Xi_{cc}^{++}$

Measuring Lifetime

Measuring Production Rates

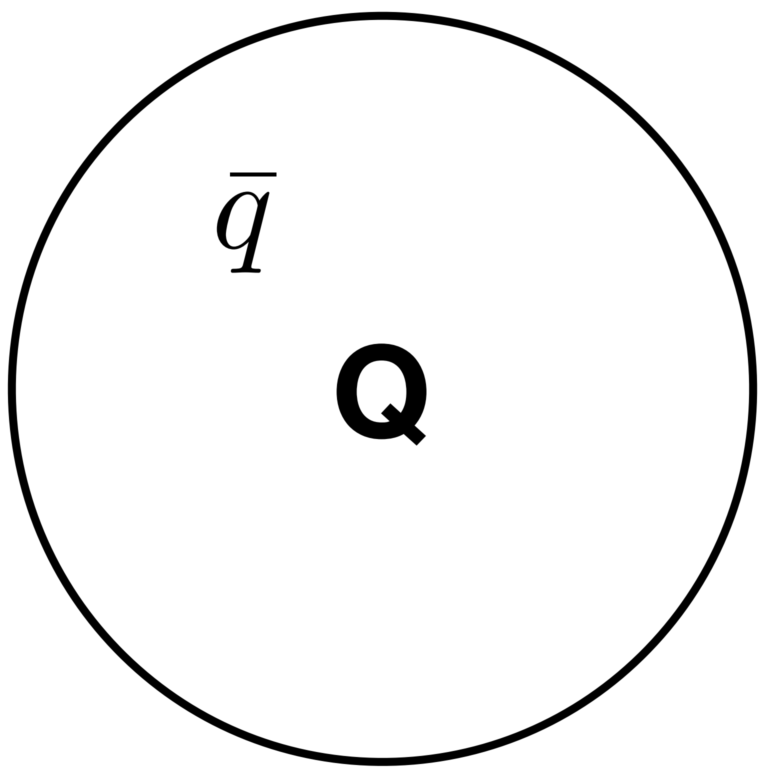
Finding Isospin Partner $\Xi_{cc}^{+}$

Observing $\Xi_{cc}^{++}$ in Other Channels

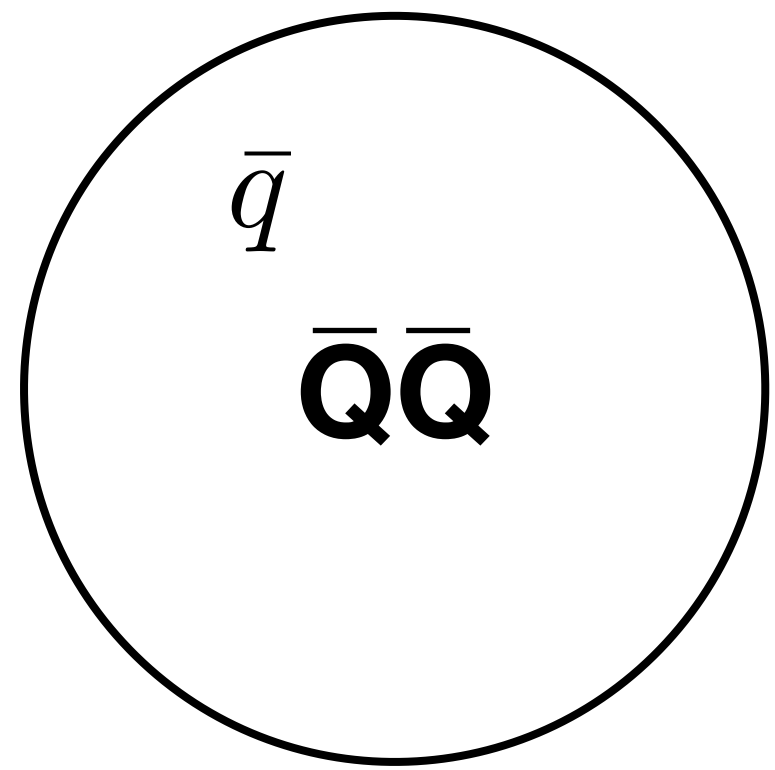
**Observing Excited Doubly Heavy Baryons**

# Heavy Quark-Diquark Symmetry

## $m_Q \to \infty$ **$\bar{Q}\bar{Q}$ is compact object in color 3**




**Singly Heavy Meson**

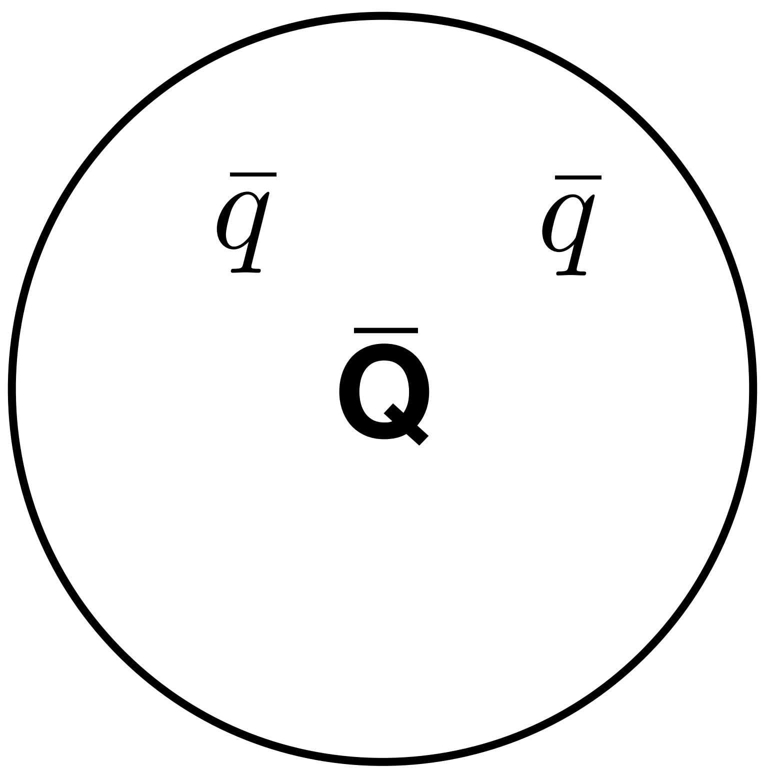



**Doubly Heavy anti-Baryon**

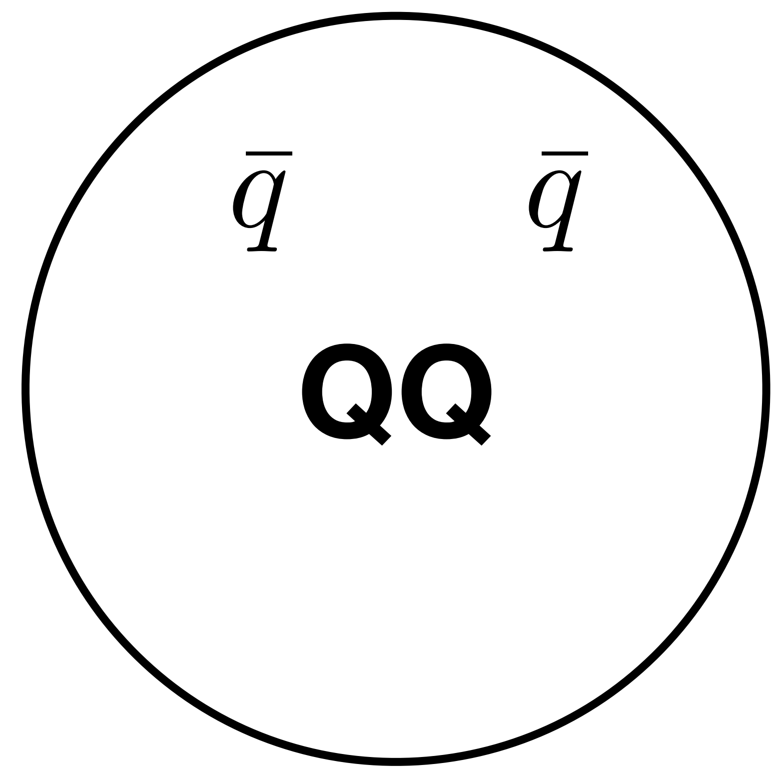
## l.d.o.f are the same in these hadrons

# Heavy Quark-Diquark Symmetry (HQDQS)

$m_Q \to \infty$ **QQ is compact object in color $\bar{\mathbf{3}}$**




**Singly Heavy anti-Baryon**

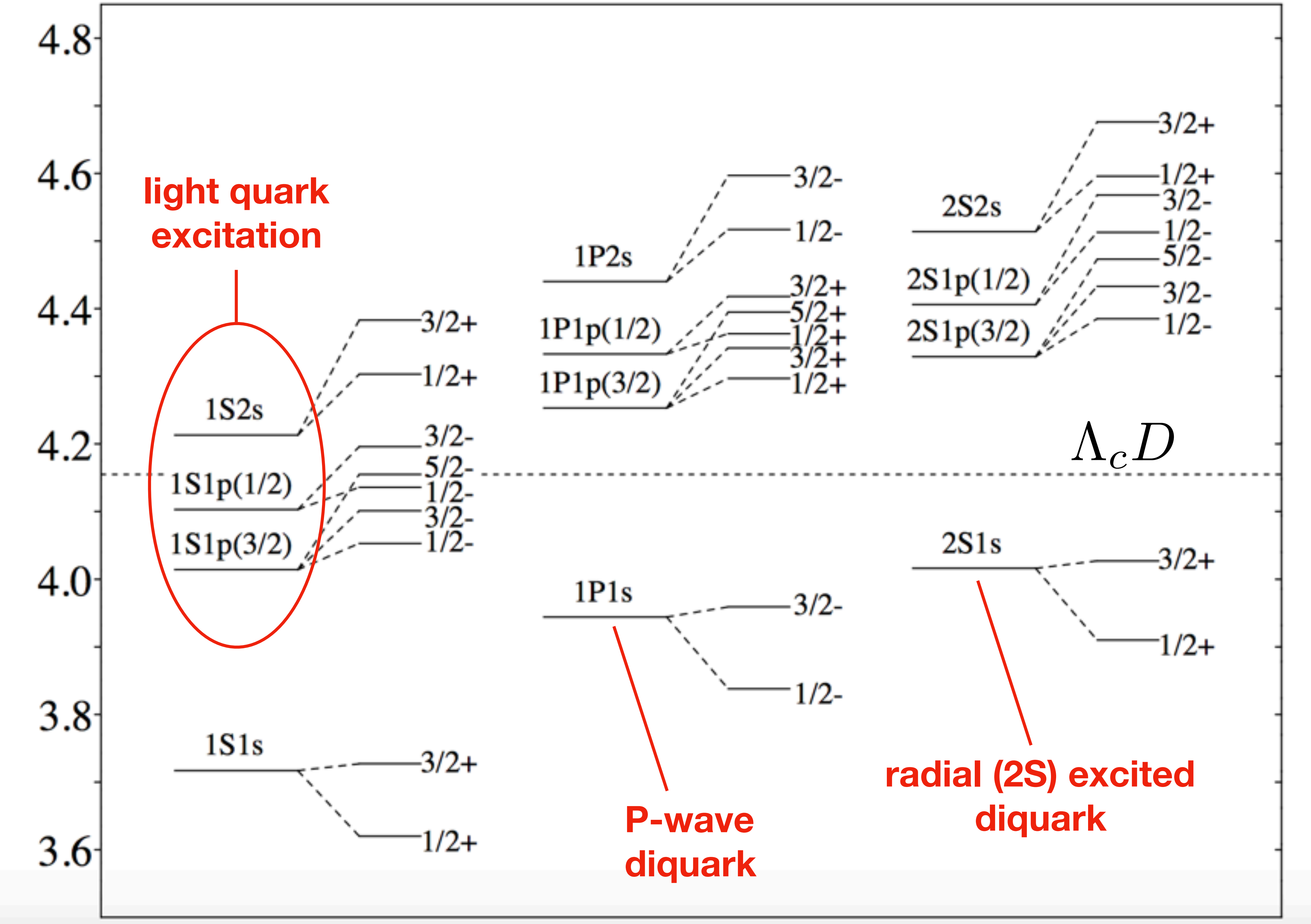



**Doubly Heavy Tetraquark**

**l.d.o.f are the same in these hadrons**

# Spectrum of Excited Doubly Charm Baryons

D. Ebert, et. al., PRD **66** (2002) 014008



# HQET for heavy quarks and diquarks

$$\mathcal{L} = h^\dagger \, iD_0 \, h + \mathbf{T}^\dagger \, iD_0 \, \mathbf{T} + \frac{g_s}{2m_Q} h^\dagger \, \boldsymbol{\sigma} \cdot \mathbf{B} \, h - \frac{ig_s}{2m_Q} \mathbf{T}^\dagger \cdot \mathbf{B} \times \mathbf{T} + \ldots \, .$$

M. Savage and M. Wise, PLB **248** (1990) 177

N. Brambilla, A. Vairo, T. Rosch, PRD **72** (2005) 034021

S. Fleming and T. Mehen, PRD **73** (2006) 034502

# Hyperfine splittings

$$m_{\Xi^*_{cc}} - m_{\Xi_{cc}} = \frac{3}{4}(m_{D^*} - m_D) \approx 106 \, \mathrm{MeV}$$

$m_{\Xi^*_{cc}} \approx 3727 \, \mathrm{MeV}$, em decays to ground state

# Chiral Lagrangian with Quark-Diquark Symmetry

J. Hu and T.M, PRD **73**, 054003 (2006)

Heavy hadron chiral perturbation theory - in rest frame $v^\mu = (1, \vec{0})$

$$\mathcal{L} = \mathrm{Tr}[H_a^\dagger (iD_0)_{ba} H_b] - g\mathrm{Tr}[H_a^\dagger H_b \,\vec{\sigma}\cdot\vec{A}_{ba}] + \frac{\Delta_H}{4}\mathrm{Tr}[H_a^\dagger \sigma^i \, H_a \, \sigma^i]$$

ground state spin-0,1 fields: $H_a = \vec{P}_a^* \cdot \vec{\sigma} + P_a$ 2 x 2 field $Q_\alpha \bar{q}_\beta$

$H_{a,\alpha\beta} \rightarrow \mathcal{H}_{a,\mu\beta} = H_{a,\alpha\beta} + T_{a,i\beta}$ $T_{a,i\beta} = \sqrt{2}\left(\Xi^*_{a,i\beta} + \frac{1}{\sqrt{3}}\Xi_{a,\gamma}\,\sigma^i_{\gamma\beta}\right)$ 3 x 2 field $(QQ)_i \bar{q}_\beta$

**Chiral Lagrangian for doubly heavy baryons**

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}[T_a^\dagger (iD_0)_{ba} T_b] - g\,\mathrm{Tr}[T_a^\dagger T_b \,\vec{\sigma}\cdot\vec{A}_{ba}] + \frac{\Delta_H}{4}\mathrm{Tr}[T_a^\dagger \,\mathcal{T}^i \, T_a \, \sigma^i] \\ &+ \frac{e\beta}{2}\mathrm{Tr}[H_a^\dagger H_b \,\vec{\sigma}\cdot\vec{B}\,Q_{ab}] + \frac{e}{2m_Q}Q'\mathrm{Tr}[H_a^\dagger \vec{\sigma}\cdot\vec{B}\,H_a]\end{aligned}$$

$(\mathcal{T}^i)_{jk} = -i\epsilon_{ijk}$

← **EM interactions**

# Electromagnetic decay of $\Xi^*$

J. Hu and T.M, PRD **73**, 054003 (2006)

$$\Gamma[P_a^* \to P_a \gamma] = \frac{\alpha}{3}\left(\beta Q_{aa} + \frac{Q'}{m_Q}\right)^2 \frac{m_P}{m_{P^*}} E_\gamma^3$$

$$\Gamma[\Xi_a^* \to \Xi_a \gamma] = \frac{4\alpha}{9}\left(\beta Q_{aa} - \frac{Q'}{m_Q}\right)^2 \frac{m_\Xi}{m_{\Xi^*}} E_\gamma^3$$

Chiral loop corrections

J. Amundson, et. al., PLB **296** (1992) 415

$$\beta Q_{11} \to \frac{2}{3}\beta - \frac{g^2 m_K}{4\pi f_K^2} - \frac{g^2 m_\pi}{4\pi f_\pi^2}$$

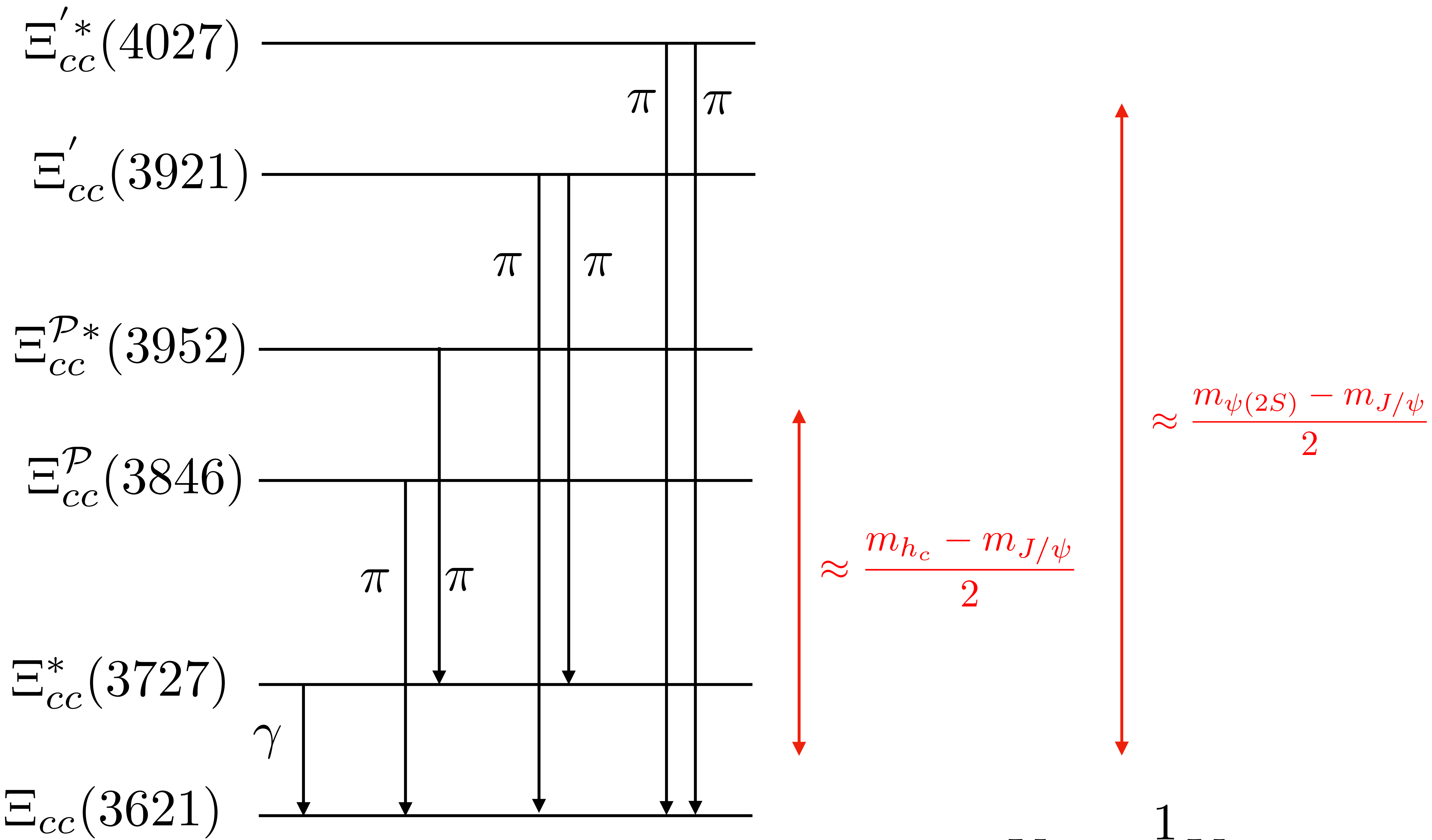
$$\beta Q_{22} \to -\frac{1}{3}\beta + \frac{g^2 m_\pi}{4\pi f_\pi^2}$$

$$\beta Q_{33} \to -\frac{1}{3}\beta + \frac{g^2 m_K}{4\pi f_K^2} .$$

**Predictions**

$$\Gamma[\Xi_{cc}^{*++}] = 5.1\,\mathrm{keV} \qquad \Gamma[\Xi_{cc}^{*+}] = 7.6\,\mathrm{keV}$$

# Excited Doubly Heavy Baryon Decays




$$V_{cc} = \frac{1}{2} V_{c\bar{c}}$$

# Predictions for Strong Decays

J. Hu and T.M, PRD **73**, 054003 (2006)

Radially excited states $\tilde{g} \sim O(1)$

$$\Gamma[\Xi'_{cc}] = \tilde{g}^2 52 \text{ MeV} \qquad \Gamma[\Xi'^{*}_{cc}] = \tilde{g}^2 391 \text{ MeV}$$

$$\frac{\Gamma[\Xi'^{*}_{cc} \to \Xi^{*}_{cc}\pi]}{\Gamma[\Xi'^{*}_{cc} \to \Xi_{cc}\pi]} = 0.46 \qquad \frac{\Gamma[\Xi'_{cc} \to \Xi^{*}_{cc}\pi]}{\Gamma[\Xi'_{cc} \to \Xi_{cc}\pi]} = 1.2,$$

P-wave excited states $\lambda_{1/2} \approx \lambda_{3/2} \sim O(\Lambda_{\rm QCD}/m_c)$

$$\Gamma[\Xi^{\mathcal{P}*}_{cc} \to \Xi^{*}_{cc}\pi] = \lambda^2_{3/2} 112 \text{ MeV}$$

$$\Gamma[\Xi^{\mathcal{P}}_{cc} \to \Xi_{cc}\pi] = \lambda^2_{1/2} 111 \text{ MeV},$$

$$1/3 < \lambda_{1/2,3/2} < 1/2$$

$$12\,\text{MeV} < \Gamma[\Xi^{\mathcal{P}(*)}_{cc}] < 28\,\text{MeV}$$

# Strange Sector

HQDQS predicts

$$m_{\Omega_{cc}} - m_{\Xi_{cc}^{+,++}} = m_{D_s} - m_{D^{0,+}} = 101 \ \text{MeV}$$

Lattice QCD

$$m_{\Omega_{cc}} - m_{\Xi_{cc}^{+,++}} = 98 \pm 9 \pm 22 \pm 12 \, \text{MeV}$$

L. Liu, et. al., et. al., PRD **81** 094505 (2010)

Low lying excitation spectrum nearly identical to non-strange

Analogous decay to ground state via kaon emission forbidden
strong decay via isospin violating decays to $\pi^0$

Narrow states, width ~ few keV

J. Hu and T.M, PRD **73**, 054003 (2006)

# Stable Doubly Heavy Tetraquarks

**quark model** M. Karliner, J. Rosner, arXiv:1707.07666

$T_{bb\bar{u}\bar{d}}$ $\quad J^P = 1^+ \quad I = 0 \quad 10389 \pm 12\,\mathrm{MeV}$

$215\,\mathrm{MeV}$ below $B\bar{B}^*$ threshold, stable to strong interaction

**heavy quark symmetry** E. Eichten, C. Quigg, arXiv:1707.09575

$10468\,\mathrm{MeV}$ $\quad 135\,\mathrm{MeV}$ below threshold

**lattice QCD**

$189 \pm 10\,\mathrm{MeV}$ below threshold A. Francis, et. al., PRL **118**, 142001 (2017)

$60^{+30}_{-38}\,\mathrm{MeV}$ below threshold P. Bicudo, et. al., PRD **95**, 142001 (2017)

no analogous prediction for $T_{cc\bar{q}\bar{q}}, T_{bc\bar{q}\bar{q}}$ tetraquarks

molecular $T_{cc} = DD^*$ D. Janc, M. Rosina, Few Body Syst. **35** (2004) 175

**arguments for stability in heavy quark limit**

J.-P. Ader, J.-M. Richard, P. Taxil, PRD **25** (1982) 2370

A. Manohar, M. Wise, NPB 399 (1993) 17

A. Czarnecki, B. Long, M. Voloshin, arXiv:1708.04594

# Chiral Lagrangians

**singly heavy baryons** P. Cho, PLB **285**, 145 (1992), NP **B396** (1993)

$$\mathcal{L}_B = \bar{\Sigma}^{\alpha,i} i D_0 \Sigma^{\alpha,i} + \bar{\Lambda} i D_0 \Lambda - (m_\Sigma - m_\Lambda)\bar{\Sigma}^{\alpha,i}\Sigma^{\alpha,i} + i\Delta\bar{\Sigma}^{\alpha,i}\epsilon_{ijk}\sigma^k\Sigma^{\alpha,j} - g_3(\bar{\Lambda}A^{\alpha,i}\Sigma^{\alpha,i} + \text{H.c.}).$$

$\Sigma^{(*)} - \Sigma$ hyperfine splitting

$\Sigma^{(*)} \to \Lambda\pi$ decay

$\Lambda$ - spin-1/2, isosinglet

$$\Sigma^{\alpha,i} = \Sigma^{*\alpha,i} + \frac{\sigma^i}{\sqrt{3}}\Sigma^\alpha$$ - vector-spinor, isovector

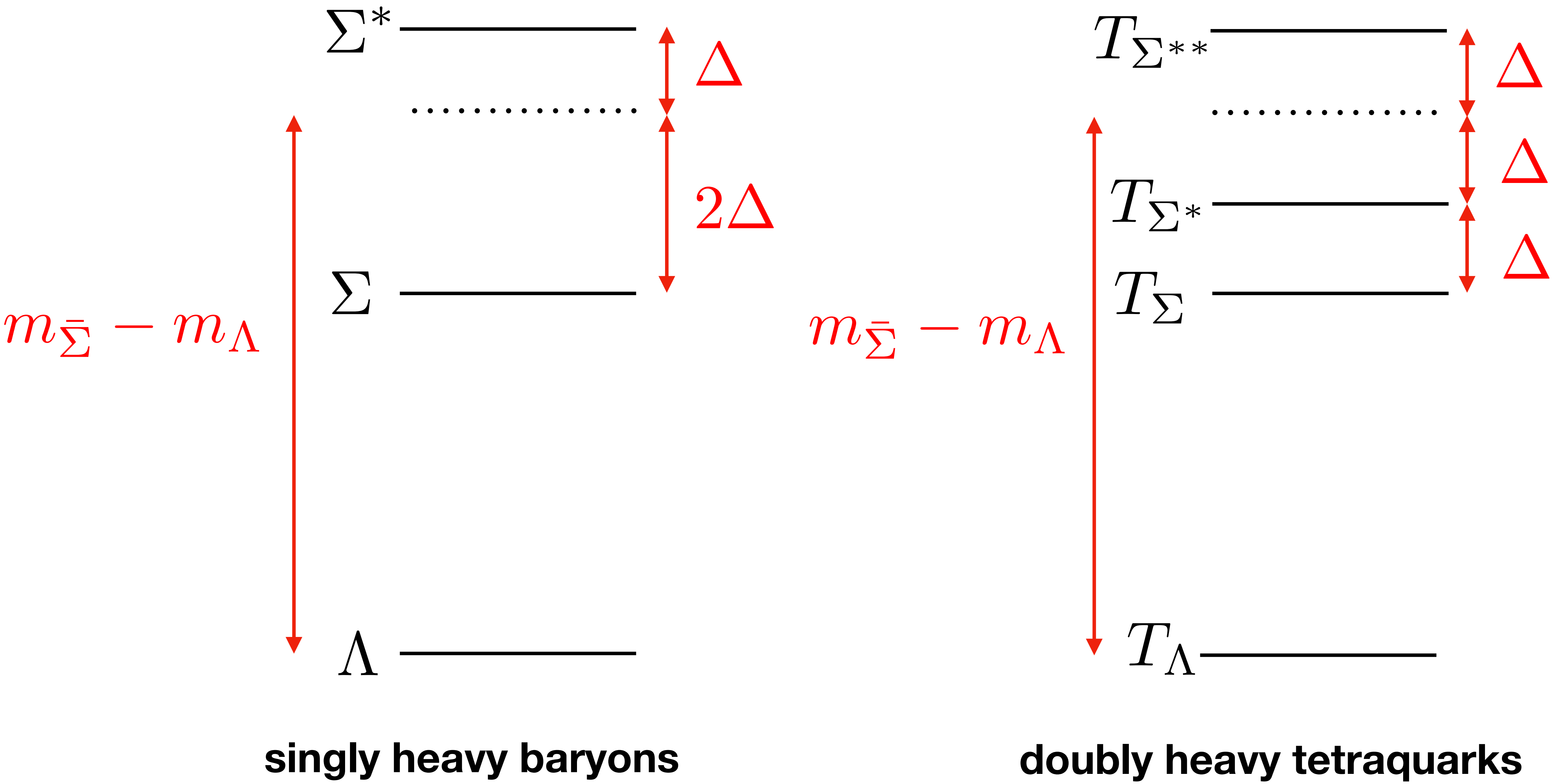
**doubly heavy tetraquarks** T. Mehen, arXiv:1708.05020

$$\Lambda \to T^j_\Lambda$$
$$\Sigma^{\alpha,i} \to T^{\alpha,ji}_\Sigma$$

$$T^{\alpha,ji}_\Sigma = T^{\alpha,ji}_{\Sigma^{**}} + \frac{1}{\sqrt{2}}\epsilon^{jik}T^k_{\Sigma^*} + \frac{\delta^{ij}}{\sqrt{3}}T_\Sigma$$

spin symmetry breaking $\sigma^i \to (\mathcal{T}^i)_{jk} = -i\epsilon_{ijk}$

# Mass Spectra



# Mass Formulae

$$m_{T_\Sigma} - m_{T_\Lambda} = m_\Sigma - m_\Lambda$$

$$m_{T_{\Sigma^{**}}} - m_{T_{\Sigma^*}} = \frac{2}{3}(m_{\Sigma^*} - m_\Sigma)$$

$$m_{T_{\Sigma^*}} - m_{T_\Sigma} = \frac{1}{3}(m_{\Sigma^*} - m_\Sigma)$$

prediction for spin-averaged mass splittings: **strong experimental support**

$$m_{T_{\bar{\Sigma}}} - m_{T_\Lambda} = m_{\bar{\Sigma}} - m_\Lambda$$

$$m_{\bar{\Sigma}_b} - m_{\Lambda_b} = 207 \text{ MeV}$$

$$m_{\bar{\Sigma}_c} - m_{\Lambda_c} = 210 \text{ MeV}$$

$$m_{\bar{\Sigma}} - m_\Lambda = 205 \text{ MeV},$$

$$I = 0, S = 0 \,\text{diquark} \rightarrow I = 1, S = 1 \,\text{diquark costs} \approx 207 \,\text{MeV}$$

similar conclusion from $m_\Delta - m_N$ R. Jaffe, Phys. Rept. 409 (2005) 1

prediction for hyperfine splittings: **small, no experimental confirmation**

$$\Delta_c = 21 \,\text{MeV} \qquad \Delta_b = 7 \,\text{MeV}$$

# Mass Formulae

$$m_{T_{\Sigma_{bb}}} = m_{T_{\Lambda_{bb}}} + 193 \text{ MeV}$$
$$m_{T_{\Sigma^*_{bb}}} = m_{T_{\Lambda_{bb}}} + 200 \text{ MeV}$$
$$m_{T_{\Sigma^{**}_{bb}}} = m_{T_{\Lambda_{bb}}} + 214 \text{ MeV},$$

If $m_{T_{\Lambda_{bb}}} < 10405\,\text{MeV}$ $T_{\Sigma^*_{bb}}$ also below open bottom threshold

# Decays

$$\Gamma[\Sigma^{(*)} \to \Lambda_c^+ \pi] = \Gamma[T_{\Sigma^{**}} \to T_\Lambda \pi] = \Gamma[T_{\Sigma^*} \to T_\Lambda \pi] = \Gamma[T_\Sigma \to T_\Lambda \pi] = \frac{g_3^2}{6\pi f^2} \frac{m_{T_\Lambda}}{m_{T_\Sigma}} p_\pi^3$$

$$\Gamma[\Sigma_c] = 1.8^{+0.10}_{-0.19}\,\text{MeV}$$
$$\Gamma[\Sigma_c^*] = 15.0^{+0.4}_{-0.5}\,\text{MeV}$$

extract $g_3^2 = 0.93$

predict

$$\Gamma[\Sigma_b] = 6.3\,\text{MeV}$$
$$\Gamma[\Sigma_b^*] = 11.3\,\text{MeV}$$

consistent with data

**Assuming** $m_{T_{\Lambda_{bb}}} = 10389\,\mathrm{MeV}$ **and mass formulae**

$$\Gamma[T_{\Sigma_{bb}^{**}} \to T_{\Lambda_{bb}}\pi] = 11.8\text{ MeV}$$
$$\Gamma[T_{\Sigma_{bb}^{*}} \to T_{\Lambda_{bb}}\pi] = 8.1\text{ MeV}$$
$$\Gamma[T_{\Sigma_{bb}} \to T_{\Lambda_{bb}}\pi] = 6.5\text{ MeV},$$

$T_{\Sigma_{bb}^{*}}(J^P = 1^+, I = 1)$ is a narrow state (< 10 MeV) below $B\bar{B}^*$ threshold

$T_{\Sigma_{bb}}, T_{\Sigma_{bb}^{**}}$ are above $B\bar{B}$ threshold, will be much broader

# Conclusions

**Heavy quark-diquark symmetry:** $Q\bar{q} \leftrightarrow \bar{Q}\bar{Q}\bar{q}$

$Q\bar{q}\bar{q} \leftrightarrow \bar{Q}\bar{Q}\bar{q}\bar{q}$

**Chiral Lagrangian for doubly heavy baryons**

Predictions for strong/em decays of low-lying excited doubly charm baryons

J. Hu and T.M., PRD 73, 054003 (2006)

**Chiral Lagrangian for doubly heavy tetraquarks**

Predictions for mass splittings, strong decays of doubly bottom tetraquarks

T.M., arXiv:1708.05020

**If** $T_{\Lambda_{bb}}$ **mass is** $< 10405\,\mathrm{MeV}$ $T_{\Sigma^*_{bb}}$ **below open bottom threshold**

$T_{\Sigma^*_{bb}} \to T_{\Lambda_{bb}}\pi$ **strong decay width < 10 MeV**