# Radiative decays $\chi_{cJ} \to V\gamma$


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PR**D96** (2017) 054007

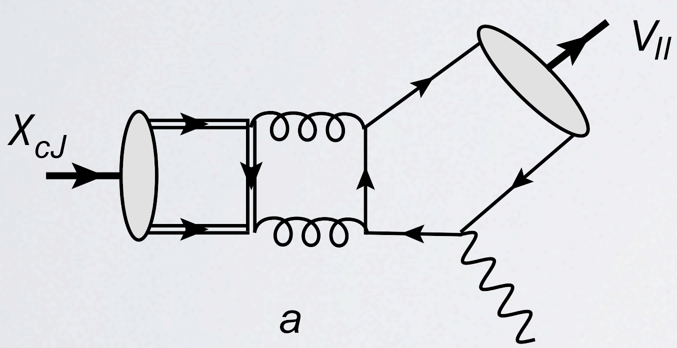

**QWG workshop, November 6-10, Beijing, China**

# Radiative decays $\chi_{cJ} \to V\gamma$

Br's in units of $10^{-6}$ Data: CLEO (2008) & BES III (20011)

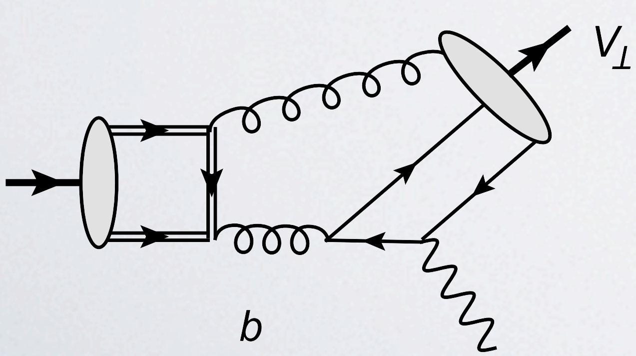
| | $\chi_{c1} \to V\gamma$ | $\chi_{c1} \to V_{\parallel}\gamma$ | $\chi_{c1} \to V_{\perp}\gamma$ | $\chi_{c0} \to V\gamma$ | $\chi_{c2} \to V\gamma$ |
|---|---|---|---|---|---|
| $\gamma\rho$ | $220 \pm 18$ | $184.8 \pm 15.7$ | $35.2 \pm 7.4$ | $< 9$ | $< 20$ |
| $\gamma\omega$ | $69 \pm 8$ | $51.8 \pm 8.9$ | $17.3 \pm 6.5$ | $< 8$ | $< 6$ |
| $\gamma\phi$ | $25 \pm 5$ | $17.7 \pm 4.9$ | $7.3 \pm 3.6$ | $< 6$ | $< 8$ |

Leading-order color-singlet contributions:



a

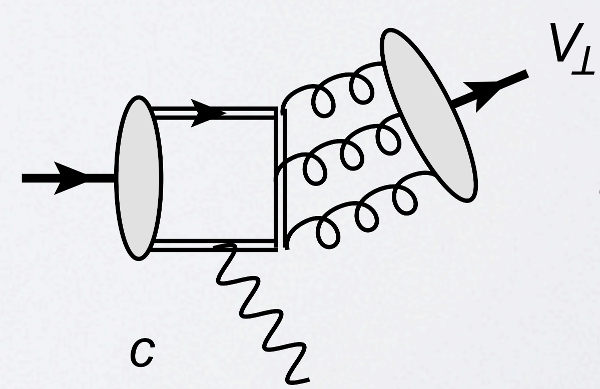
$$\sim \alpha_s^2 v^4 \Lambda/m_c$$



b

$$\sim \alpha_s v^4 (\Lambda/m_c)^2$$

only I=0 mesons



c

$$\sim \alpha_s v^4 (\Lambda/m_c)^2$$

# Radiative decays $\chi_{cJ} \to V\gamma$

Br's in units of $10^{-6}$ Data: CLEO & BES III

| | $\chi_{c1} \to V\gamma$ | $\chi_{c1} \to V_{\parallel}\gamma$ | $\chi_{c1} \to V_{\perp}\gamma$ | $\chi_{c0} \to V\gamma$ | $\chi_{c2} \to V\gamma$ |
|---|---|---|---|---|---|
| $\gamma\rho$ | $220 \pm 18$ | $184.8 \pm 15.7$ | $35.2 \pm 7.4$ | $< 9$ | $< 20$ |
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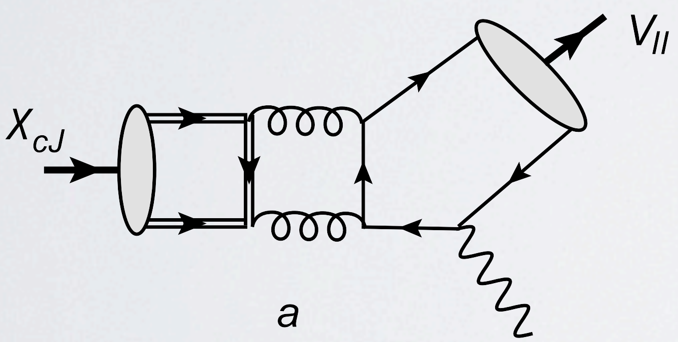
## Previous calculations

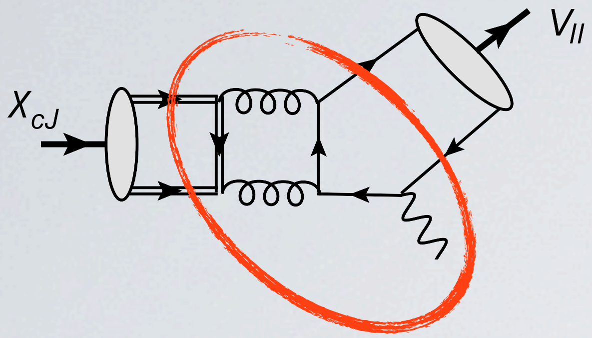



Gao, Zhang, Chao 2006, 2007

quark model,
nonrelativistic wave functions
for vector meson V

$$10^6 Br[\chi_{c1} \to \rho\gamma] = 41 \qquad 10^6 Br[\chi_{c1} \to \omega\gamma] = 4.6 \qquad 10^6 Br[\chi_{c1} \to \phi\gamma] = 1.1$$

# Radiative decays $\chi_{cJ} \to V_{||}\,\gamma$




$$A[\chi_{cJ} \to V_{\|}\gamma] \sim \frac{R'_{21}(0)}{m_c^{5/2}}\,\sqrt{4\pi\alpha}\,\int_0^1 dx\,\frac{f_V}{m_c}\phi_V^{\|}(x)\,\alpha_s^2(\mu_h)T_J(x)$$

decay const. $f_\rho = 221\,\mathrm{MeV}, \quad f_\omega = 198\,\mathrm{MeV}, \quad f_\phi = 161\,\mathrm{MeV},$

$\phi_V^{\|}(x)$ Light-cone distribution amplitude

**describes the momentum-fraction distribution of partons at zero transverse separation in a 2-particle Fock state**

This function is known in literature

Model $\phi_V(x,\mu) = 6x\bar{x}\left\{1 + a_2^V(\mu)C_2^{3/2}(2x-1)\right\}$

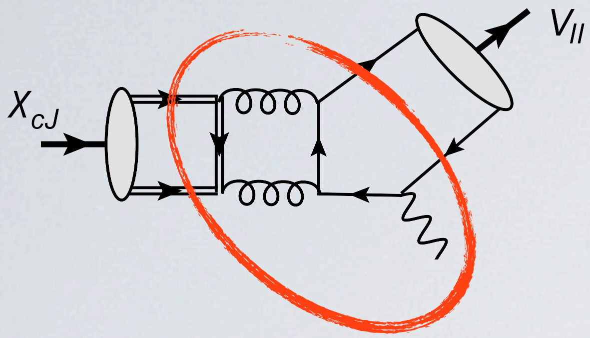
$\mu = 1\,\mathrm{GeV}$ $\quad a_2^\rho = a_2^\omega = 0.15 \pm 0.07,\ a_2^\phi = 0.18 \pm 0.08$

QCD SR

Ball, Braun 1996, 1999

Ball, Braun, Lenz 2006

# Radiative decays $\chi_{cJ} \to V_{||}\,\gamma$




$$A[\chi_{cJ} \to V_{\|}\gamma] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \sqrt{4\pi\alpha} \int_0^1 dx\, \frac{f_V}{m_c} \phi_V^{\|}(x)\, \alpha_s^2(\mu_h) T_J(x)$$

$$\begin{aligned}\mathrm{Re}\, T_1(x) \;=\;& -\frac{\pi^2}{12}\frac{1}{\bar{x}^3} - \frac{x}{4\bar{x}^3}\ln^2 2 + \left(-\frac{1}{\bar{x}^2} - \frac{1}{4\bar{x}} - \frac{3}{4x}\right)\ln 2 + \left(-\frac{3}{4\bar{x}} + \frac{1}{4x} - \frac{1}{2x-1}\right)\ln\bar{x} \\ & + \frac{x}{\bar{x}^3}\ln x \ln\bar{x} + \left(-\frac{1}{2\bar{x}^2} + \frac{3}{4\bar{x}} - \frac{1}{4x} + \frac{1}{2x-1}\right)\ln x - \frac{3x}{4\bar{x}^3}\ln^2 x - \frac{x}{2\bar{x}^3}\ln x \ln 2 \\ & -\frac{1}{2}\frac{x}{\bar{x}^3}\left(\mathrm{Li}\left[1-\frac{1}{2x}\right] + \mathrm{Li}\,[1-2x] + \mathrm{Li}\,[1-x] + \mathrm{Li}\,[-\bar{x}/x] - \mathrm{Li}\,[2x-1]\right) + (x \to \bar{x}),\end{aligned}$$

$$\mathrm{Im}\, T_1(x) = \frac{\pi}{4x\bar{x}^3}\left(\bar{x}(1+x(2x-1)) + 2x^2\ln[x]\right) + (x \to \bar{x})$$

endpoint behavior $\quad \mathrm{Re}\, T_i(x) \overset{x\to 1}{\sim} \frac{\ln\bar{x}}{\bar{x}},\ \ \mathrm{Im}\, T_i(x) \overset{x\to 1}{\sim} \frac{1}{\bar{x}}$

$$\phi_V(x,\mu) = 6x\bar{x}\left\{1 + a_2^V(\mu) C_2^{3/2}(2x-1)\right\},$$

# Radiative decays $\chi_{cJ} \to V_{||}\,\gamma$

Theory vs. experiment : only the color-singlet contribution

$m_c = 1.5$ GeV $\quad |R'_{21}(0)| = 0.75\text{GeV}^5$ B-T potential Eichten, Quigg 1995

Br's in units $10^{-6}$ $\qquad \alpha_s(2m_c^2) = 0.29 \quad m_c^2 < \mu_h^2 < 4m_c^2$

| | $\gamma\rho$ | $\gamma\omega$ | $\gamma\phi$ |
|---|---|---|---|
| $\chi_{c1} \to V_{\parallel}\gamma$ <br> exp. | $153.1^{+18.2+103.7}_{-16.7-70.5}$ <br> $184.8 \pm 15.7$ | $13.6^{+1.6+9.2}_{-1.5-6.3}$ <br> $51.8 \pm 8.9$ | $31.3^{+4.2+21.4}_{-3.8-14.5}$ <br> $17.7 \pm 4.9$ |
| $\chi_{c2} \to V_{\parallel}\gamma$ <br> exp. | $4.8^{+0.2+3.1}_{-0.2-2.1}$ <br> $< 20$ | $0.43^{+0.02+0.27}_{-0.02-0.19}$ <br> $< 6$ | $0.9^{+0.05+0.59}_{-0.04-0.41}$ <br> $< 8$ |

Theory $\quad \dfrac{Br\left[\chi_{c1} \to \omega_{\parallel}\gamma\right]}{Br\left[\chi_{c1} \to \rho_{\parallel}\gamma\right]} \simeq \dfrac{1}{9}$ (0.28+/-0.06) ?

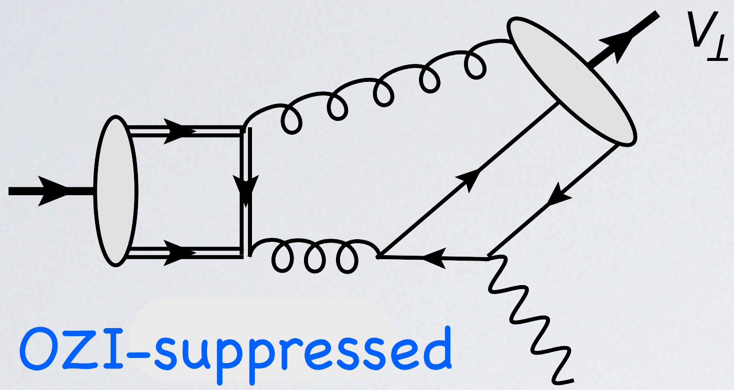
clear indication about significance of the color-octet mechanism

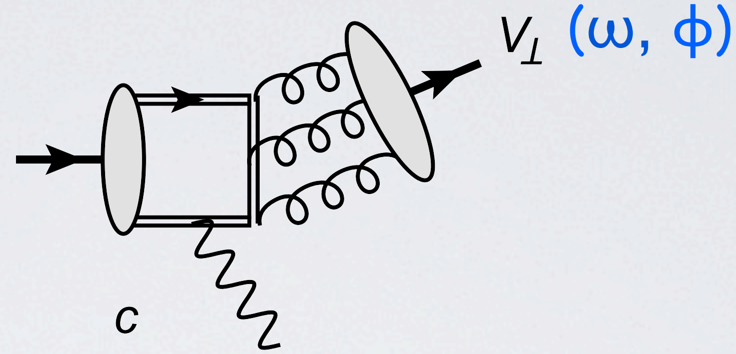
$$\langle V(p)|J^{\mu}_{em}|\chi_{cJ}\rangle = \langle V(p)|\sum_{u,d,s} e_q\ \bar{q}\gamma^{\mu}q|\chi_{cJ}\rangle + \langle V(p)|e_c\ \bar{c}\gamma^{\mu}c|\chi_{cJ}\rangle$$

color-octet mechanism

# Radiative decays $\chi_{cJ} \to V_T \gamma$

color singlet




the collinear integrals don't have endpoint IR-divergencies

## Twist-3 DAs

$$A(\alpha_i) = 360\zeta_3\alpha_1\alpha_2\alpha_3^2\left(1+\omega_3^A\frac{1}{2}(7\alpha_3-3)\right) \qquad G(\alpha_i) = 5040\,\zeta_3\omega_3^G\alpha_1^2\alpha_2^2\alpha_3^2$$

$$V(\alpha_i) = 540\zeta_3\omega_3^V\alpha_1\alpha_2\alpha_3^2(\alpha_2-\alpha_1).$$

## QCD sum rules

Ball, Braun 1996, 1999
Ball, Braun, Koike, Tanaka 1998
Ball, Braun, Lenz 2006

$$\mu = 1\,\mathrm{GeV}$$

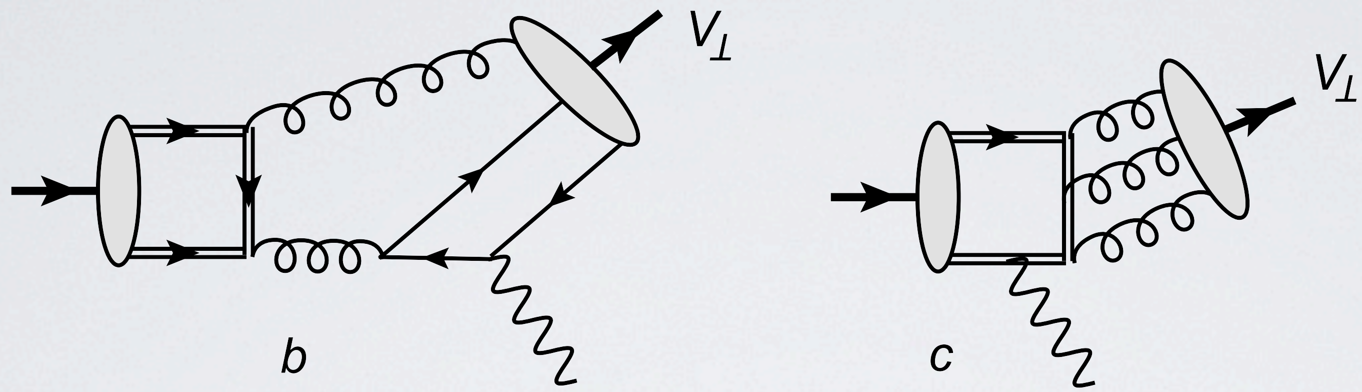
$$\begin{aligned} \rho \text{ and } \omega\text{-mesons} \quad &: \quad \zeta_3 = 0.030 \pm 0.010, \quad \omega_3^A = -3.0 \pm 1.4, \quad \omega_3^V = 5.0 \pm 2.4 \\ \phi\text{-meson} \quad &: \quad \zeta_3 = 0.024 \pm 0.008, \quad \omega_3^A = -2.6 \pm 1.3, \quad \omega_3^V = 5.3 \pm 3.0 \end{aligned}$$

qq-mesons have small overlap with gluons $\qquad |\omega_3^G(\mu = 1\mathrm{GeV})| \ll 1$

# Radiative decays $\chi_{cJ} \to V_T \gamma$

only the color singlet contribution

OZI-suppressed




$(\omega, \phi)$

$$\Gamma_\rho^\perp = 222.4\ \zeta_3^2(-9.82 + 4.78\omega_3^A + 3.31\omega_3^V)^2$$

$$\Gamma_\phi^\perp = 168.7\ \zeta_3^2(6.5 - 3.3\ \omega_3^A + 2.9\ \omega_3^V + 733.6\ \omega_3^G)^2$$

$$\Gamma_\omega^\perp = 181.2\ \zeta_3^2(-3.3 + 1.4\ \omega_3^A + 8.3\ \omega_3^V + 735.6\ \omega_3^G)^2$$

large numerical coefficient in front of small $\omega_3^G$

# Radiative decays $\chi_{cJ} \to V_T \gamma$

DAs parameters (QCD Sum rules)

$$\zeta_3 = 0.030 \pm 0.010, \quad \omega_3^A = -3.0 \pm 1.4, \quad \omega_3^V = 5.0 \pm 2.4$$

$$|\omega_3^G(\mu = 1\mathrm{GeV})| \ll 1$$

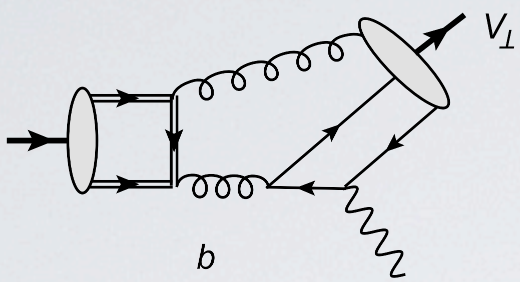
only the color singlet    branching fractions in units $10^{-4}$

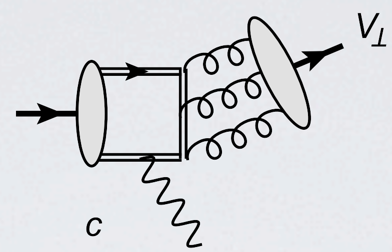
| | | | | $\chi_{c1} \to V_\perp \gamma$ | | |
|---|---|---|---|---|---|---|
| $\zeta_3$ | $\omega_3^A$ | $\omega_3^V$ | $\omega_3^G$ | $\rho$ | $\omega$ | $\phi$ |
| 0.03 | -2.2 | 2.8 | -0.037 | $(35.2 \pm 7.4)$ 29.6 | $(17.3 \pm 6.5)$ 20.8 | $(7.3 \pm 3.6)$ 4.8 |
| 0.03 | -4.4 | 5.9 | -0.043 | 30.0 | 13.9 | 8.5 |
| 0.04 | -2.5 | 3.7 | -0.041 | 39.2 | 13.5 | 6.5 |
| 0.04 | -3.4 | 5.1 | -0.038 | 33.2 | 14.2 | 6.2 |

Data can be described including small 3g contribution

| $\chi_{c2} \to V\gamma$ | | | $\chi_{c0} \to V\gamma$ | | |
|---|---|---|---|---|---|
| $\rho$ | $\omega$ | $\phi$ | $\rho$ | $\omega$ | $\phi$ |
| $< 20$ 3.4 | $< 6$ 0.18 | $< 8$ 2.6 | $< 9$ 2.0 | $< 8$ 0.17 | $< 6$ 0.66 |
| 17.2 | 3.7 | 6.5 | 0.40 | 0.05 | 0.15 |
| 7.1 | 0.40 | 5.6 | 1.9 | 0.16 | 0.54 |
| 16.3 | 3.6 | 6.50 | 0.62 | 0.09 | 0.23 |

# Radiative decays $\chi_{cJ} \to V_T \gamma$




$(\omega, \phi)$

only the color-singlet contributions

Data can only be described including small 3g contributions

$$Br[\chi_{c1} \to \gamma\rho] > Br[\chi_{c2} \to \gamma\rho] > Br[\chi_{c0} \to \gamma\rho]$$

$$Br[\chi_{c2} \to \gamma\rho] > Br[\chi_{c2} \to \gamma\phi] > Br[\chi_{c2} \to \gamma\omega]$$

$$Br[\chi_{c2} \to \gamma\phi] \geq Br[\chi_{c1} \to \gamma\phi]$$

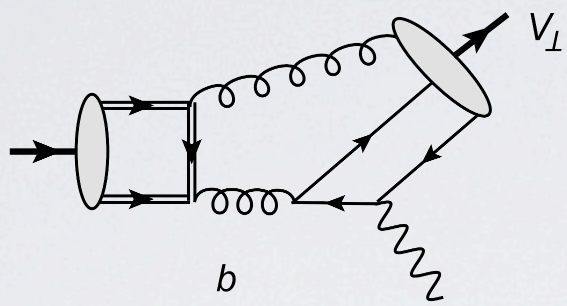
More accurate measurements of $Br[\chi_{c2} \to \gamma\rho]$

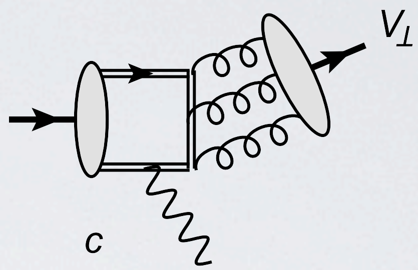
can help to reduce theoretical ambiguity

# Color-octet contributions $\chi_{cJ} \to V_T\,\gamma$

color-singlet    tw.-3 DAs

the collinear integrals
don't have endpoint
IR-divergencies

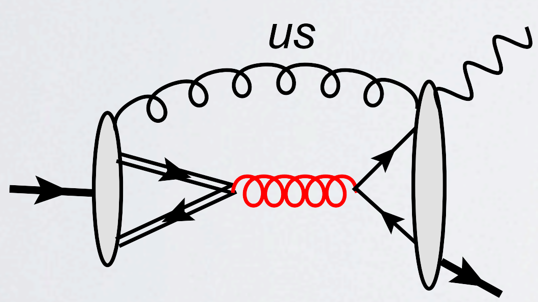



$\sim \alpha_s v^4 (\Lambda/m_c)^2$

color-octet    power counting    $mv^2 \sim \Lambda \qquad mv \sim \sqrt{m\Lambda}$

$$\chi_{cJ} \to g_{us} + c\bar{c}_8({}^3S_1) \to g_{us} + \{q_c\,\bar{q}_c\}_8 \to \gamma + V_\|$$




$\sim \alpha_s v^4 (\Lambda/m_c)^2$

color-octet is of the
same order as the color-singlet

$\Rightarrow$ large theoretical uncertainty!

# Radiative decays $\chi_{cJ} \to V_{||}\, \gamma$

Theory vs. experiment : only the color-singlet contribution

branching fractions in units $10^{-4}$ $\quad m_c < \mu_h < 2m_c$

| | $\gamma\rho$ | $\gamma\omega$ | $\gamma\phi$ |
|---|---|---|---|
| $\chi_{c1} \to V_{\parallel}\gamma$ <br> exp. | $153.1^{+18.2+103.7}_{-16.7-70.5}$ <br> $184.8 \pm 15.7$ | $13.6^{+1.6+9.2}_{-1.5-6.3}$ <br> $51.8 \pm 8.9$ | $31.3^{+4.2+21.4}_{-3.8-14.5}$ <br> $17.7 \pm 4.9$ |
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Theory $\quad \dfrac{Br\left[\chi_{c1} \to \omega_{\parallel}\gamma\right]}{Br\left[\chi_{c1} \to \rho_{\parallel}\gamma\right]} \simeq \dfrac{1}{9} \quad$ 0.28(6) ?

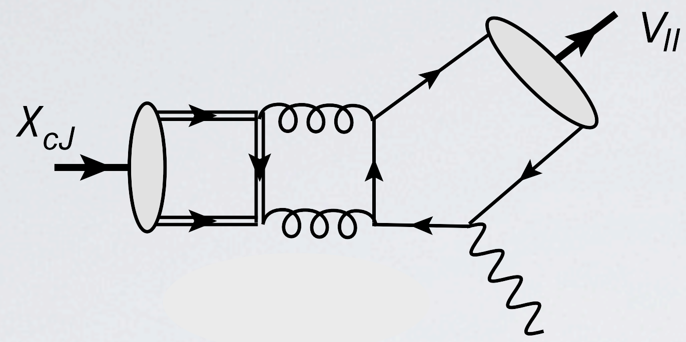
clear indication about significance of the color-octet mechanism

$$\langle V(p)|J^{\mu}_{em}|\chi_{cJ}\rangle = \langle V(p)|\sum_{u,d,s} e_q\, \bar{q}\gamma^{\mu}q|\chi_{cJ}\rangle + \langle V(p)|e_c\, \bar{c}\gamma^{\mu}c|\chi_{cJ}\rangle$$

color-octet mechanism

# Color-octet contributions $\chi_{cJ} \to V_T\, \gamma$

color-singlet

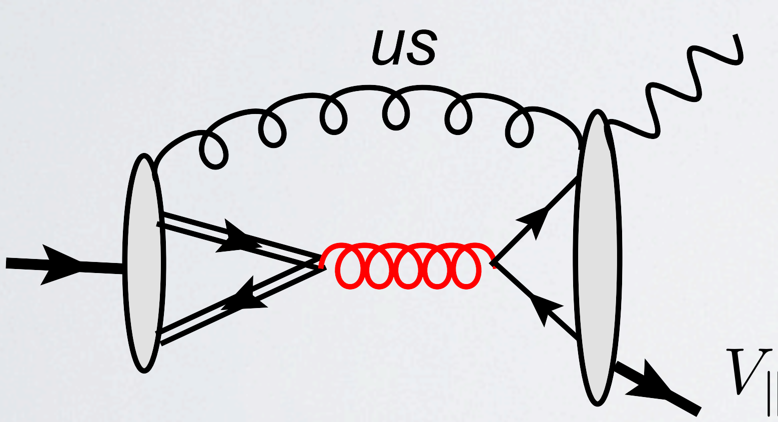



tw.-2

$$\sim \alpha_s^2 v^4 \Lambda/m_c$$

color-octet  power counting  $mv^2 \sim \Lambda \quad mv \sim \sqrt{m\Lambda}$

$$\chi_{cJ} \to g_{us} + c\bar{c}_8(^3S_1) \to g_{us} + \{q_c\, \bar{q}_c\}_8 \to \gamma + V_{\|}$$




tw2

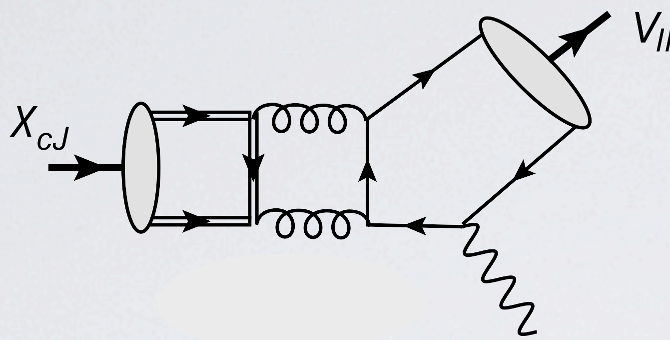
$$\sim \alpha_s v^4 (\Lambda/m_c)^2 \qquad \Rightarrow \frac{octet}{singlet} \sim \frac{v^2}{\alpha_s}$$

$$v_c^2 \sim 0.3 \qquad \alpha_2(\mu_h) \simeq 0.25 - 0.3$$

$\Rightarrow$ large theor. uncertainty but can not explain large $\dfrac{Br[\chi_{c1} \to \omega_{\|}\gamma]}{Br[\chi_{c1} \to \rho_{\|}\gamma]}$

# Color-octet contributions $\chi_{cJ} \to V_T\,\gamma$

color-singlet



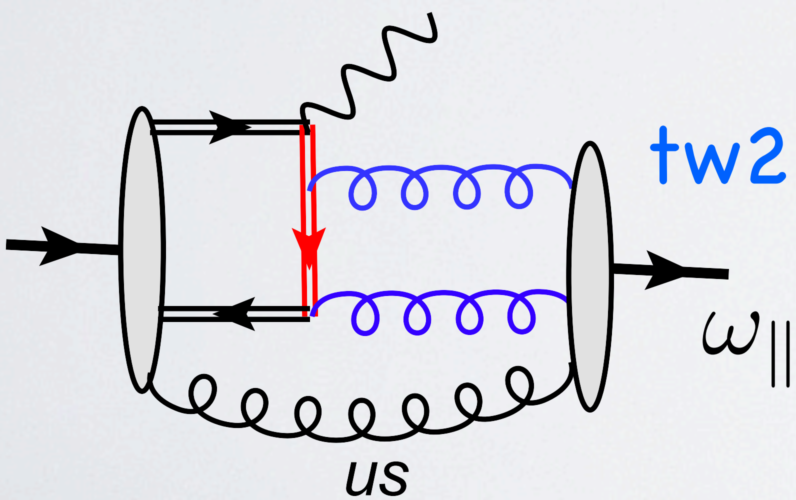

tw.-2

$$\sim \alpha_s^2 v^4 \Lambda/m_c$$

color-octet

power counting $\quad mv^2 \sim \Lambda \quad mv \sim \sqrt{m\Lambda}$

$$\chi_{cJ} \to g_{us} + c\bar{c}_8(^3S_1) \to \underbrace{g_{us} + (g_{hc} + g_{hc})_8}_{\bar{\xi}_c \xi_c(^1S_0)} + \gamma \to \omega_{\|} + \gamma$$



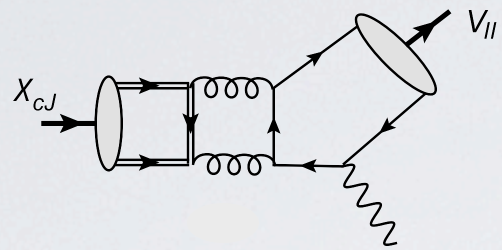

$$\sim \alpha_s v^5 (\Lambda/m_c)^2 \quad \Rightarrow \quad \frac{octet}{singlet} \sim \frac{v^3}{\alpha_s}$$

$$v_c^2 \sim 0.3 \qquad \alpha_2(\mu_h) \simeq 0.25 - 0.3$$

soft gluon fusion into longit. meson might explain the large ratio

# Conclusions

- The dominant contribution in $\chi_{c1} \to V_{\parallel}\gamma$ is the color-singlet term suppressed as

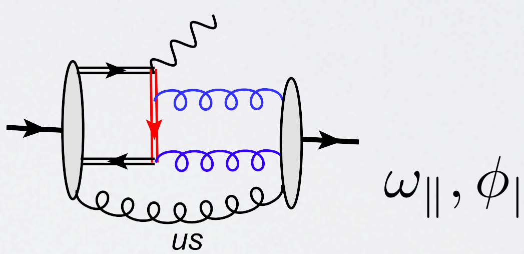



$$\sim \alpha_s^2 v^4 \Lambda / m_c$$

- The discrepancy with data for the ratio $\dfrac{Br\left[\chi_{c1} \to \omega_{\parallel}\gamma\right]}{Br\left[\chi_{c1} \to \rho_{\parallel}\gamma\right]} \simeq \dfrac{1}{9}$ (exp: 0.28+/-0.06)

  indicates about a large contribution beyond OZI-suppressed configuration

  Possible mechanism:
  color-octet gluon fusion




$$\sim \alpha_s v^5 (\Lambda/m_c)^2$$

- The octet and singlet contributions in $\chi_{c1} \to V_{\perp}\gamma$ are of the same order.
  Observation: the data can be described by color-singlet mechanism including small 3g coupling for I=0 mesons. More accurate data for hadronic DAs are important!

- Further measurements for $\chi_{c0,2} \to V\gamma$ will help to clarify the decay mechanism

# Thank you!

# Radiative decays $\chi_{cJ} \to V\gamma$

## Helicity amplitudes

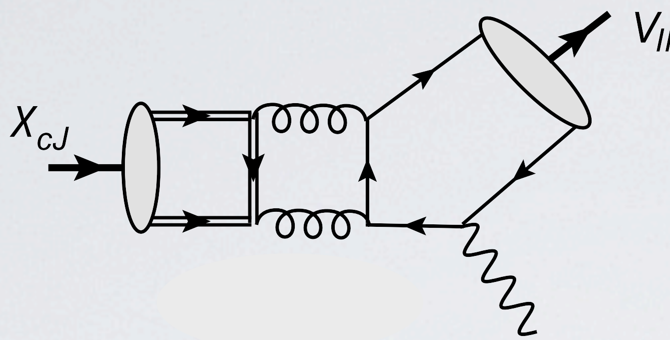
$$A_{0V}^{\perp} : \chi_{c0} \to V(\lambda_V = \pm 1)\gamma(\lambda_\gamma = \pm 1)$$

$$\begin{aligned} A_{1V}^{\perp} &: \quad \chi_{c1}(\lambda_\chi = 0) \to V(\lambda_V = \pm 1)\gamma(\lambda_\gamma = \pm 1) \\ A_{1V}^{\parallel} &: \quad \underline{\chi_{c1}(\lambda_\chi = \pm 1) \to V(\lambda_V = 0)\gamma(\lambda_\gamma = \pm 1)} \end{aligned}$$

$$\begin{aligned} A_{2V}^{\perp} &: \quad \chi_{c2}(\lambda_\chi = 0) \to V(\lambda_V = \pm 1)\gamma(\lambda_\gamma = \pm 1) \\ A_{2V}^{\parallel} &: \quad \underline{\chi_{c2}(\lambda_\chi = \pm 1) \to V(\lambda_V = 0)\gamma(\lambda_\gamma = \pm 1)} \\ T_{2V}^{\perp} &: \quad \chi_{c2}(\lambda_\chi = \pm 2) \to V(\lambda_V = \mp 1)\gamma(\lambda_\gamma = \pm 1) \end{aligned}$$

# Color-octet contributions $\chi_{cJ} \to V_T\,\gamma$

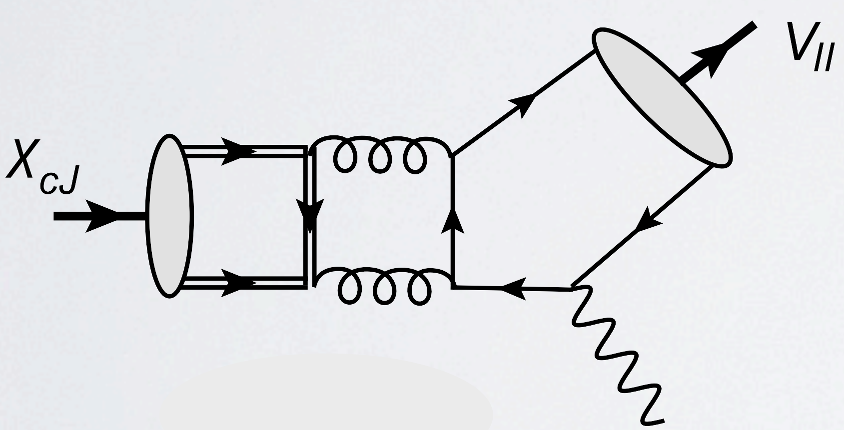
color-singlet




tw.-2

$$\sim \alpha_s^2 v^4 \Lambda/m_c$$

higher Fock state of the light meson




tw.-4

$$\sim \alpha_s^2 v^4 (\Lambda/m_c)^3$$ small

# Radiative decays $\chi_{cJ} \to V_{||}\,\gamma$

Theory vs. experiment : only the color singlet contribution

branching fractions in units $10^{-4}$

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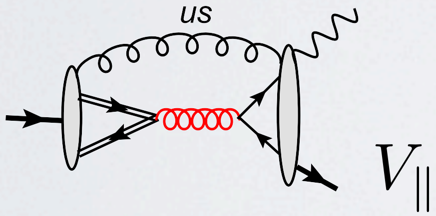
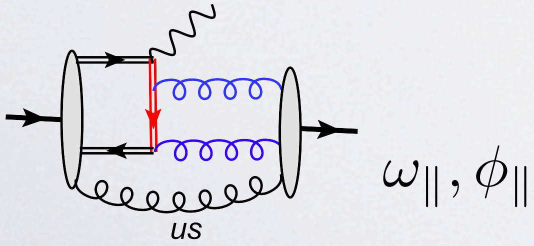
|---|---|---|---|
| $\chi_{c1} \to V_{\parallel}\gamma$ <br> exp. | $153.1^{+18.2+103.7}_{-16.7-70.5}$ <br> $184.8 \pm 15.7$ | $13.6^{+1.6+9.2}_{-1.5-6.3}$ <br> $51.8 \pm 8.9$ | $31.3^{+4.2+21.4}_{-3.8-14.5}$ <br> $17.7 \pm 4.9$ |
| $\chi_{c2} \to V_{\parallel}\gamma$ <br> exp. | $2.11^{+0.09+1.3}_{-0.08-0.9}$ <br> $< 20$ | $0.19^{+0.008+0.12}_{-0.007-0.08}$ <br> $< 6$ | $0.41^{+0.02+0.26}_{-0.02-0.18}$ <br> $< 8$ |

color octet contributions c-quark $v^2 \simeq 0.3$ $\alpha_s(2m_c^2) = 0.29$




$$\sim \alpha_s v^4 (\Lambda/m_c)^2 \quad \Rightarrow \quad \frac{octet}{singlet} \sim \frac{v^2}{\alpha_s}$$




$$\sim \alpha_s v^5 (\Lambda/m_c)^2 \quad \Rightarrow \quad \frac{octet}{singlet} \sim \frac{v^3}{\alpha_s}$$