RENORMALONS IN INCLUSIVE η_Q DECAY

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OUTLINE

- Resummation of perturbative corrections in η_Q decay
- Emergence of renormalon ambiguities in resummation
- Cancellation of renormalons in factorization formula
- Resummed predictions for the η_c and η_b decay rate
- Summary

η_Q DECAY RATE

- Decay rate of η_Q is written in NRQCD factorization formalism at leading order in v as

Long-distance matrix element

$$\Gamma_{\eta_Q} = \frac{\pi C_F \alpha_s^2}{N_c m^2}[1 + O(\alpha_s)]\langle \eta_Q | \mathcal{O}_1(^1S_0) | \eta_Q \rangle$$

Short-distance coefficient

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- The short-distance coefficient is known to have large corrections from radiative corrections that are related to the running of α_s, hindering precise prediction.

- NLO correction by **Hagiwara, Kim, Yoshino**

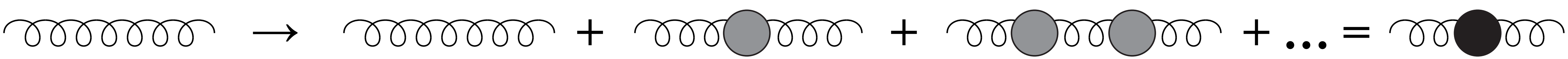
K. Hagiwara, C. B. Kim, T. Yoshino, NPB 177, 461 (1981)

- NNLO correction recently by **Feng, Jia and Sang** *Next talk by Yu Jia*

F. Feng, Y. Jia, W. L. Sang, arXiv:1707.05758

η_Q DECAY RATE

- We resum QCD corrections in the ratio $R=\Gamma(\eta_Q)/\Gamma(\eta_{Q\to\gamma\gamma})$ in the large-n_f limit, which is given by bubble-chain resummation.



- LDME at leading order in v cancels in the ratio R.
- Effect of resummation cancels for gluons in the initial state, so we consider resummation in the final-state gluons only.
- We use dimensional regularization to regulate IR divergences and regulate UV divergences in NRQCD with a hard cutoff.
- Bubble-chain resummation involves renormalon ambiguities in the inverse Borel transform. We show their cancellation in the decay rate.

η_Q DECAY RATE

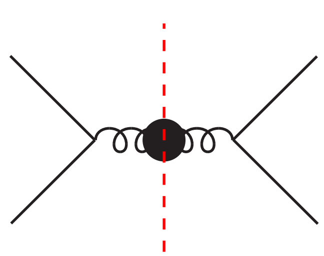
- Factorization formula with lowest-dimensional operators :

$$\Gamma_{\eta_Q} = 2\mathrm{Im}\left[\frac{f_1({}^1S_0)}{m^2}\right]\langle \eta_Q|\mathcal{O}_1({}^1S_0)|\eta_Q\rangle + 2\mathrm{Im}\left[\frac{f_8({}^3S_1)}{m^2}\right]\langle \eta_Q|\mathcal{O}_8({}^3S_1)|\eta_Q\rangle$$

- Basic strategy : compute QCD amplitude on the left and NRQCD LDMEs on the right for perturbative $Q\bar{Q}$ states, and obtain short-distance coefficients by comparison.

- Color-octet short-distance coefficient is computed from the decay of color-octet spin-triplet $Q\bar{Q}$ state :

Scalar part of vacuum polarization at $Q\bar{Q}$ threshold

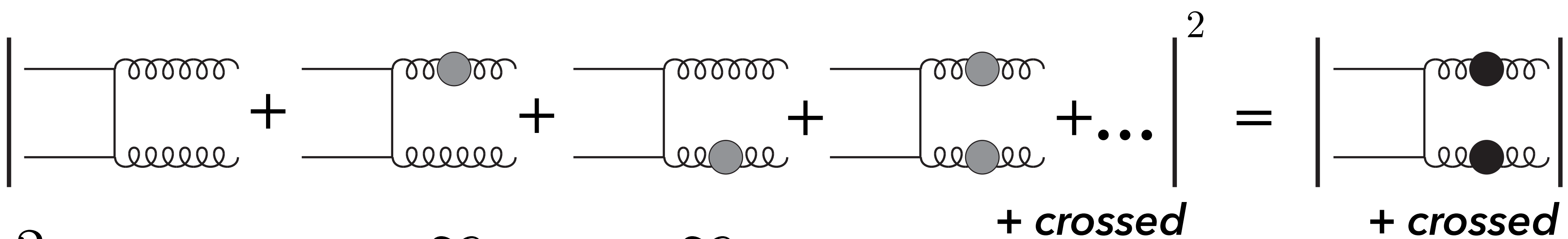


$$\rightarrow \quad 2\mathrm{Im}\left[\frac{f_8({}^3S_1)}{m^2}\right] = -\frac{2\pi\alpha_s}{m^2}\frac{\alpha_s\beta_0\pi}{(1-\boxed{\alpha_s\beta_0 d})^2+\pi^2\alpha_s^2\beta_0^2}$$

η_Q DECAY RATE : QCD CALCULATION

- Resummation of QCD corrections to final-state gluons :

$$\Gamma_{Q\bar{Q}_1(^1S_0)} =$$



$$= \frac{2\pi C_F \alpha_s^2}{m^2} \frac{1}{(\alpha_s \beta_0)^2} \int_0^\infty dt \int_0^\infty d\tau \, e^{-\omega(t+\tau)} T(t,\tau)$$

$$\omega = \frac{1}{\alpha_s \beta_0} - d$$

- Renormalon singularities appear at $t, \tau = 1, 3/2, \ldots$

$$\lim_{t \to 1} (1-t) T(t,\tau) = -\frac{3}{\pi} \sin(\pi\tau) \qquad T(t,\tau) = T(\tau,t)$$

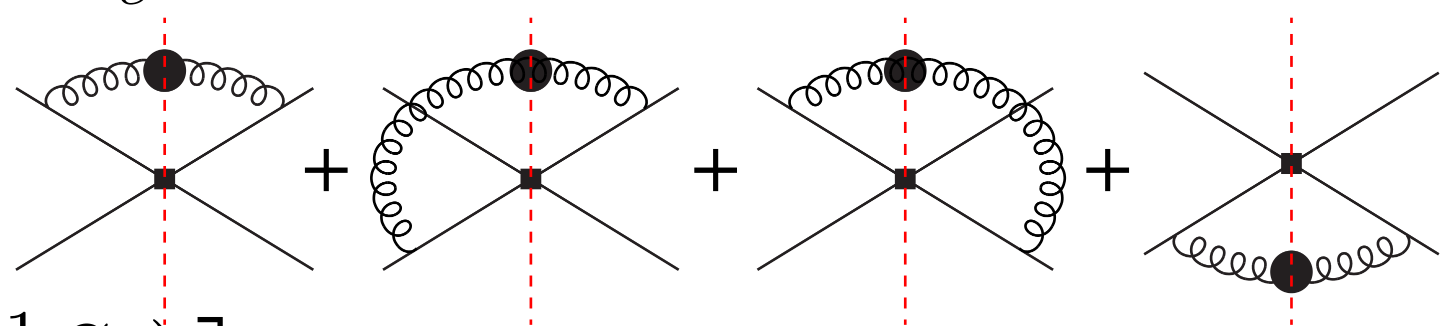
- Renormalon ambiguities from subleading singularities are negligible.

η_Q DECAY RATE : NRQCD CALCULATION

- Calculation of NRQCD LDMEs at LO in α_s

$$\langle Q\bar{Q}_1({}^1S_0)|\mathcal{O}_1({}^1S_0)|Q\bar{Q}_1({}^1S_0)\rangle = 2N_c$$

$$\langle Q\bar{Q}_1({}^1S_0)|\mathcal{O}_8({}^3S_1)|Q\bar{Q}_1({}^1S_0)\rangle =$$



$$\Gamma_{Q\bar{Q}_1({}^1S_0)} = 2N_c \times 2\mathrm{Im}\left[\frac{f_1({}^1S_0)}{m^2}\right] + \frac{2\pi C_F\alpha_s^2}{m^2}\frac{1}{(\alpha_s\beta_0)^2}\int_0^\infty dt\int_0^\infty d\tau\, e^{-w(t+\tau)}T_8^{(\Lambda)}(t,\tau)$$

- Same renormalon singularity appears at $t,\ \tau = 1$:

$$\lim_{t\to 1}(1-t)T_8^{(\Lambda)}(t,\tau) = -\frac{3}{\pi}\sin(\pi\tau) \qquad T_8^{(\Lambda)}(t,\tau) = T_8^{(\Lambda)}(\tau,t)$$

η_Q DECAY RATE

- Leading renormalon ambiguity in the QCD amplitude is exactly reproduced by the LDME : short-distance coefficients are free of renormalon ambiguities.

$$2\mathrm{Im}\left[\frac{f_1(^1S_0)}{m^2}\right] = \frac{\pi C_F \alpha_s^2}{N_c m^2}\frac{1}{(\alpha_s\beta_0)^2}\int_0^\infty dt\int_0^\infty d\tau\, e^{-w(t+\tau)}[T(t,\tau) - T_8^{(\Lambda)}(t,\tau)]$$

- NRQCD cutoff dependence cancels between short-distance coefficient and the color-octet matrix element.

- Reproduces known NLO and recent NNLO corrections in the large-n_f limit.

K. Hagiwara, C. B. Kim, T. Yoshino, NPB 177, 461 (1981)

F. Feng, Y. Jia, W. L. Sang, arXiv:1707.05758

η_Q DECAY : COMPARISON WITH FIXED-ORDER CALCULATION

- NNLO correction to the η_Q decay rate has been computed recently by F. Feng, Y. Jia, W. L. Sang, arXiv:1707.05758.
- We resum QCD corrections in the large-n_f limit to all orders in α_s. The fixed-order calculation by Feng, Jia and Sang includes contributions beyond the large-n_f limit but only up to NNLO in α_s.
- We also resum QCD corrections related to α_s evolution at the one-loop level, and the resummed result has mild dependence on the renormalization scale.
- We can combine our resummed result with fixed-order calculations so that the result is valid beyond the large-n_f limit up to NNLO in α_s.

η_Q DECAY : COMPARISON WITH PREVIOUS CALCULATION

- Resummation of the η_c decay rate has been done by G. T. Bodwin and Y.-Q. Chen in PRD 60, 054008 (1999).
- Bodwin and Chen used IR cutoff. We use dimensional regularization so that the cancellation of renormalons can be seen explicitly.
- Bodwin and Chen used fixed IR cutoff. We consider dependence on the NRQCD UV cutoff, which will cancel in the decay rate once the color-octet LDME is included.
- We also combine our results with the recently calculated NNLO corrections by Feng, Jia and Sang. F. Feng, Y. Jia, W. L. Sang, arXiv:1707.05758
- We also give prediction for the η_b decay rate.

η_Q DECAY RATE RATIO

- We can combine the resummed result and known perturbative corrections up to NNLO to make predictions for the ratio R.

Fixed-order calculation

$$R^{\mathrm{Resum}+\Delta\mathrm{N}^n\mathrm{LO}} = R^{\mathrm{Resum}} + R^{N^n LO} - \delta R^{\mathrm{Resum}}$$

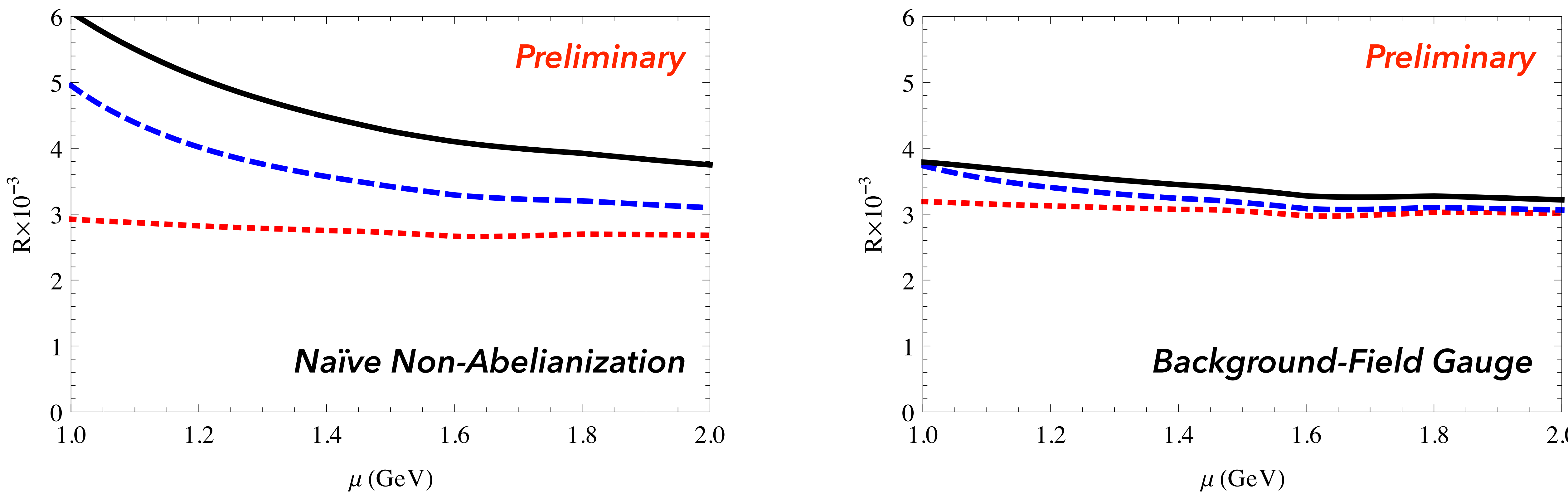
Resummation

Resummed contribution already included in the fixed-order calculation

- It is not known how to compute the color-octet LDME, which is suppressed by $O(v^3)$. We neglect the color-octet contribution and include its effect in the uncertainties.

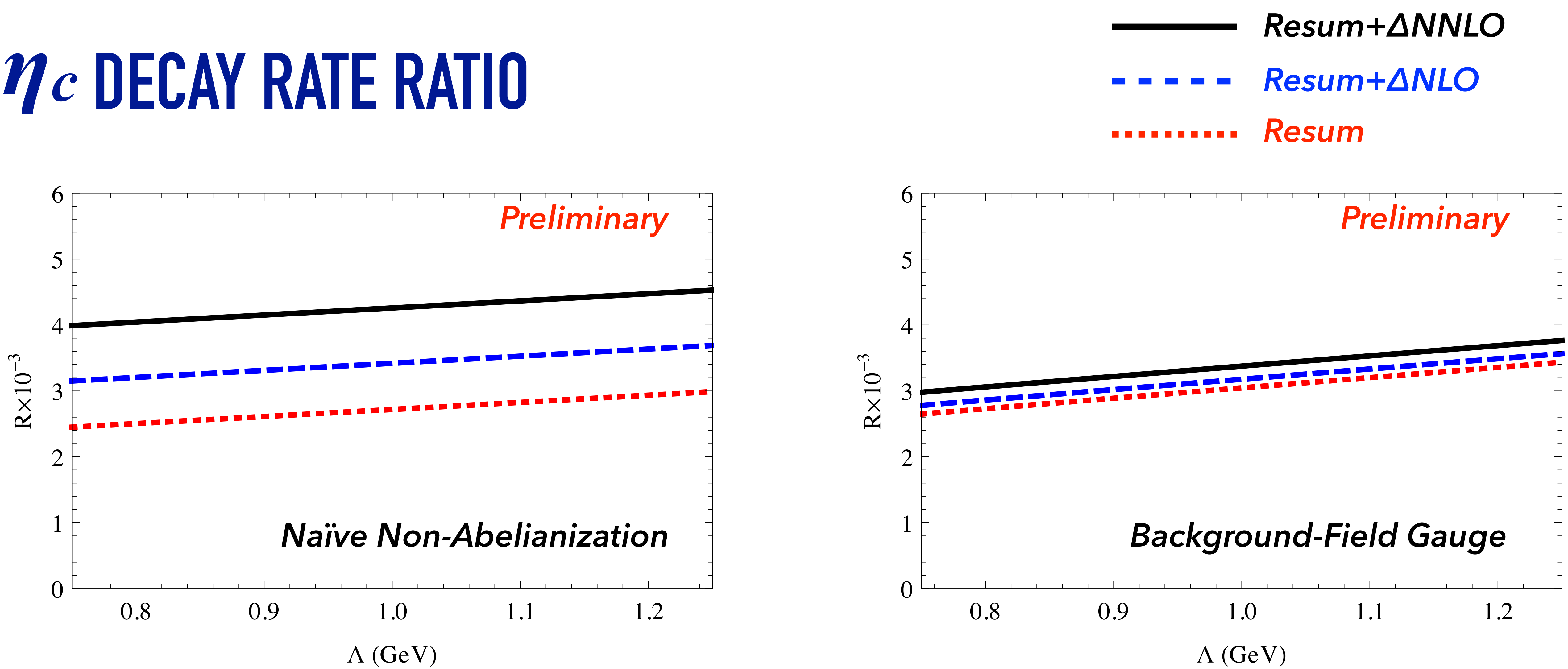
- We present our preliminary results for η_c and η_b. We use Naïve Non-Abelianization (NNA) and Background-Field Gauge (BFG) method to resum bubble chains.

η_c DECAY RATE RATIO



- Strong renormalization-scale dependence for NNA, especially for μ below the heavy quark mass m.
- Mild dependence on μ for BFG.
- NRQCD UV cutoff $\Lambda = 1$ GeV ($\approx mv$)

η_c DECAY RATE RATIO



- Mild dependence on NRQCD UV cutoff
- Renormalization scale $\mu = 1.5$ GeV

η_c DECAY RATE RATIO

- Our preliminary numerical results for η_c are

Preliminary, *NNA*

$$R = (4.3^{+1.8}_{-0.5} \pm 0.3 \pm 0.4) \times 10^3 = (4.3^{+1.9}_{-0.7}) \times 10^3$$

Preliminary, *BFG*

$$R = (3.4^{+0.1}_{-0.4} \pm 0.4 \pm 0.5) \times 10^3 = (3.4^{+0.6}_{-0.8}) \times 10^3$$

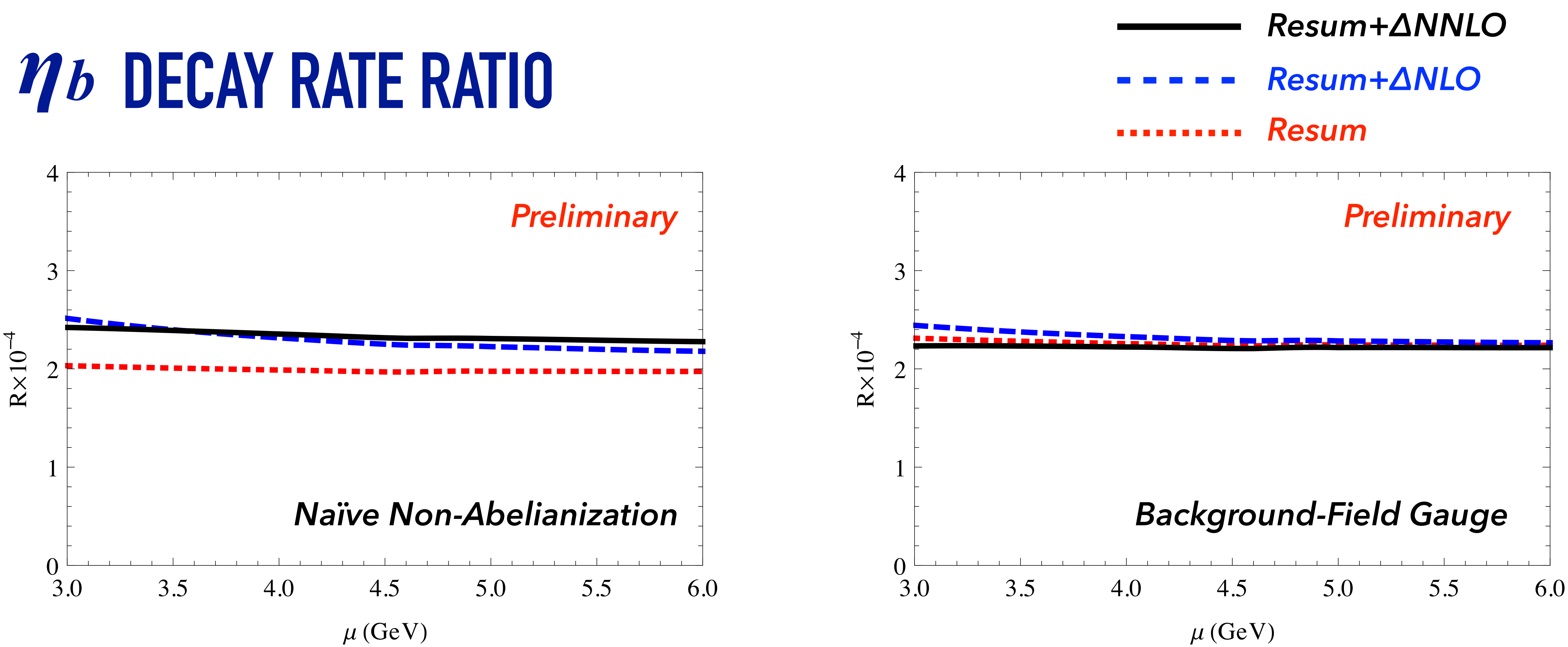
Uncertainties from μ Λ *CO LDME neglected*

- Large uncertainty from renormalization-scale dependence for NNA. Large uncertainty from neglect of CO LDME.

- Compatible with NNLO prediction by Feng, Jia and Sang, but there is tension between PDG value *see next talk by Yu Jia*

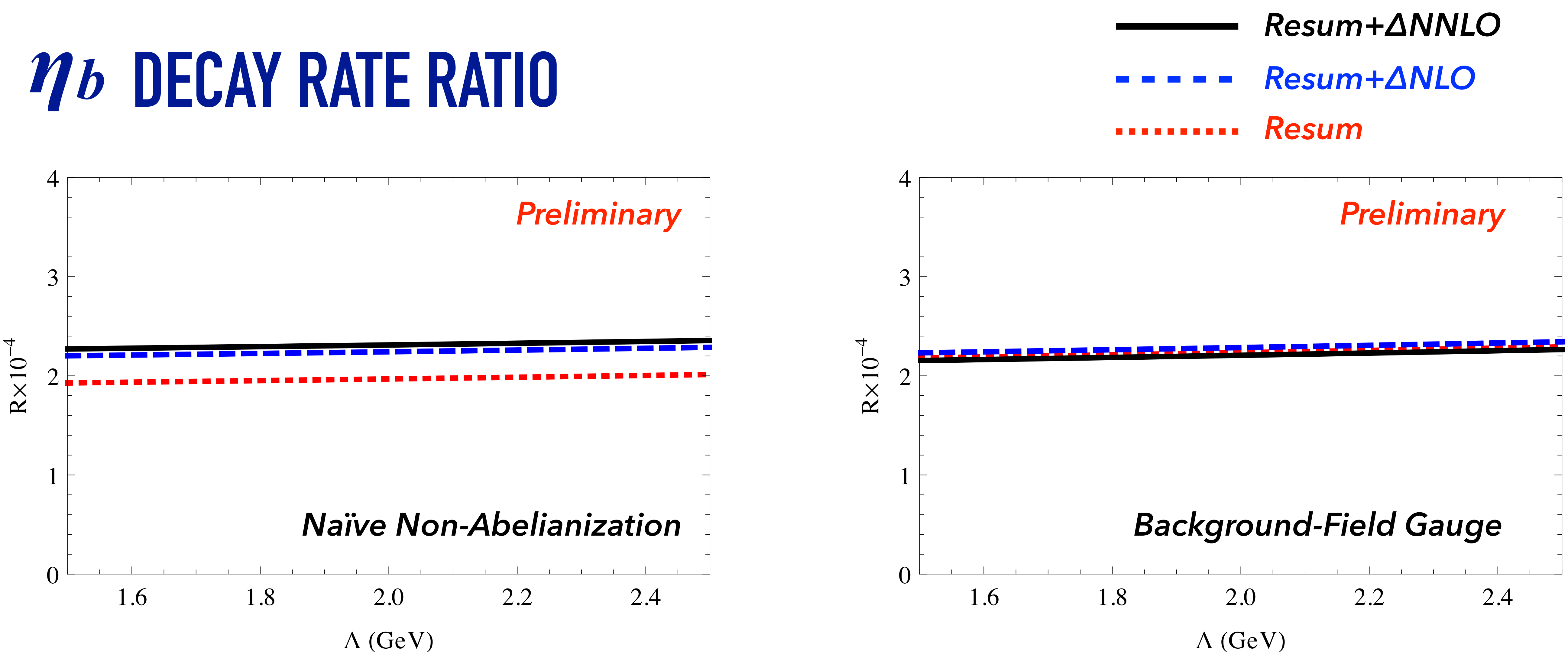
  $1/\mathrm{Br}(\eta_c \to \gamma\gamma) = (6.3^{+0.6}_{-0.5}) \times 10^3$ from constrained fit.

η_b DECAY RATE RATIO



- Very mild renormalization-scale dependence
- NRQCD UV cutoff $\Lambda = 2\ \mathrm{GeV}\ (\approx mv\ )$

η_b DECAY RATE RATIO



- Very mild dependence on NRQCD UV cutoff
- Renormalization scale $\mu = 4.6$ GeV

η_b DECAY RATE RATIO

- Our preliminary numerical results for η_b are

Preliminary, *NNA*

$$R = (2.31^{+0.11}_{-0.03} \pm 0.04 \pm 0.06) \times 10^4 = (2.31^{+0.13}_{-0.08}) \times 10^4$$

Preliminary, *BFG*

$$R = (2.21^{+0.03}_{-0.00}{}^{+0.06}_{-0.05} \pm 0.07) \times 10^4 = (2.21 \pm 0.09) \times 10^4$$

Uncertainties from μ Λ *CO LDME neglected*

- Good agreement between NNA and BFG methods. Large uncertainty from neglect of CO LDME.

- Compatible with NNLO prediction by Feng, Jia and Sang, but uncertainties are reduced by resummation. *see next talk by Yu Jia*

η_b DECAY RATE

- We can combine our results with predictions for $\Gamma(\eta_b \to \gamma\gamma)$

$\Gamma(\eta_b \to \gamma\gamma) = 0.54 \pm 0.15 \text{ keV}$ Y. Kiyo, A. Pineda, A. Signer, NPB 841, 231 (2010)

$\Gamma(\eta_b \to \gamma\gamma) = 0.512^{+0.096}_{-0.094} \text{ keV}$ **HSC**, J. Lee, C. Yu, PLB697, 48 (2011)

NNA Resum **× Kiyo, Pineda, Signer** $\Gamma_{\eta_b} = 12.5 \pm 3.5 \text{ MeV}$

BFG Resum **× Kiyo, Pineda, Signer** $\Gamma_{\eta_b} = 11.9 \pm 3.3 \text{ MeV}$

NNA Resum **× HSC, Lee, Yu** $\Gamma_{\eta_b} = 11.8^{+2.3}_{-2.2} \text{ MeV}$

BFG Resum **× HSC, Lee, Yu** $\Gamma_{\eta_b} = 11.3^{+2.2}_{-2.1} \text{ MeV}$

Preliminary

- These predictions are compatible with each other. PDG value $\Gamma_{\eta_b} = 10^{+5}_{-4} \text{ MeV}$ has large uncertainties.

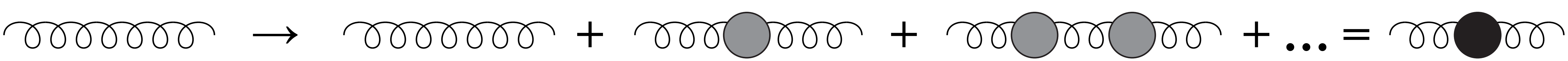
SUMMARY

- We performed resummation of perturbative corrections to the η_c and η_b decay rate.
- We showed explicit cancellation of renormalon ambiguities by calculating QCD and NRQCD amplitudes.
- Resummation leads to solid prediction of the η_b decay rate.
- In the η_c decay rate, the residual perturbative corrections are still large after resummation, and there is tension with data.
- Color-octet contribution is necessary to reduce theoretical uncertainties.

BACKUP

BUBBLE-CHAIN RESUMMATION

- The perturbation series can be resummed in the large-n_f limit by resumming bubble chains as a Borel sum.



- Naïve Non-Abelianization (NNA) : compute effect of light quarks in vacuum polarization, promote n_f to the QCD beta function.

M. Beneke and V.M. Braun, PLB348, 513 (1995)

- Background-Field Gauge : compute vacuum polarization against a background field in R_ξ gauge.

B.S. Dewitt, Phys. Rev. 162, 1195 (1967)

- Two methods give same results in the large-n_f limit

BUBBLE-CHAIN RESUMMATION

- The resummation is conveniently rewritten using Borel transform.

$$\mathcal{F}(\alpha_s) = \sum_{n=1}^{\infty} a_n \alpha_s^n \xrightarrow{\text{Borel transform}} \tilde{\mathcal{F}}(t) = \sum_{n=1}^{\infty} \frac{a_n}{(n-1)!} t^{n-1}$$

Inverse Borel transform

$$\boxed{\mathcal{F}(\alpha_s) = \int_0^{\infty} dt\, e^{-t/\alpha_s} \tilde{F}(t)}$$

- **Renormalons** : In *dimensional regularization*, we encounter poles in the Borel plane on the positive real axis, and the *integral is ambiguous*.

BUBBLE-CHAIN RESUMMATION

- Renormalons appear in dimensional regularization because loop integrals contain contributions from arbitrarily soft gluon momenta where perturbation theory can break down.
- Factorization formulae are free of renormalon ambiguities because long-distance effects are separated from short-distance contributions.

E. Braaten and Y.-Q. Chen, PRD 57, 4236 (1998)
G. T. Bodwin and Y.-Q. Chen, PRD 60, 054008 (1999)

- We show the cancellation of renormalon ambiguities by explicit calculations of short-distance coefficients.