Quarkonia production at the LHC in NRQCD with k_T-factorization

Sergey Baranov
P.N.Lebedev Institute of Physics, Moscow

Artem Lipatov
D.V.Skobeltsyn Institute of Nuclear Physics, MSU, Moscow

P L A N O F T H E T A L K

0. Introduction: experimental observables
1. First acquaintance with k_t-factorization
2. Implementing the Quarkonium physics
3. Numerical results
4. Conclusions

EXPERIMENTAL OBSERVABLES

– **Differential cross sections for charm and bottom families**
– **Differential cross sections for ground states and excited states**
– **Direct to indirect production ratios (feed-down from χ_c and χ_b)**
– P**-wave production ratios** $\sigma(\chi_{c1})/\sigma(\chi_{c2})$, $\sigma(\chi_{b1})/\sigma(\chi_{b2})$
– **Polarization**

THEORETICAL APPROACHES

Several approaches are competing: Color-Singlet versus Color-Octet model; both may be extended to NLO or tree-level NNLO*; both may be incorporated with collinear or k_T-factorization. This talk is devoted to k_T-factorization.

– **Deep theory: a method to calculate high-order contributions (ladder-type diagrams enhanced with "large logarithms").**

– **Practice: making use of so called k_T-dependent parton densities. (Unusual properties: nonzero k_T and longitudinal polarization.)**

First acquaintance with k_T-factorization

PARTON OFF-SHELLNESS AND NONZERO k_T

QED

Weizsäcker-Williams approximation (collinear on-shell photons)

$$F_\gamma(x) = \frac{\alpha}{2\pi}\left[1 + (1 - x^2)\right] \log \frac{s}{4m^2}$$

Equivalent Photon approximation

$$F_\gamma(x, Q^2) = \frac{\alpha}{2\pi}\frac{1}{Q^2}\left[1 + (1 - x^2)\right]$$

$$Q^2 \approx k_t^2/(1 - x)$$

Photon spin density matrix

$$L^{\mu\nu} \approx p^\mu p^\nu$$

use $k = xp + k_t$, then do gauge shift

$$\epsilon \longrightarrow \epsilon - k/x$$

QCD

Conventional Parton Model (collinear gluon density)

$$x\, G(x, \mu^2)$$

Unintegrated gluon density

$$\mathcal{F}(x, k_t^2, \mu^2)$$

$$\int \mathcal{F}(x, k_t^2, \mu^2) dk_t^2 = x\, G(x, \mu^2)$$

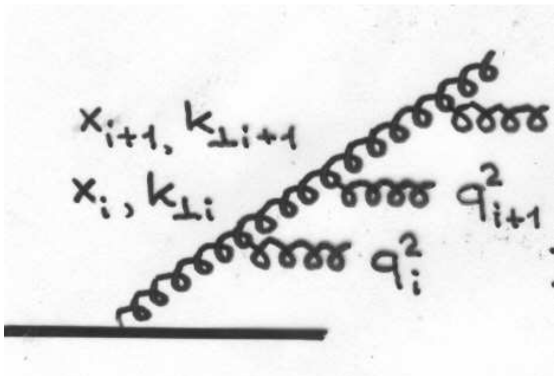
Gluon spin density matrix

$$\epsilon^\mu \epsilon^{\nu *} = k_t^\mu k_t^\nu / |k_T|^2$$

so called nonsense polarization with longitudinal components

Looks like Equivalent Photon Approximation extended to strong interactions.

The underlying theory: Initial State Radiation cascade



Every elementary emission gives $\alpha_s \cdot 1/x \cdot 1/q^2$
x = *longitudinal momentum fraction*
q^2 = *gluon virtuality*

Integration over the phase space yields
$\alpha_s \cdot \ln x \cdot \ln q^2$ *so called large logarithms, the reason to focus on this type of diagrams*

Random walk in the k_T-plane: $\ldots\ \langle k_{T\,i-1}\rangle \ll \langle k_{T\,i}\rangle \ll \langle k_{T\,i+1}\rangle\ \ldots$
$\ldots\ \langle x_{i-1}\rangle\ >\ \langle x_i\rangle\ >\ \langle x_{i+1}\rangle\ \ldots$

Technical method of summation: integro-differential QCD equations

BFKL **E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 45, 199 (1977); Ya. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978);**

or CCFM

S.Catani, F.Fiorani, G.Marchesini, Phys.Lett.B 234, 339 (1990); Nucl.Phys. B336, 18 (1990); G.Marchesini, Nucl.Phys. B445, 49 (1995); M.Ciafaloni, Nucl.Phys. B296, 49 (1998);

CCFM is more convenient for programming because of strict angular ordering $\ldots\theta_{i-1} < \theta_{i-1} < \theta_{i+1}\ldots \Rightarrow$ **A step-by-step solution.**

k_t-factorization is:

A method to collect contributions of the type $\alpha_s^n[\ln(1/x)]^n[\ln(Q^2)]^n$ up to infinetly high order. Sometimes it may be better than conventional calculations to a fixed order. (None of the methods is compete.)

The evolution cascade is part of the hard interaction process; it affects both the kinematics (initial k_T) and the polarization (off-shell spin density matrix). The corrections always have the same (ladder) structure, irrespective of the 'central' part of hard interaction, and can be conveniently absorbed into redefined parton densities
$\Rightarrow$ k_T-dependent = "unintegrated" distribution functions $\mathcal{F}(x, k_t^2, \mu^2)$

Advantages:

With the LO matrix elements for 'central' subprocess we get access to effects requiring complicated next-to-leading order calculations in the collinear scheme. Many important results have been obtained in the k_t-factorization approach much earlier than in the collinear case.

Includes effects of soft resummation and makes predictions applicable even to small p_t region.

Implementing the Quarkonium production

Color-Singlet mechanism

Perturbative production of a heavy quark pair within QCD; standard rules except gluon polarization vectors: $\epsilon_g^\mu = k_T^\mu/|k_T|$

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 45, 199 (1977);
Ya. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978);
L.V. Gribov, E.M. Levin, M. G. Ryskin, Phys. Rep. 100, 1 (1983).

Spin projection operators to guarantee the proper quantum numbers:

for Spin-triplet states $\mathcal{P}(^3S_1) = \not\epsilon_V(\not p_Q + m_Q)/(2m_Q)$

for Spin-singlet states $\mathcal{P}(^1S_0) = \gamma_5(\not p_Q + m_Q)/(2m_Q)$

Probability to form a bound state is determined by the wave function:
for S-wave states $|R_S(0)|^2$ is known from leptonic decay widths;
for P-wave states $|R'_P(0)|^2$ is taken from potential models.

E. J. Eichten, C. Quigg, Phys. Rev. D 52, 1726 (1995)

If $L \neq 0$ and $S \neq 0$ we use the Clebsch-Gordan coefficients to reexpress the $|L, S\rangle$ states in terms of $|J, J_z\rangle$ states, namely, the χ_0, χ_1, χ_2 mesons.

What was wrong with Color-Singlet Model?

– Wrong p_t dependence of the cross sections $\Rightarrow$ an indication that something important is missing. Partly corrected by k_T-factorization.
– Quarkonium formation is assumed to be completed already at the perturbative stage; but what if the $Q\bar{Q}$ pair is produced in the color octet state? Treating soft gluons in a perturbative manner is wrong: soft gluons cannot resolve the $Q\bar{Q}$ pair into quarks. Need another language to describe the emission of gluons by the entire $Q\bar{Q}$ system, not by individual quarks. Come to NRQCD.

What was wrong with Color-Octet Model (NRQCD)?

– Assumes that soft gluons can change the color and other quantum numbers of a $Q\bar{Q}$ system without changing the energy-momentum. An obvious conflict with confinement that prohibits radiation of infinitely soft colored quanta. Need to consider not infinitely small energy-momentum exchange. Not only a kinematric correction!
– Long-distance matrix elements (LDMEs) for ${}^3S_1^{[8]} \to J/\psi$ transitions are treated as spin-blind numbers. Need to replace them with amplitudes showing well defined spin structure.

Modified Color-Octet mechanism

– Step 1: use perturbative QCD to create a heavy quark pair $Q\bar{Q}$ in the hard gluon-gluon fusion subprocess.
– Step 2: use multipole expansion for soft gluon radiation. Another perturbation theory where the small parameter is the relative quark velocity (or the size of the $Q\bar{Q}$ system over the gluon wavelength)

Both steps are combined into a single amplitude: $Q\bar{Q}$ spin density matrix is contracted with E1 transition amplitudes (same as for real χ_c decays).

Color-Electric Dipole transitions

$$\mathcal{A}(\chi_{c0}(p) \to J/\psi(p-k) + \gamma(k)) \propto k_\mu p^\mu \varepsilon^\nu_{(J/\psi)} \varepsilon^{(\gamma)}_\nu$$

$$\mathcal{A}(\chi_{c1}(p) \to J/\psi(p-k) + \gamma(k)) \propto \epsilon^{\mu\nu\alpha\beta} k_\mu \varepsilon^{(\chi_{c1})}_\nu \varepsilon^{(J/\psi)}_\alpha \varepsilon^{(\gamma)}_\beta$$

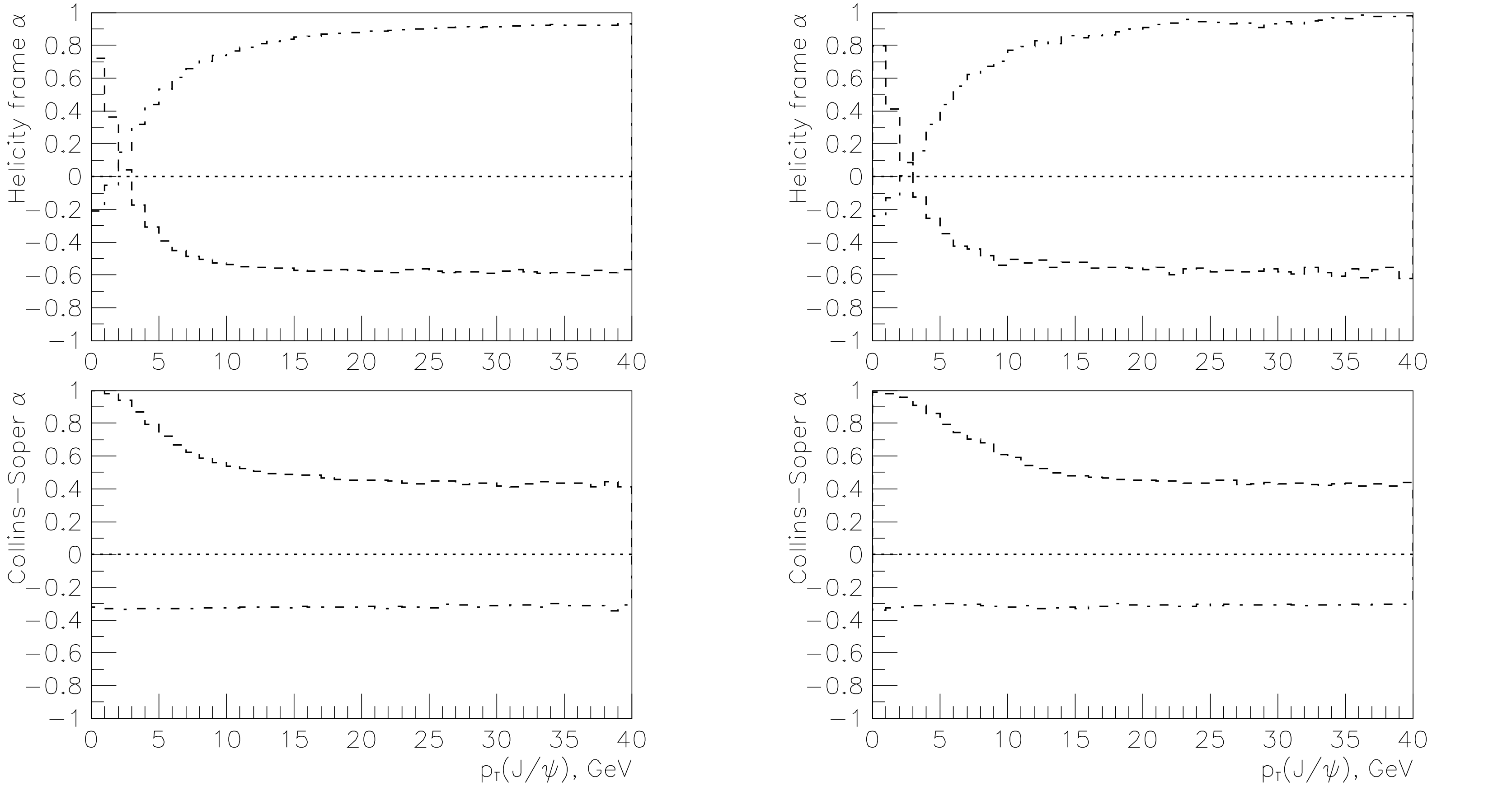
$$\mathcal{A}(\chi_{c2}(p) \to J/\psi(p-k) + \gamma(k)) \propto p^\mu \varepsilon^{\alpha\beta}_{(\chi_{c2})} \varepsilon^{(J/\psi)}_\alpha [k_\mu \varepsilon^{(\gamma)}_\beta - k_\beta \varepsilon^{(\gamma)}_\mu]$$

A.V.Batunin, S.R.Slabospitsky, Phys.Lett.B 188, 269 (1987)
P.Cho, M.Wise, S.Trivedi, Phys. Rev. D 51, R2039 (1995)

One or two subsequent transitions to convert a color octet into J/ψ:
$${}^3P_J^{[8]} \to J/\psi + g \quad \text{or} \quad {}^3S_1^{[8]} \to {}^3P_J^{[8]} + g,\ {}^3P_J^{[8]} \to J/\psi + g,\ J = 0, 1, 2.$$

J/ψ from $^3P_J^{[8]}$ polarization

k_T-factorization | **LO collinear factorization**

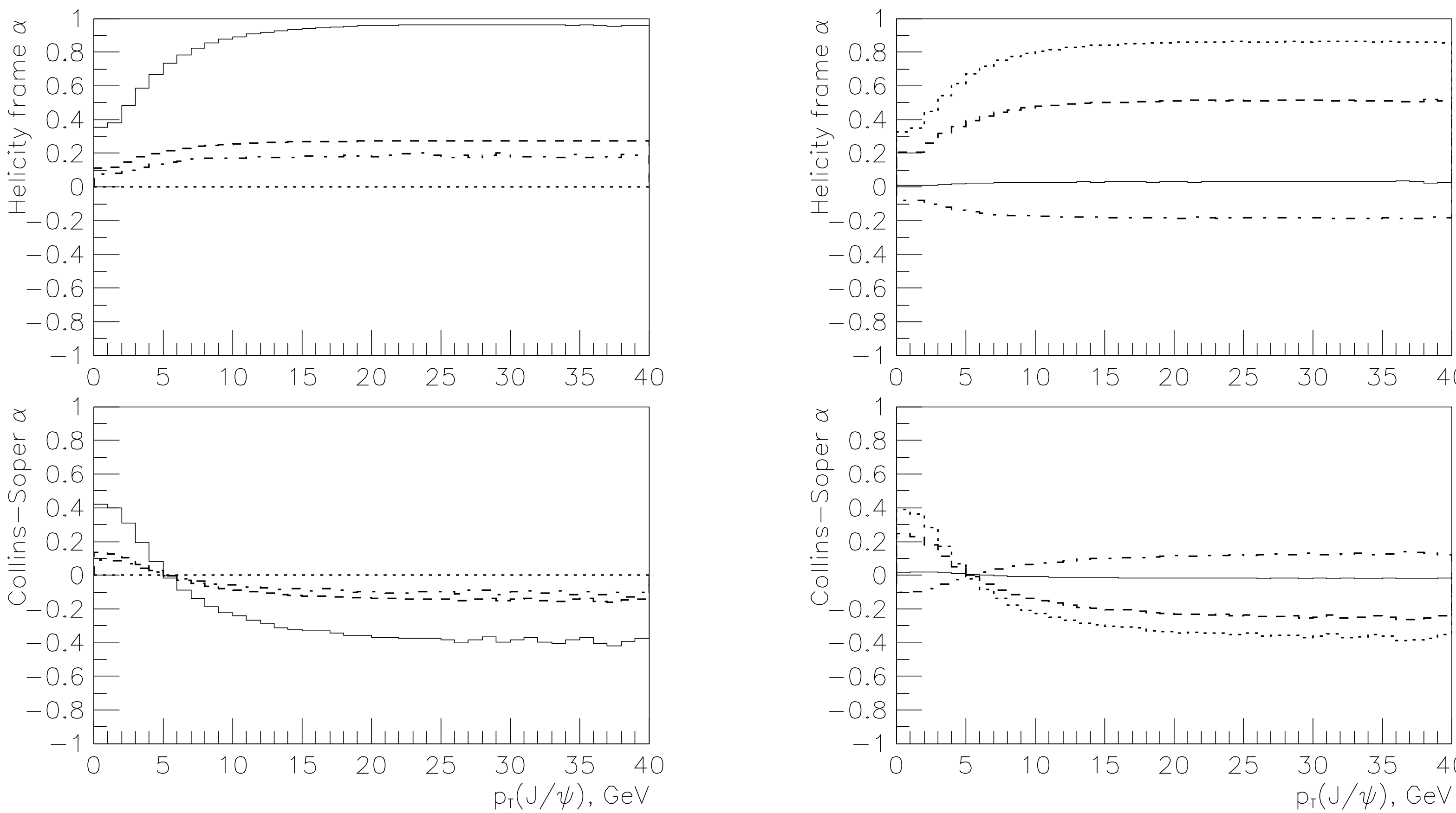


Dashed = $^3P_2$; dash-doted = $^3P_1$; doted = $^3P_0$

Approximate cancellation between $^3P_1^{[8]}$ and $^3P_2^{[8]}$ channels.

J/ψ from ${}^3S_1^{[8]}$ polarization

Pure ${}^3P_J^{[8]}$ states | **Interfering channels**

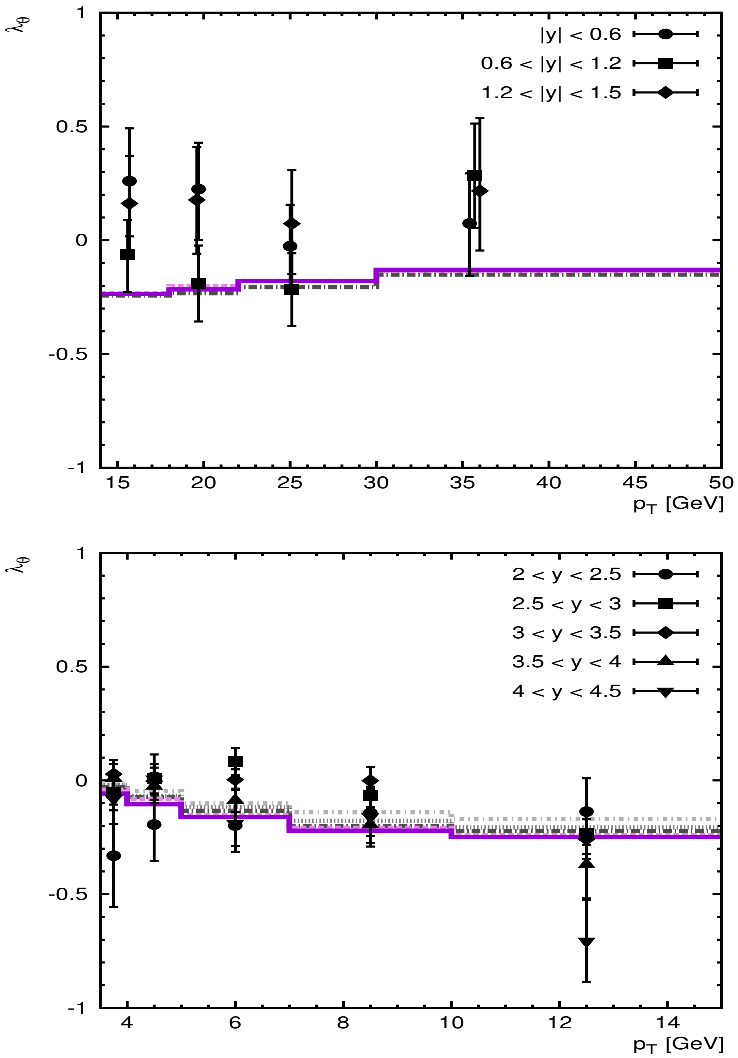


Dash=${}^3P_2$; dash-dot=${}^3P_1$; dot=${}^3P_0$;
Solid = ${}^3S_1^{[8]}$ spin preserved.

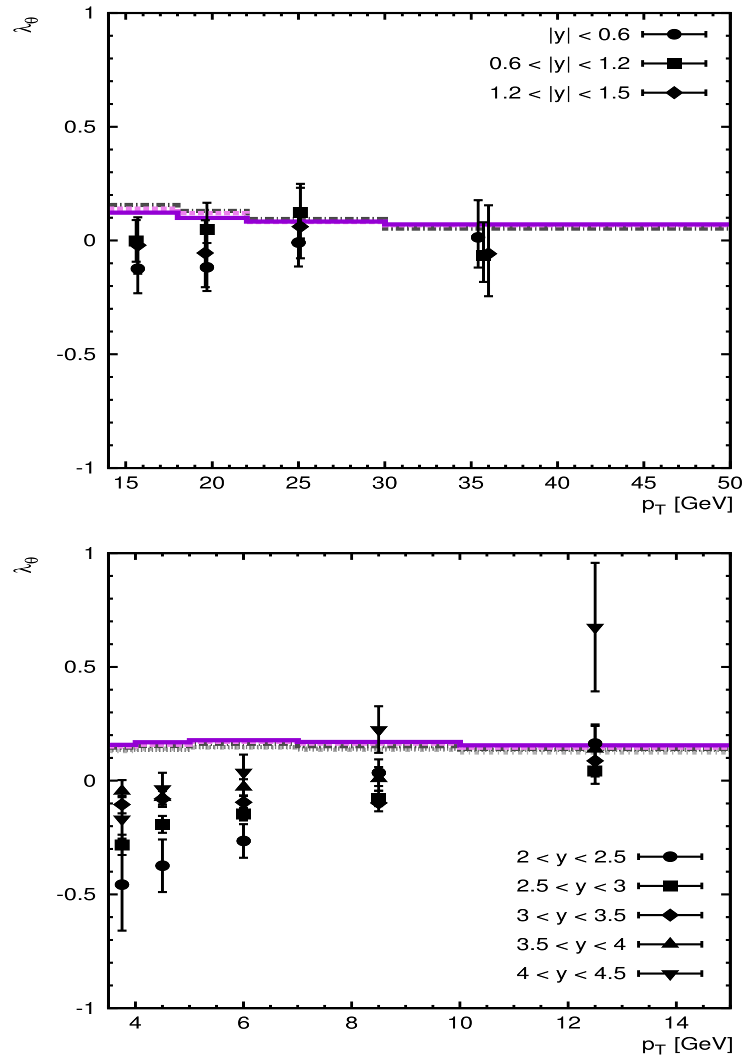
Solid=$\left|{}^3P_2^{[8]} + {}^3P_2^{[8]} + {}^3P_2^{[8]}\right|^2$
normalized to $\sqrt{2J+1}$

Model versus CMS and LHCb data

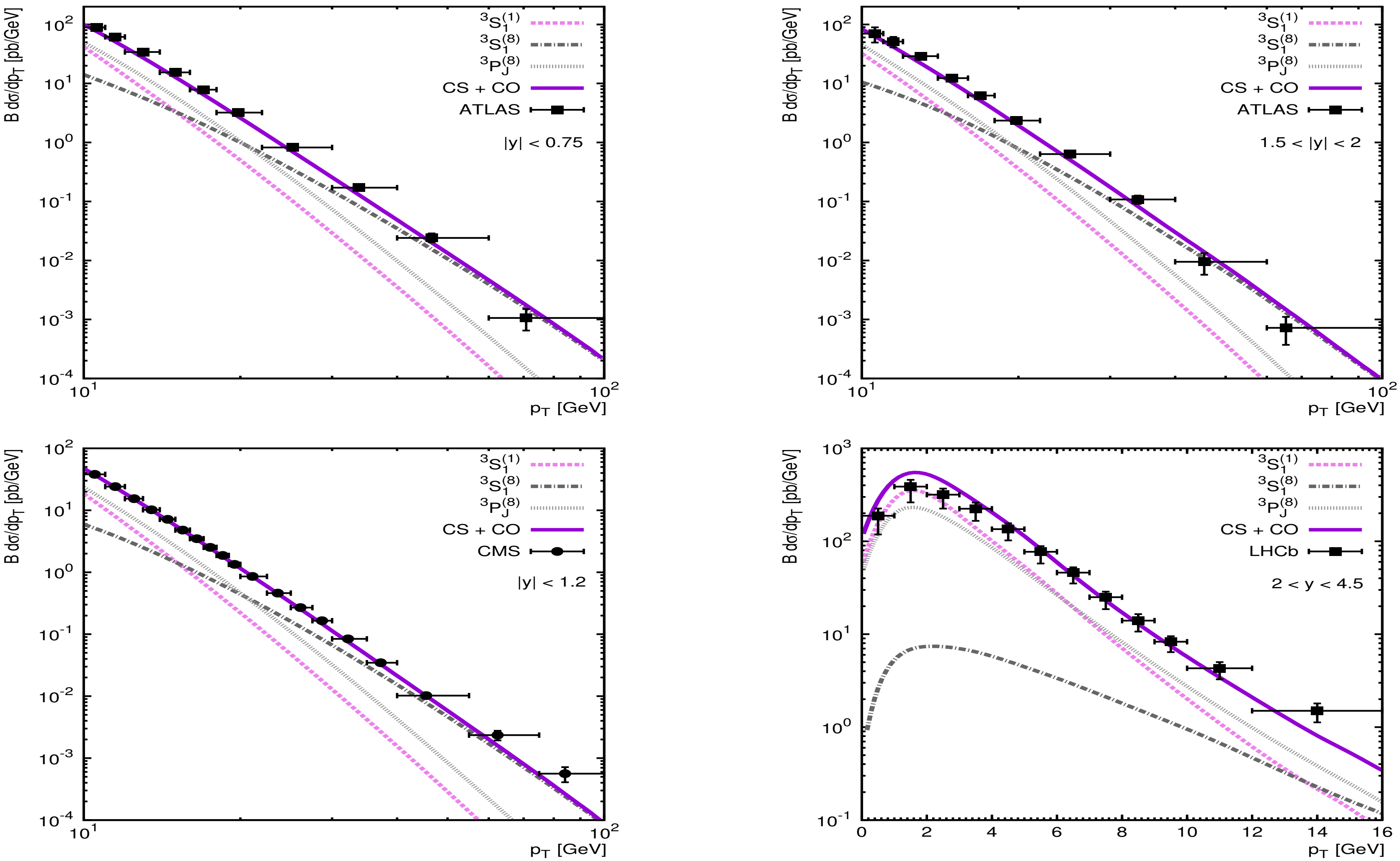
Helicity frame $\psi(2S)$



Collins-Soper frame $\psi(2S)$

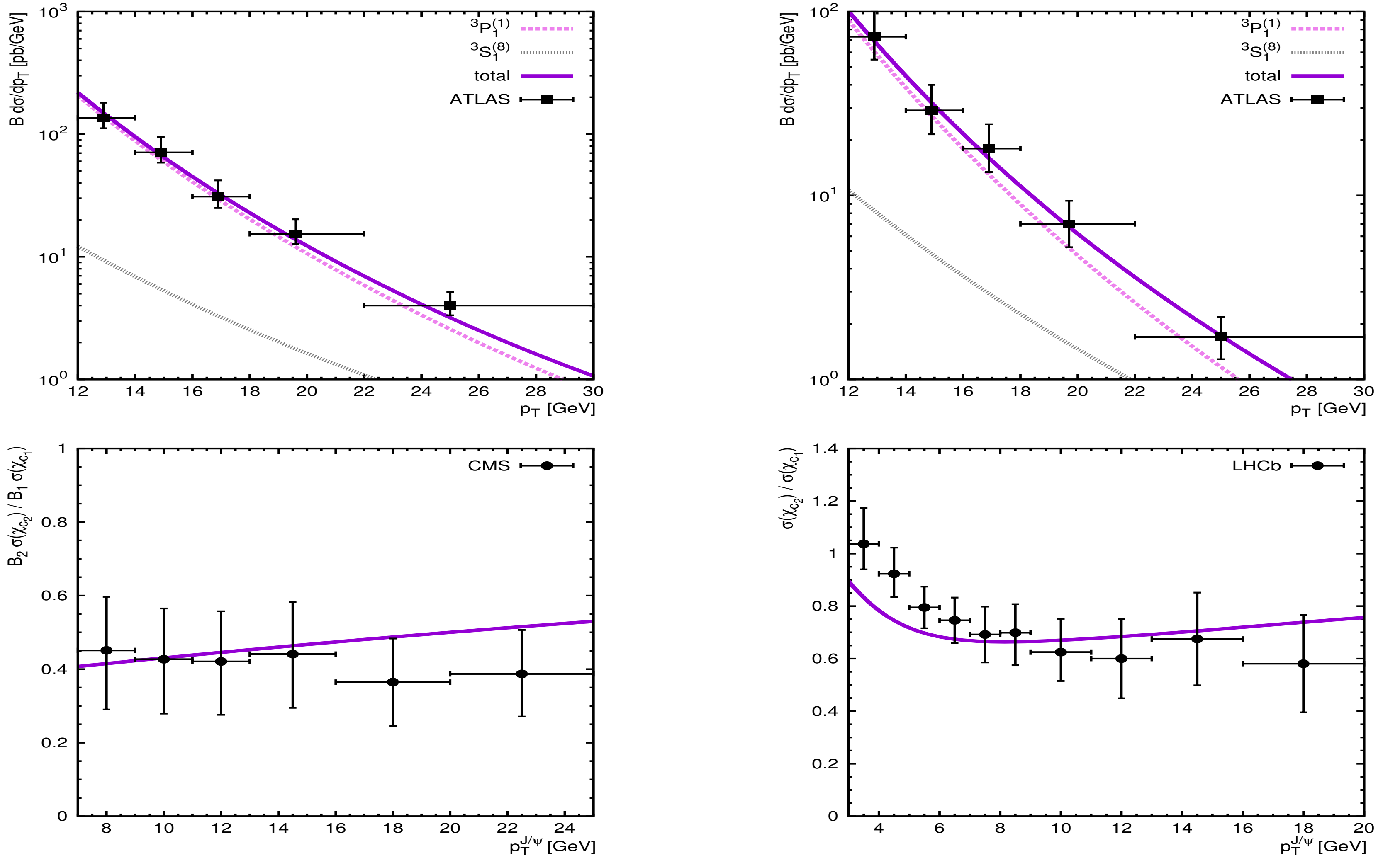


$\psi(2S)$ production at ATLAS, CMS, LHCb



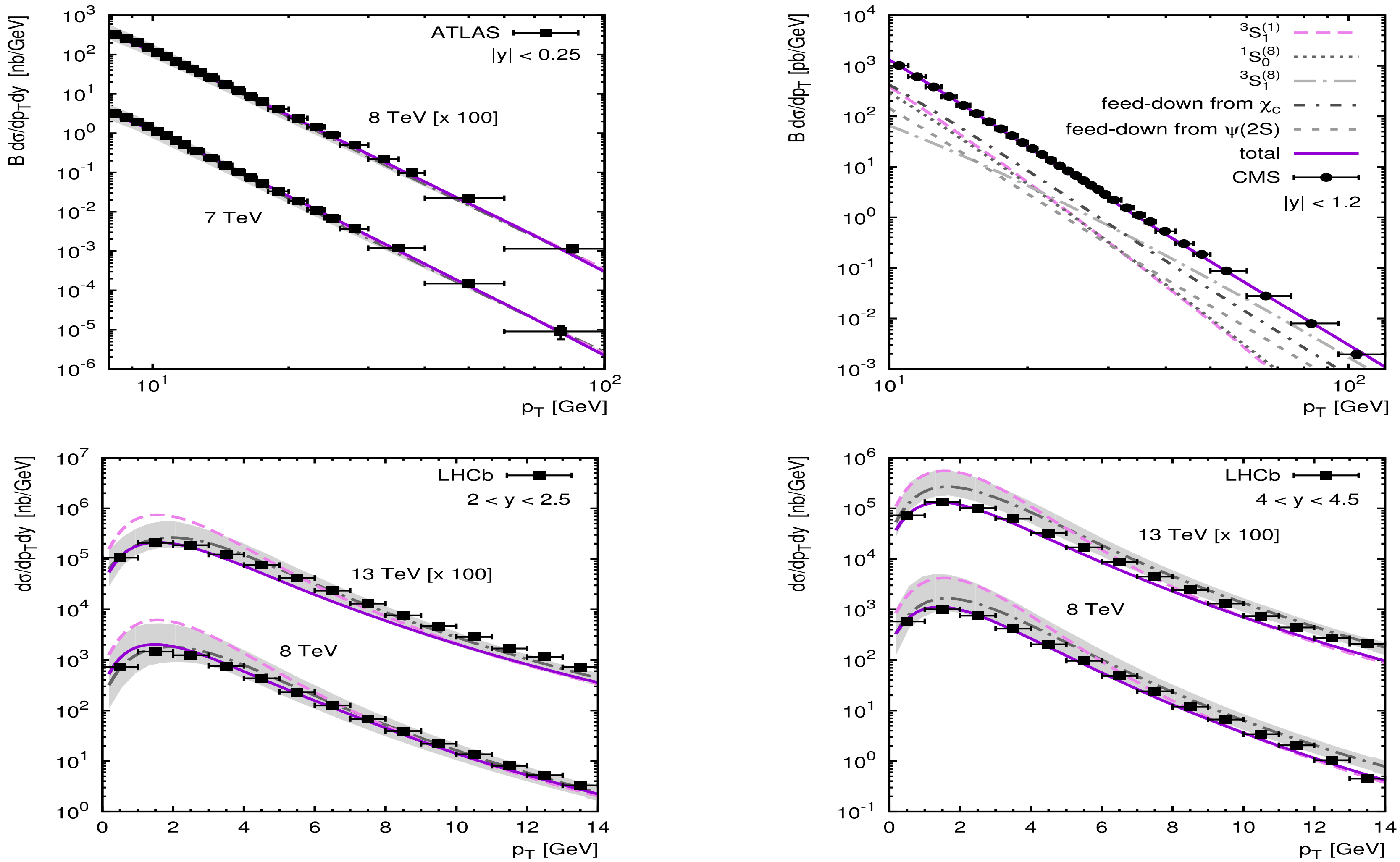
Artem Lipatov *et al.*, Eur. Phys. J. C 75, 455 (2015)

χ_{c1} and χ_{c2} production at ATLAS, CMS, LHCb



Artem Lipatov *et al.*, **Phys. Rev. D 93, 094012 (2016)**

J/ψ production at ATLAS, CMS, LHCb



Artem Lipatov *et al.*, Phys. Rev. D 96, 034019 (2017)

Fitted $\psi(2S)$ LDME values

	$\langle \mathcal{O}\left[{}^3S_1^{(1)}\right]\rangle/\mathrm{GeV}^3$	$\langle \mathcal{O}\left[{}^1S_0^{(8)}\right]\rangle/\mathrm{GeV}^3$	$\langle \mathcal{O}\left[{}^3S_1^{(8)}\right]\rangle/\mathrm{GeV}^3$	$\langle \mathcal{O}\left[{}^3P_0^{(8)}\right]\rangle/\mathrm{GeV}^5$
A0	7.04×10^{-1}	0.0	5.64×10^{-4}	3.71×10^{-3}
JH	7.04×10^{-1}	0.0	3.19×10^{-4}	7.14×10^{-3}
KMR	7.04×10^{-1}	8.14×10^{-3}	2.58×10^{-4}	1.19×10^{-3}
[1]	6.50×10^{-1}	7.01×10^{-3}	1.88×10^{-3}	-2.08×10^{-3}
[2]	5.29×10^{-1}	-1.20×10^{-4}	3.40×10^{-3}	9.45×10^{-3}

[1] M.Butenschön, B.A.Kniehl, Phys. Rev. Lett. 106, 022003 (2011)
[2] B.Gong, L.-P.Wan, J.-X.Wang, H.-F.Zhang, Phys. Rev. Lett. 110, 042002 (2013)

Color-singlet wave function was taken as a free parameter;
the fitted value is consistent with $\Gamma(\psi(2S) \to \mu^+\mu^-)$ *width.*

Color-octet matrix elements are typically much smaller than in collinear fits.

Fitted χ_{c1} and χ_{c2} LDME values

	$\lvert\mathcal{R}'^{(1)}_{\chi_{c1}}(0)\rvert^2/\mathrm{GeV}^5$	$\lvert\mathcal{R}'^{(1)}_{\chi_{c2}}(0)\rvert^2/\mathrm{GeV}^5$	$\langle\mathcal{O}^{\chi}\left[{}^3S_1^{(8)}\right]\rangle/\mathrm{GeV}^3$
A0	3.85×10^{-1}	6.18×10^{-2}	8.28×10^{-5}
JH	5.23×10^{-1}	9.05×10^{-2}	4.78×10^{-5}
KMR	3.07×10^{-1}	6.16×10^{-2}	1.40×10^{-4}
[3]	7.50×10^{-2}	7.50×10^{-2}	2.01×10^{-3}
[4]	3.50×10^{-1}	3.50×10^{-1}	4.40×10^{-4}

[3] H.-F.Zhang, L.Yu, S.-X.Zhang, L.Jia, Phys. Rev. D 93, 054033 (2016)
[4] A.K.Likhoded, A.V.Luchinsky, S.V.Poslavsky, Phys. Rev. D 90, 074021 (2014)

Color-singlet χ_{c1} *and* χ_{c2} *wave functions were taken as independent free parameters; the fitted value is consistent with* $\Gamma(\chi_{c2}\to\gamma\gamma)$ *width.*

Color-octet matrix elements are typically much smaller than in collinear fits.

Fitted J/ψ LDME values

	$\langle\mathcal{O}\left[{}^3S_1^{(1)}\right]\rangle/\mathrm{GeV}^3$	$\langle\mathcal{O}\left[{}^1S_0^{(8)}\right]\rangle/\mathrm{GeV}^3$	$\langle\mathcal{O}\left[{}^3S_1^{(8)}\right]\rangle/\mathrm{GeV}^3$	$\langle\mathcal{O}\left[{}^3P_0^{(8)}\right]\rangle/\mathrm{GeV}^5$
A0	1.97	0.0	9.01×10^{-4}	0.0
JH	1.62	1.71×10^{-2}	2.83×10^{-4}	0.0
KMR	1.58	8.35×10^{-3}	2.32×10^{-4}	0.0
[1]	1.32	3.04×10^{-2}	1.68×10^{-3}	-9.08×10^{-3}
[2]	1.16	9.7×10^{-2}	-4.6×10^{-3}	-2.14×10^{-2}

[1] M.Butenschön, B.A.Kniehl, Phys. Rev. Lett. 106, 022003 (2011)
[2] B.Gong, L.-P.Wan, J.-X.Wang, H.-F.Zhang, Phys. Rev. Lett. 110, 042002 (2013)

Color-singlet wave function was taken as a free parameter;
the fitted value is consistent with $\Gamma(J/\psi \to \mu^+\mu^-)$ *width.*

Color-octet matrix elements are typically much smaller than in collinear fits.

CONCLUSIONS

Two important innovations:

- **k_T-factorization to describe the initial state**
  Includes initial state radiation and soft gluon resummation; makes the required CO LDMEs smaller than in the collinear case

- **Multipole radiation formalism to describe the final state**
  Explicit spin-dependent expresions for transition amplitudes; probably solves the quarkonium polarization problem.

After all, a reasonably good agreement is achieved with the data:

J/ψ, χ_{c1}, χ_{c2}, $\psi(2S)$ $\boldsymbol{p_t}$ **spectra;** J/ψ, $\psi(2S)$ **polarization;**
ATLAS, CMS, LHCb data in the whole kinamatic range.

Thank You!