NLO Effects for Doubly Heavy Baryon in QCD Sum Rules

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Based on *Wang, Meng, Ma, and Chao, arXiv:1708.04563* and works in preperation.

Outline

QCD Sum Rules

Introduction

- Developed by Shifman, Vainshtein and Zakharov, to study various properties of mesons. *Shifman, Vainshtein, and Zakharov 1979a,b*
- Extended to baryon in 1981. *Ioffe 1981*
- Applications:
  - masses
  - decay constants
  - form factors
  - hadronic structure functions and matrix elements
  - ...

QCD Sum Rules

Correlation Function

- The central object in QCD sum rule is

$$\Pi(q^2) = i\int d^4x\, e^{iqx}\langle\Omega|T\{\eta(x)\bar{\eta}(0)\}|\Omega\rangle = \Pi_1(q^2)\not{q} + \Pi_2(q^2)\,. \quad (1)$$

- $\Pi(q^2)$ can be calculated in two different ways:
    - Källén-Lehmann spectral decomposition ($q^2 > 0$)

$$\Pi(q^2) = \frac{1}{\pi}\int_0^\infty ds\, \frac{\Im\Pi(s+i\epsilon)}{s-q^2} = \int_0^\infty ds\, \frac{\rho(s+i\epsilon)}{s-q^2}\,, \quad (2)$$

    - Operator product expansion ($q^2 \to -\infty$)

$$\Pi(q^2) = C_1(q^2) + \sum_i C_i(q^2)\langle O_i\rangle\,. \quad (3)$$

OPE is controlled by the dimension of operators O_i, higher dimension operators contribute less. Vacuum condensates $\langle O_i\rangle$ are universal.

- Cauchy integral formula relates these two results and generates an equation.

QCD Sum Rules

Sum Rule

- For a typical spectral density

$$\rho(q^2) = \lambda_H^2(\not{q} + m_H)\delta(q^2 - m_H^2) + \rho_c(q^2)\theta(q^2 - s_{th}) \,, \quad (4)$$

- QCD sum rule corresponds to $\Pi_1(q^2)$ is

$$\lambda_H^2 e^{-\frac{m_H^2}{m_B^2}} = \int_{s_{th}}^{s_0} ds\, \rho_1(s) e^{-\frac{s}{m_B^2}} + \sum_i \langle O_i \rangle \int_{s_{th}}^{\infty} ds\, \rho_i(s) e^{-\frac{s}{m_B^2}} \,, \quad (5)$$

  where s_0 is the threshold parameter, m_B is the Borel parameter, and $\rho_k = \mathrm{Tr}(\mathfrak{I} C_k \not{q})/(\pi q^2)$ for $k = 1, i$.
- Each Wilson coefficient (i.e. ρ_1, ρ_i) can be calculated to desired order of α_s.
- Contribution from higher dimension operators is suppressed by their dimension $\sim |\Lambda_{QCD}/m_B|^{d-2}$.
- Physical m_H and λ_H should not depend on m_B and s_0.

Motivation

Doubly Heavy Baryon

- Many works have devoted to the study of doubly heavy baryons within QCD sum rules. *Aliev, Azizi, and Savci 2012; Bagan, Chabab, and Narison 1993; H.-X. Chen, Mao, et al. 2017; Kiselev and Onishchenko 2000; Tang et al. 2012; Z.-G. Wang 2010; Zhang and M.-Q. Huang 2008*
- No loop correction is avaliable.
- Large theoretical uncertainty at LO, such as scale choice and quark mass choice.
- For baryon with 0 or 1 massive quark, loop correction has significant contribution to ρ_1. *Groote, Korner, and Pivovarov 2008; Ovchinnikov, Pivovarov, and Surguladze 1991*

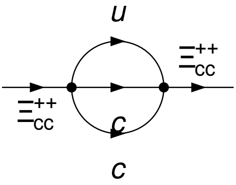


Figure: Feynman diagrams of ρ_1 for Ξ_{cc}^{++}.

Motivation

Importance of Loop Correction

- No NLO correction to various hadron currents of phenomenology interests:
  - LHCb collaboration observed $\Xi_{cc}^{++}(ccu)$ with mass $3621 \pm 0.72 \pm 0.27 \pm 0.14$ MeV. *Aaij et al. 2017*
  - BESIII and Belle collaboration observed various $Z_c(\bar{c}c\ldots)$ states. *Ablikim et al. 2013; Z. Q. Liu et al. 2013*
  - LHCb collaboration observed two $P_c^+(\bar{c}cuud)$ with mass $4380 \pm 8 \pm 29$ MeV and $4449.8 \pm 1.7 \pm 2.5$ MeV. *Aaij et al. 2015*
- In some cases, loop correction has significant contribution to ρ_1, larger than higher dimension operators in OPE. *Groote, Korner, and Pivovarov 2008; Ovchinnikov, Pivovarov, and Surguladze 1991*
- NLO renormalization scheme fixes heavy quark mass scheme, reduces uncertainties from quark mass parameter.
- Stability of renormalization scale μ provides us an additional criterion for choosing parameters.

Loop Calculation

Baryon Current

- For a $J^P = \frac{1}{2}^+$ baryon, there are only two possible currents

$$\eta_1 = \epsilon^{abc}\left(Q^a C\gamma_\mu Q^b\right)\gamma^\mu\gamma^5 q^c\,, \tag{6}$$

$$\eta_2 = \epsilon^{abc}\left(Q^a C\sigma_{\mu\nu} Q^b\right)\sigma^{\mu\nu} i\gamma^5 q^c\,. \tag{7}$$

In general, the operator can be a linear combination of them

$$\eta = \eta_1 + \theta\eta_2\,. \tag{8}$$

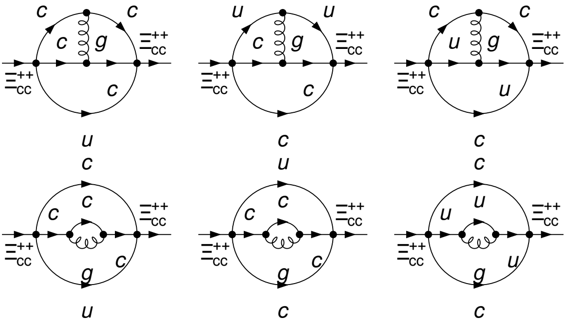


Figure: NLO Feynman diagrams for Ξ_{cc}^{++}.

Loop Calculation

Integration By Parts

- We first apply integration by parts (IBP) reduction, expressing our amplitude as combination of a set of master integrals I_k

$$C_1^{\mathrm{NLO}}(\varepsilon, q^2, m_Q) = \sum_k c_k(\varepsilon, q^2, m_Q) I_k(\varepsilon, \nu)\,, \qquad (9)$$

  where $\nu = \sqrt{1 - \frac{4m_Q^2}{q^2}}$.
- Direct evaluation of master integrals I_k:
  - Generalized hypergeometric function ${}_pF_q$.
  - Not systematic.

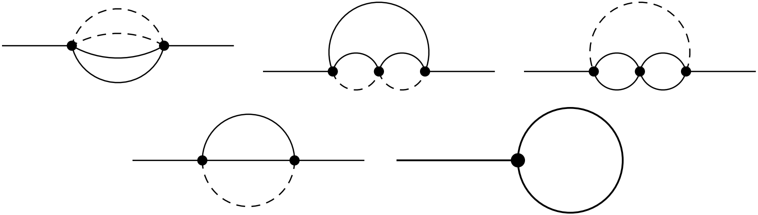


Figure: Topologies of master integrals.

Loop Calculation

Differential Equation

- To obtain $I_k(\varepsilon, \nu)$, we set up a system of differential equations with respect to ν
$$\frac{\partial}{\partial \nu}\vec{I} = A(\varepsilon, \nu)\vec{I}\,. \tag{10}$$
- We transform our equation into so-called ε-form *Henn 2013; Lee 2015*
$$\frac{\partial}{\partial \nu}\vec{I}' = \varepsilon A'(\nu)\vec{I}'\,. \tag{11}$$
The boundary value at $\nu = 1$ (i.e. $m_Q = 0$) is simple to calculate.
- We eventually get analytical results at NLO level. Our NLO result of ρ_1 confirms the massless result in the limit of $m_Q \to 0$. *Jamin 1988; Ovchinnikov, Pivovarov, and Surguladze 1991*

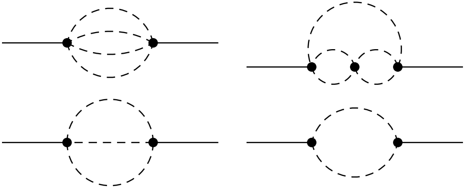


Figure: Topologies of master integrals at boundary $\nu = 1$ (i.e. $m_Q = 0$).

Doubly Heavy Baryon

Hadron Mass & Window

- Hadron mass can be obtained from our sum rule Eq. (5)

$$m_H^2 = \frac{\int_{s_{th}}^{s_0} ds\, \rho_1(s) s e^{-\frac{s}{m_B^2}} + \sum_i \langle O_i \rangle \int_{s_{th}}^{\infty} ds\, \rho_i(s) s e^{-\frac{s}{m_B^2}}}{\int_{s_{th}}^{s_0} ds\, \rho_1(s) e^{-\frac{s}{m_B^2}} + \sum_i \langle O_i \rangle \int_{s_{th}}^{\infty} ds\, \rho_i(s) e^{-\frac{s}{m_B^2}}} \,. \qquad (12)$$

- To obtain a reliable result, we need to ensure that contributions from OPE and continuum spectrum are small.
- Define relative contributions r_i and $r_{\text{cont.}}$

$$r_i = \frac{\langle O_i \rangle \int_{s_{th}}^{\infty} ds\, \rho_i(s) e^{-\frac{s}{m_B^2}}}{\int_{s_{th}}^{\infty} ds\, \rho_1(s) e^{-\frac{s}{m_B^2}}} \,, \quad r_{\text{cont.}} = \frac{\int_{s_0}^{\infty} ds\, \rho_1(s) e^{-\frac{s}{m_B^2}}}{\int_{s_{th}}^{\infty} ds\, \rho_1(s) e^{-\frac{s}{m_B^2}}} \,, \qquad (13)$$

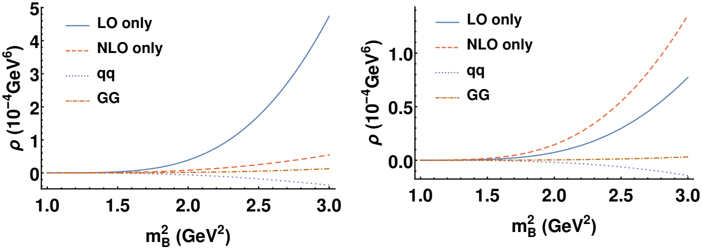
and impose the following constraints

$$|r_i| \leqslant 30\% \,, \quad \left| \sum_i r_i \right| \leqslant 30\% \,, \quad |r_{\text{cont.}}| \leqslant 30\% \,. \qquad (14)$$

Doubly Heavy Baryon

Parameters & Relative Contributions

- We choose $\theta = 0.018i$, which ensures that m_H depends weakly on m_B^2 and s_0.
- We adopt standard QCD parameters. *Bagan, Chabab, and Narison 1993; Dominguez, Gluckman, and Paver 1994; Dominguez, Hernandez, and Schilcher 2015; Patrignani et al. 2016*
  - $m_c^{\overline{\text{MS}}}(m_c) = 1.28 \pm 0.03$ GeV and $m_c^{\text{on-shell}} = 1.46 \pm 0.07$ GeV.
  - Condensates are $\langle \bar{q} q \rangle (2\ \text{GeV}) = -(0.287 \pm 0.019\ \text{GeV})^3$ and $\langle g_s^2 G G \rangle = 4\pi^2 (0.037 \pm 0.015)\ \text{GeV}^4$.



(a) $m_Q^{\overline{\text{MS}}}$ scheme

(b) $m_Q^{\text{on-shell}}$ scheme

Figure: Contributions of various terms on the right hand side of the sum rule.

Doubly Heavy Baryon

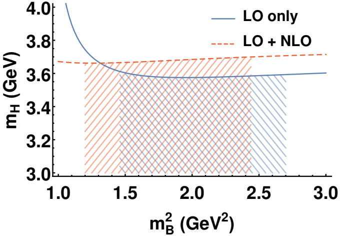


Figure: Prediction of $m_{\Xi_{cc}^{++}}$ as a function of m_B^2 and s_0 in $m_Q^{\overline{MS}}$ scheme.

- The dependence of m_B^2 and s_0 is weaker when NLO correction is included.

Order	m_B^2 (GeV^2)	s_0 (GeV^2)	$m_{\Xi_{cc}^{++}}$ (GeV)
LO	2.0 ± 0.3	17 ± 2	$3.58^{+0.09}_{-0.11}$
NLO	1.7 ± 0.3	17 ± 2	$3.67^{+0.09}_{-0.10}$

Table: Parameters of plateau and estimations for $m_{\Xi_{cc}^{++}}$ in $m_Q^{\overline{MS}}$ scheme.

Doubly Heavy Baryon

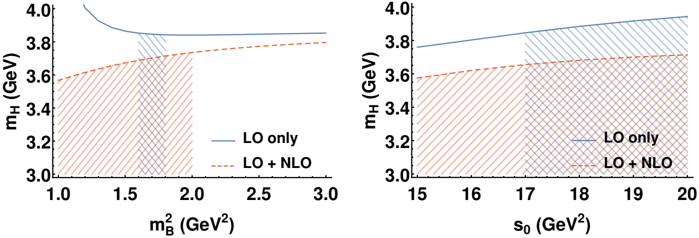


Figure: Prediction of $m_{\Xi_{cc}^{++}}$ as a function of m_B^2 and s_0 in $m_Q^{\text{on-shell}}$ scheme.

- The window of LO + NLO is much wider compared with LO only.
- Consistent with $m_Q^{\overline{\text{MS}}}$ scheme only in LO + NLO case.

Order	m_B^2 (GeV^2)	s_0 (GeV^2)	$m_{\Xi_{cc}^{++}}$ (GeV)
LO	1.7 ± 0.3	17 ± 2	$3.85^{+0.16}_{-0.14}$
NLO	1.4 ± 0.3	17 ± 2	$3.66^{+0.12}_{-0.14}$

Table: Parameters of plateau and estimations for $m_{\Xi_{cc}^{++}}$ in $m_Q^{\text{on-shell}}$ scheme.

Doubly Heavy Baryon

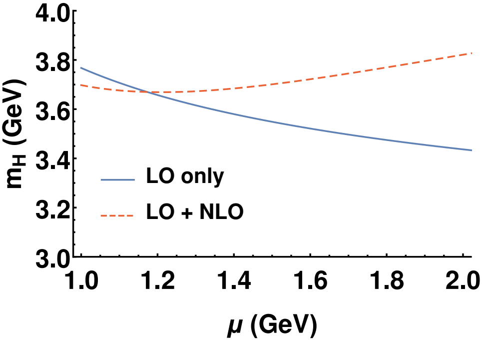


Figure: Estimation of $m_{\Xi_{cc}^{++}}$ as a function of μ.

- Renormalization scale μ dependence is reduced in $m_Q^{\overline{\text{MS}}}$ scheme.
- In $m_Q^{\text{on-shell}}$ scheme, LO is independent of μ, thus no meaningful comparison can be made.

Doubly Heavy Baryon

Summary

m_Q scheme	Order	m_B^2 (GeV^2)	s_0 (GeV^2)	$m_{\Xi_{cc}^{++}}$ (GeV)
$\overline{\text{MS}}$	LO	2.0 ± 0.3	17 ± 2	$3.58^{+0.09}_{-0.11}$
	NLO	1.7 ± 0.3	17 ± 2	$3.67^{+0.09}_{-0.10}$
on-shell	LO	1.7 ± 0.3	17 ± 2	$3.85^{+0.16}_{-0.14}$
	NLO	1.4 ± 0.3	17 ± 2	$3.66^{+0.12}_{-0.14}$

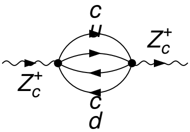
Table: Parameters of plateau and estimations for $m_{\Xi_{cc}^{++}}$ in different mass renormalization schemes.

- The difference between two mass schemes is reduced when NLO correction is added.
- Renormalization scale μ dependence is reduced in $m_Q^{\overline{\text{MS}}}$ scheme.

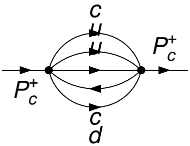
Outlook

Tetraquark & Pentaquark

- Numerous QCD sum rules analyses of singly and doubly heavy tetraquark and pentaquark have been carried out. *Agaev, Azizi, and Sundu 2017a,b; Azizi, Sarac, and Sundu 2017a,b; H.-X. Chen, Cui, et al. 2016; W. Chen, H.-X. Chen, et al. 2017; W. Chen, Steele, and Zhu 2014; Du et al. 2013; Z.-W. Huang and J. Liu 2012; Z.-G. Wang 2017; Z.-G. Wang, Xu, and H.-J. Wang 2011; Z.-G. Wang and Yan 2017*
- Perturbative correction to ρ_1
  - Tetraquark with massless quarks: $\mathcal{O}(\alpha_s)$ *Groote, Körner, and Niinepuu 2014*
  - Pentaquark with massless quarks: $\mathcal{O}(\alpha_s)$ *Groote, Korner, and Pivovarov 2012*
- Full NLO correction to ρ_1 is absent.



(a) Z_c^+



(b) P_c^+

Figure: Feynman diagrams of ρ_1 for typical exotic states.

Outlook

Tetraquark & Pentaquark

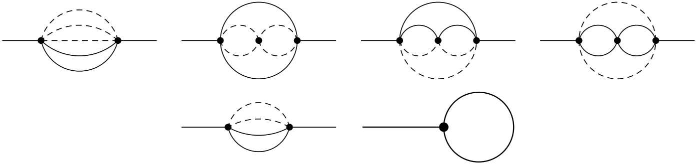


Figure: Topologies of master integrals in tetraquark case.

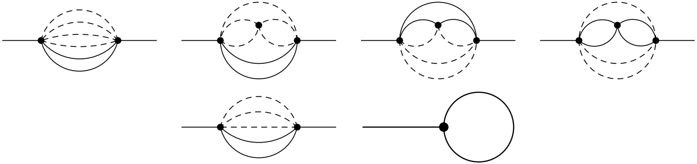


Figure: Topologies of master integrals in pentaquark case.

- Analytical results of master integrals have been obtained.
- Phenomenology analysis is under way.

Summary

- The QCD sum rules estimation of $m_{\Xi_{cc}^{++}}$ is $3.67^{+0.09}_{-0.10}$ GeV, which is consistent with the LHCb measurement within uncertainties.
- NLO perturbative correction substantially reduces m_Q renormalization scheme dependence and renormalization scale μ dependence.
- Multi-loop calculation techniques improve the predictive power of QCD sum rule in studying new hadronic states.
- Thank you!

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