Soft gluon factorization for heavy quarkonium production

Yan-Qing Ma (马滟青)

Peking University

Based on: YQM, Chao, 1703.08402

The 12th International Workshop on Heavy Quarkonium PKU, Nov. 8th, 2017 **NRQCD** Factorization

See R. Vogt's talk for ICEM

> Factorization formula

PKU, Nov. 8th, 2017

Bodwin, Braaten, Lepage, 9407339

 $(2\pi)^{3}2P_{H}^{0}\frac{d\sigma_{H}}{d^{3}P_{H}} = \sum_{n} d\hat{\sigma}_{n}(P_{H})\langle \mathcal{O}_{n}^{H}\rangle$ Production of a heavy quark pair
Hadronization (LDMEs)

• n: quantum numbers of the pair: color, spin, orbital angular

momentum, total angular momentum, spectroscopic notation ${}^{2S+1}L_{I}^{[c]}$

Achievement: explain ψ' surplus

\succ Nicely explain ψ' surplus by CO contributions

States		p _⊤ behavior at LO
³ S ₁ ^[1]		р _т -8
³ S ₁ ^[8]		p _⊤ -4
¹ S ₀ ^[8]		p _T ⁻ ⁶
³ P _J ^[8]		p _T ⁻ ⁶
10 ⁴ 10 ³ 10 ² (лэр/qц) ¹ цр/ор 10 ⁻¹	$\sqrt{S} = 7 \text{ TeV}$ $0 < V w < 0.75$	Prompt J/// ATLAS Data
10 ⁻³	10 20 30 <i>p_T</i>	40 50 60 70 (GeV)

YQM, Wang, Chao, 1012.1030

PKU, Nov. 8th, 2017



Butenschoen, Kniehl, 1105.0820

Achievement: prediction for χ_{cI} production

 $\succ \chi_{cJ} \text{ production:} \ d\sigma_{\chi_{cJ}} \approx d\hat{\sigma}_{3P_{I}^{[1]}} \langle O\left({}^{3}P_{0}^{[1]}\right) \rangle + (2J+1)d\hat{\sigma}_{3S_{1}^{[8]}} \langle O\left({}^{3}S_{1}^{[8]}\right) \rangle$



YQM, Wang, Chao, 1002.3987

> Agree with new data



PKU, Nov. 8th, 2017



Difficulty: polarization puzzle

> LO NRQCD

PKU, Nov. 8th, 2017

• Dominated by ${}^{3}S_{1}^{[8]}$, LO NRQCD predicts transversely

polarized $\psi(nS)$, contradicts with CDF data



FIG. 4 (color online). Prompt polarizations as functions of p_T : (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T -factorization model [9]).

Difficulty: polarization puzzle

> J/ψ : transverse polarization cancelled between ${}^{3}S_{1}^{[8]}$ and ${}^{3}P_{I}^{[8]}$ channel, ${}^{1}S_{0}^{[8]}$ may dominate







Chao,YQM,Shao,Wang, Zhang,1201.2675



Faccioli, Knunz, Lourenco, Seixas, Wohri, 1403.3970

$\flat \psi(2S): still hard$ to understand

Shao, Han, YQM, Meng, Zhang, Chao, 1411.3300

PKU, Nov. 8th, 2017



Difficulty: hierarchy problem

> Best fit of J/ψ yield data at high p_T

YQM, Wang, Chao, 1009.3655

 $M_{0} = \langle O\left({}^{1}S_{0}^{[8]} \right) \rangle + 3.9 \langle O\left({}^{3}\boldsymbol{P}_{0}^{[8]} \right) \rangle / m_{c}^{2} \approx 0.074 \text{ GeV}^{3}$ $M_{1} = \langle O\left({}^{3}S_{1}^{[8]} \right) \rangle - 0.56 \langle O\left({}^{3}\boldsymbol{P}_{0}^{[8]} \right) \rangle / m_{c}^{2} \approx 0.0005 \text{ GeV}^{3}$

Velocity scaling rule of NRQCD

 $\langle O\left({}^{1}S_{0}^{[8]}\right)\rangle \sim \langle O\left({}^{3}S_{1}^{[8]}\right)\rangle \sim \langle O\left({}^{3}P_{0}^{[8]}\right)\rangle /m_{c}^{2}$ Thus

 $M_0 \sim M_1$

> Two orders difference: unnatural

Difficulty: universality problem

- Necessary condition for NRQCD
 - LDMEs, like M_0 and M_1 , are process independent
- > Upper bound of M_0 set by e^+e^- collision

Zhang, YQM, Wang, Chao, 0911.2166 $M_0 < 0.02 \text{GeV}^3$

- Comparing with $M_0 \approx 0.074 \text{ GeV}^3$ from pp collison
- Solution Set in the set of the s
- Data cannot be described consistently!



> Looks like a rigorous theory

- EFT of QCD
- Factorization has been tested to NNLO Nayak, Qiu, Sterman, 0509021

But why does not it work for quarkonium production?

An important effect overlooked

> Soft gluon emission in the hadronization process

- P_{ψ} is different from $P, P = P_{\psi}[1 + O(\lambda)]$
- **NRQCD** approximate *P* by P_{ψ}

PKU, Nov. 8th, 2017

> An over simplified model of NRQCD expansion



• Cross section approximately $\propto P^{-4} = P_{\psi}^{-4} [1 + O(\lambda)]^{-4}$

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42$$

With $\lambda \approx v^2 \approx 0.3$
= 1 - 4 λ + 40/3 λ^2 - 40 λ^3 + ...
= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + ...

Soft gluon factorization (SGF)

\succ Keep momentum difference between $Q\overline{Q}$ and H

Expect no further large relativistic corrections

> SGF for quarkonium *H* production:

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} = \sum_n \int \frac{d^4 P}{(2\pi)^4} d\hat{\sigma}_n(P) F_n^H(P, P_H) \qquad n = {}^{2S+1} L_J^{[c]}$$

- $d\hat{\sigma}$: perturbatively calculable hard part
- F_n^H : nonperturabtive soft gluon distribution
- UV cutoff scale is suppressed

Soft gluon distributions (SGDs)

> Operator definition

PKU, Nov. 8th, 2017

Expectation values of bilocal operators in QCD vacuum

 $F_n^H(P,P_H) = \int d^4x e^{iP \cdot x} S_l(x) \langle 0 | \bar{\psi}(0) \Gamma'_n \Phi_l^{\dagger}(0) \psi(0) a_H^{\dagger} a_H \bar{\psi}(x) \Gamma_n \Phi_l(x) \psi(x) | 0 \rangle$

Gauge links to ensure gauge invariance

$$\Phi_l(x) = \mathcal{P} \exp\left\{-ig_s \int_0^\infty d\lambda \, l \cdot A(x+\lambda \, l)\right\}$$

- Soft factor $S_l(x)$ to absorb additional IR divergences
- A different choice of *l*: *S*_{*l*}(*x*) will be changed by a gauge invariant Wilson loop

> Set $P \approx P_H$ in hard part: "reproduce" NRQCD

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} \approx \sum_n d\hat{\sigma}_n (P_H) \langle \mathcal{O}_n^H \rangle_l S_l(0)$$

12/20

$\textbf{CO} \rightarrow \textbf{TMD v.s. NRQCD} \rightarrow \textbf{SGF}$

> CO factorization v.s. TMD factorization

- TMD factorization: both longitudinal-momentum dependence and transverse-momentum dependence
- CO factorization: integrated out transverse momentum, and leaving only longitudinal-momentum dependence

> NRQCD v.s. SGF

- SGF: have relative-momentum dependence between momentum of $Q\overline{Q}$ pair and that of quarkonium
- NRQCD: integrated out relative momentum, no momentum dependence

Conclusion

PKU, Nov. 8th, 2017

SGF is a "TMD version" of NRQCD

TMD v.s. SGF

> Similarity: gauge links, soft factors

- Study of TMD can help to understand SGF, and vice versa
- Soft factor vanishes in CO factorization; does it vanish in NRQCD factorization?

> When to use TMD?

- In small p_T region where higher-twist contributions are significant
- TMD resums a series of higher-twist contributions in CO factorization

> When to use SGF?

- In any region where relativistic corrections are significant: all p_T regions in hadroproduction, Xsection changes fast
- SGF resums a series of relativistic contributions in NRQCD

Simplification: 1d form

>SGF-4d hard to use in practice

- Hard to extract four-dimensional SGDs
- Hard to do perturbative calculation

Property of SGDs

PKU, Nov. 8th, 2017

• At the rest frame of *H*, dominant region (with $P^2 = M^2$) $P_{rest}^{\mu} = (M + O(\lambda^2), O(\lambda), O(\lambda), O(\lambda))$

> Expanding $O(\lambda)$ terms in hard part: SGF-1d

 $(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} \approx \sum_n \int dz \, d\hat{\sigma}_n (P_H/z) F_n^H(z) \quad \text{with} \quad z = \frac{m_H}{M}$

Very similar to CO factorization

Comparison: relating momentum of χ_{cJ} and its decaying J/ψ

$$p_{J/\psi} \approx \frac{m_{J/\psi}}{m_{\chi_{cJ}}} p_{\chi_{cJ}}$$
 • Deviation less than 8%

> "SGF-1d expansion"

PKU, Nov. 8th, 2017

 $\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42 \qquad \text{With } \lambda = 0.3$

$$=\frac{1}{(1+\lambda)^4} \left(1 + \frac{10}{3}\lambda^2 - \frac{20}{3}\lambda^3 + 17\lambda^4 + \cdots\right)$$

 $= 0.350 + 0.105 - 0.063 + 0.048 - 0.035 + \cdots$

> Comparing with "NRQCD expansion"

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42$$
$$= 1 - 4\lambda + 40/3\lambda^2 - 40\lambda^3 + \cdots$$
$$= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

Gluon fragmenting to quarkonium

Numerical comparison between SGF & NRQCD Model input

$$F_{3S_1^{[8]}}^H(P, P_H) = a \, k^2 \exp(-\frac{k_0^2 + k^2}{\Lambda^2})$$

Beneke, Schuler, Wolf, 0001062

- $\Lambda \sim m_0 v^2$, choose 500 MeV
- Conclusion independent of the model



 SGF-4d and -1d have different shape, but have the same accumulated value

$$\int_{0}^{1} dx \, D_{g \to H}^{4d}(x) = \int_{0}^{1} dx \, D_{g \to H}^{1d}(x)$$

-1d⁽⁰⁾: leading term of expansion m_Q around M/2, very close to -1d.

17/20

PKU, Nov. 8th, 2017

Cross section ratio

> Assume SGF-4d is exact



- 1d very close to 4d, deviation less than 6%
- Expanding m_Q results in about 10% uncertainty
- Od with $z_0 = 0.86$ well reproduce 4d
- NRQCD overshoots 4d by a factor of 4 —not reliable

Rough explanation

PKU, Nov. 8th, 2017

$$0.86^9 \approx 1/4 \sim (1 - v^2/2)^9$$

Similar effects exist in many EFTs

- > NRQCD factorization: polarization puzzle, hierarchy problem, universality problem
 - Possible reason: convergence of v^2 expansion is too bad because of soft gluon emission

> Soft gluon factorization (SGF)

• Soft gluons effects are considered, should have much better convergence of v^2 expansion

> Two important expansions

- From 4d to 1d, with small v^2 correction
- Expansion m_Q around M/2, good convergence

Outlook

Proof of SGF to all order in perturbation theory

- Similar to the proof of NRQCD to all order in α_s and v^2
- One-loop proof is available; two-loop should not be hard

> Phenomenological study

PKU, Nov. 8th, 2017

- Complexity is similar to NRQCD, thanks to the two expansions
- Most established codes reuse directly (FDC, Helac-Onia,...)
- Many NRQCD results should be redone, a lot of works

> May resolve difficulties in NRQCD

- Universality problem: importance of v^2 correction depends on process
- Hierarchy problem: relative importance of different channels changed in SGF
- Polarization puzzle: may also have large v^2 correction



What is new?

- Factorization in full QCD but not NRQCD effective field theory
 - More convenient to deal with power corrections in full QCD than EFT
- Momentum difference between QQ and H considered
 - No further large power corrections

> Difference between shape function models

- External $Q\bar{Q}$ in hard part are on mass shell, gauge invariant for hard part
- Operator definition of SGDs with gauge links to ensure gauge invariance
- Soft factor in SGDs

Simplification: Od form

> SGF-0d

- If $F_n^H(z)$ peaks around $z = z_n \sim 1 O(\lambda/m_H)$
- Approximate $F_n^H(z) \approx \delta(z z_n) \langle O_n^H \rangle$

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} \approx \sum_n d\hat{\sigma}_n (P_H/z_n) \langle \mathcal{O}_n^H \rangle_l S_l(0)$$

> Roughly recover NRQCD if choosing $z_n = 1$

• May result in large corrections

Simplification: expansion of m_Q

> At least two hard scales in short distance

- Invariant mass of $Q\bar{Q}$ pair M and quark mass m_Q
- **Relation:** $M = 2m_Q + O(\lambda)$

Expansion

PKU, Nov. 8th, 2017

- $d\hat{\sigma}(m_Q, M) = d\hat{\sigma}\left(\frac{M}{2}, M\right) + \left(m_Q \frac{M}{2}\right)d\hat{\sigma}'\left(\frac{M}{2}, M\right) + \cdots$
- Good convergence

> Comparing with NRQCD expansion

- $d\hat{\sigma}(m_Q, M) = d\hat{\sigma}(m_Q, 2m_Q) + (M 2m_Q)d\hat{\sigma}'(m_Q, 2m_Q) + \cdots$
- Bad convergence: $d\hat{\sigma}(m_Q, M) \max \propto M^{-5}$

J/ψ production via gluon fragmentation



Easy to calculate

$$d\sigma_H(p_T) = \int dx \, d\hat{\sigma}_g(p_T/x) D_{g \to H}(x)$$

- $d\hat{\sigma}_g$: well known
- $D_{g \rightarrow H}$: calculated by using NRQCD or SGF