Messages from Lineshapes of near threshold states

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Hadronic Molecules

→ are few-hadron states, bound by the strong force

→ do exist: light nuclei.
e.g. deuteron as pn & hypertriton as Λd bound state

→ are located typically close to relevant continuum threshold;
e.g., for $E_B = m_1 + m_2 - M$

- $E_B^{\rm deuteron} = 2.22$ MeV
- $E_B^{\rm hypertriton} = (0.13 \pm 0.05)$ MeV (to Λd)

→ can be identified in observables (Weinberg compositeness):

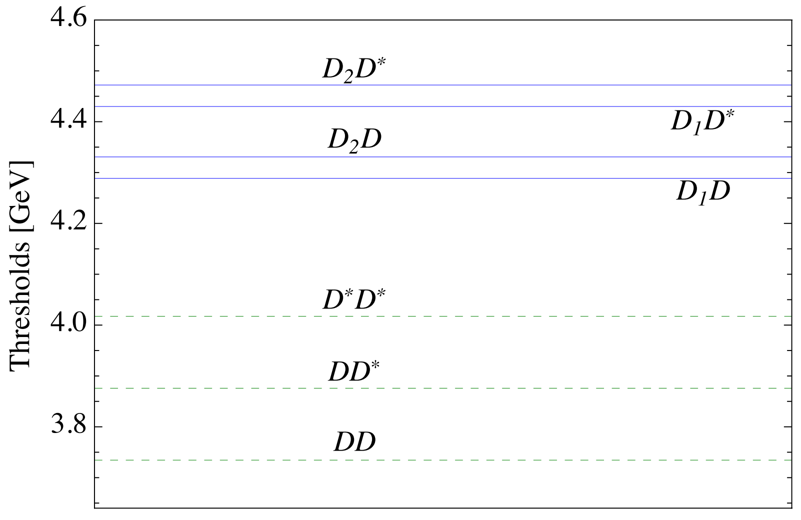
$$\frac{g_{\rm eff}^2}{4\pi} = \frac{4M^2\gamma}{\mu}(1-\lambda^2) \;\rightarrow\; a = -2\left(\frac{1-\lambda^2}{2-\lambda^2}\right)\frac{1}{\gamma}\,; \quad r = -\left(\frac{\lambda^2}{1-\lambda^2}\right)\frac{1}{\gamma}$$

where $(1-\lambda^2)$=probability to find molecular component in bound state wave function

Are there mesonic molecules?

General considerations

Example: $1/2^+$ multiplet $\{D, D^*\}$ and $3/2^-$ multiplet $\{D_1, D_2\}$ $\rightarrow$



$3^{-\pm}$: D^*D_2
$0^{-\pm}$: D^*D_1
$2^{-\pm}$: $D^*D_1 - D^*D_2 - DD_2$
$1^{-\pm}$: $DD_1 - D^*D_1 - D^*D_2$ ($Y(4260), Y(4360)$ $(I{=}0)$)

2^{++}: D^*D^*
1^{++}: DD^* ($X(3872)$ $(I{=}0)$)
1^{+-}: $DD^* - D^*D^*$ ($Z_c(3900)^+, Z_c(4020)^+$ $(I{=}1)$)
0^{++}: $DD - D^*D^*$;

$\rightarrow$ Explains mass gap between $J^P = 1^+$ and 1^- states:
$M_{Y(4260)} - M_{X(3872)} = 388$ MeV $\simeq M_{D_1(2420)} - M_{D^*} = 410$ MeV

$\rightarrow$ Predicts, e.g., $M(0^-) - M(1^-) \simeq M_{D^*} - M_D \simeq +100$ MeV,
if it exists

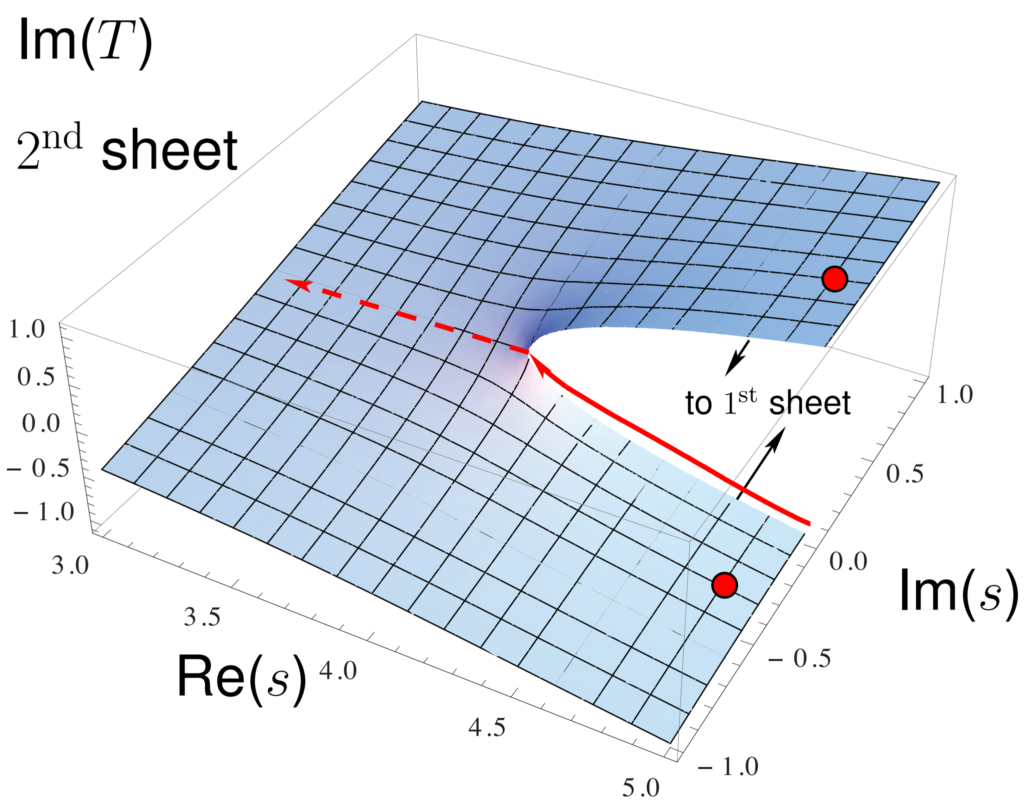
Note: for hadrocharmonium: $M(0^-) - M(1^-) \simeq -100$ MeV

M. Cleven et al., PRD 92 (2015) 014005

Remarks on S-matrix

→ For real $s < s_{\rm min}^{\rm thres}$, S is real → Branchpoint at $s = s^{\rm thres}$

→ $S(s^*) = S^*(s) \longrightarrow$ pole at s implies pole at s^*



For narrow resonances:

In resonance region:
only lower pole matters

At threshold:
both poles important!

For broad resonances:

always both important

Keep track of the cuts!

Poles on real axis are called virtual ($2^{\rm nd}$) or bound ($1^{\rm st}$) states

Constructing parameterizations

For shallow bound/virtual states

$$T_{\mathrm{NR}}(E) = \frac{g_0^2}{E + E_B + g_0^2\mu/(2\pi)(ik \pm \gamma)}, \quad g_0^2 = \frac{2\pi\gamma}{\mu^2}\left(\frac{1}{\lambda^2} - 1\right)$$

where $k = \sqrt{2\mu E}$ and $\gamma = \sqrt{2\mu E_B}$.

Where λ^2=Prob. to find compact comp. in wf.

$\rightarrow$ $\lambda^2 = 1 \Longrightarrow$ Compact state with $g_0^2 = 0$

$\rightarrow$ $\lambda^2 = 0 \Longrightarrow$ Molecular state with $g_0^2 = \infty$

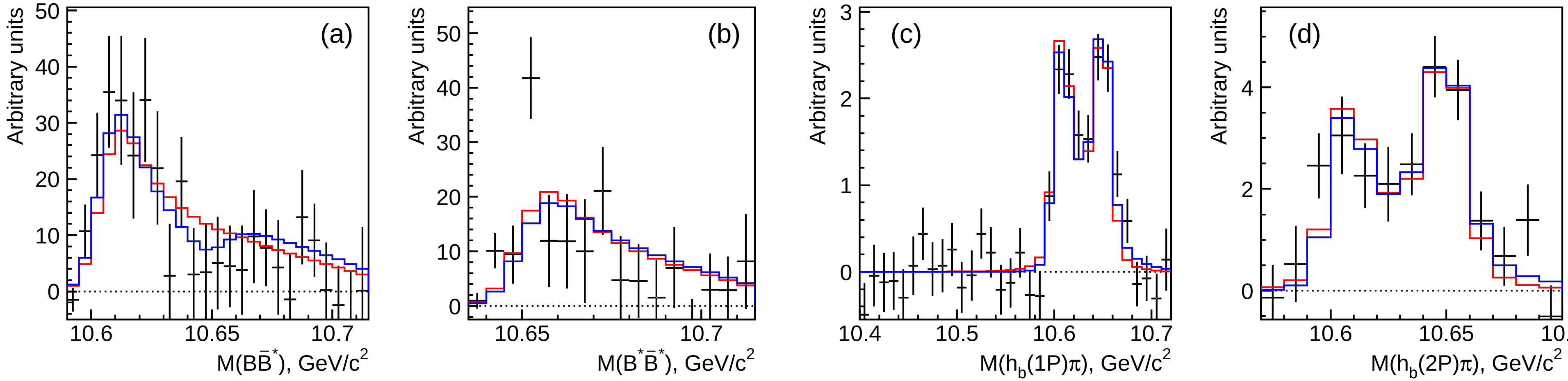
dimensional analysis: $g_0^2 \sim 2\pi\beta/\mu^2$ with $\beta = {}^1/_{\text{range of forces}} \gg \gamma$

Importance of two-body cut measures molecular admixture

This information is contained in the line shapes ...

Application: Z_b states

Inclusion of inel. channels: C.H. et al., PRL115(2015)202001



Calcs.: Guo et al., PRD93(2016)074031; Data: Garmash *et al.* [Belle], PRL116(2016)212001

$\rightarrow$ from fit with HQSS constraints

$\varepsilon_B(Z_b) = (1.51^{+0.76}_{-0.61} \pm i0.11^{+0.03}_{-0.03})$ MeV, $\varepsilon_B(Z_b') = (1.50^{+1.76}_{-0.60} \pm i0.15^{+0.04}_{-0.04})$ MeV,

$\rightarrow$ from fit without HQSS constraints

$\varepsilon_B(Z_b) = (0.85^{+0.80}_{-0.54} \pm i0.04^{+0.02}_{-0.02})$ MeV, $\varepsilon_B(Z_b') = (1.27^{+0.91}_{-0.69} \pm i1.20^{+0.51}_{-0.43})$ MeV,

all on the unphysical sheets$\rightarrow$two–hadron states

Result converted from BW-fit R. Mizuk, private communication

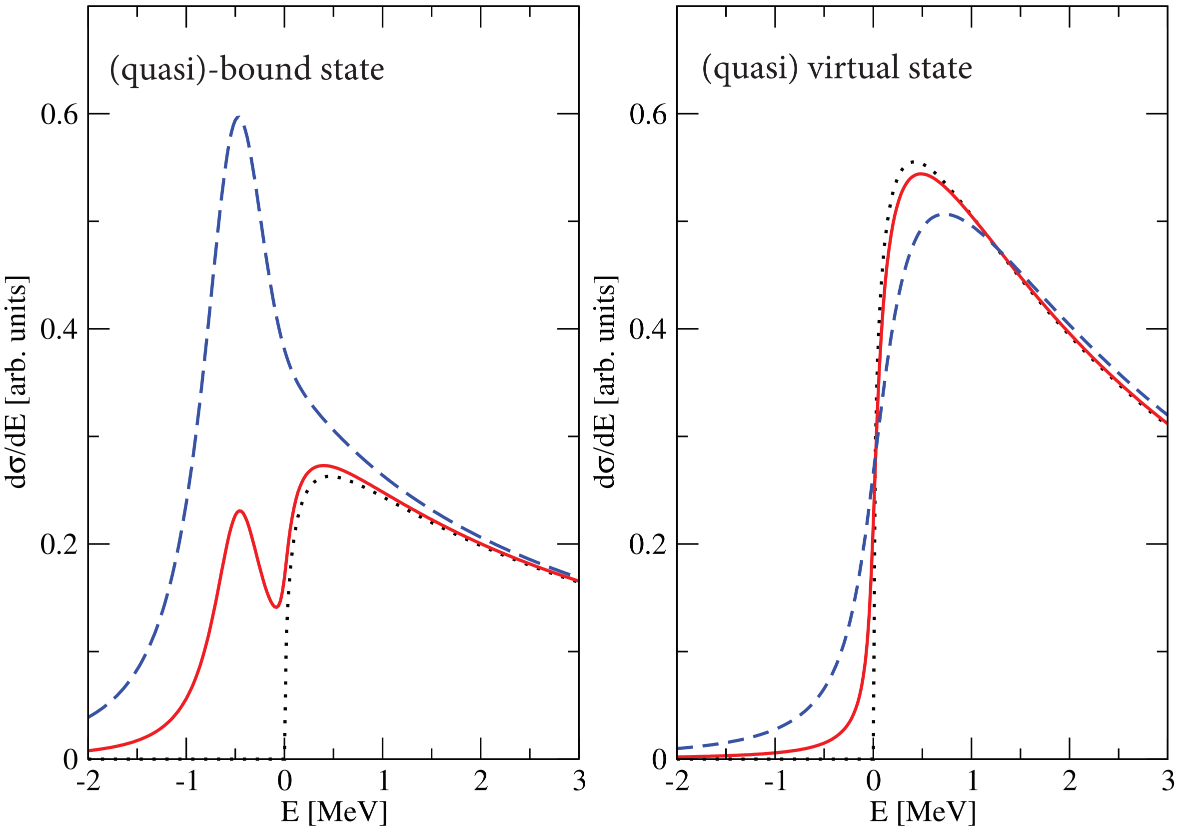
$\varepsilon_B(Z_b) = (-2.6 \pm 2.1 - i(9.2 \pm 1.2))$ MeV, $\varepsilon_B(Z_b') = (-1.8 \pm 1.7 - i(5.8 \pm 1.1))$ MeV,

Unstable constituents

Braaten & Lu PRD76 (2007) 094028; C.H. et al., PRD81 (2010) 094028

$$k \to \sqrt{\mu}\sqrt{\sqrt{E^2+\Gamma^2/4}+E} + i\sqrt{\mu}\sqrt{\sqrt{E^2+\Gamma^2/4}-E} + \mathcal{O}(\Gamma/2E_r)\ ,$$

$$\gamma \to \pm\sqrt{\mu}\sqrt{\sqrt{E_r^2+\Gamma^2/4}-E_r} + \mathcal{O}(\Gamma/2E_r)\ ,$$



for $Y \to AB \to [cd]B$ with

$E_r = -0.5$ MeV

$\Gamma_0 = 1.5$ MeV

$g_0^2 = 0.2$ GeV^{-1}

natural value for molecular state

and $\Gamma = 0, 0.1, 1$ MeV

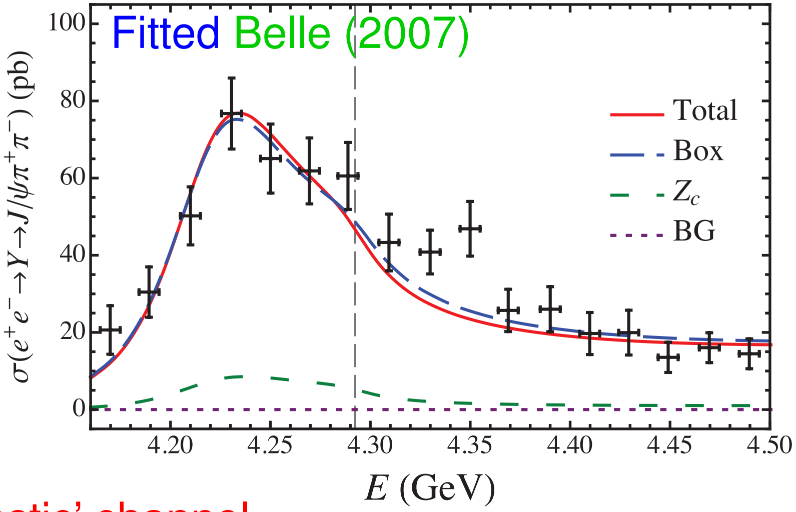
non-Breit-Wigner shapes emerge unavoidably!

Lineshapes of $Y(4260)$

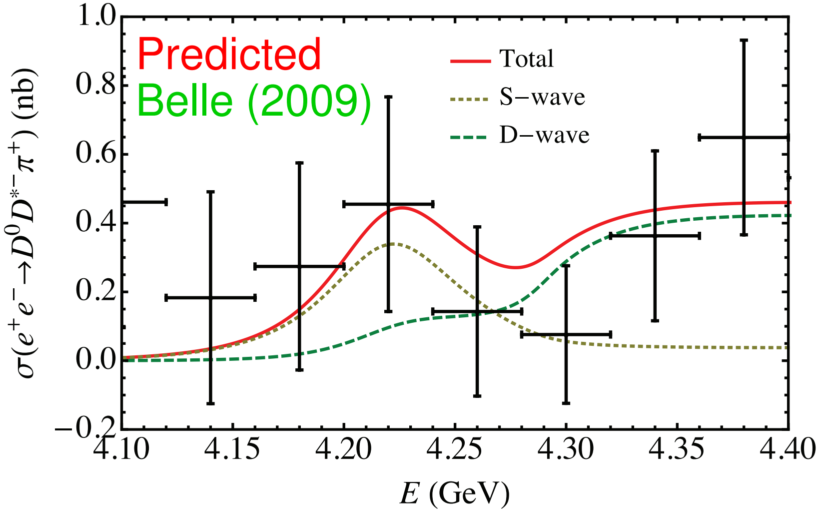
IF the $Y(4260)$ is a $D_1\bar{D}$ molecule it MUST have a large coupling to this channel $\Longrightarrow$ great impact on lineshapes

Cleven et al., PRD90 (2014) 074039; see also Qin et al. PRD94(2016)054035

Inelastic channel



'elastic' channel



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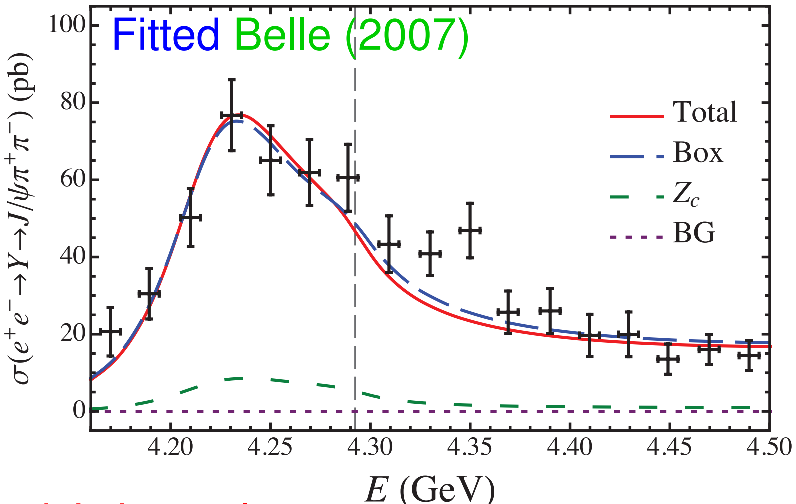

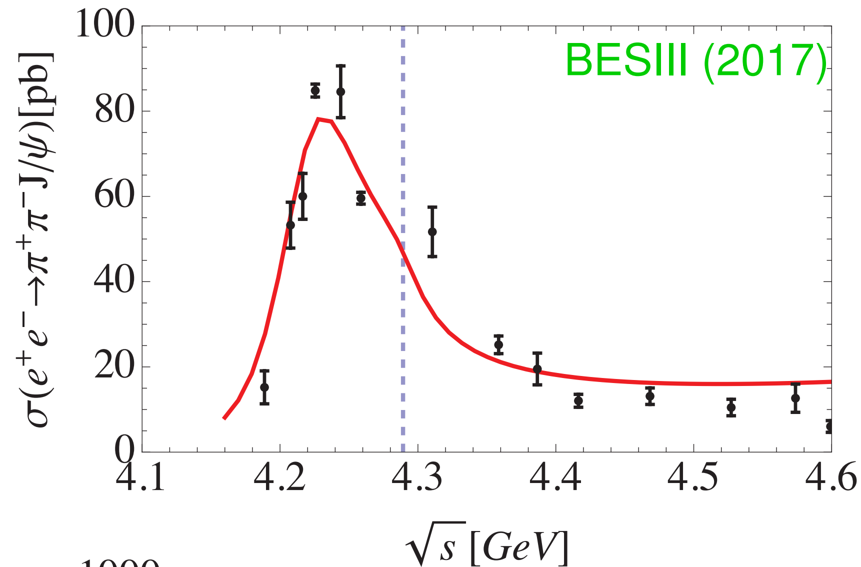

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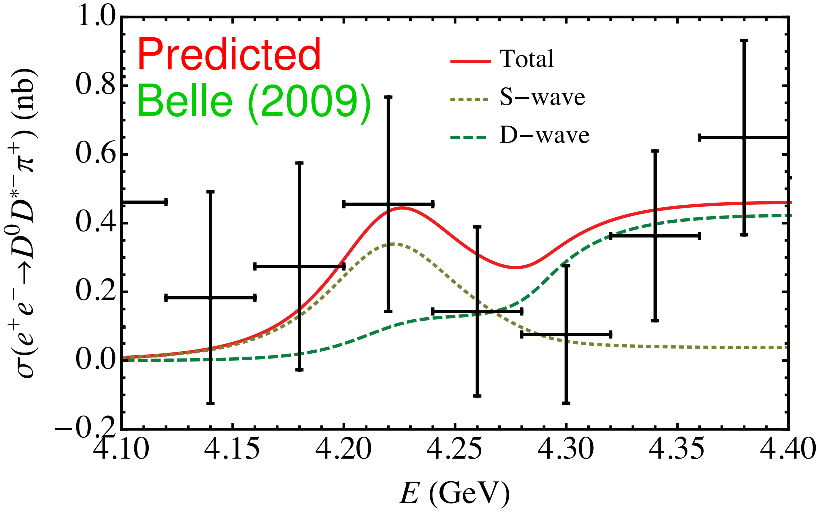

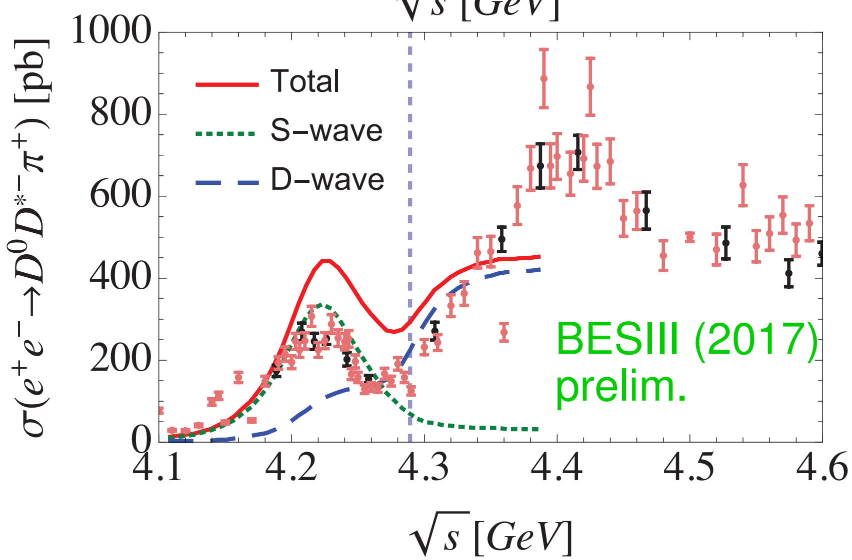

Summary and Perspectives

The molecular picture for 'near' threshold states

$\rightarrow$ provides a natural explanation for ordering of positive and negative parity near threshold exotics

$\rightarrow$ and for the line shapes of $Y(4260)$ (number of states?)

$\rightarrow$ allows one predict spin partners — talk by A. Nefediev tomorrow

$\Longrightarrow$ specific patterns emerge

M. Cleven et al., PRD 92 (2015) 014005

$\rightarrow$ allows one to predict new transitions (e.g. $Y(4260) \rightarrow X(3872)\gamma$)

$\rightarrow$ is consistent with production data — talk by F.-K. Guo today

Thank you very much for your attention