# (Lattice) NRQCD at $T > 0$

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in collaboration with P. Petreczky(BNL) and A. Rothkopf(Heidelberg)

work in progress

# Outline

# Summary of the previous works

• quarkonium is a “thermometer” for Quark-Gluon Plasma

• quantitative understanding on the in-medium modification of quarkonim properties based on first principle calculation (i.e. lattice QCD)

• study on quarkonium at $T \neq 0$ using lattice NRQCD and applying Bayesian reconstruction method for the quarkonium spectral functionss

• FASTSUM collaboration (G. Aarts, C. Allton, M.-P. Lombardo, SK, S. Ryan, D.K. Sinclair, J.-I. Skullerud + many others)

• KPR (SK(Sejong), P. Petreczky(BNL), A. Rothkopf(Heidelberg))

$$G(\tau) = \int \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega')$$

# Summary of the previous works

- $m_Q$ is "integrated out" and focus on the scale of "binding" $\rightarrow$ avoid large scale separation problem

- small statistical errors in quarkonium correlators ($\sim \mathcal{O}(10^{-4})$)

- avoid the known zero mode problem (cf. T. Umeda, PRD75 (2007) 094502 and A. Mocsy and P. Petreczky, PRD77 (2008) 014501)

- intial value problem $\rightarrow$ larger $\tau$ range

- continuum limit can't be taken ($m_Q a \sim \mathcal{O}(1)$)

# Summary of the previous works

- in QCD,

$$
\begin{aligned}
G_\Lambda(\tau) &= \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x})\Gamma\psi(\tau,\vec{x})\overline{\psi}(0,\vec{0})\Gamma\psi(0,\vec{0})\rangle \\
&= \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Gamma(\omega,\vec{p})
\end{aligned}
$$

and

$$
K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}.
$$

- In NRQCD, with $\omega = 2M + \omega'$ and $T/M << 1$, $K(\tau,\omega) \to e^{-\omega\tau}$

$$
G(\tau) = \int_0^\infty \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega')
$$

- inverse Laplace transform problem

# Bayesian methods

- given $G(\tau)$ which is calculated on lattice, what is the spectral function, $\rho(\omega)$ ?

$$G(\tau) = \int_0^\infty \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega')$$

- Bayes theorm

$$P[X|Y] = P[Y|X]P[X]/P[Y]$$

- in other words

$$P[\rho|D,H] \propto P[D|\rho,H]P[\rho|H]$$

- systematic inclusion of prior knowledge ($H$)

$$P[D|\rho,H] = e^{-L}, \quad L = \frac{1}{2}\sum_i (D_i - D_i^\rho)^2/\sigma_i^2$$

and

$$P[\rho|H] = e^{-S}, \quad S = S[\rho(\omega), m(\omega)]$$

where $S$ is the prior and $m(\omega)$ is default model

# Bayesian methods

• Shannon-Jaynes entropy for $S$ (cf. Asakawa, Hatsuda, Nakahara, Prog. Part.Nucl.Phys. 45 (2001) 459)

$$S_{SJ} = \alpha \int d\omega \left(\rho - m - \rho \log(\frac{\rho}{m})\right)$$

• new prior (cf. Y.Burnier, A. Rothkopf, PRL111 (2013) 182003)

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log(\frac{\rho}{m})\right)$$

• modification of new prior (cf. A. Rothkopf, work in progress)

$$S_{\mathrm{newBR}} = \alpha \int d\omega \left[(\frac{\partial\rho}{\partial\omega})^2 + 1 - \frac{\rho}{m} + \log(\frac{\rho}{m})\right]$$

• in large data limit (vanishing statistical error + infinite $N_\tau$ limi), the form of the prior goes to Shannon-Jaynes entropy (cf. Asakawa et al)

# Summary of the previous result (FASTSUM)

• FASTSUM(most recently, JHEP07 (2014) 097), G. Aarts, C. Allton, T. Harris, S.K., M.P. Lombardo, S.M. Ryan, J-I. Skullerud) : NRQCD + MEM

• anisotropic lattices with fixed scale, $T$ change by $N_\tau$:

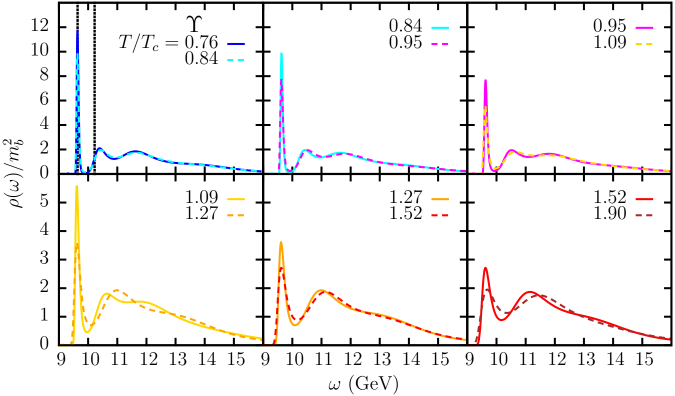
1st Gen ($12^3 \times N_\tau, a_s/a_\tau = 6.0, N_f = 2$),
2nd Gen ($24^3 \times N_\tau, a_s/a_\tau = 3.5, N_f = 2+1$),
3rd Gen ($32^3 \times N_\tau, a_s/a_\tau \sim 7, N_f = 2+1$) (currently progressing)

• S-wave bottomonium (PRL106 (2011) 061602, JHEP11 (2011) 013, JHEP03 (2013) 084), P-wave bottomonium (JHEP12 (2013) 064), and S- and P-wave (JHEP07 (2014) 097) + proceedings

• detailed systematic errors study (default-model dependence, energy window, number of configurations, euclidean time window)

# Summary of the previous result (FASTSUM)

- FASTSUM, JHEP1407 (2014) 097 : S-wave



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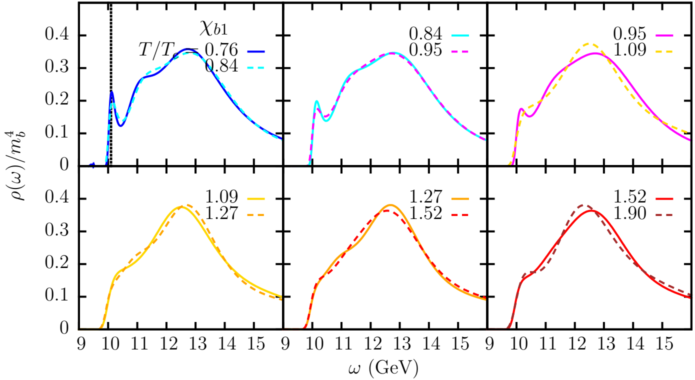


# Summary of the previous result (KPR)

• S.K., A. Rothkopf, P. Petreczky: NRQCD + (MEM, new Bayesian)

• isotropic lattices from HotQCD ($48^3 \times 12$, $N_f = 2+1$ light $m_\pi \sim 160$ MeV), $T$ change by changing $a$ (needs accompanying $T = 0$ calculation)

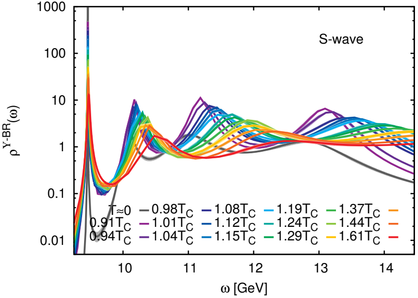
• S.K. A. Rothkopf, P. Petreczky, PRD 91 (2015) 054511

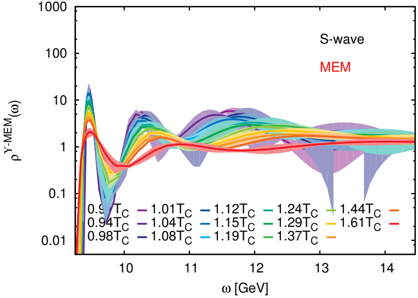
• detailed systematic errors study (default-model dependence, energy window, number of configurations, euclidean time window)

• investigation on the prior dependences (MEM vs new Bayesian)

# Summary of the previous result (KPR)

- S.K. A. Rothkopf, P. Petreczky, PRD 91 (2015) 054511 : S-wave



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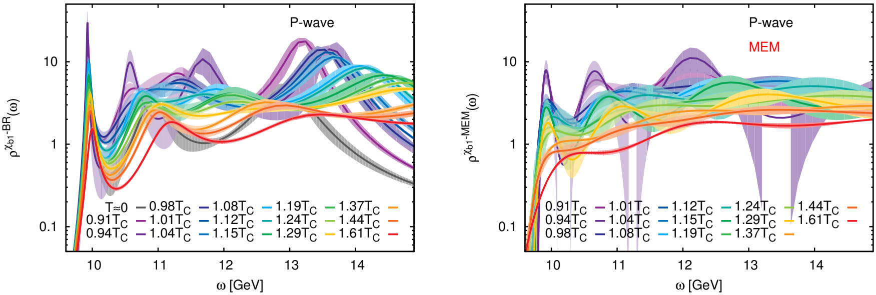


# Short summary

- qualitatively shown sequential suppression of $\Upsilon(1S, 2S, 3S)$ upto $1.9T_c$ (FASTSUM) or upto $1.6T_c$ (KPR) based on "first principle" calculation

- melting of $\chi_{b1}$ immediately above $T_c$ (FASTSUM) or binding of $\chi_{b1}$ upto $1.6T_c$ (KPR) depending on the choice of the prior

- which prior?

- BR prior has a problem of its own (ringing)

# Further improvements

- the same setup: isotropic lattices from HotQCD ($48^3 \times 12$, $N_f = 2+1$ light $m_\pi \sim 160$ MeV), $T$ change by changing lattice spacing $a$ (needs accompanying $T = 0$ calculation) with

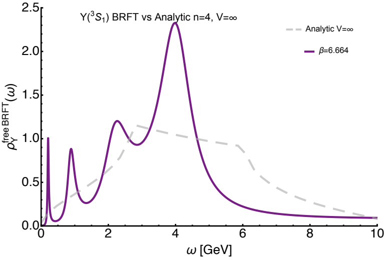
  - increased statistics: $\mathcal{O}(4000)$ correlators at some temperatures and 400 $T = 0$ correlators
  - higher temperatures: upto $T = 401\mathrm{MeV}$ (or $T = 2.60\,T_c$)
  - different Bayesian regulators
  - different basis functions for the SVD space

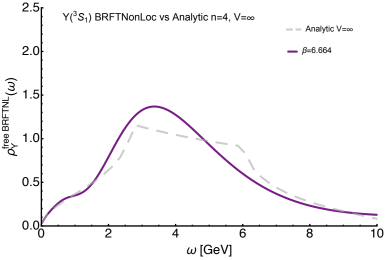
# Ringing problem with BR prior

- peaks in the free spectral function!

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log(\frac{\rho}{m})\right)$$

$$S_{\mathrm{newBR}} = \alpha \int d\omega \left[(\frac{\partial\rho}{\partial\omega})^2 + 1 - \frac{\rho}{m} + \log(\frac{\rho}{m})\right]$$



# $T = 0$ spectral functions

- comparison of $\Upsilon$ spectral functions from BR prior and that from new BR prior at $T = 0$

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log(\frac{\rho}{m})\right)$$

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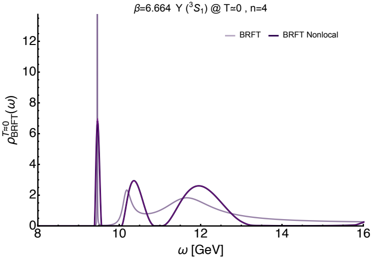


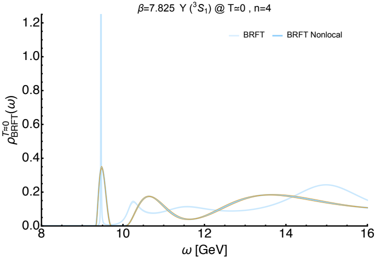

# $T = 0$ spectral functions

- comparison of $\chi_{b1}$ spectral functions from BR prior and that from new BR prior at $T = 0$

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log(\frac{\rho}{m})\right)$$

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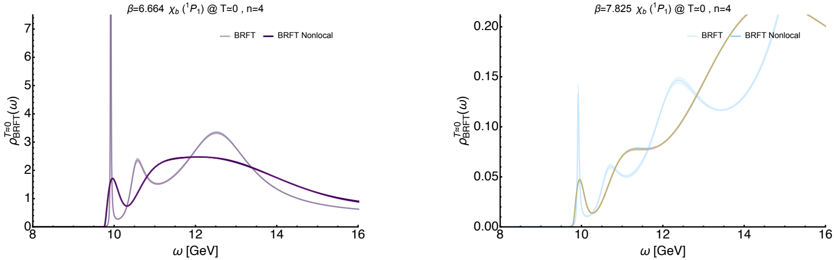
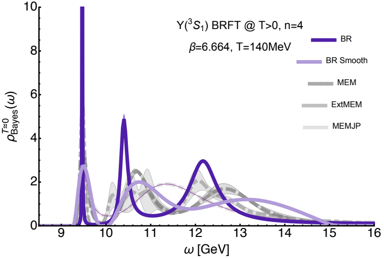
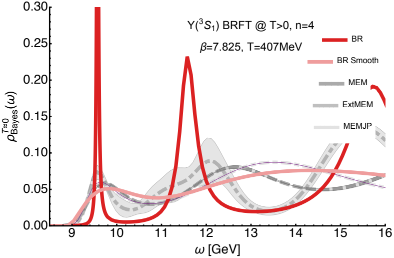


# $T \neq 0$ spectral functions

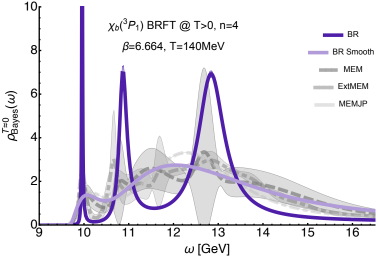
- comparison of $\Upsilon$ spectral functions from various Bayesian considerations at $T \neq 0$

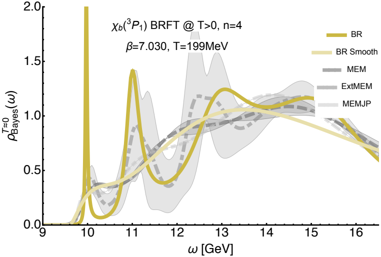


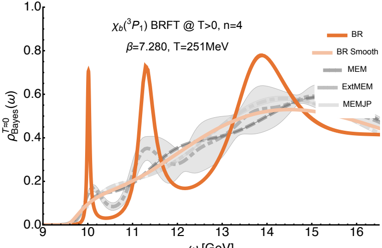



# $T \neq 0$ spectral functions

- comparison of $\chi_{b1}$ spectral functions from various Bayesian considerations at $T \neq 0$



# Summary and Outlook

• lattice NRQCD provides us quite accurate and fast estimation quarkonium correlators in thermal medium by separating large heavy quark mass $M$ from the problem (relative error of $O(10^{-4})$ in the correlators).

• obtaining in-medium spectral function of quarkonium relies on Bayesian method.

• An importance issue from the prior dependence of Bayesian method is undertood better.

• Different priors in Bayesian reconstruction method suggest a converging result.