

Quarkonium suppression in a fireball

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Open Quantum Systems

Density matrix

An arbitrary statistical ensemble of quantum states can be represented by a **density matrix** ρ , which is

- **Hermitian**: $\rho^\dagger = \rho$;
- **positive**: $\langle \psi | \rho | \psi \rangle \geq 0$ for all nonzero states $|\psi\rangle$;
- and can be **normalized** to have unit trace: $\text{Tr}\{\rho\} = 1$.

The time evolution of the density matrix is described by the **von Neumann equation**:

$$i \frac{d\rho}{dt} = [H, \rho]$$

which follows from the Schrödinger equation for $|\psi\rangle$. The evolution equation

- is **linear** in ρ ;
- **preserves the trace** of ρ ;
- is **Markovian**.

Open quantum system

In quantum information theory, one separates the full system into a **subsystem** of interest and its **environment**. A density matrix ρ for the subsystem can be obtained from the density matrix ρ_{full} for the full system by the partial trace over the environment states:

$$\rho = \text{Tr}_{\text{environment}} \{ \rho_{\text{full}} \}$$

In general the evolution of ρ is non-Markovian.

The evolution is Markovian if the time during which the subsystem is observed is much larger than the time scale for correlations between the subsystem and the environment. We must also restrict to the low-frequency behavior of the subsystem, which can be accomplished by smoothing out over times larger than the correlation time scale.

Lindblad equation

The density matrix ρ for the subsystem necessarily satisfies the three basic properties: it is **Hermitian**, **positive**, and it can be **normalized**.

If further the time evolution is **linear** in ρ , **preserves the trace** of ρ , is **Markovian** and the linear operator that determines the time evolution of ρ is **completely positive**
 \Rightarrow then this require the time evolution equation to have the **Lindblad form**

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2} \{C_i^\dagger C_i, \rho\})$$

where H is a Hermitian operator and the C_n 's are an additional set of operators called **collapse operators**.

○ Lindblad CMP 48 (1976) 119

Gorini Kossakowski Sudarshan JMP 17 (1976) 821

Numerical solutions: QuTiP

There exist numerical toolboxes for open quantum systems. An example is QuTiP.



○ Johansson, Nation, Nori, CPC 183 (2012) 1760, 184 (2013) 1234

Quarkonium in medium as an open quantum system

Quarkonium as a multiscale system

Quarkonium being a composite system is characterized by several energy scales:

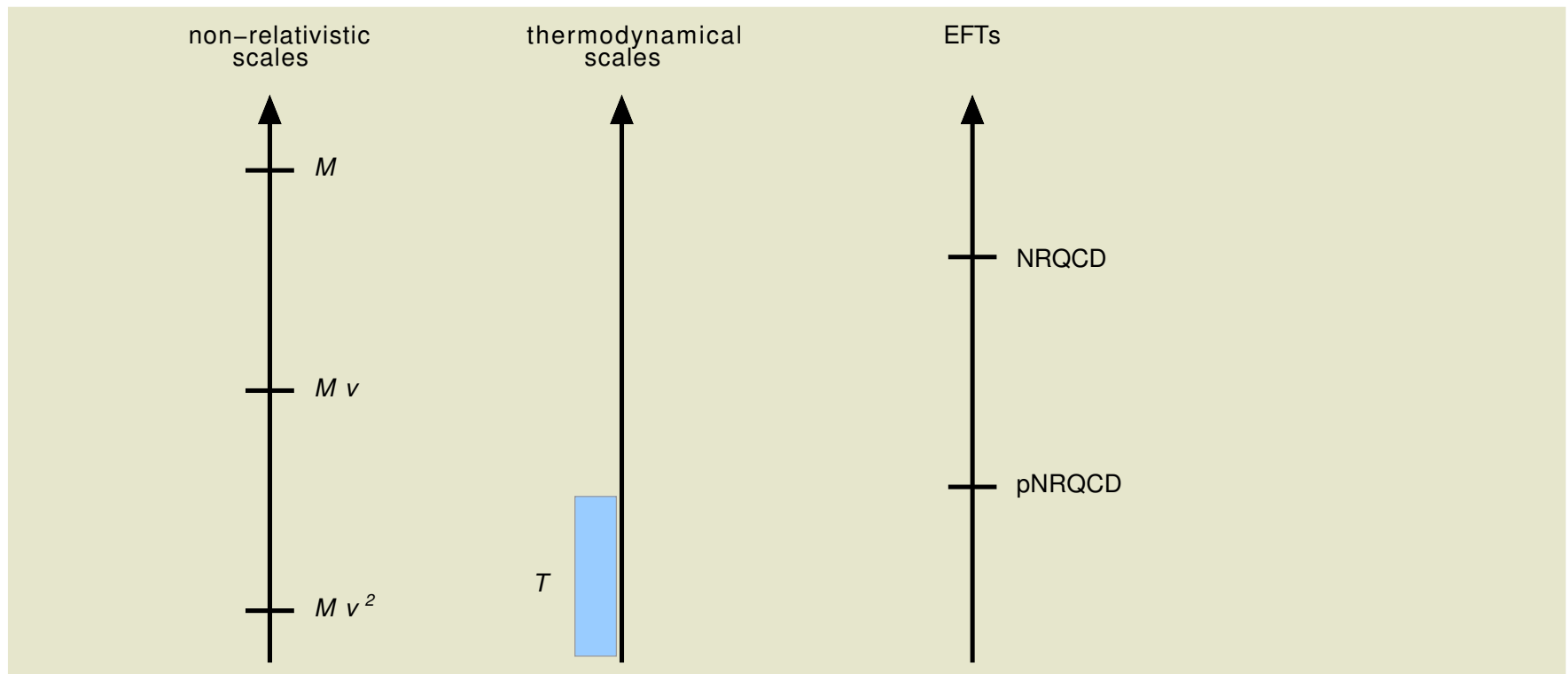
- the scales of a **non-relativistic** bound state
(v is the relative heavy-quark velocity; $v \sim \alpha_s$ for a Coulombic bound state):
 M (mass),
 Mv (momentum transfer, inverse distance),
 Mv^2 (kinetic energy, binding energy, potential V), ...
- the **thermodynamical** scales:
 T (temperature), ...

T may stand for the inverse correlation length between system and environment.
For definiteness we will assume that the system is locally in thermal equilibrium so that a **slowly varying time-dependent temperature** can be defined.

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$

Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system ([quarkonium at rest in a thermal bath](#)) in terms of a hierarchy of EFTs.



○ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

pNRQCD

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \sum_{i=1}^{n_f} \bar{q}_i i \not{D} q_i + \int d^3r \text{Tr} \left\{ S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right\}$$

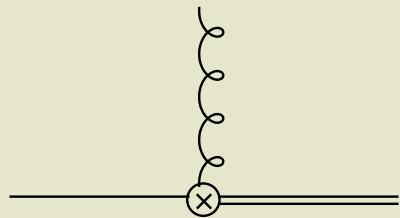
• LO in r

$$\overline{\hspace{2cm}} \qquad \underline{\hspace{2cm}}$$

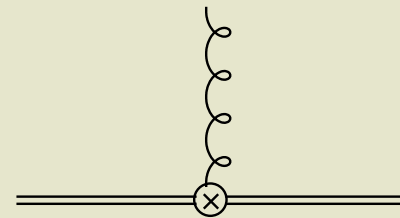
$$\theta(T) e^{-iTh_s} \qquad \theta(T) e^{-iTh_o} \left(e^{-i \int dt A^{\text{adj}}} \right)$$

$$+ V_A \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} S + S^\dagger \mathbf{r} \cdot g\mathbf{E} O \right\} + \frac{V_B}{2} \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g\mathbf{E} O + O^\dagger O \mathbf{r} \cdot g\mathbf{E} \right\}$$

• NLO in r



$$O^\dagger \mathbf{r} \cdot g\mathbf{E} S$$



$$O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \}$$

+ ...

Quarkonium in a fireball

- After the heavy-ion collisions, heavy quark-antiquarks propagate freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t_F they propagate in the medium.
- We assume the medium infinite, homogeneous and isotropic.
- We assume the heavy quarks comoving with the medium.
- We assume the medium to be locally in thermal equilibrium,

i.e., the temperature T of the medium changes (slowly) with time:

$$T = T_0 \left(\frac{t_0}{t} \right)^{v_s^2}, \quad t_0 = 0.6 \text{ fm}, \quad v_s^2 = \frac{1}{3} \text{ (sound velocity)}$$

○ Bjorken PRD 27 (1983) 140

Fireball's initial temperature

The initial temperature T_0 may account for different centralities

centrality (%)	$\langle b \rangle$ (fm)	T_0 (MeV)
0 – 20	4.76	466
0 – 10	3.4	471
10 – 20	6.0	461
20 – 90	11.6	360
20 – 30	7.8	449
30 – 40	9.35	433
40 – 50	10.6	412
30 – 50	9.9	425
50 – 70	12.2	366
50 – 100	13.6	304

○ CMS PRC 84 (2011) 024906

Miller Reygers Sanders Steinberg ARNPS 57 (2007) 205

Quarkonium as a Coulombic bound state

The lowest quarkonium states (1S bottomonium and charmonium, 2S bottomonium) are the most tightly bound. For these we assume the hierarchy of energy scales

$$M \gg \frac{1}{r} \sim M\alpha_s \gg T \sim gT \gg \text{any other scale}, \quad v \sim \alpha_s$$

This qualifies the bound state as **Coulombic**:

- quark-antiquark **color singlet** Hamiltonian = $h_s = \frac{\mathbf{p}^2}{M} - \frac{4}{3} \frac{\alpha_s}{r}$
- quark-antiquark **color octet** Hamiltonian = $h_o = \frac{\mathbf{p}^2}{M} + \frac{\alpha_s}{6r}$

The attractive factor $-4/3$ and repulsive factor $1/6$ come from the color quantum numbers. The octet potential describes an unbound quark-antiquark pair.

Density matrices

- **Subsystem:** heavy quarks/quarkonium
- **Environment:** quark gluon plasma

We may define a **density matrix** in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

$$\begin{aligned}\langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \text{Tr}\{\rho_{\text{full}}(t_0) S^\dagger(t, \mathbf{r}, \mathbf{R}) S(t', \mathbf{r}', \mathbf{R}')\} \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{N_c^2 - 1} &\equiv \text{Tr}\{\rho_{\text{full}}(t_0) O^{a\dagger}(t, \mathbf{r}, \mathbf{R}) O^b(t', \mathbf{r}', \mathbf{R}')\}\end{aligned}$$

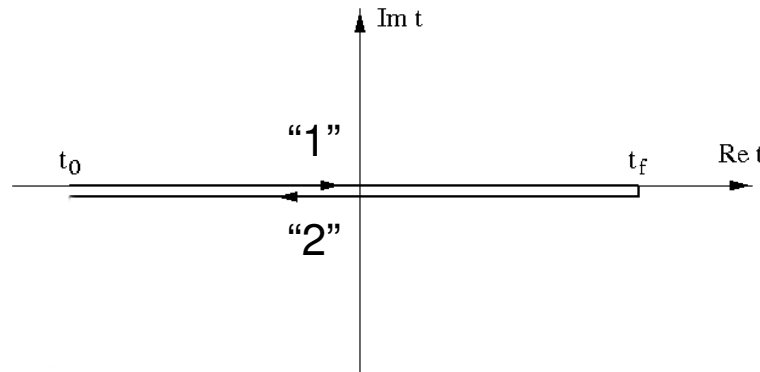
$t_0 \approx 0.6$ fm is the time formation of the plasma.

The system is in **non-equilibrium** because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although **the number of heavy quarks is conserved**: $\text{Tr}\{\rho_s\} + \text{Tr}\{\rho_o\} = 1$.

Closed-time path formalism

In the **closed-time path formalism** we can represent the density matrices as 12 propagators on a closed time path:

$$\begin{aligned}\langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &= \langle S_1(t', \mathbf{r}', \mathbf{R}') S_2^\dagger(t, \mathbf{r}, \mathbf{R}) \rangle \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{N_c^2 - 1}, &= \langle O_1^b(t', \mathbf{r}', \mathbf{R}') O_2^{a\dagger}(t, \mathbf{r}, \mathbf{R}) \rangle\end{aligned}$$



Differently from the thermal equilibrium case 12 propagators are relevant (in thermal equilibrium they are exponentially suppressed).

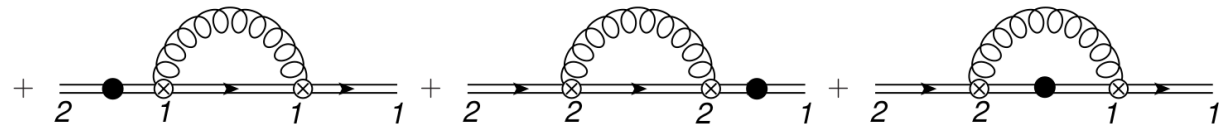
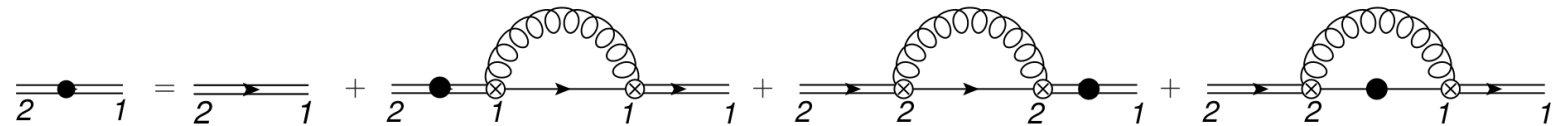
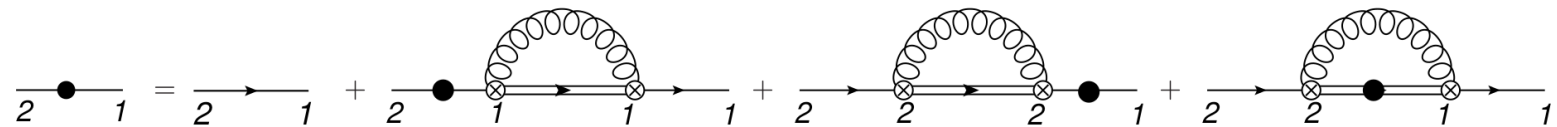
12 propagators are not time ordered, while 11 and 22 operators select the forward time direction $\propto \theta(t - t'), \theta(t' - t)$.

Assumptions

- The density of heavy quarks is much smaller than the one of light quarks: we expand at **first order in the heavy quark-antiquark density**.
- We consider **T much smaller than the Bohr radius** of the quarkonium: we expand up to **order r^2 in the multipole expansion**.

Resummation

Resumming $(t - t_0) \times$ self-energy contributions à la Schwinger–Dyson ...



The resummation is accurate at order r^2 and consistent with unitary evolution at leading order.

Evolution equations

... and differentiating over time we obtain the **coupled evolution equations**:

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

$$\begin{aligned} \frac{d\rho_o(t;t)}{dt} &= -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) \\ &\quad + \Xi_{oo}(\rho_o(t;t), t) \end{aligned}$$

- Brambilla Escobedo Soto Vairo PRD 96 (2017) 034021

Interactions with the medium

The functions Σ and Ξ are **linear in the densities** and depend on **three independent operators** characterizing the interactions with the medium

$$\Sigma_s(t) = r^i A_i^{s_o \dagger}(t)$$

$$\Sigma_o(t) = \frac{1}{N_c^2 - 1} r^i A_i^{o_s \dagger}(t) + \frac{N_c^2 - 4}{2(N_c^2 - 1)} r^i A_i^{o_o \dagger}(t)$$

$$\Xi_{so}(\rho_o(t; t), t) = \frac{1}{N_c^2 - 1} \left(A_i^{o_s \dagger}(t) \rho_o(t; t) r^i + r^i \rho_o(t; t) A_i^{o_s}(t) \right)$$

$$\Xi_{os}(\rho_s(t; t), t) = A_i^{s_o \dagger}(t) \rho_s(t; t) r^i + r^i \rho_s(t; t) A_i^{s_o}(t)$$

$$\Xi_{oo}(\rho_o(t; t), t) = \frac{N_c^2 - 4}{2(N_c^2 - 1)} \left(A_i^{o_o \dagger}(t) \rho_o(t; t) r^i + r^i \rho_o(t; t) A_i^{o_o}(t) \right)$$

where

$$A_i^{s_o}(t) = \frac{g^2}{2N_c} \int_{t_0}^t dt_2 e^{ih_s(t_2-t)} r^j e^{ih_o(t-t_2)} \langle E^{a,j}(t_2, \mathbf{0}) E^{a,i}(t, \mathbf{0}) \rangle$$

Interpretation

- The self energies Σ_s and Σ_o provide the **in-medium induced mass shifts**, $\delta m_{s,o}$, and **widths**, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$-i\Sigma_{s,o}(t) + i\Sigma_{s,o}^\dagger(t) = 2 \operatorname{Re}(-i\Sigma_{s,o}(t)) = 2\delta m_{s,o}(t)$$

$$\Sigma_{s,o}(t) + \Sigma_{s,o}^\dagger(t) = -2 \operatorname{Im}(-i\Sigma_{s,o}(t)) = \Gamma_{s,o}(t)$$

- Ξ_{so} accounts for the **production of singlets through the decay of octets**, and Ξ_{os} and Ξ_{oo} account for the **production of octets through the decays of singlets and octets** respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.
- The conservation of the trace of the sum of the densities, i.e., the **conservation of the number of heavy quarks**, follows from

$$\operatorname{Tr} \left\{ \rho_s(t; t) \left(\Sigma_s(t) + \Sigma_s^\dagger(t) \right) \right\} = \operatorname{Tr} \left\{ \Xi_{os}(\rho_s(t; t), t) \right\}$$

$$\operatorname{Tr} \left\{ \rho_o(t; t) \left(\Sigma_o(t) + \Sigma_o^\dagger(t) \right) \right\} = \operatorname{Tr} \left\{ \Xi_{so}(\rho_o(t; t), t) + \Xi_{oo}(\rho_o(t; t), t) \right\}$$

From the evolution equations to the Lindblad equation

- We observe the evolution for **large times** (\propto freeze-out time).
Hence for $t - t_0$ larger than any other time scale that appears in the problem

$$\int_{t_0}^t dt_2 f(t_2) \approx \int_0^\infty ds f(t - s)$$

- We assume the evolution to be **quasistatic**: $1/T \times dT/dt \sim 1/t \ll E$.
The time dependence of the temperature can be neglected at leading order

$$\langle E^{a,i}(t, \mathbf{0}) E^{a,j}(t - s, \mathbf{0}) \rangle \approx \langle E^{a,i}(s, \mathbf{0}) E^{a,j}(0, \mathbf{0}) \rangle \approx \langle E^{a,i}(0, \mathbf{0}) E^{a,j}(-s, \mathbf{0}) \rangle$$

Under the above conditions the evolution equations can be written in the **Lindblad form**.

The strongly coupled case $m_D \sim T \gg E$

If $E \ll T \sim m_D$ the Lindblad equation for a strongly coupled plasma reads

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2} \{C_i^\dagger C_i, \rho\})$$
$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$
$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$
$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \quad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

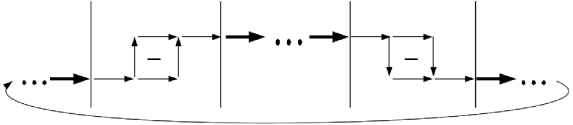
Results:

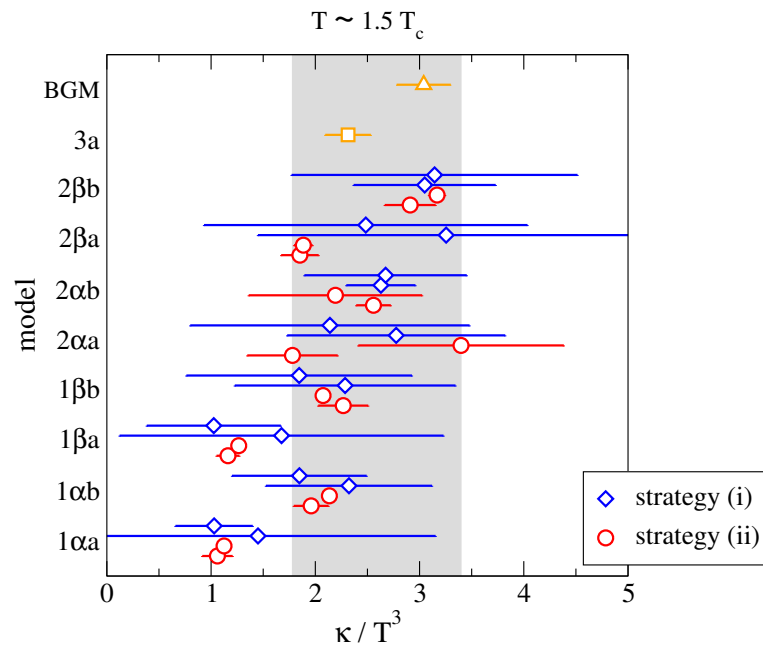
The Out of Equilibrium Evolution

κ

Low energy parameters may be determined by numerical calculations in lattice QCD.

κ is the heavy-quark **momentum diffusion coefficient**:

$$\kappa = \frac{g^2}{6 N_c} \text{Re} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle =$$




γ

$$\gamma = \frac{g^2}{6 N_c} \text{Im} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle$$

γ is known only in perturbation theory:

$$\gamma = -3\zeta(3)C_F \frac{\alpha_s(m_D)}{\pi} T m_D^2 + \frac{4}{3}\zeta(3)N_c C_F \alpha_s^2(\mu_T) T^3$$

A value that at leading order is negative.

- Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

Initial conditions

- The production of singlets is α_s suppressed compared to that of octets.
- Cho Leibovich PRD 53 (1996) 6203
- Our choice at $t = 0$ is

$$\rho_s = N|\mathbf{0}\rangle\langle\mathbf{0}|, \quad \rho_o = \frac{\delta}{\alpha_s(M)}\rho_s$$

N is fixed by $\text{Tr}\{\rho_s\} + \text{Tr}\{\rho_o\} = 1$

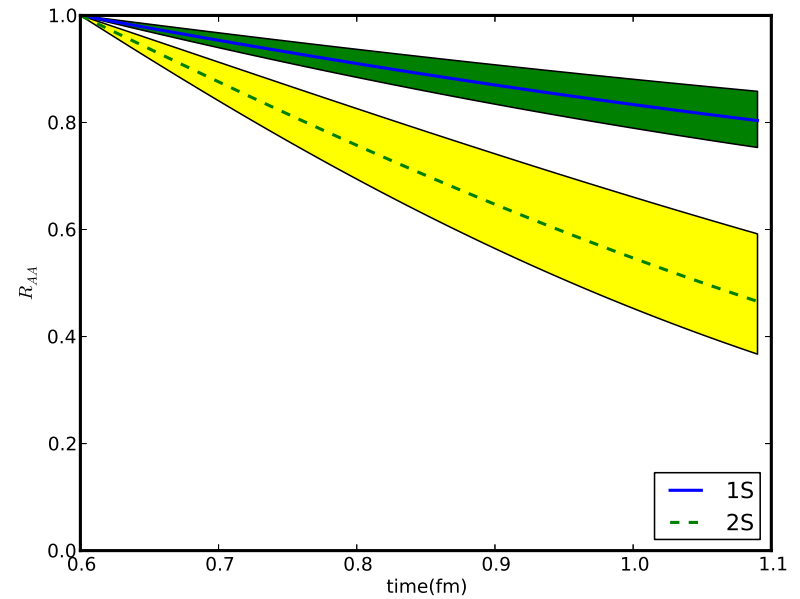
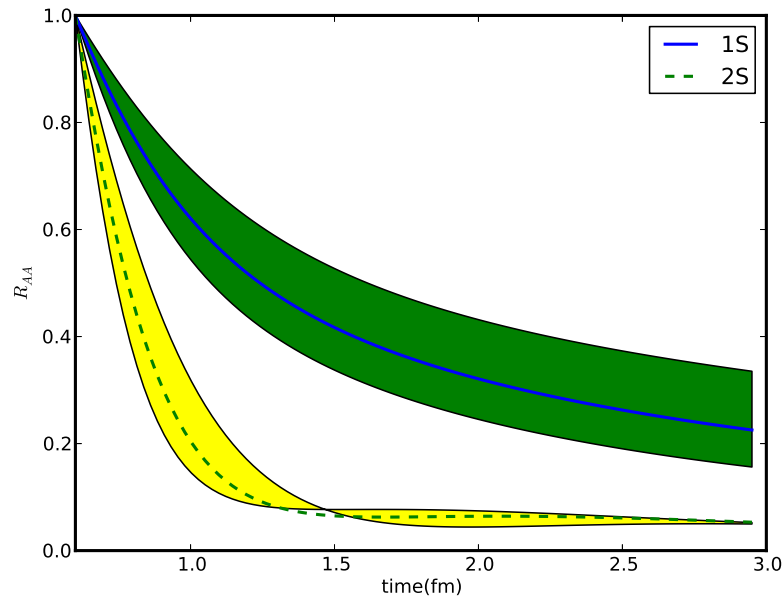
δ fixes the octet fraction with respect to the singlet.

Nuclear modification factor

We compute the nuclear modification factor R_{AA} :

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$

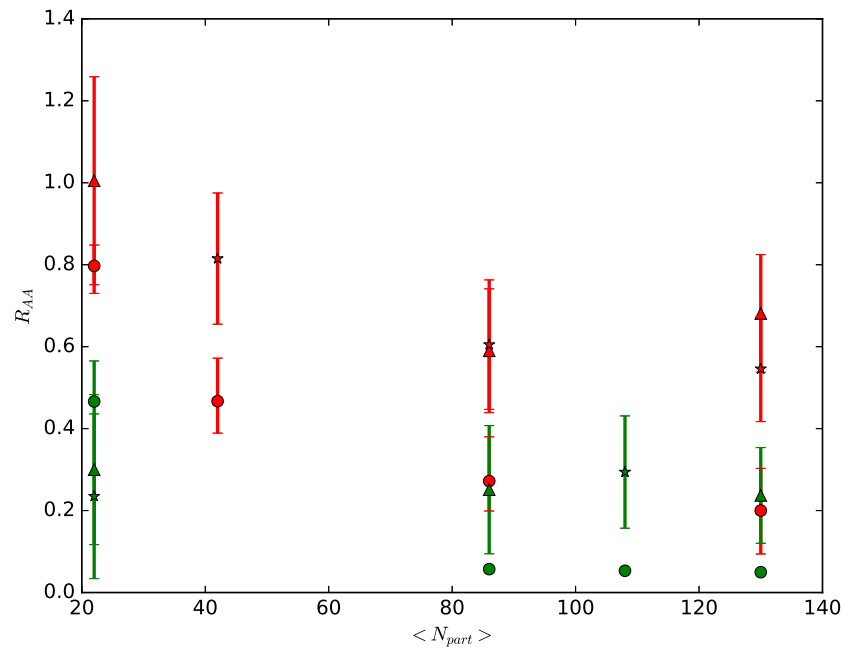
Time evolution of R_{AA} for $1S$ and $2S$ bottomonium



30 – 50% centrality (left) and 50 – 100% centrality (right)

$\gamma = 0$ and $\delta = 1$

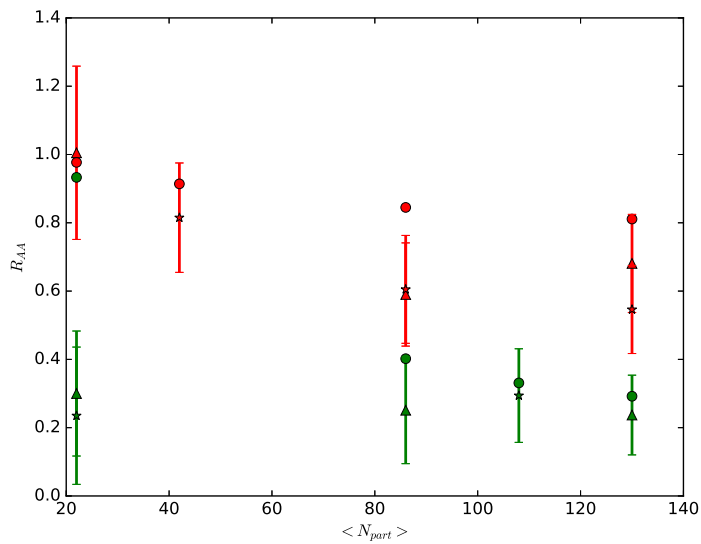
Bottomonium nuclear modification factor vs CMS data



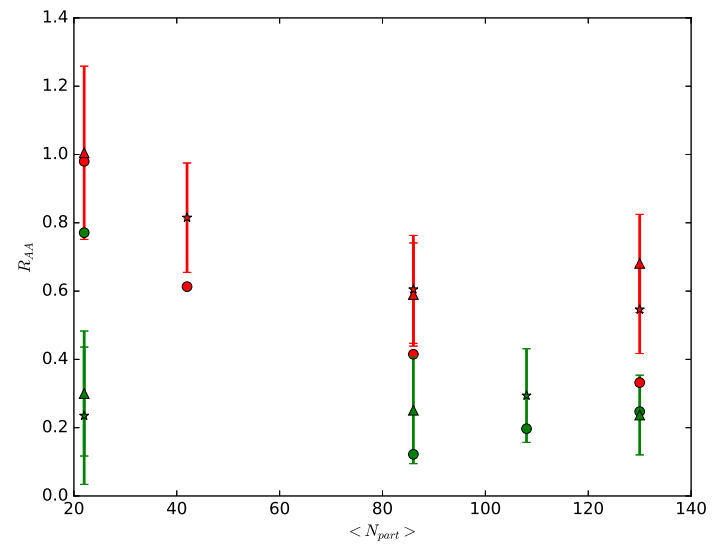
○: theory results; △: CMS data; red: $\Upsilon(1S)$; green: $\Upsilon(2S)$.

○ CMS coll. PRL 109 (2012) 222301 and PLB 770 (2017) 357
Brambilla Escobedo Soto Vairo PRD 96 (2017) 034021

Bottomonium nuclear modification factor vs CMS data



$\kappa/T^3 = 0.25$ and $\gamma = 0$;



$\kappa/T^3 = 2.6$ and $\gamma/T^3 = 6$.

○ Brambilla Escobedo Soto Vairo (2017) 1711.xxxxx

Conclusions

We have shown how the heavy quark-antiquark pair **out-of-equilibrium evolution** can be treated in the framework of pNRQCD. With respect to previous determinations:

- the medium may be a **strongly-coupled plasma** (not necessarily a quark-gluon plasma) whose characteristics are determined by lattice calculations;
- the **total number of heavy quarks**, i.e., $\text{Tr}\{\rho_s\} + \text{Tr}\{\rho_o\}$, **is preserved** by the evolution equations;
- the **non-abelian** nature of QCD is fully accounted for;
- the treatment does **not rely on classical approximations**.

The evolution equations follow from assuming the **inverse size of the quark-antiquark system to be larger than any other scale** of the medium and from being accurate at **first non-trivial order in the multipole expansion** and at **first order in the heavy-quark density**.

Under some conditions (**large time, quasistatic evolution, temperature much larger than the inverse evolution time of the quarkonium**) the evolution equations are of the **Lindblad form**. Their numerical solution provides $R_{AA}(nS)$ close to experimental data.

Outlook

Several improvements are possible starting from this first analysis, e.g.,

- use more refined models for the **hydrodynamical evolution** than the Bjorken one;
- use more refined initial conditions accounting for **higher-order production mechanisms**, or for state of matters, like the color glass condensate;
- include the **momentum dependence** of the quark-antiquark pairs;
- determine κ and γ on unquenched lattices for wide ranges of temperature;
- relax the conditions leading to the Lindblad form and **solve directly the non-Lindblad evolution equations**;
- ...