Quarkonium production and open quantum systems

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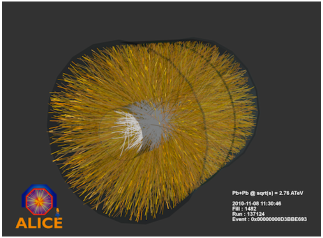
Refs: Akamatsu-Rothkopf (12), Akamatsu (15), Kajimoto et al (17)

Quarkonia in the Quark-Gluon Plasma

QGP fireball in heavy-ion collisions

- LHC: Pb+Pb $\sqrt{s_{NN}} = 5.5$ TeV, RHIC: Au+Au $\sqrt{s_{NN}} = 0.2$ TeV
- Number of produced particles $\sim$ 20000, Temperature ~ 0.5 GeV

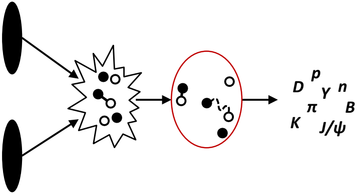


Heavy quark production mostly in initial hard collisions ($M_{c,b} \gg T$)

	$\sqrt{s_{NN}}$ [TeV]	$N_{c\bar{c}}$	$N_{b\bar{b}}$	T [GeV]
RHIC	0.2	10	-	0.4
LHC	5.5	100	3	0.8

Quarkonia are impurity probes for QGP

Quarkonium production out of hot QGP



Classic scenarios for quarkonium production in heavy-ion collisions

1. Suppression due to Debye screened potential in QGP [Matsui-Satz (86)]

$$V(r) = -\frac{\alpha}{r} e^{-m_D r}$$

2. Enhanced statistical production from initially uncorrelated pairs

   [BraunMunzinger-Stachel (00), Thews-Schroedter-Rafelski (01)]

   - Important for J/ψ at the LHC? ($N_{c\bar{c}} \sim 100$)

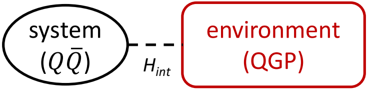
How do quarkonia and heavy quarks propagate in QGP?

Why open quantum systems?

Production rate is calculated with reduced density matrix of a quarkonium

$$N_{\Upsilon}(t_F) = \int_{x,y} \rho_{b\bar{b}}(x,y,t_F)\, \underbrace{P_{\Upsilon}(y,x)}_{\text{projection}}$$

Open quantum system (quarkonium + QGP)



1. Total density matrix & von Neumann equation:

$$\rho_{\text{tot}}(t) = \sum_{\Psi \in \mathcal{H}_{\text{tot}}} w_{\Psi} |\Psi(t)\rangle\langle\Psi(t)|, \quad i\frac{d}{dt}\rho_{\text{tot}} = [H_{\text{tot}}, \rho_{\text{tot}}]$$

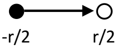
2. Reduced density matrix & Master equation:

$$\rho_{Q\bar{Q}}(t) \equiv \text{Tr}_{\text{QGP}}\, \rho_{\text{tot}}(t), \quad i\frac{d}{dt}\rho_{Q\bar{Q}} = [\text{potential} + \text{fluctuation} + \text{dissipation}]$$

Quarkonia probe in-medium forces in the QGP

Stochastic Potential Model

Stochastic potential model [Akamatsu-Rothkopf (12)]



Energy of a quarkonium in a thermal bath

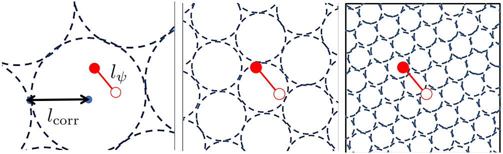
$$H = -\frac{\nabla^2}{M} + \underbrace{V(r)}_{\text{screening}} + \underbrace{\theta(t, r/2)}_{\text{noise for } Q} - \underbrace{\theta(t, -r/2)}_{\text{noise for } \bar{Q}}$$

$$\langle \theta(t_1, x_1)\theta(t_2, x_2)\rangle = \delta(t_1 - t_2) D(x_1 - x_2)$$

Debye screened potential and noise with finite correlation length

$$V(r) = -\frac{\alpha_{\mathsf{eff}}}{r} e^{-m_D r}, \quad D(r) = \gamma \exp[-r^2/l_{\mathsf{corr}}^2]$$

$$\alpha_{\mathsf{eff}} \sim \alpha_s, \quad m_D \sim gT, \quad \gamma \sim \alpha_s T, \quad l_{\mathsf{corr}} \sim 1/gT$$



$l_{\text{corr}} \gg l_\psi$ $\qquad$ $l_{\text{corr}} \sim l_\psi$ $\qquad$ $l_{\text{corr}} \ll l_\psi$

l_{corr} controls the effectiveness of wavefunction decoherence

Relation to the complex potential

Real-time quarkonium propagator with $M = \infty$

- Averaged wave function in the stochastic potential model

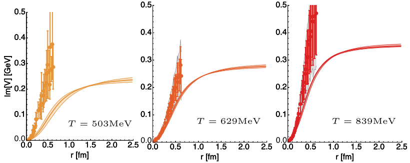
$$G_>(t,0;r) = \langle \Psi(t,r)\rangle_\theta = \exp[-it \underbrace{\{V(r) - i(D(0) - D(r))\}}_{\text{complex potential } V_{\mathsf{Re}} + iV_{\mathsf{Im}}}]$$

- Perturbation theory for $r \sim 1/gT$ (Landau damping)

[Laine et al (07), Beraudo et al (08), Brambilla et al (08)]

$$V_{\rm Im}(r) = -\frac{g^2 T C_F}{4\pi}\phi(m_D r), \quad \phi(x) = 2\int_0^\infty \frac{dz\ z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right],$$

- Lattice QCD simulation [Rothkopf et al (17,. . .,12)]



$$\to V_{\mathsf{Im}}(r) \sim -\gamma\left[1 - e^{-r^2/l_{\mathsf{corr}}^2}\right]$$

Noise correlation function is extracted from the imaginary potential

1-dim simulation of quarkonia in a Bjorken-expanding medium

1. Temperature in a Bjorken expansion

$$T(t) = T_0 \left(\frac{t_0}{t_0 + t} \right)^{1/3}, \quad T_0 = 0.4 \text{ GeV}, \quad t_0 = 1 \text{ fm}$$

2. Temperature dependent parametrization

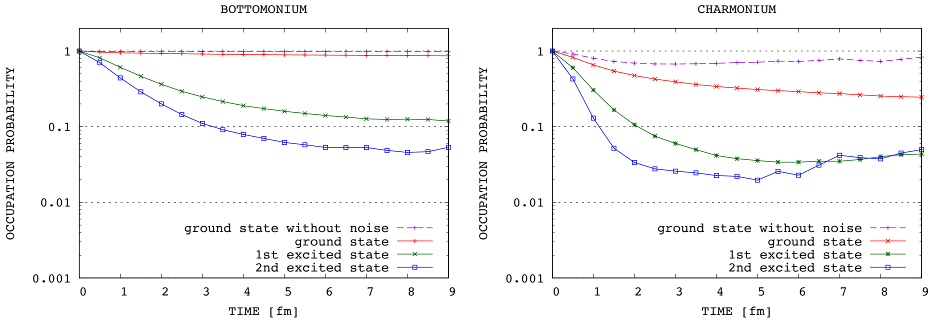
$$V(r) = -\frac{\alpha_{\mathsf{eff}}}{r} e^{-m_D r}, \quad D(r) = \gamma \exp[-r^2/l_{\mathsf{corr}}^2]$$

m [GeV]	α_{eff}	m_D	γ	l_{corr}
4.8 / 1.18	0.3	T	$0.3T$	$1/T$

3. Start from vacuum eigenstates and calculate their survival probability

$$V_{\mathsf{vac}}(r) = -\frac{\alpha_{\mathsf{eff}}}{r} + \sigma r, \quad \sigma = 1 \text{ GeV/fm}, \quad N_{\Upsilon}(t) = \langle \| \Psi_{\Upsilon}^* \cdot \Psi_{b\bar{b}}(t) \|^2 \rangle$$

Quarkonia in a Bjorken-expanding QGP [Kajimoto et al (17)]



- Decoherence more effective for larger radius l_Ψ

$$l_\Psi^{b\bar{b}} < l_\Psi^{c\bar{c}},\ l_\Psi^{\text{ground}} < l_\Psi^{\text{excited}} \to R_{AA}^{b\bar{b}} > R_{AA}^{c\bar{c}},\ R_{AA}^{\text{ground}} > R_{AA}^{\text{excited}}$$

- Noise enhances quarkonium dissociation rate

$$(R_{AA}^{\text{complex}} <) R_{AA}^{\text{stochastic}} < R_{AA}^{\text{Debye}}$$

Decoherence gives an additional dynamical dissociation mechanism

Theoretical Developments

Stochastic potential and Lindblad master equation

Stochastic potential does not describe dissipation and overheats quarkonia

$$i\frac{d}{dt}\rho_{Q\bar{Q}} = [\text{potential} + \text{fluctuation} + \cancel{\text{dissipation}}]$$

Lindblad form master equation [Lindblad (76)]

1. General form ensuring positivity of the density matrix

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i[H, \rho_{Q\bar{Q}}] + \sum_{i=1}^{N}\left(L_i\rho_{Q\bar{Q}}L_i^\dagger - \frac{1}{2}L_i^\dagger L_i\rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}}L_i^\dagger L_i\right)$$

2. Lindblad operators for stochastic potential and dissipation [Akamatsu (15)]

$$L_k = \underbrace{\sqrt{D(k)}e^{ikx}}_{\Delta p_Q = k} \underbrace{\times(1 \text{ or } t^a)}_{\text{U(1) or SU}(N_c)} \to \sqrt{D(k)}e^{ikx/2}\left[1 + \underbrace{\frac{ik\cdot\nabla_x}{4MT}}_{\Delta x_Q \sim k/MT}\right]e^{ikx/2}$$

- Dissipation is given by introducing recoil of heavy quark in a collision

Quantum dissipation will become important in phenomenological studies

Recent developments of open quantum systems for quarkonia

Color singlet-octet transitions in QGP in the Lindblad form [Brambilla et al (17)]

- Based on potential NRQCD effective field theory (visit Antonio's talk)

$$L_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix}, \quad L_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- Same structure found in SU(N_c) stochastic potential at short distance

Generalized Langevin dynamics for a heavy quark pair [Blaizot et al (16)]

- Correlation not only the potential but also drag and random forces
  - Interference between $Q+g \to Q+g$ and $\bar{Q}+g \to \bar{Q}+g$

$$M\ddot{r} + \frac{\beta g^2}{2}(\mathcal{H}(0)\dot{r} - \mathcal{H}(s)\dot{\bar{r}}) - g^2\nabla V(s) = \xi(s,t)$$

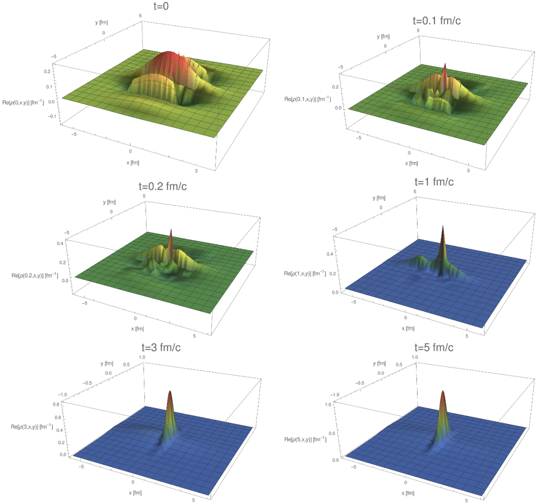
$$M\ddot{\bar{r}} + \frac{\beta g^2}{2}(\mathcal{H}(0)\dot{\bar{r}} - \mathcal{H}(s)\dot{r}) + g^2\nabla V(s) = \bar{\xi}(s,t)$$

$$\langle \xi(s,t)\xi(s,t')\rangle = g^2\mathcal{H}(0)\delta(t-t'), \quad \langle \xi(s,t)\bar{\xi}(s,t')\rangle = -g^2\mathcal{H}(s)\delta(t-t')$$

First simulation of quantum dissipation of quarkonia

1-dim simulation of quantum master equation [DeBoni (17)]

- Time evolution of the density matrix



Conclusion and outlook

- Quarkonium in QGP is described as open quantum system
  1. Stochastic potential is simulated for a Bjorken expanding background
  2. Developments in the Lindblad master equation

- Sophisticate modeling and numerics toward realistic phenomenology
  1. Simulation of master equation in $1 \rightarrow 3$ dimensions with quantum dissipation and colors
     - Numerical simulation with an extended stochastic Schrödinger equation [Akamatsu et al, in progress]
  2. Classicalization: Matching the overlapping regimes of decoherence and classical dissipation

Backup sildes

Open quantum system from microscopic theory

1. Path integral formula

$$\rho_{\mathrm{tot}}(t, \underbrace{x, y}_{\in Q\bar{Q}}, \underbrace{X, Y}_{\in \mathrm{QGP}}) = \int dx_0 dy_0 dX_0 dY_0 \int_{x_0,y_0,X_0,Y_0}^{x,y,X,Y} \mathcal{D}[\bar{x}, \bar{y}, \bar{X}, \bar{Y}]$$

$$\times \underbrace{\rho_{\mathrm{tot}}(0, x_0, y_0, X_0, Y_0)}_{\text{factorizable } \rho_{Q\bar{Q}}(0) \otimes \rho^{\mathrm{eq}}_{\mathrm{QGP}}} e^{iS_{\mathrm{tot}}[\bar{x},\bar{X}] - iS_{\mathrm{tot}}[\bar{y},\bar{Y}]}$$

2. Influence functional S_{IF} [Feynman-Vernon (63)]
   - Average over the QGP environment by tracing out ρ_{tot}

$$\rho_{Q\bar{Q}}(t, x, y) = \underbrace{\int dX dY \delta(X - Y)}_{\text{trace out QGP = path closed at } t} \rho_{\mathrm{tot}}(t, x, y, X, Y)$$

$$= \int dx_0 dy_0 \rho_{Q\bar{Q}}(0, x_0, y_0) \int_{x_0,y_0}^{x,y} \mathcal{D}[\bar{x}, \bar{y}] e^{iS_{Q\bar{Q}}[\bar{x}] - iS_{Q\bar{Q}}[\bar{y}] + iS_{\mathrm{IF}}[\bar{x},\bar{y}]}$$

Influence functional contains all the information of the open system

Coarse graining in the influence functional

1. Influence functional up to quadratic order:

$$iS_{\mathsf{IF}}[x,y] = -\frac{1}{2}\int_0^t dt'dt''(x,y)_{(t')}\underbrace{\begin{pmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22}\end{pmatrix}_{(t',t'')}}_{\text{correlation functions of } X, Y}\begin{pmatrix} x \\ y \end{pmatrix}_{(t'')} + \dots$$

2. Time coarse graining by derivative expansion [Diosi (93), Akamatsu (15)]

$$iS_{\mathsf{IF}} = \underbrace{iS_{\mathsf{fluct}}}_{\propto xx} + \underbrace{iS_{\mathsf{diss}}}_{\propto x\dot{x}} + \underbrace{iS_{\mathsf{L}}}_{\propto \dot{x}\dot{x}}$$

3. Lindblad operators for heavy quarks for $r \sim 1/gT$ [Akamatsu (15)]
   - Recoil of heavy quark in a collision

$$L_k = \underbrace{\sqrt{D(k)}e^{ikx}}_{S_{\mathsf{fluct}}\text{ only}} \xrightarrow{+S_{\mathsf{diss}}+S_{\mathsf{L}}} \sqrt{D(k)}e^{ikx/2}\left[1 + \underbrace{\frac{ik\cdot\nabla_x}{4MT}}_{\Delta x_Q \sim k/MT}\right]e^{ikx/2}$$

Dissipation is introduced by time coarse graining the influence functional