Heavy Quark Symmetry and Stable Tetraquarks

Estia Eichten

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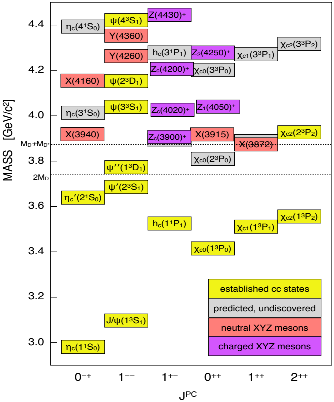
EE & Chris Quigg, arXiv:1707.09575 $\rightarrow$ *PRL*

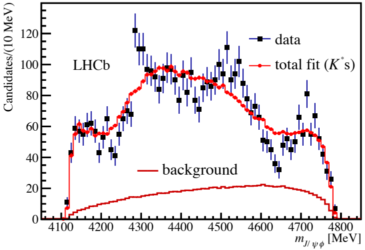
The XYZ states

- $Q\bar{Q}q_i\bar{q}_j$
  - X(3872)
  - $Z_c^{\pm}(3885)$, $Z_c^{\pm}(4025)$, $Z_b^{\pm}(10610)$, $Z_b^{\pm}(10650)$
  - ...
- All these states at or above the associated heavy-light meson pair threshold.



More $c\bar{c}q_i\bar{q}_j$ states

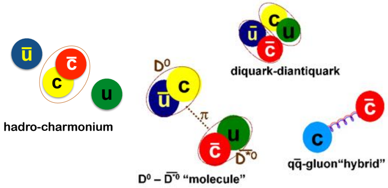
Particle	J^P	Signif-icance	Mass (MeV)	Γ (MeV)	Fit Fraction (%)
X(4140)	1^+	8.4 σ	$4146.5 \pm 4.5^{+4.6}_{-2.8}$	$83 \pm 21^{+21}_{-14}$	$13.0 \pm 3.2^{+4.8}_{-2.0}$
X(4274)	1^+	6.0 σ	$4273.3 \pm 8.3^{+17.2}_{-3.6}$	$56 \pm 11^{+8}_{-11}$	$7.1 \pm 2.5^{+3.5}_{-2.4}$
X(4500)	0^+	6.1 σ	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	$6.6 \pm 2.4^{+3.5}_{-2.3}$
X(4700)	0^+	5.6 σ	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$12 \pm 5^{+9}_{-5}$



- Thresholds: $D_s\bar{D}_s^*$ (4081), $D_s^*\bar{D}_s^*$ (4225), $D_s(1P_0)\bar{D}_s(1P_0)(4636)$
- SU(3) symmetry $\to$ $c\bar{c}u\bar{s}$, $c\bar{c}d\bar{s}$ states

QCD dynamics of charmonium-like states

- Many models
  - diquark-diantiquark
  - molecule
  - hadro-charmonium
  - hybrid
  - cusp effect



- Complicated dynamics
- LQCD approaches
- The systematic variation with heavy quark mass may help to distinguish models. $(c\bar{c}), (c\bar{b}), (b\bar{b})$

HQS implies stable heavy tetraquark mesons $Q_iQ_j\bar{q}_k\bar{q}_l$

- In the limit of very heavy quarks Q, novel narrow doubly heavy tetraquark states must exist.
- HQS relates the mass of a doubly heavy tetraquark state to combination of the masses of a doubly heavy baryon, a singly heavy baryon and a heavy-light meson.
- The lightest double-beauty states composed of $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ will be stable against strong decays.
- Heavier $bb\bar{q}_k\bar{q}_l$ states, double-charm states $cc\bar{q}_k\bar{q}_l$, mixed $bc\bar{q}_k\bar{q}_l$ states, will dissociate into pairs of heavy-light mesons.
- Observing a weakly decaying double-beauty state would establish the existence of tetraquarks and illuminate the role of heavy color-antitriplet diquarks as hadron constituents.

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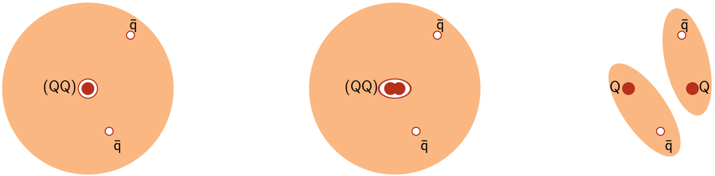
Systematics of doubly heavy tetraquarks

- Ground states - S waves.
  - $Q_i \bar{Q}_j$ color $(1, 8)$ spin $(0, 1)$ (Quarkonium-like)
  - $\{Q_i Q_j\}$ color $\bar{3}$ spin 1 or color 6 spin 0 (flavor symmetic)
  - $[Q_i Q_j]$ color $\bar{3}$ spin 0 or color 6 spin 1 (flavor antisymmetic)
- $m(Q_i) > \Lambda_{\mathrm{QCD}} > m(q_j)$
- The static energy between the heavy quarks is a $(2x2)$ matrix in color. As the separation, R, is varied:
  - Energy varies.
  - Color admixture varies.



Dynamics

- For small $Q_i - Q_j$ separation the interaction is attractive in the color $\bar{3}$ and repulsive for the color 6.
  - The effective potential for color $\bar{3}$ is given by $\frac{1}{2}V_{Q\bar{Q}}(R)$. (LQCD)
  - In a half-strength Cornell potential, rms core radii are small on tetraquark scale: $\langle r^2\rangle^{1/2} = 0.28$ fm (cc); 0.24 fm (bc); 0.19 fm (bb).
- For large $Q_i - Q_j$ separation the light quarks mostly shield the color and the system rearranges into two heavy-light mesons.
- As $m(Q_i), m(Q_j) \to \infty$ the ground state of $Q_i Q_j \bar{q}_k \bar{q}_l$ has the properties:
  - The two heavy quarks are attracted close together in a color $\bar{3}$
  - The tetraquark state becomes STABLE to decay into two heavy-light mesons.
    (eg. $m(Q_i Q_i \bar{q}_k \bar{q}_k) - 2m(Q_i \bar{q}_k) = \Delta - \frac{1}{2}(\frac{2}{3}\alpha_s)^2 m(Q_i) + O(\frac{1}{m(Q_i)})$ with Δ fixed)

Heavy quark symmetry relations

- In the heavy limit, the color of the core Q_iQ_j is $\bar{3}$ the same as a $\bar{Q}_x$. Hence in leading order of $\mathcal{M}^{-1}$ the light degrees of freedom have the same dynamics in the two systems leading to the following relations

$$
\begin{aligned}
m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y)\,.
\end{aligned}
$$

- Finite mass corrections for all the states in these relations:

$$\delta m = \mathcal{S}\frac{\vec{S}\cdot\vec{j_\ell}}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}}$$

Stability

- Stable against decay to two heavy-light mesons.
- Decay to doubly heavy baryon and light antibaryon?

$$(Q_i Q_j \bar{q}_k \bar{q}_l) \to (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m)$$

  - Starting from the HQS relation
$$m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m)$$
    stability requires
$$m(Q_x q_k q_l) - m(Q_x \bar{q}_m) < m(q_k q_l q_m)$$
  - $\mathcal{M} \to \infty$ does not systematically improve the stability.
  - $m(Q_x q_k q_l) - m(Q_x \bar{q}_m)$ has form $\Delta_0 + \Delta_1 / M_{Q_x}$.
    $m(\Lambda_c) - m(D) = 416.87$ MeV and $m(\Lambda_b) - m(B) = 340.26$ MeV,
    $\Delta_0 \approx 330$ MeV
  - $m(q_k q_l q_m) > 938$ MeV

$$\boxed{\text{As } \overline{M} \to \infty\text{, stable } Q_i Q_j \bar{q}_k \bar{q}_l \text{ mesons must exist}}$$

Known ground-state hadrons containing heavy quarks

- The spin dependent corrections can be directly calculated from the known mass spectrum.

State	j_ℓ	Mass $(j_\ell + \frac{1}{2})$	Mass $(j_\ell - \frac{1}{2})$	Centroid	Spin Splitting	$\mathcal{S}$ [GeV^2]
$D^{(*)}$ $(c\bar{d})$	$\frac{1}{2}$	2010.26	1869.59	1975.09	140.7	0.436
$D_s^{(*)}$ $(c\bar{s})$	$\frac{1}{2}$	2112.1	1968.28	2076.15	143.8	0.446
Λ_c $(cud)_{\bar{\mathbf{3}}}$	0	2286.46	–	–		–
Σ_c $(cud)_{\mathbf{6}}$	1	2518.41	2453.97	2496.93	64.44	0.132
Ξ_c $(cus)_{\bar{\mathbf{3}}}$	0	2467.87	–	–		–
Ξ_c' $(cus)_{\mathbf{6}}$	1	2645.53	2577.4	2622.82	68.13	0.141
Ω_c $(css)_{\mathbf{6}}$	1	2765.9	2695.2	2742.33	70.7	0.146
Ξ_{cc} $(ccu)_{\bar{\mathbf{3}}}$	0	3621.40	–		–	
$B^{(*)}$ $(b\bar{d})$	$\frac{1}{2}$	5324.65	5279.32	5313.32	45.33	0.427
$B_s^{(*)}$ $(b\bar{s})$	$\frac{1}{2}$	5415.4	5366.89	5403.3	48.5	0.459
Λ_b $(bud)_{\bar{\mathbf{3}}}$	0	5619.58	–		–	
Σ_b $(bud)_{\mathbf{6}}$	1	5832.1	5811.3	5825.2	20.8	0.131
Ξ_b $(bds)_{\bar{\mathbf{3}}}$	0	5794.5	–		–	
Ξ_b' $(bds)_{\mathbf{6}}$	1	5955.33	5935.02	5948.56	20.31	0.128
Ω_b $(bss)_{\mathbf{6}}$	1		6046.1			
B_c $(b\bar{c})$	$\frac{1}{2}$	*6329*	6274.9	*6315.4*	*54*	*0.340*

Determining $\mathcal{K}$

$$m = m_0 + \mathcal{S}\frac{\vec{S}\cdot\vec{j_\ell}}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}} + O(\frac{1}{\mathcal{M}})$$

- Kinetic-energy shift differs in $Q\bar{q}$ mesons and Qqq baryons.

$$\delta\mathcal{K} \equiv \mathcal{K}_{(ud)} - \mathcal{K}_d$$

- Using known cog mass splittings:

$$\begin{aligned}[m((cud)_{\bar{\mathbf{3}}}) - m(c\bar{d})] - [m((bud)_{\bar{\mathbf{3}}}) - m(b\bar{d})] \\ = \delta\mathcal{K}\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) = 5.11\text{ MeV}\end{aligned}$$

yields $\delta\mathcal{K} = 0.0235\text{ GeV}^2$

($m_c = m(J/\psi)/2$ and $m_b = m(\Upsilon)/2$ used)

Putting it all together

1. The RHS of the HQS relation:

$$m(Q_iQ_j\bar{q}_k\bar{q}_l) - m(Q_iQ_jq_m) = m(Q_xq_kq_l) - m(Q_x\bar{q}_m)$$

has been determined by data and $\mathcal{K}$ and $\mathcal{S}$ are known.

2. Knowing $\mathcal{K}$ allows determining the kinetic-energy shifts for the double heavy quark systems.

$$m(\{cc\}(\bar{u}\bar{d})) - m(\{cc\}d): \qquad \frac{\delta\mathcal{K}}{4m_c} = 2.80 \text{ MeV}$$

$$m((bc)(\bar{u}\bar{d})) - m(\{bc\}d): \qquad \frac{\delta\mathcal{K}}{2(m_c+m_b)} = 1.87 \text{ MeV}$$

$$m(\{bb\}(\bar{u}\bar{d})) - m(\{bb\}d): \qquad \frac{\delta\mathcal{K}}{4m_b} = 1.24 \text{ MeV}$$

(only slightly larger than isospin-breaking effects we neglected)

Expectations for ground-state tetraquark masses

State	J^P	j_ℓ	$m(Q_iQ_jq_m)$	HQS relation	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay Channel	$\mathcal{Q}$ [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	0	3663	$m(\{cc\}u)+315$	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	0	3764	$m(\{cc\}s)+392$	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	3663	$m(\{cc\}u)+526$	4146, 4167, 4210	D^+D^0, D^+D^{*0} 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	0	6914	$m([bc]u)+315$	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	0	7010	$m([bc]s)+392$	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	1	6914	$m([bc]u)+526$	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	0	6957	$m(\{bc\}u)+315$	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	0	7053	$m(\{bc\}s)+392$	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	6957	$m(\{bc\}u)+526$	7461, 7472, 7493	BD/B^*D 7146/7190	317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	0	10176	$m(\{bb\}u)+306$	10482	$B^-\bar{B}^{*0}$ 10603	$\boxed{-121}$
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	0	10252	$m(\{bb\}s)+391$	10643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10695/10691	$\boxed{-48}$
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	10176	$m(\{bb\}u)+512$	10674, 10681, 10695	B^-B^0, B^-B^{*0} 10559, 10603	115, 78, 136

RHS+all shifts

Estimating ground-state tetraquark masses

- Decay thresholds
  - Strong decays $(Q_i Q_j \bar{q}_k \bar{q}_l) \not\to (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m)$
  - Must consider decays to a pair of heavy–light mesons case-by-case

- Doubly heavy baryons
  - One doubly heavy baryon observed, Ξ_{cc}

    LHC*b*: $M(\Xi_{cc}^{++}) = 3621.40 \pm 0.78$ MeV
  - At present others must come from model calculations:
    We adopt Karliner & Rosner, *PRD* **90**, 094007 (2014)
  - Future: Experiment or LQCD doubly heavy baryon calculations

Expectations for ground-state tetraquark masses

State	J^P	$m(Q_i Q_j \bar{q}_k \bar{q}_l)$	Decay Channel	$\mathcal{Q}$ [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	3978	$D^+ D^{*0}$ 3876	102
$\{cc\}[\bar{q}_k \bar{s}]$	1^+	4156	$D^+ D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	4146, 4167, 4210	$D^+ D^0$, $D^+ D^{*0}$ 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	7229	$B^- D^+ / B^0 D^0$ 7146	83
$[bc][\bar{q}_k \bar{s}]$	0^+	7406	$B_s D$ 7236	170
$[bc]\{\bar{q}_k \bar{q}_l\}$	1^+	7439	$B^* D / B D^*$ 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	7272	$B^* D / B D^*$ 7190/7290	82
$\{bc\}[\bar{q}_k \bar{s}]$	1^+	7445	$D B_s^*$ 7282	163
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$\{bb\}[\bar{u}\bar{d}]$	1^+	10482	$B^- \bar{B}^{*0}$ 10603	$\boxed{\mathbf{-121}}$
$\{bb\}[\bar{q}_k \bar{s}]$	1^+	10643	$\bar{B}\bar{B}_s^* / \bar{B}_s \bar{B}^*$ 10695/10691	$\boxed{\mathbf{-48}}$
$\{bb\}\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	10674, 10681, 10695	$B^- B^0$, $B^- B^{*0}$ 10559, 10603	115, 78, 136

Comments

- Denote $\mathcal{T}$ for tetraquark states. So $\mathcal{T}^{\{bb\}}_{[\bar{u}\bar{d}]} = \{bb\}[\bar{u}\bar{d}]$.
- No excited states of doubly heavy tetraquark systems will be stable.
- The assumption of the core $Q_i Q_j$ being dominately a color $\bar{3}$, becomes less reliable as we approach the lowest two heavy-light meson threshold.
- Unstable doubly heavy tetraquarks near thresholds might be observable as resonances in wrong sign BB, BD, DD modes. Prime examples:
  - $\mathcal{T}^{\{bb\}}_{\{\bar{q}_k\bar{q}_l\}}(10681)$ $J^P = 1^+$ with $\mathcal{Q} = 78$ MeV
  - $\mathcal{T}^{[bc]}_{[\bar{q}_k\bar{s}]}(7272)$ $J^P = 1^+$ with $\mathcal{Q} = 82$ MeV
  - $\mathcal{T}^{\{bc\}}_{[\bar{u}\bar{d}]}(7229)$ $J^P = 0^+$ with $\mathcal{Q} = 83$ MeV
  - $\mathcal{T}^{\{cc\}}_{[\bar{u}\bar{d}]}(3978)$ $J^P = 1^+$ with $\mathcal{Q} = 102$ MeV
- Karliner & Rosner model results, arXiv:1707.07666.
  $Q(\{bb\}[\bar{u}\bar{d}]) = -215$ MeV

Observing stable tetraquarks

Opportunities at ATLAS, CMS, LHCb. Ideal for a Tera Z^0 factory.

$J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$ meson, bound by 121 MeV

(77 MeV below $B^-\bar{B}^0\gamma$)

$$\mathcal{T}^{\{bb\}}_{[\bar{u}\bar{d}]}(10482)^- \to \Xi^0_{bc}\bar{p},\ B^-D^+\pi^-, \text{ and } \underbrace{B^-D^+\ell^-\bar{\nu}}_{\text{weak!}}$$

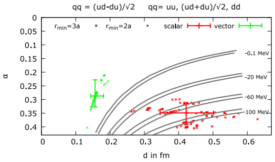
$J^P = 1^+$ $\{bb\}[\bar{u}\bar{s}]$ and $\{bb\}[\bar{d}\bar{s}]$ mesons, bound by 48 MeV

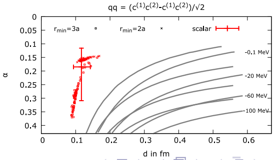
(3 MeV below $BB_s\gamma$)

$$\mathcal{T}^{\{bb\}}_{[\bar{u}\bar{s}]}(10643)^- \to \Xi^0_{bc}\overline{\Sigma}^- \qquad \mathcal{T}^{\{bb\}}_{[\bar{d}\bar{s}]}(10643)^0 \to \Xi^0_{bc}(\bar{\Lambda}, \overline{\Sigma}^0)$$

Generalizing results

- All heavy quarks implies perturbative QCD applies:
  $\{Q_iQ_j\}[\bar{Q}_k\bar{Q}_l]$, $\{Q_iQ_j\}\{\bar{Q}_k\bar{Q}_l\}$, $[Q_iQ_j][\bar{Q}_k\bar{Q}_l]$, $[Q_iQ_j]\{\bar{Q}_k\bar{Q}_l\}$ with
  $m(Q_i) = m(Q_j) = M_1 \geq m(Q_k) = m(Q_l) = M_2 >> \Lambda_{\mathrm{QCD}}$
  A. Czarnecki, B. Leng & M. Voloshin model results, arXiv:1708.04595
  One state (w_{++}) bound for $M_2/M_1 < 0.152$
- $bb\bar{b}\bar{b}$ not bound.
  C. Hughes, E. E., & C. Davies LQCD calculation (see Ciaran's talk here), arXiv:1710.03236
- Can one map out the general region of stability using LQCD?
  Calculate the static energy of the heavier quarks and then use the SE.
  P. Bicudo, K. Cichy, A. Peters, B. Wagenbach &M. Wagner PRD.92.014507
  Fitted $V(r) = -\frac{\alpha}{r}\exp\left(-(\frac{r}{d})^p\right) + V_0$ (with $p = 1.5\ldots2$)





Heavy quark limit for other systems

Likely no other stable states in the heavy-quark limit (or for $Q = c, b$)

- $Q_i \bar{Q}_j q_k \bar{q}_l$
  - By a similar argument as applied to $Q_i Q_j \bar{q}_k \bar{q}_l$
    Can't decay to a pair of heavy flavor hadrons.
    (Ortho-quarkonium)
  - Stability would requires $m(Q_i \bar{Q}_j q_k \bar{q}_l) < m(Q_i \bar{Q}_j) + m(q_k \bar{q}_l)$.
    No argument.
- $Q_i Q_j \bar{q}_k \bar{q}_l q_n$. Same as above.
- $Q_i \bar{q}_k \bar{q}_l q_n$
  - Stability would requires $m(Q_i q_j \bar{q}_k \bar{q}_l) < m(Q_i \bar{q}_k) + m(q_j \bar{q}_l)$.
    No argument.
- $Q_i Q_j \bar{Q}_n q_k q_l$?

Summary

- In the limit of very heavy quarks Q, novel narrow doubly heavy tetraquark states must exist.
- HQS relates the mass of a doubly heavy tetraquark state to combination of the masses of a doubly heavy baryon, a singly heavy baryon and a heavy-light meson.
- The lightest double-beauty states composed of $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ will be stable against strong decays.
- Heavier $bb\bar{q}_k\bar{q}_l$ states, double-charm states $cc\bar{q}_k\bar{q}_l$, mixed $bc\bar{q}_k\bar{q}_l$ states, will dissociate into pairs of heavy-light mesons.
- Observing a weakly decaying double-beauty state would establish the existence of tetraquarks and illuminate the role of heavy color-antitriplet diquarks as hadron constituents.