Exotic Hadrons - A Diquarkonium Perspective

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DESY, Hamburg

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12th Workshop on Heavy Quarkonium, Beijing

- Experimental Evidence for Multiquark states X, Y, Z and P_c
- The Diquark model of Tetraquarks
- Mass Spectrum of the low-lying S and P Wave Tetraquark States
- A New Look at the excited Ω_c and the Υ States in the Diquark Model
- Summary

$X(3872)$ - the poster Child of the X, Y, Z Mesons

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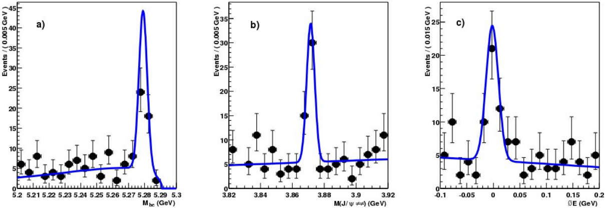
Observation of a Narrow Charmoniumlike State in Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays

S.-K. Choi,[5] S. L. Olsen,[6] K. Abe,[7] T. Abe,[7] I. Adachi,[7] Byoung Sup Ahn,[14] H. Aihara,[43] K. Akai,[7] M. Akatsu,[20] M. Akemoto,[7] Y. Asano,[48] T. Aso,[47] V. Aulchenko,[1] T. Aushev,[11] A. M. Bakich,[38] Y. Ban,[31] S. Banerjee,[39] A. Bondar,[1] A. Bozek,[25] M. Bračko,[18,12] J. Brodzicka,[25] T. E. Browder,[6] P. Chang,[24] Y. Chao,[24] K.-F. Chen,[24] B. G. Cheon,[37] R. Chistov,[11] Y. Choi,[37] Y. K. Choi,[37] M. Danilov,[11] L. Y. Dong,[9] A. Drutskoy,[11] S. Eidelman,[1] V. Eiges,[11] J. Flanagan,[7] C. Fukunaga,[45] K. Furukawa,[7] N. Gabyshev,[7] T. Gershon,[7] B. Golob,[17,12] H. Guler,[6] R. Guo,[22] C. Hagner,[50] F. Handa,[42] T. Hara,[29] N. C. Hastings,[7] H. Hayashii,[21] M. Hazumi,[7] L. Hinz,[16] Y. Hoshi,[41] W.-S. Hou,[24] Y. B. Hsiung,[24,*] H.-C. Huang,[24] T. Iijima,[20] K. Inami,[20] A. Ishikawa,[20] R. Itoh,[7] M. Iwasaki,[43] Y. Iwasaki,[7] J. H. Kang,[52] S. U. Kataoka,[21] N. Katayama,[7] H. Kawai,[2] T. Kawasaki,[27] H. Kichimi,[7] E. Kikutani,[7] H. J. Kim,[52] Hyunwoo Kim,[14] J. H. Kim,[37] S. K. Kim,[36] K. Kinoshita,[3] H. Koiso,[7] P. Koppenburg,[7] S. Korpar,[18,12] P. Križan,[17,12] P. Krokovny,[1] S. Kumar,[30] A. Kuzmin,[1] J. S. Lange,[4,33] G. Leder,[10] S. H. Lee,[36] T. Lesiak,[25] S.-W. Lin,[24] D. Liventsev,[11] J. MacNaughton,[10] G. Majumder,[39] F. Mandl,[10] D. Marlow,[32] T. Matsumoto,[45] S. Michizono,[7] T. Mimashi,[7] W. Mitaroff,[10] K. Miyabayashi,[21] H. Miyake,[29] D. Mohapatra,[50] G. R. Moloney,[19] T. Nagamine,[42] Y. Nagasaka,[8] T. Nakadaira,[43] T. T. Nakamura,[7] M. Nakao,[7] Z. Natkaniec,[25] S. Nishida,[7] O. Nitoh,[46] T. Nozaki,[7] S. Ogawa,[40] Y. Ogawa,[7] K. Ohmi,[7] Y. Ohnishi,[7] T. Ohshima,[20] N. Ohuchi,[7] K. Oide,[7] T. Okabe,[20] S. Okuno,[13] W. Ostrowicz,[25] H. Ozaki,[7] H. Palka,[25] H. Park,[15] N. Parslow,[38] L. E. Piilonen,[50] H. Sagawa,[7] S. Saitoh,[7] Y. Sakai,[7] T. R. Sarangi,[49] M. Satapathy,[49] A. Satpathy,[7,3] O. Schneider,[16] A. J. Schwartz,[3] S. Semenov,[11] K. Senyo,[20] R. Seuster,[6] M. E. Sevior,[19] H. Shibuya,[40] T. Shidara,[7] B. Shwartz,[1] V. Sidorov,[1] N. Soni,[30] S. Stanič,[48,†] M. Starič,[12] A. Sugiyama,[34] T. Sumiyoshi,[45] S. Suzuki,[51] F. Takasaki,[7] K. Tamai,[7] N. Tamura,[27] M. Tanaka,[7] M. Tawada,[7] G. N. Taylor,[19] Y. Teramoto,[28] T. Tomura,[43] K. Trabelsi,[6] T. Tsukamoto,[7] S. Uehara,[7] K. Ueno,[24] Y. Unno,[2] S. Uno,[7] G. Varner,[6] K. E. Varvell,[38] C. C. Wang,[24] C. H. Wang,[23] J. G. Wang,[50] Y. Watanabe,[44] E. Won,[14] B. D. Yabsley,[50] Y. Yamada,[7] A. Yamaguchi,[42] Y. Yamashita,[26] H. Yanai,[27] Heyoung Yang,[36] J. Ying,[31] M. Yoshida,[7] C. C. Zhang,[9] Z. P. Zhang,[35] and D. Žontar[17,12]

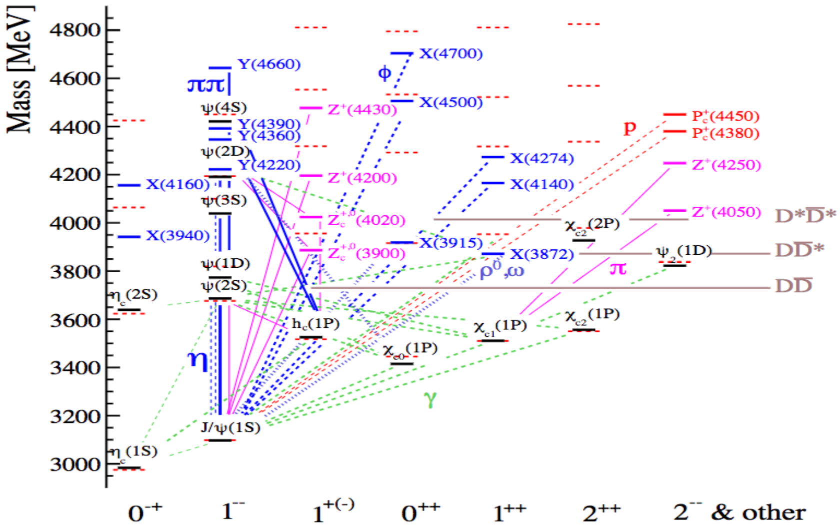
(Belle Collaboration)



- Discovery Mode : $B \to J/\psi\pi^+\pi^-K$
- $M = 3872.0 \pm 0.6 \pm 0.5$ MeV
- $\Gamma < 2.3$ MeV
- $J^{PC} = 1^{++}$ [LHCb] [PRL110, 22201(2013)]

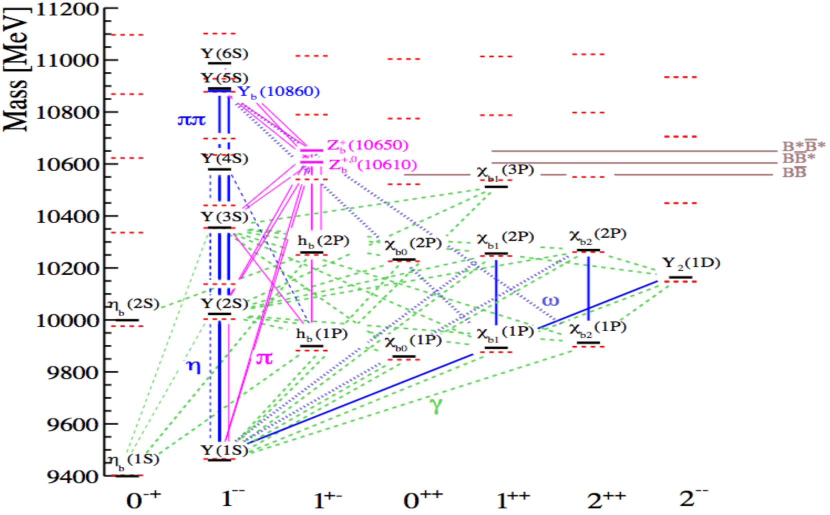
X, Y, Z, P_c and Charmonium States

[S.L. Olsen, T. Skwarnicki, D. Zieminska, arxiv: 1708.04012]



Bottomonium and Bottomonium-like States

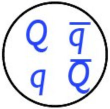
[S.L. Olsen, T. Skwarnicki, D. Zieminska, arxiv: 1708.04012]



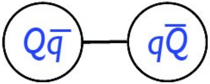
Models for *XYZ* Mesons

Quarkonium Tetraquarks

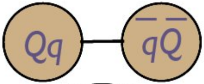
- compact tetraquark



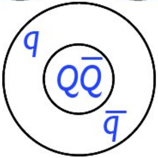
- meson molecule



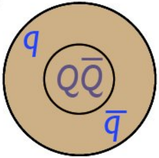
- diquark-onium



- hadro-quarkonium



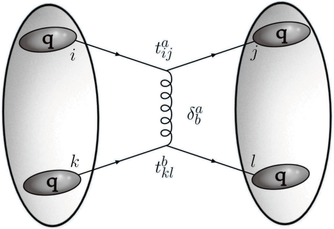
- quarkonium adjoint meson



Diquarks: Color Representation

- One gluon exchange model [Jaffe,Phys.Rept.(2005)]

  ↪ Color factor determines binding:
  Negative sign → Attractive



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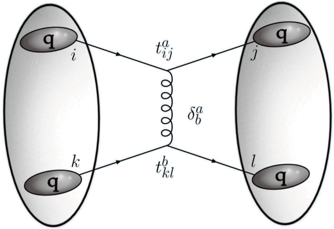

- Quarks in diquark transform as:

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

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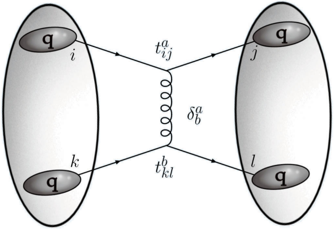

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- qq bound state color factor:

$$t^a_{ij} t^a_{kl} = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{\mathbf{3}}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } \mathbf{6}}$$

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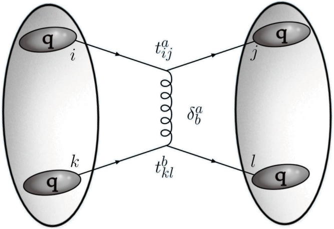

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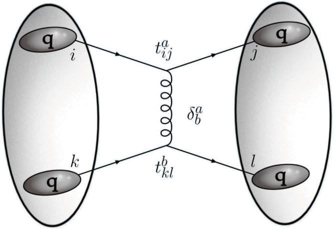

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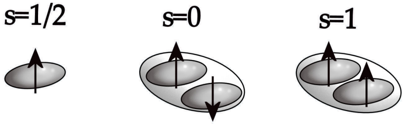
Diquarks: Spin representation



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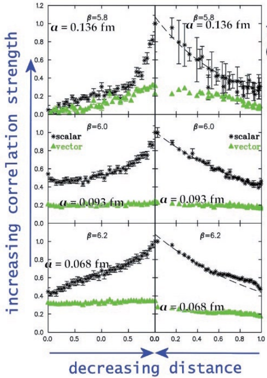

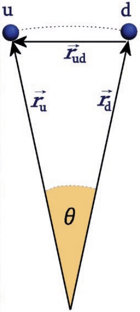

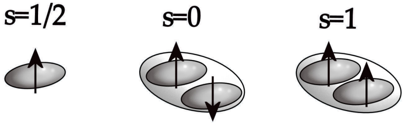

Lattice simulations for light quarks
[Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Calculation of 2 quark correlation strength
- Decreasing distance
  ↪ Increasing strength for "good" diquarks
- Diquark size $\mathcal{O}(1\text{fm})$

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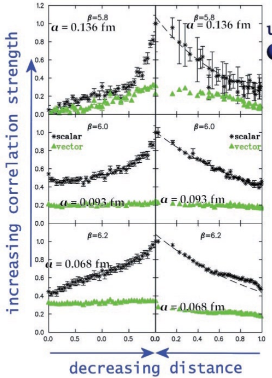

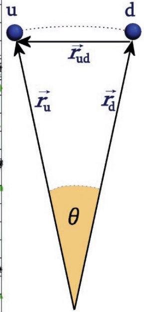

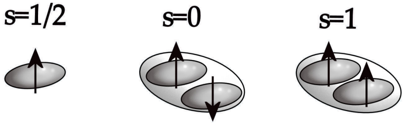

Lattice simulations for light quarks [Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Binding for "good" spin 0 diquarks
- No binding for "bad" spin 1 diquarks

- Calculation of 2 quark correlation strength
- Decreasing distance
  ↪ Increasing strength for "good" diquarks
- Diquark size $\mathcal{O}(1\text{fm})$

Spin decoupling in HQ-Limit

↪ "Bad" diquarks in b-sector might bind

Diquark Model of Tetra- and Pentaquarks

Diquarks and Anti-diquarks are the building blocks of Tetraquarks
Color representation: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive; $C_{\bar{3}} = 1/2\, C_3$

Interpolating diquark operators for the two spin-states of diquarks

$$\begin{array}{rccl} \text{Scalar:} & 0^+ & \mathcal{Q}_{i\alpha} = & \epsilon_{\alpha\beta\gamma}(\bar{c}_c^{\beta}\gamma_5 q_i^{\gamma} - \bar{q}_{i_c}^{\beta}\gamma_5 c^{\gamma}) \\ \text{Axial-Vector:} & 1^+ & \vec{\mathcal{Q}}_{i\alpha} = & \epsilon_{\alpha\beta\gamma}(\bar{c}_c^{\beta}\vec{\gamma} q_i^{\gamma} + \bar{q}_{i_c}^{\beta}\vec{\gamma} c^{\gamma}) \end{array}$$

α, β, γ: $SU(3)_C$ indices

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NR limit: States parametrized by Pauli matrices :

$$\begin{array}{rccl} \text{Scalar:} & 0^+ & \Gamma^0 & = \frac{\sigma_2}{\sqrt{2}} \\ \text{Axial-Vector:} & 1^+ & \vec{\Gamma} & = \frac{\sigma_2\vec{\sigma}}{\sqrt{2}} \end{array}$$

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Diquark spin $s_\mathcal{Q}$ ➡ tetraquark total angular momentum J:

$$\left| Y_{[bq]} \right\rangle = \left| s_\mathcal{Q}, s_{\bar{\mathcal{Q}}};\, J \right\rangle$$

➥ Tetraquarks:

$$\begin{aligned} \left|0_\mathcal{Q}, 0_{\bar{\mathcal{Q}}};\, 0_J\right\rangle &= \Gamma^0 \otimes \Gamma^0 \\ \left|1_\mathcal{Q}, 1_{\bar{\mathcal{Q}}};\, 0_J\right\rangle &= \frac{1}{\sqrt{3}} \Gamma^i \otimes \Gamma_i \ldots \\ \left|0_\mathcal{Q}, 1_{\bar{\mathcal{Q}}};\, 1_J\right\rangle &= \Gamma^0 \otimes \Gamma^i \end{aligned}$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X,Y,Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ}\left[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}\right] + b_Y\frac{S_{12}}{4}$$

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constituent mass

with

$$= b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

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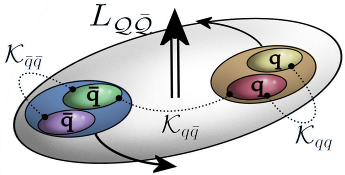
qq spin coupling

$q\bar{q}$ spin coupling

with

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$



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LS coupling *LL* coupling

with

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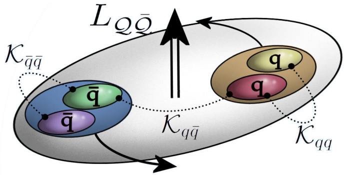
$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L})$$

$$H_{LL} = B_{\mathcal{Q}} \frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)}{2}$$

$$= b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$



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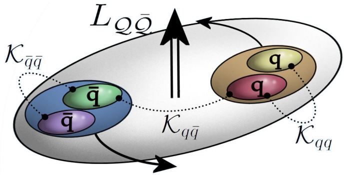
$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L})$$

$$H_{LL} = B_{\mathcal{Q}} \frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)}{2}$$



$$H_T = b_Y \frac{S_{12}}{4} = b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2} L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ}\left[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}\right] + b_Y \frac{S_{12}}{4}$$

Low-lying S-Wave Tetraquark States

- In the $|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$and $|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$bases, the positive parity S-wave tetraquarks are listed below; $M_{00} = 2m_{\mathcal{Q}}$

Label	J^{PC}	$\|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$\|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
X_0	0^{++}	$\|0,0;0,0\rangle_0$	$\left(\|0,0;0,0\rangle_0 + \sqrt{3}\|1,1;0,0\rangle_0\right)/2$	$M_{00} - 3\kappa_{qQ}$
X_0'	0^{++}	$\|1,1;0,0\rangle_0$	$\left(\sqrt{3}\|0,0;0,0\rangle_0 - \|1,1;0,0\rangle_0\right)/2$	$M_{00} + \kappa_{qQ}$
X_1	1^{++}	$\left(\|1,0;1,0\rangle_1 + \|0,1;1,0\rangle_1\right)/\sqrt{2}$	$\|1,1;1,0\rangle_1$	$M_{00} - \kappa_{qQ}$
Z	1^{+-}	$\left(\|1,0;1,0\rangle_1 - \|0,1;1,0\rangle_1\right)/\sqrt{2}$	$\left(\|1,0;1,0\rangle_1 - \|0,1;1,0\rangle_1\right)/\sqrt{2}$	$M_{00} - \kappa_{qQ}$
Z'	1^{+-}	$\|1,1;1,0\rangle_1$	$\left(\|1,0;1,0\rangle_1 + \|0,1;1,0\rangle_1\right)/\sqrt{2}$	$M_{00} + \kappa_{qQ}$
X_2	2^{++}	$\|1,1;2,0\rangle_2$	$\|1,1;2,0\rangle_2$	$M_{00} + \kappa_{qQ}$

- The spectrum of these states depends on just two parameters, $M_{00}(Q)$and κ_{qQ}, $Q = c, b$, hence very predictive
- Some of the states, such as X_0, X_0', X_2, still missing and are being searched for at the LHC

Parameters in the Mass Formula

	charmonium-like	bottomonium-like
M_{00} [MeV]	3957	10630
κ_{qQ} [MeV]	67	22.5

		charmonium-like		bottomonium-like	
Label	J^{PC}	State	Mass [MeV]	State	Mass [MeV]
X_0	0^{++}	—	3756	—	10562
X_0'	0^{++}	—	4024	—	10652
X_1	1^{++}	$X(3872)$	3890	—	10607
Z	1^{+-}	$Z_c^+(3900)$	3890	$Z_b^{+,0}(10610)$	10607
Z'	1^{+-}	$Z_c^+(4020)$	4024	$Z_b^+(10650)$	10652
X_2	2^{++}	—	4024	—	10652

A new look at the Y tetraquarks and the excited Ω_c states in the Diquark model

- Observation of 5 narrow excited Ω_c baryons in $\Omega_c \to \Xi_c^+ K^-$ [LHCb, PRL 118, 182001 (2017)]
- Measured masses (in MeV) [LHCb] and plausible J^P quantum numbers, assuming diquark model $\Omega_c (= css) = c[ss]$ [M. Karliner, J.L. Rosner, PR D95, 114012 (2017)]

$$\begin{aligned}
M(\Omega_c(3000)) &= 3000.4 \pm 0.2 \pm 0.1;\ J^P = 1/2^- \\
M(\Omega_c(3050)) &= 3050.2 \pm 0.1 \pm 0.1;\ J^P = 1/2^- \\
M(\Omega_c(3066)) &= 3065.6 \pm 0.1 \pm 0.3;\ J^P = 3/2^- \\
M(\Omega_c(3090)) &= 3090.2 \pm 0.3 \pm 0.5;\ J^P = 3/2^- \\
M(\Omega_c(3119)) &= 3119.1 \pm 0.3 \pm 0.9;\ J^P = 5/2^-
\end{aligned}$$

- For the P states, important to take into account the tensor couplings

$$\begin{aligned}
H_{\text{eff}} &= m_c + m_{[ss]} + \kappa_{ss} S_s \cdot S_s + \frac{B_{\mathcal{Q}}}{2} L^2 + V_{SD}, \\
V_{SD} &= a_1 L \cdot S_{[ss]} + a_2 L \cdot S_c + b \frac{\langle S_{12} \rangle}{4} + c\, S_{[ss]} \cdot S_c
\end{aligned}$$

Analysis of the excited Ω_c states in the Diquark-Quark model

- $b\langle S_{12}\rangle/4$ represents the matrix element of the tensor interaction

$$\frac{S_{12}}{4} = Q(S_1, S_2) = 3(S_1 \cdot n)(S_2 \cdot n) - (S_1 \cdot S_2)$$

- $\mathbf{S_1 = S_{[ss]}}$ and $\mathbf{S_2 = S_c}$ are the spins of the diquark and the charm quark, respectively, $\vec{n} = \vec{r}/r$ is the unit vector along the radius vector
- The scalar operator above can be expressed as the convolution $3S_1^i S_2^j N_{ij}$

$$N_{ij} = n_i n_j - \frac{1}{3}\delta_{ij}$$

- Need matrix elements of this operator between the states with the same fixed value L of the angular momentum operator L, using an identity from Landau and Lifshitz :

$$\langle N_{ij}\rangle = a(L)(L_i L_j + L_j L_i - \frac{2}{3}\delta_{ij} L(L+1))$$

  where $a(L) = \frac{-1}{(2L-1)(2L+3)}$
- $\longrightarrow \langle Q(S_X, S_X)\rangle = -\frac{3}{5}\langle[2(L \cdot S_X)^2 + (L \cdot S_X) - \frac{4}{3}(S_X \cdot S_X)]\rangle$
  where $S_X = S_{[ss]}, S_c, S = S_{[ss]} + S_c$

Analysis of the excited Ω_c states in the Diquark-Quark model-contd.

- For $L = 1$, and $S_{[ss]} = 1$, all three terms are non-zero, as opposed to the charmonium case, and one has to calculate the matrix element

$$\langle L, S'; J | L \cdot S_X | L, S; J \rangle$$

$$\frac{\langle S_{12} \rangle}{2} = \langle 2Q(S_{[ss]}, S_c) \rangle = \langle Q(S, S) - Q(S_c, S_c) - Q(S_{[ss]}, S_{[ss]}) \rangle$$

- Tensor operator mixes the two $J = 1/2$, and the two $J = 3/2$ states

$$J = 1/2 : \quad \frac{1}{4} \langle S_{12} \rangle = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{pmatrix}$$

$$J = 3/2 : \quad \frac{1}{4} \langle S_{12} \rangle = \begin{pmatrix} 0 & -\frac{1}{2\sqrt{5}} \\ -\frac{1}{2\sqrt{5}} & \frac{4}{5} \end{pmatrix}$$

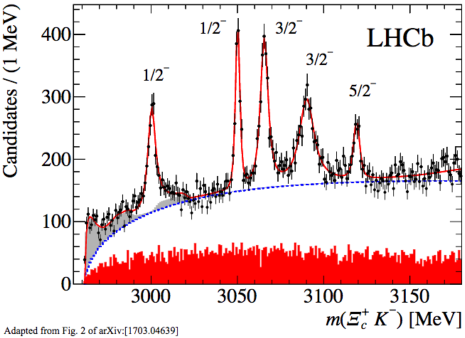
$$J = 5/2 : \quad \frac{1}{4} \langle S_{12} \rangle = -\frac{1}{5}$$

Analysis of the excited Ω_c states in the Diquark-Quark model- contd.

- Coeffs. determined from the masses of the J^P states (in MeV)

a_1	a_2	b	c	M_0
26.95	25.75	13.52	4.07	3079.94

$$M_0 \equiv m_c + m_{[ss]} + 2\kappa_{ss} + B_{\mathcal{Q}}$$



Adapted from Fig. 2 of arXiv:[1703.04639]

Analysis of the tetraquark Y states in the diquark model

$$H_{\rm eff} = 2m_{\mathcal{Q}} + \frac{B_{\mathcal{Q}}}{2}L^2 - 3\kappa_{cq} + 2a_Y L\cdot S + b_Y\frac{\langle S_{12}\rangle}{4}$$
$$+ \kappa_{cq}\left[2(S_q\cdot S_c + S_{\bar{q}}\cdot S_{\bar{c}}) + 3\right]$$

$$\frac{1}{4}\langle S_{12}\rangle = \begin{pmatrix} 0 & 2/\sqrt{5} \\ 2/\sqrt{5} & -7/5 \end{pmatrix}$$

- There are four $L=1$ and one $L=3$ tetraquark states with $J^{PC}=1^{--}$
- Tensor couplings non-vanishing only for the states with $S_{\mathcal{Q}} = S_{\bar{\mathcal{Q}}} = 1$

P-wave ($J^{PC}=1^{--}$) states

Label	J^{PC}	$\lvert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	Mass
Y_1	1^{--}	$\lvert 0,0;0,1\rangle_1$	$M_{00} - 3\kappa_{qQ} + B_Q \equiv \tilde{M}_{00}$
Y_2	1^{--}	$\left(\lvert 1,0;1,1\rangle_1 + \lvert 0,1;1,1\rangle_1\right)/\sqrt{2}$	$\tilde{M}_{00} + 2\kappa_{qQ} - 2A_Q$
Y_3	1^{--}	$\lvert 1,1;0,1\rangle_1$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_+$
Y_4	1^{--}	$\lvert 1,1;2,1\rangle_1$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_-$
Y_5	1^{--}	$\lvert 1,1;2,3\rangle_1$	$M_{Y_2} + 2\kappa_{qQ} - 14A_Q + 5B_Q - 8/5b_Y$

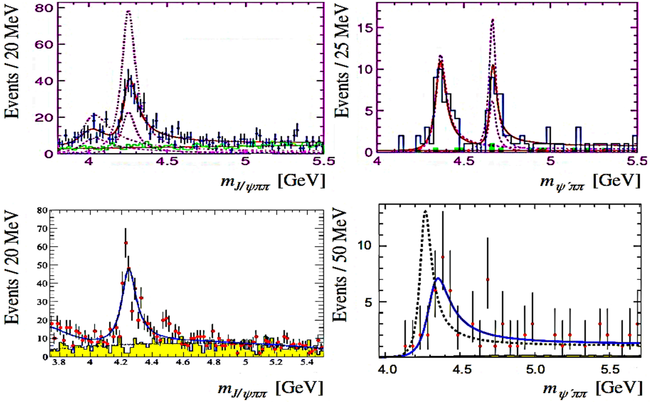
$$E_\pm = \frac{1}{10}(-30A_Q - 7b_Y \mp \sqrt{3}\sqrt{300A_Q^2 + 140A_Q b_Y + 43b_Y^2})$$

Experimental situation with the tetraquark Y states rather confusing

- Summary of the Y states observed in Initial State Radiation (ISR) processes in e^+e^- annihilation [BaBaR, Belle, CLEO]

$$e^+e^- \to \gamma_{\rm ISR}\ \ J/\psi\pi^+\pi^-;\ \ \gamma_{\rm ISR}\ \ \psi'\pi^+\pi^-$$

$$\Longrightarrow\ Y(4008),\ Y(4260),\ Y(4360),\ Y(4660)$$

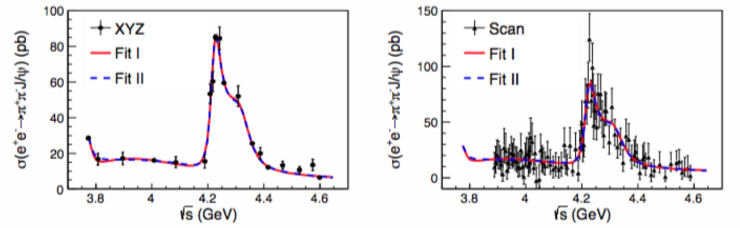


$e^+e^- \to J/\psi\pi^+\pi^-$ cross section at $\sqrt{s} = (3.77 - 4.60)$ GeV

(BESIII, PRL 118, 092001 (2017))

- Y(4008) is not confirmed; Y(4260) is split into 2 resonances: Y(4220) and Y(4320), with the Y(4220) probably the same as Y(4260)

Parameters	Fit result
$M(R_1)$	$3812.6^{+61.9}_{-96.6}$ $(\cdots)$
$\Gamma_{\rm tot}(R_1)$	$476.9^{+78.4}_{-64.8}$ $(\cdots)$
$M(R_2)$	4222.0 ± 3.1 (4220.9 ± 2.9)
$\Gamma_{\rm tot}(R_2)$	44.1 ± 4.3 (44.1 ± 3.8)
$M(R_3)$	4320.0 ± 10.4 (4326.8 ± 10.0)
$\Gamma_{\rm tot}(R_3)$	$101.4^{+25.3}_{-19.7}$ $(98.2^{+25.4}_{-19.6})$

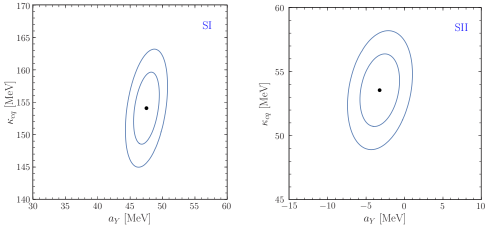


Two Experimental Scenarios for the Y States

[AA, L. Maiani, A. Borisov, I. Ahmed, A. Rehman, M.J. Aslam, A. Parkhomenko, A.D. Polosa, arxiv:1708.04650]

- SI (Based on CLEO, BaBaR, Belle): Y(4008), Y(4260), Y(4360), Y(4660)
- SII (BESIII, PRL 118, 092001 (2017): Y(4220), Y(4320), with Y(4390), Y(4660) the same as in SI

a_Y - κ_{cq} Correlations



- SII (based on BESIII data) is favored, with a_Y and κ_{cq} values similar to the Ω_c analysis

Correlations (Contd.)

- Fixing $\kappa_{cq} = 67$ MeV(from the S states); fitted the two scenarios $\Longrightarrow$ clear preference for SII, with the following parameters (in MeV)

Scenario	M_{00}	a_Y	b_Y	$\chi^2_{\rm min}/$n.d.f.
SI	4321 ± 79	2 ± 41	-141 ± 63	$12.8/1$
SII	4421 ± 6	22 ± 3	-136 ± 6	$1.3/1$

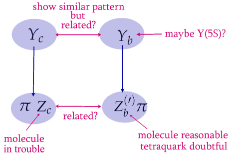
- SII: $M_{00} \equiv 2m_{\mathcal{Q}} + B_{\mathcal{Q}} \Longrightarrow B_{\mathcal{Q}} = 442$ MeV
- Comparable to the orbital angular momentum excitation energy in charmonia

$$B_{\mathcal{Q}}(c\bar{c}) = M(h_c) - \frac{1}{4}\left[3M(J/\psi) + M(\eta_c)\right] = 457\ \text{MeV}$$

- κ_{cq} and a_Y for Y states similar to the ones in (X, Z) and Ω_c
- Precise data on the Y-states is needed to confirm or refute the diquark picture

Summary

- A new facet of QCD is opened by the discovery of the exotic X, Y, Z, and the pentaquark states $\mathbb{P}(4380)$ and $\mathbb{P}(4450)$, but a dynamical theory still lacking
- A very rich spectrum of tetraquark and pentaquark states is anticipated, including the ones with a single c, or a single b quark, as well as those with multiple heavy quarks
- Important puzzles remain in the complex:



- What is the nature of $Y_c(4260)$? A tetraquark? or a $c\bar{c}g$ hybrid? Is $Y_c(4260)$split? How many P states are there? We do expect a tower of radial and orbital excited states in the diquark picture!
- What exactly is $\Upsilon(10888)$? Is it just $\Upsilon(5S)$? Does $Y_b(10890)$ still exist?
- We look forward to decisive experimental results from BESIII, Belle-II, and the LHC