Relativistic corrections to the exclusive $\chi_{cJ} + \gamma$ production from e^+e^- annihilation

Vladyslav Shtabovenko [1,2]

in collaboration with N. Brambilla [2], W. Chen [3], Y. Jia [3] and A. Vairo [2]

[1] Zhejiang University Hangzhou, China
[2] Technische Universität München, Germany
[3] Institute of High Energy Physics Beijing, China

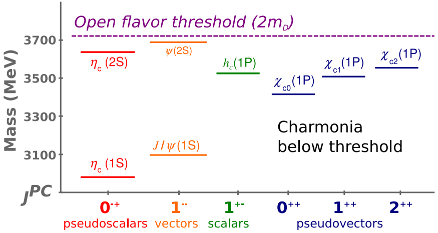
The 12^{th} International Workshop on Heavy Quarkonium
November 5 - 10, 2017
Peking University, Beijing, China

Technische Universität München

中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

1. Theoretical framework
   - Heavy quarkonia
   - Nonrelativistic QCD
2. Relativistic corrections to the exclusive production $e^+e^- \to \chi_{cJ} + \gamma$
   - Overview of the existing results
   - Contributions from the higher Fock state $|Q\bar{Q}g\rangle$
   - Perturbative matching between QCD and NRQCD
   - New results
3. Numerical analysis
   - Challenges and uncertainties
   - Preliminary results
4. Summary and Outlook

- Heavy quarkonia are an ideal laboratory to test our understanding of QCD
  - ⋆ Nonrelativistic system
  - ⋆ Rich phenomenology
  - ⋆ Creation/annihilation of $Q\bar{Q}$-pairs in short-distance processes
  - ⋆ Formation of $Q\bar{Q}$-bound states in long-distance processes



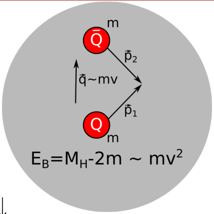
- First heavy quarkonia (J/ψ and $\psi(2s)$) discovered over 40 years ago [Aubert et al., 1974], [Augustin et al., 1974]
- Crucial for the establishment of QCD as the correct theory of strong interactions (November revolution of 1974, evidence for the existence of the charm quark, ...)
- Early attempts to develop a theoretical description of heavy quarkonia: phenomenological models
  - ⋆ Spectra from potential models
  - ⋆ Color singlet model [Einhorn & Ellis, 1975, Ellis et al., 1976, Carlson & Suaya, 1976]
  - ⋆ Color evaporation model [Fritzsch, 1977, Halzen, 1977, Halzen & Matsuda, 1978, Gluck et al., 1978]
- Effective Field Theory (EFT) methods [Weinberg, 1979, Wilson, 1974]: the modern way to treat $Q\bar{Q}$-bound states

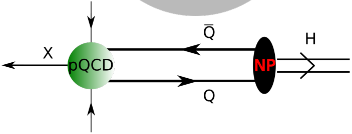
- Relevant dynamical scales of a heavy quarkonium

$$\underbrace{m}_{hard} \gg \underbrace{mv}_{soft} \gg \underbrace{mv^2}_{ultrasoft}$$

with $v_c^2 \sim 0.3$, $v_b^2 \sim 0.1$

- Relativistic corrections are very important for charmonia!
- The formation of a $Q\bar{Q}$-pair occurs within a distance $1/m$ (short distance process)





- The formation of a heavy quarkonium happens over distances of order $1/(mv)$ or larger in the quarkonium rest frame (long distance process).
- A suitable EFT for studying quarkonium production is Nonrelativistic QCD (NRQCD) [Caswell & Lepage, 1986, Bodwin et al., 1995]
  - ⋆ Starting from the full QCD, all scales above *mv* are integrated out.
  - ⋆ The effects of the high-energy contributions are encoded in the matching coefficients $c_n(\alpha_s(m), \mu)$ multiplying NRQCD operators $O_n(\mu)$.
  - ⋆ $\mathcal{L}_{\text{NRQCD}} = \sum_n \frac{c_n(\alpha_s(m), \mu)}{m^{d_n - 4}} O_n(\mu)$ is an expansion in α_s and v.
  - ⋆ ∞-number of operators with increasing mass dimension.
  - ⋆ Contributions to a process at the given accuracy estimated by velocity scaling rules.

- Long distance matrix elements (LDME) $\langle O_n(\mu)\rangle$ encode nonperturbative contributions. They can be extracted from experiment or lattice calculations.
- Predictive power of NRQCD: LDMEs do not depend on the short-distance process (universality).
- First, one can write down an NRQCD factorized production cross section

$$\sigma(H) = \sum_n \frac{F_n(\alpha_s(m), \mu)}{m^{d_n - 4}} \langle 0 | \mathcal{O}_n^H(\mu) | 0 \rangle \, .$$

- Then, one can use perturbative matching to determine the matching coefficients

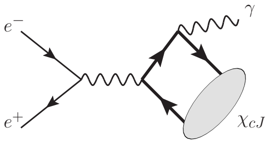
$$\sigma(Q\bar{Q})\big|_{\text{pert. QCD}} \stackrel{!}{=} \sum_n \frac{F_n(\alpha_s(m), \mu)}{m^{d_n - 4}} \langle 0 | \mathcal{O}_n^{Q\bar{Q}}(\mu) | 0 \rangle \, \big|_{\text{pert. NRQCD}} .$$

- Fock-state expansion of a heavy quarkonium according to NRQCD

$$|H\rangle \sim a_0 \, |Q\bar{Q}\rangle + a_1 \, |Q\bar{Q}g\rangle + a_2 \, |Q\bar{Q}gg\rangle + \ldots$$

- Higher order Fock states with $Q\bar{Q}$-pairs in the color octet (CO) configuration are suppressed by power of v.
- Still, they must be included in higher order relativistic or radiative corrections.
- Tests of NRQCD: Study the role of the CO mechanism in the phenomenology.

- Electromagnetic spin triplet P-wave quarkonium production in e^+e^--annihilation: virtual photon decays into a hard ($\mathbf{k}_p \sim m$) on-shell photon and χ_{cJ}:

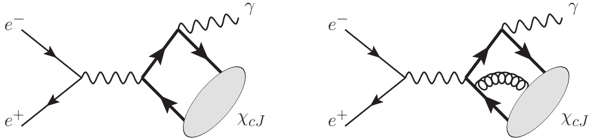


- This process can be measured at a B-factory with a sufficiently high CM energy.
- An early study [Chung et al., 2008] based on $\mathcal{O}(\alpha_s^0 v^0)$ results predicted following cross sections for $\sqrt{s} = 10.6$ GeV:
  - ⋆ $\sigma(e^+e^- \to \chi_{c0} + \gamma) = 1.3$ fb
  - ⋆ $\sigma(e^+e^- \to \chi_{c1} + \gamma) = 13.7$ fb
  - ⋆ $\sigma(e^+e^- \to \chi_{c2} + \gamma) = 5.3$ fb
- Subsequently, corrections of order $\mathcal{O}(\alpha_s v^0)$ [Sang & Chen, 2010], [Li et al., 2009], $\mathcal{O}(\alpha_s^0 v^2)$ [Li et al., 2014, Chao et al., 2013] and finally $\mathcal{O}(\alpha_s v^2)$ [Xu et al., 2014] were obtained as well.
- Some studies also employed the light cone formalism [Braguta, 2010], [Wang & Yang, 2014]

- So far, all the previous NRQCD studies were concerned with the operators that contribute through the dominant Fock state $|Q\bar{Q}\rangle$. Yet this is not the full story

$$|\chi_{cJ}(^3P_J)\rangle \sim \mathcal{O}(1)\,|c\bar{c}(^3P_J)\rangle + \mathcal{O}(v)\,|c\bar{c}(^3S_1)g\rangle + \mathcal{O}(v^2),$$

- To be consistent with the power-counting and the CO mechanism, at $\mathcal{O}(v^2)$ we must include operators that contribute through the subleading Fock state $|Q\bar{Q}g\rangle$.



- The external gluon is not hard ($|\mathbf{p}_g| << m$) and must be treated in NRQCD.
- In this case the perturbative quarkonium becomes a 3-body system

$$Q(p_1)\bar{Q}(p_2) \to Q(p_1)\bar{Q}(p_2)g(p_g)$$

- The availability of the CS $\mathcal{O}(\alpha_s v^2)$ corrections suggests that CO $\mathcal{O}(\alpha_s^0 v^2)$ corrections should be determined as well.

Exclusive production of χ_{cJ} in NRQCD at $\mathcal{O}(v^2)$ including the new operators

$$\sigma(e^+e^- \to \chi_{c0} + \gamma) = \frac{F_1(^3P_0)}{m^4} \langle 0|\mathcal{O}_1(^3P_0)|0\rangle + \frac{G_1(^3P_0)}{m^6} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle + \frac{T_8(^3P_0)}{m^5} \langle 0|\mathcal{T}_8(^3P_0)|0\rangle$$

$$\sigma(e^+e^- \to \chi_{c1} + \gamma) = \frac{F_1(^3P_1)}{m^4} \langle 0|\mathcal{O}_1(^3P_1)|0\rangle + \frac{G_1(^3P_1)}{m^6} \langle 0|\mathcal{P}_1(^3P_1)|0\rangle + \frac{T_8(^3P_1)}{m^5} \langle 0|\mathcal{T}_8(^3P_1)|0\rangle$$

$$\sigma(e^+e^- \to \chi_{c2} + \gamma) = \frac{F_1(^3P_2)}{m^4} \langle 0|\mathcal{O}_1(^3P_2)|0\rangle + \frac{G_1(^3P_2)}{m^6} \langle 0|\mathcal{P}_1(^3P_2)|0\rangle + \frac{T_8(^3P_2)}{m^5} \langle 0|\mathcal{T}_8(^3P_2)|0\rangle$$

- Production LDMEs

$$\langle 0|\mathcal{O}_1(^3P_0)|0\rangle \equiv \frac{1}{3} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|\chi_{c0}\rangle \times \langle \chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\chi|0\rangle$$

$$\langle 0|\mathcal{P}_1(^3P_0)|0\rangle \equiv \frac{1}{6}\left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|\chi_{c0}\rangle \times \langle \chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle + \text{h.c.}\right)$$

$$\langle 0|\mathcal{T}_8(^3P_0)|0\rangle \equiv \frac{1}{6}\left(\langle 0|\chi^\dagger(\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|\chi_{c0}\rangle \times \langle \chi_{c0}|\psi^\dagger(g\mathbf{E} \cdot \boldsymbol{\sigma})\chi|0\rangle + \text{h.c.}\right)$$

- Scaling
  - ★ In the power-counting of [Bodwin et al., 1995] we have

$$\langle 0|\mathcal{O}_1(^3P_J)|0\rangle \sim v^5$$
$$\langle 0|\mathcal{P}_1(^3P_J)|0\rangle \sim \langle 0|\mathcal{T}_8(^3P_J)|0\rangle \sim v^7$$

  - ★ Once we include contributions from $\langle 0|\mathcal{P}_1(^3P_J)|0\rangle$, we *must* also include $\langle 0|\mathcal{T}_8(^3P_J)|0\rangle$!
- Notation
  - ★ **bold font** denotes Cartesian 3-vectors
  - ★ ψ (χ) annihilates (creates) a heavy quark (antiquark)
  - ★ $\boldsymbol{\sigma}$ is the Pauli vector
  - ★ $\mathbf{D} \equiv \boldsymbol{\nabla} - ig\mathbf{A}$, $\psi^\dagger \overleftrightarrow{\mathbf{D}} \chi \equiv \psi^\dagger(\mathbf{D}\chi) - (\mathbf{D}\psi)^\dagger\chi$
  - ★ $\mathbf{A}$ is the gluon field, $\mathbf{E}$ is the chromoelectric field

- For this calculation the amplitude-level matching [Braaten & Chen, 1998, Braaten & Lee, 2003] is very advantageous
  - ★ The matching calculation is simpler, no need to square QCD amplitudes
  - ★ Need to deal with a smaller number of terms on the QCD side
  - ★ Avoids the nonrelativistic expansion of the phase space measure
- NRQCD-factorized amplitudes for $e^+e^- \to \chi_{cJ} + \gamma$ at $\mathcal{O}(v^2)$

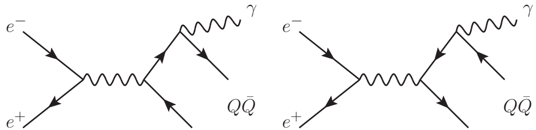
$$
\begin{aligned}
\mathcal{A}^{J=0}_{\text{NRQCD}} &= \frac{c_1^{J=0}}{m^2} \langle \chi_{c0} | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma} \right) \chi | 0 \rangle \\
&+ \frac{c_3^{J=0}}{m^4} \langle \chi_{c0} | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma} \right) \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \chi | 0 \rangle + \frac{d_1^{J=0}}{m^3} \langle \chi_{c0} | \psi^\dagger g\mathbf{E} \cdot \boldsymbol{\sigma} \chi | 0 \rangle ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{J=1}_{\text{NRQCD}} &= \frac{(c_1^{J=1})^i}{m^2} \langle \chi_{c1} | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma} \right)^i \chi | 0 \rangle \\
&+ \frac{(c_3^{J=1})^i}{m^4} \langle \chi_{c1} | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma} \right)^i \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \chi | 0 \rangle + \frac{(d_1^{J=1})^i}{m^3} \langle \chi_{c1} | \psi^\dagger (g\mathbf{E} \times \boldsymbol{\sigma})^i \chi | 0 \rangle ,
\end{aligned}
$$

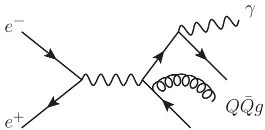
$$
\begin{aligned}
\mathcal{A}^{J=2}_{\text{NRQCD}} &= \frac{(c_1^{J=2})^{ij}}{m^2} \langle \chi_{c2} | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}}^{(i} \boldsymbol{\sigma}^{j)} \right) \chi | 0 \rangle \\
&+ \frac{(c_3^{J=2})^{ij}}{m^4} \langle \chi_{c2} | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}}^{(i} \boldsymbol{\sigma}^{j)} \right) \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \chi | 0 \rangle + \frac{(d_1^{J=})^{ij}}{m^3} \langle \chi_{c2} | \psi^\dagger g\mathbf{E}^{(i} \boldsymbol{\sigma}^{j)} \chi | 0 \rangle
\end{aligned}
$$

- Need to determine the short distance coefficients $d_2^{J=0}$, $(d_2^{J=1})^i$ and $(d_2^{J=1})^{ij}$.

- Operators that contain only **D** and $\boldsymbol{\sigma}$ contribute already through $|Q\bar{Q}\rangle$.
- Operators with one power of **E** or **B** require the inclusion of $|Q\bar{Q}g\rangle$.
- What does this mean for the QCD side of the matching?
  - ⋆ Contributions from $|Q\bar{Q}\rangle \Rightarrow$ 2 QCD diagrams to produce an on-shell $Q\bar{Q}$-system



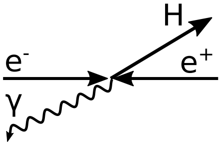
  - ⋆ Contributions from $|Q\bar{Q}g\rangle \Rightarrow$ 6 QCD diagrams to produce an on-shell $Q\bar{Q}g$

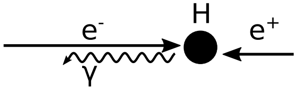


+ permutations

- Tree-level but NLO in v (complexity is the main challenge)
- First explicit calculation of matching coefficients multiplying chromoelectric LDMEs in a heavy quarkonium production process.

- Nowadays, matching calculations on the QCD side are usually carried out using the covariant projector technique [Bodwin & Petrelli, 2002].
- Manifest Lorentz covariance greatly facilitates automatic calculations.
- Here we use the threshold expansion method of Braaten and Chen [Braaten & Chen, 1996].
- Nonrelativistic calculations are more challenging to automatize, but this is still possible (c. f. my talk about FEYNONIUM this Friday)
- To cross-check our results, we carried out the matching in two different frames: the laboratory frame and the quarkonium rest frame.





- Our perturbative QCD and NRQCD amplitudes exactly match order by order in v

Final full $\mathcal{O}(v^2)$ results (including the $\mathcal{O}(\alpha_s v^0)$ [Sang & Chen, 2010, Li & Chao, 2009] correction to $F_1(^3P_J)$ and our new corrections)

$$\sigma(e^+e^- \to \chi_{c0} + \gamma) = \frac{(4\pi\alpha)^3 e_Q^4 (1-3r)^2}{18\pi m^3 s^2 (1-r)} \left\{ \left(1 + \frac{\alpha_s}{\pi} C_0^0(r)\right) \langle 0|\mathcal{O}_1(^3P_0)|0\rangle - \frac{(13 - 18r + 25r^2)}{10m^2(1-4r+3r^2)} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle + \frac{2r(2-3r)}{m(1-4r+3r^2)} \langle 0|\mathcal{T}_8(^3P_0)|0\rangle \right\},$$

$$\sigma(e^+e^- \to \chi_{c1} + \gamma) = \frac{(4\pi\alpha)^3 e_Q^4 (1+r)}{3\pi m^3 s^2 (1-r)} \left\{ \left(1 + \frac{\alpha_s}{\pi} \frac{C_1^0(r) + rC_1^1(r)}{1+r}\right) \langle 0|\mathcal{O}_1(^3P_1)|0\rangle - \frac{(11 - 20r - 11r^2)}{10m^2(1-r^2)} \langle 0|\mathcal{P}_1(^3P_1)|0\rangle - \frac{(3-3r-4r^2)}{2m(1-r^2)} \langle 0|\mathcal{T}_8(^3P_1)|0\rangle \right\},$$

$$\sigma(e^+e^- \to \chi_{c2} + \gamma) = \frac{(4\pi\alpha)^3 e_Q^4 (1+3r+6r^2)}{9\pi m^3 s^2 (1-r)} \left\{ \left(1 + \frac{\alpha_s}{\pi} \frac{C_2^0(r) + 3rC_2^1(r) + 6r^2 C_2^2(r)}{1+3r+6r^2}\right) \langle 0|\mathcal{O}_1(^3P_2)|0\rangle - \frac{(1+4r-30r^2)}{10m^2(1+3r+6r^2)} \langle 0|\mathcal{P}_1(^3P_2)|0\rangle - \frac{(3+r-6r^2-18r^3)}{2m(1-r)(1+3r+6r^2)} \langle 0|\mathcal{T}_8(^3P_2)|0\rangle \right\}$$

- Explicit values of $C_j^i(r)$ can be found in the Appendix B of [Sang & Chen, 2010]
- e_Q is the heavy quark charge
- m is the heavy quark mass
- s is the CM energy
- $r = 4m^2/s$

- One may naturally wonder about the numerical impact of the new chromoelectric LDMEs on the total cross sections
- Unfortunately, numerical predictions for this process are very challenging due to the large uncertainties from poorly known input parameters.
  - ⋆ Each cross section depends on three LDMEs

$$\langle 0|\mathcal{O}_1(^3P_J)|0\rangle\,, \quad \langle 0|\mathcal{P}_1(^3P_J)|0\rangle\,, \quad \langle 0|\mathcal{T}_8(^3P_J)|0\rangle$$

  - ⋆ No reliable determinations (experiment or lattice) for any of these 9 LDMEs.
- The same matrix elements also enter NRQCD predictions for $\chi_{c0,2} \to \gamma\gamma$ [Barbieri et al., 1976, Barbieri et al., 1980, Bodwin et al., 1995, Ma & Wang, 2002, Brambilla et al., 2006]
- The subleading LDMEs can be simplified using the heavy-quark spin symmetry

$$\langle 0|\mathcal{P}_1(^3P_J)|0\rangle = \langle 0|\mathcal{P}_1(^3P_0)|0\rangle \left(1+\mathcal{O}(v^2)\right)\,,$$
$$\langle 0|\mathcal{T}_8(^3P_J)|0\rangle = \langle 0|\mathcal{T}_8(^3P_0)|0\rangle \left(1+\mathcal{O}(v^2)\right)$$

- Gremm-Kapustin [Gremm & Kapustin, 1997] relations reduce the number of independent LDMEs

$$\langle 0|\mathcal{P}_1(^3P_J)|0\rangle = mE_{\chi_{cJ}}\,\langle 0|\mathcal{O}_1(^3P_J)|0\rangle + m\,\langle 0|\mathcal{T}_8(^3P_J)|0\rangle$$

- However, the results will be very sensitive to the value of the binding energy $E_{\chi_{cJ}}$.

- Our starting point are these two sets of linear equations

$$\Gamma(\chi_{c0} \to \gamma\gamma) = \frac{6\alpha^2 e_Q^4 \pi}{m^4}\left(1 + \frac{3\pi^2 - 28}{9}\frac{\alpha_s}{\pi}\right)\langle 0|\mathcal{O}_1(^3P_0)|0\rangle - \frac{14\alpha^2 e_Q^4 \pi}{m^6}\langle 0|\mathcal{P}_1(^3P_0)|0\rangle - \frac{3\alpha^2 e_Q^4 \pi}{m^5}\langle 0|\mathcal{T}_8(^3P_0)|0\rangle\,,$$
$$\langle 0|\mathcal{P}_1(^3P_0)|0\rangle = mE_{\chi_{c0}}\langle 0|\mathcal{O}_1(^3P_0)|0\rangle + m\langle 0|\mathcal{T}_8(^3P_0)|0\rangle\,.$$

$$\Gamma(\chi_{c2} \to \gamma\gamma) = \frac{8\alpha^2 e_Q^4 \pi}{5m^4}\left(1 - \frac{16}{3}\frac{\alpha_s}{\pi}\right)\langle 0|\mathcal{O}_1(^3P_2)|0\rangle - \frac{16\alpha^2 e_Q^4 \pi}{5m^6}\langle 0|\mathcal{P}_1(^3P_0)|0\rangle\,,$$
$$\langle 0|\mathcal{P}_1(^3P_2)|0\rangle = mE_{\chi_{c2}}\langle 0|\mathcal{O}_1(^3P_0)|0\rangle + m\langle 0|\mathcal{T}_8(^3P_0)|0\rangle\,.$$

- We can trade the charm pole mass for the hadron mass and the binding energy via $M_{\chi_{cJ}} = 2m + E_{\chi_{cJ}}$
- Solving each set for the subleading LDMEs and expanding up to $\mathcal{O}(v^2)$ yields

$$\langle 0|\mathcal{P}_1(^3P_0)|0\rangle = \frac{3}{34}M_{\chi_{c0}}\langle 0|\mathcal{O}_1(^3P_0)|0\rangle\left(M_{\chi_{c0}} + \frac{(3\pi^2-28)}{9\pi}M_{\chi_{c0}}\frac{\alpha_s}{\pi} + 5E_{\chi_{c0}}\right) - \frac{M_{\chi_{c0}}^6}{1088\pi\alpha^2 e_Q^4}\Gamma(\chi_{c0}\to\gamma\gamma),$$
$$\langle 0|\mathcal{T}_8(^3P_0)|0\rangle = \frac{3}{17}\langle 0|\mathcal{O}_1(^3P_0)|0\rangle\left(M_{\chi_{c0}} + \frac{(3\pi^2-28)}{9\pi}M_{\chi_{c0}}\frac{\alpha_s}{\pi} - \frac{2}{3}E_{\chi_{c0}}\right) - \frac{M_{\chi_{c0}}^5}{544\pi\alpha^2 e_Q^4}\Gamma(\chi_{c0}\to\gamma\gamma).$$

$$\langle 0|\mathcal{P}_1(^3P_0)|0\rangle = \frac{1}{8}M_{\chi_{c2}}\langle 0|\mathcal{O}_1(^3P_2)|0\rangle\left(M_{\chi_{c2}} - \frac{16}{3\pi}M_{\chi_{c2}}\frac{\alpha_s}{\pi} + 4E_{\chi_{c2}}\right) - \frac{5M_{\chi_{c2}}^6}{1024\pi\alpha^2 e_Q^4}\Gamma(\chi_{c2}\to\gamma\gamma),$$
$$\langle 0|\mathcal{T}_8(^3P_0)|0\rangle = \frac{1}{4}M_{\chi_{c2}}\langle 0|\mathcal{O}_1(^3P_2)|0\rangle\left(1 - \frac{16}{3\pi}\frac{\alpha_s}{\pi}\right) - \frac{5M_{\chi_{c2}}^5}{512\pi\alpha^2 e_Q^4}\Gamma(\chi_{c2}\to\gamma\gamma).$$

- The unknown input parameters are $\langle 0|\mathcal{O}_1(^3P_{0,2})|0\rangle$ and $E_{\chi_{c0,2}}$
- No information regarding χ_{c1}, as it cannot decay into two photons.
- Even for $\chi_{c0,2}$ we have no precise predictions neither for the binding energy, nor for the LO LDMEs …

- Although we are not in a position to make a rigorous numerical prediction, we still would like to somehow estimate the size of the new correction.
- LO LDME: adopt the value from a Buchmüller-Tye potential model calculation [Eichten & Quigg, 1995] and assign an $\mathcal{O}(v^2)$ uncertainty to account for the spin-symmetry breaking effects

$$\langle 0|\mathcal{O}_1(^3P_J)|0\rangle = (0.107 \pm 0.032)\,\text{GeV}^5.$$

- Binding energy: Parametrize $E_{\chi_{cJ}}$ according to the NRQCD power-counting

$$E_{\chi_{cJ}} = 0.3\, x_J \frac{M_{\chi_{cJ}}}{2}, \quad \text{with} \quad x_J = 1.0 \pm 1.0$$

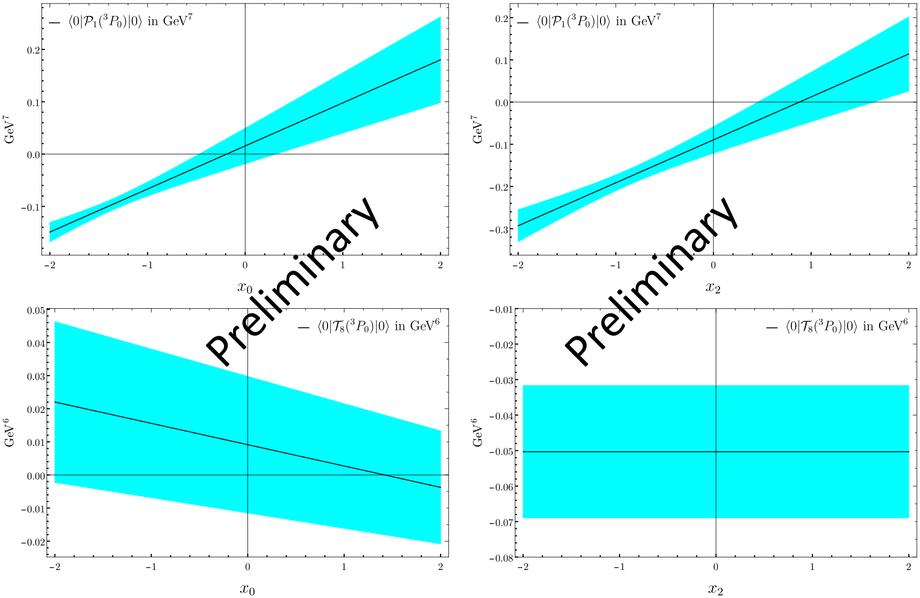
- Now we can determine the subleading LDMEs using $\langle 0|\mathcal{P}_1(^3P_0)|0\rangle$ and $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$ from the experimental data for χ_{c0} or χ_{c2}.
- We perform both determinations and compare the results.
- Input parameters for determining the LDMEs from $\chi_{c0,2} \to \gamma\gamma$ (using BES III data [Ablikim et al., 2012])

$$\Gamma(\chi_{c0} \to \gamma\gamma) = (2.33 \pm 0.20 \pm 0.22) \times 10^{-3}\,\text{MeV},$$
$$\Gamma(\chi_{c2} \to \gamma\gamma) = (0.63 \pm 0.04 \pm 0.06) \times 10^{-3}\,\text{MeV},$$

$$M_{\chi_{c0}} = (3414.75 \pm 0.31)\,\text{MeV}, \quad M_{\chi_{c2}} = (3556.20 \pm 0.09)\,\text{MeV},$$

$$\alpha(M_{\chi_{c0,2}}/2) = 1/133, \quad \alpha_s(M_{\chi_{c0}}/2) = 0.285, \quad \alpha_s(M_{\chi_{c2}}/2) = 0.280,$$

Dependence of the subleading LDMEs on x_J



- Average of the two determinations

Preliminary

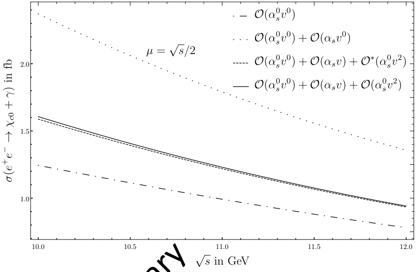
$$\langle 0|\mathcal{P}_1({}^3P_0)|0\rangle = (0.055 \pm 0.058 \pm 0.092 \pm 0.024)\,\text{GeV}^7,$$
$$\langle 0|\mathcal{T}_8({}^3P_0)|0\rangle = (-0.024 \pm 0.016 \pm 0.003 \pm 0.014)\,\text{GeV}^6,$$

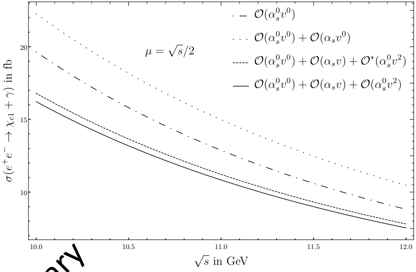
- Error sources: uncertainty in $\langle 0|\mathcal{O}_1({}^3P_J)|0\rangle$, in x_J and in the experimental determination of $M_{\chi_{cJ}}$ and $\Gamma(\chi_{cJ} \to \gamma\gamma)$.
- To reduce the uncertainties in the total cross sections, we compute them by replacing $\langle 0|\mathcal{P}_1({}^3P_0)|0\rangle$ and $\langle 0|\mathcal{T}_8({}^3P_0)|0\rangle$ with the corresponding formulas, not the fixed numerical values.
- Input parameters for the calculation of the cross sections

$$\sqrt{s} = 10.6\,\text{GeV}, \quad \alpha(10.6\,\text{GeV}) = 1/131$$

$$\alpha_s(\sqrt{s}) = 0.171, \quad \alpha_s(\sqrt{s}/2) = 0.200, \quad \alpha_s(M_{\chi_{c0}}/2) = 0.226, \quad \alpha_s(M_{\chi_{c2}}/2) = 0.223,$$

Cross sections obtained from the determination using experimental data for χ_{c0} (just the central values)

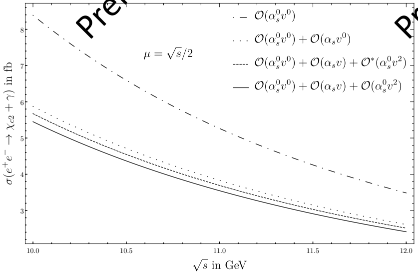




Preliminary

Preliminary

$$\sigma(e^+e^- \to \chi_{c0} + \gamma) = \frac{(4\pi\alpha)^3 e_Q^4 (1-3r)^2}{18\pi m^3 s^2 (1-r)}$$
$$\times \Bigg\{ \underbrace{\left(1 + \frac{\alpha_s}{\pi} C_0^0(r)\right) \langle 0|\mathcal{O}_1(^3P_0)|0\rangle}_{\mathcal{O}(\alpha_s^0 v^0) + \mathcal{O}(\alpha_s v^0)}$$
$$- \underbrace{\frac{(13 - 18r + 25r^2)}{10m^2(1-4r+3r^2)} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle}_{\mathcal{O}^*(\alpha_s^0 v^2)}$$
$$+ \underbrace{\frac{2r(2-3r)}{m(1-4r+3r^2)} \langle 0|\mathcal{T}_8(^3P_0)|0\rangle}_{\mathcal{O}(\alpha_s^0 v^2)} \Bigg\},$$



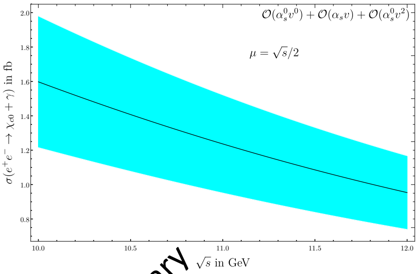
Cross sections obtained from the determination using experimental data for χ_{c0} at $\sqrt{s} = 10.6$

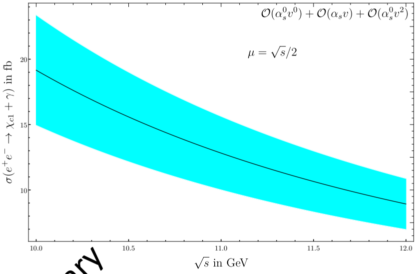
Preliminary

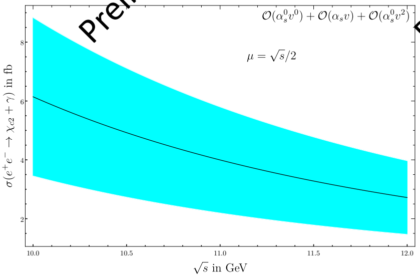
	$\mathcal{O}(\alpha_s^0 v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$ and $\mathcal{O}(\alpha_s v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}^*(\alpha_s^0 v^2)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}(\alpha_s^0 v^2)$	$\frac{\sigma_8}{\sigma_1}$
$\sigma(\chi_{c_0})$	1.09 ± 0.33	2.00 ± 0.60 $\pm 0.75 \pm 0.03$ ± 0.00	1.36 ± 0.23 $\pm 0.21 \pm 0.03$ ± 0.08	1.37 ± 0.28 $\pm 0.19 \pm 0.03$ ± 0.05	1.01 ± 0.04 $\pm 0.02 \pm 0.00$ ± 0.02
$\sigma(\chi_{c_1})$	15.16 ± 4.53	17.47 ± 5.22 $\pm 5.87 \pm 0.69$ ± 0.01	13.12 ± 2.71 $\pm 2.30 \pm 0.70$ ± 0.52	12.66 ± 0.76 $\pm 3.01 \pm 0.72$ ± 1.29	0.97 ± 0.14 $\pm 0.06 \pm 0.00$ ± 0.06
$\sigma(\chi_{c_2})$	6.30 ± 1.88	4.52 ± 1.35 $\pm 2.01 \pm 0.73$ ± 0.00	4.36 ± 1.26 $\pm 1.88 \pm 0.74$ ± 0.02	4.19 ± 0.52 $\pm 2.15 \pm 0.74$ ± 0.31	0.96 ± 0.16 $\pm 0.08 \pm 0.01$ ± 0.07

- σ_8/σ_1 is the ratio of the two final cross sections, when $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$ is included (σ_8) or omitted (σ_1).
- There are four main sources of uncertainties
  - ⋆ The uncertainty in $\langle 0|\mathcal{O}_1(^3P_0)|0\rangle$ (1^{st})
  - ⋆ The uncertainty in x_J (2^{nd})
  - ⋆ The uncertainty in the renormalization scale μ (3^{rd})
  - ⋆ The uncertainties in the experimental values of $M_{\chi_{cJ}}$ and $\Gamma(\chi_{cJ} \to \gamma\gamma)$ (4^{th})

Final cross sections in the energy region of Belle II from averaging the cross sections calculated using the data from χ_{c0} and χ_{c2}.







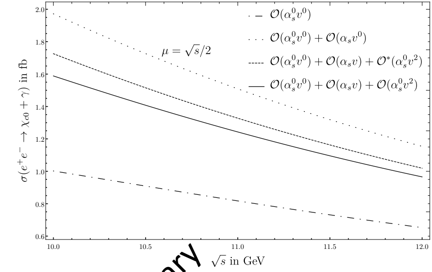
- Our preliminary predictions from the combined determination

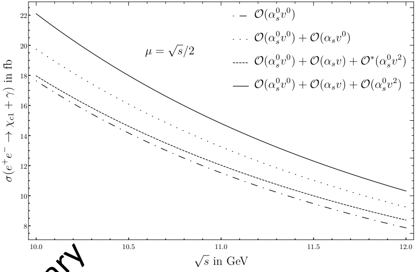
$$\sigma(e^+e^- \to \chi_{c0} + \gamma) = (1.37 \pm 0.32) \text{ fb},$$
$$\sigma(e^+e^- \to \chi_{c1} + \gamma) = (14.98 \pm 3.29) \text{ fb},$$
$$\sigma(e^+e^- \to \chi_{c2} + \gamma) = (4.72 \pm 2.11) \text{ fb}.$$

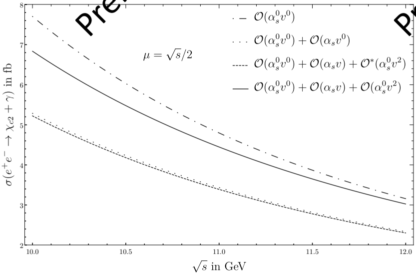
Conclusions

- We studied the $\mathcal{O}(\alpha_s^0 v^2)$ corrections to the electromagnetic quarkonium production process $e^+e^- \to \chi_{cJ} + \gamma$ using NRQCD factorization at the amplitude level and threshold expansion method.
- We determined the previously unknown matching coefficients multiplying $\langle 0|\mathcal{T}_8(^3P_J)|0\rangle$, which where the last missing piece to have the full $\mathcal{O}(v^2)$ NRQCD result for this process.
- The operator $\mathcal{T}_8(^3P_J)$ depends on the chromoelectric field **E** and hence contributes only through the subleading Fock states $|Q\bar{Q}g\rangle$. To our knowledge, this a first production study where the matching for such operators was worked out explicitly.
- We tried to estimate the numerical impact of the new contribution, but our analysis suffers from large uncertainties due to the poor knowledge of the *P*-wave electromagnetic LDMEs (even at LO in v).
- The technicalities behind this calculation (noncovariant matching) forced us to develop extensive self-written codes. This expertise will be made available to the (NR)EFT community (project FeynOnium).

Cross sections obtained from the determination using experimental data for χ_{c2} (just the central values)







$$
\sigma(e^+e^- \to \chi_{c0} + \gamma) = \frac{(4\pi\alpha)^3 e_Q^4 (1-3r)^2}{18\pi m^3 s^2 (1-r)}
\times \Bigg\{ \underbrace{\left(1 + \frac{\alpha_s}{\pi} C_0^0(r)\right) \langle 0|\mathcal{O}_1(^3P_0)|0\rangle}_{\mathcal{O}(\alpha_s^0 v^0)+\mathcal{O}(\alpha_s v^0)}
- \underbrace{\frac{(13-18r+25r^2)}{10m^2(1-4r+3r^2)} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle}_{\mathcal{O}^*(\alpha_s^0 v^2)}
+ \underbrace{\frac{2r(2-3r)}{m(1-4r+3r^2)} \langle 0|\mathcal{T}_8(^3P_0)|0\rangle}_{\mathcal{O}(\alpha_s^0 v^2)} \Bigg\},
$$

Cross sections obtained from the determination using experimental data for χ_{c2} at $\sqrt{s} = 10.6$

Preliminary

	$\mathcal{O}(\alpha_s^0 v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$ and $\mathcal{O}(\alpha_s v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}^*(\alpha_s^0 v^2)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}(\alpha_s^0 v^2)$	$\frac{\sigma_8}{\sigma_1}$
$\sigma(\chi_{c_0})$	0.89 ± 0.27	1.68 ± 0.50 $\pm 0.64 \pm 0.03$ ± 0.00	1.48 ± 0.18 $\pm 0.13 \pm 0.08$ ± 0.10	1.37 ± 0.24 $\pm 0.13 \pm 0.06$ ± 0.07	0.93 ± 0.05 $\pm 0.01 \pm 0.01$ ± 0.02
$\sigma(\chi_{c_1})$	13.58 ± 4.06	15.46 ± 4.62 $\pm 5.18 \pm 0.61$ ± 0.00	14.05 ± 2.39 $\pm 1.63 \pm 0.24$ ± 0.69	17.29 ± 0.62 $\pm 1.63 \pm 0.32$ ± 1.74	1.23 ± 0.17 $\pm 0.03 \pm 0.04$ ± 0.06
$\sigma(\chi_{c_2})$	5.75 ± 1.72	4.06 ± 1.21 $\pm 1.77 \pm 0.64$ ± 0.00	4.00 ± 1.13 $\pm 1.64 \pm 0.63$ ± 0.03	5.25 ± 0.45 $\pm 1.64 \pm 0.41$ ± 0.43	1.31 ± 0.26 $\pm 0.13 \pm 0.10$ ± 0.10

- σ_8/σ_1 is the ratio of the two final cross sections, when $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$ is included (σ_8) or omitted (σ_1).
- There are four main sources of uncertainties
  - ★ The uncertainty in $\langle 0|\mathcal{O}_1(^3P_0)|0\rangle$ (1^{st})
  - ★ The uncertainty in x_J (2^{nd})
  - ★ The uncertainty in the renormalization scale μ (3^{rd})
  - ★ The uncertainties in the experimental values of $M_{\chi_{cJ}}$ and $\Gamma(\chi_{cJ} \to \gamma\gamma)$ (4^{th})

Exclusive production of χ_{cJ} in NRQCD at $\mathcal{O}(v^2)$

$$
\begin{aligned}
\sigma(e^+e^- \to \chi_{c_0} + \gamma) = \frac{F_1(^3P_0)}{3m^2} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\psi|\chi_{c0}\rangle\langle\chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\chi|0\rangle \\
+ \frac{G_1(^3P_0)}{6m^4}\left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\psi|\chi_{c0}\rangle\langle\chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle + \text{h.c.}\right) \\
+ \frac{iT_8(^3P_0)}{3m^3}\left(\langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\psi|\chi_{c0}\rangle\langle\chi_{c0}|\psi^\dagger(g\mathbf{E}\cdot\boldsymbol{\sigma})\chi|0\rangle + \text{h.c.}\right) \\
\equiv \frac{F_1(^3P_0)}{m^2}\langle 0|\mathcal{O}_1(^3P_0)|0\rangle + \frac{G_1(^3P_0)}{m^4}\langle 0|\mathcal{P}_1(^3P_0)|0\rangle + \frac{T_8(^3P_0)}{m^3}\langle 0|\mathcal{T}_8(^3P_0)|0\rangle
\end{aligned}
$$

- **bold font** denotes Cartesian 3-vectors
- ψ (χ) annihilates (creates) a heavy quark (antiquark)
- $\boldsymbol{\sigma}$ is the Pauli vector
- $\mathbf{D} \equiv \boldsymbol{\nabla} - ig\mathbf{A}$
- $\mathbf{A}$ is the gluon field
- $\mathbf{E}$ is the chromoelectric field
- $\psi^\dagger \overleftrightarrow{\mathbf{D}} \chi \equiv \psi^\dagger(\mathbf{D}\chi) - (\mathbf{D}\psi)^\dagger\chi$

$$\sigma(e^+e^- \to \chi_{c1} + \gamma) = \frac{F_1(^3P_1)}{2m^2} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\psi|\chi_{c1}\rangle \cdot \langle \chi_{c1}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\chi|0\rangle$$
$$+ \frac{G_1(^3P_1)}{4m^4}\left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\psi|\chi_{c1}\rangle \cdot \langle \chi_{c1}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle + \text{h.c.}\right)$$
$$+ \frac{iT_8(^3P_1)}{2m^3}\left(\langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\psi|\chi_{c1}\rangle \cdot \langle \chi_{c1}|\psi^\dagger(g\mathbf{E} \times \boldsymbol{\sigma})\chi|0\rangle + \text{h.c.}\right)$$
$$\equiv \frac{F_1(^3P_1)}{m^2}\langle 0|\mathcal{O}_1(^3P_1)|0\rangle + \frac{G_1(^3P_1)}{m^4}\langle 0|\mathcal{P}_1(^3P_1)|0\rangle + \frac{T_8(^3P_1)}{m^3}\langle 0|\mathcal{T}_8(^3P_1)|0\rangle$$

$$\sigma(e^+e^- \to \chi_{c2} + \gamma) = \frac{F_1(^3P_2)}{m^2} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\psi|\chi_{c2}\rangle \langle \chi_{c2}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\chi|0\rangle$$
$$+ \frac{G_1(^3P_2)}{2m^4}\left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\psi|\chi_{c2}\rangle \langle \chi_{c2}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle + \text{h.c.}\right)$$
$$+ \frac{iT_8(^3P_2)}{m^3}\left(\langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\psi|\chi_{c2}\rangle \langle \chi_{c2}|\psi^\dagger(g\mathbf{E}^{(i}\boldsymbol{\sigma}^{j)})\chi|0\rangle + \text{h.c.}\right)$$
$$\equiv \frac{F_1(^3P_2)}{m^2}\langle 0|\mathcal{O}_1(^3P_2)|0\rangle + \frac{G_1(^3P_2)}{m^4}\langle 0|\mathcal{P}_1(^3P_2)|0\rangle + \frac{T_8(^3P_2)}{m^3}\langle 0|\mathcal{T}_8(^3P_2)|0\rangle$$

- $\mathbf{a}^{(i}\mathbf{b}^{j)} \equiv \frac{\mathbf{a}^i\mathbf{b}^j + \mathbf{a}^j\mathbf{b}^i}{2} - \frac{1}{3}\delta^{ij}(\mathbf{a}\cdot\mathbf{b})$.

- Exclusive production (such as $e^+e^- \to \chi_{cJ} + \gamma$): NRQCD factorization at the amplitude level [Braaten & Chen, 1998, Braaten & Lee, 2003]

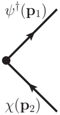
$$\mathcal{A}_{\text{pert. QCD}} \stackrel{!}{=} \sum_n c_n^{i_1 \dots i_k} \langle Q\bar{Q} | \, \psi^\dagger \mathcal{K}_n^{i_1 \dots i_k} \chi \, | 0 \rangle \equiv \mathcal{A}_{\text{pert. NRQCD}}$$

  - ★ $\mathcal{K}_n^{i_1 \dots i_k}$ are polynomials in **D**, **E**, **B** and contain a spin matrix and a color matrix
  - ★ $c_n^{i_1 \dots i_k}$ are short-distance coefficients
- The matching condition equates the on-shell amplitude to produce a heavy $Q\bar{Q}$-pair in pQCD to the sum of quarkonium-to-vacuum matrix elements multiplied or contracted with short-distance coefficients in pert. NRQCD.
- The c_n are then substituted into NRQCD-factorized production amplitudes

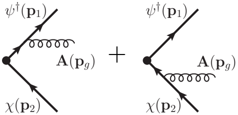
$$\mathcal{A}_{\text{NRQCD}} = \sum_n c_n^{i_1 \dots i_k} \langle H | \, \psi^\dagger \mathcal{K}_n^{i_1 \dots i_k} \chi \, | 0 \rangle$$

- Squaring $\mathcal{A}_{\text{NRQCD}}$ and integrating over the phase space of the physical quarkonium we obtain previously shown NRQCD production cross sections
- In general, not all matrix elements from $\mathcal{A}_{\text{pert. NRQCD}}$ will also appear in $\mathcal{A}_{\text{NRQCD}}$
- QCD amplitudes do not have a definite angular momentum $J \Rightarrow$ in the matching we will determine more short-distance coefficients than required.
- Important crosscheck: $\mathcal{A}_{\text{pert. QCD}} - \mathcal{A}_{\text{pert. NRQCD}} \stackrel{!}{=} 0$ order by order in v

- Emission of an ultrasoft gluon from an external heavy quark line leads to IR singularities
- More specifically, the amplitude contains terms singular in the limit $|\mathbf{p}_g| \to 0$
- We are not working with cross sections $\Rightarrow$ not a singularity in the phase-space integration.
- Such terms cancel in the matching: QCD and NRQCD share the same IR behavior
- The cancellation requires inclusion of NRQCD operators with Lagrangian insertions



$$= i \langle Q(\mathbf{p}_{1,R}) \bar{Q}(\mathbf{p}_{2,R}) | \psi^\dagger \chi | 0 \rangle$$

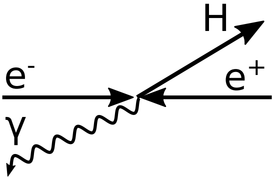


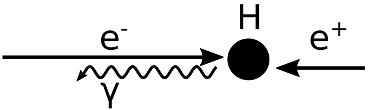
$$= i \langle Q(\mathbf{p}_{1,R}) \bar{Q}(\mathbf{p}_{2,R}) g(\mathbf{p}_{g,R}) | \psi^\dagger \chi | 0 \rangle_{\mathcal{L}_{2-f}},$$

$$\mathcal{L}_{2-f} = \psi^\dagger \left(\frac{\mathbf{D}^2}{2m} + \frac{\boldsymbol{\sigma} \cdot g\mathbf{B}}{2m} + \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \frac{i\boldsymbol{\sigma} \cdot [\mathbf{D}\times, g\mathbf{E}]}{8m^2} + \frac{\mathbf{D}^4}{8m^3} + \frac{\{\mathbf{D}^2, \boldsymbol{\sigma} \cdot g\mathbf{B}\}}{8m^3} \right) \psi$$
$$+ \chi^\dagger \left(-\frac{\mathbf{D}^2}{2m} - \frac{\boldsymbol{\sigma} \cdot g\mathbf{B}}{2m} + \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \frac{i\boldsymbol{\sigma} \cdot [\mathbf{D}\times, g\mathbf{E}]}{8m^2} - \frac{\mathbf{D}^4}{8m^3} + \frac{\{\mathbf{D}^2, \boldsymbol{\sigma} \cdot g\mathbf{B}\}}{8m^3} \right) \chi$$

Choice of the frame

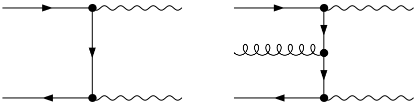
- The NRQCD side of the matching is usually evaluated in the frame where the heavy quarkonium is at rest (rest frame).
- The QCD side can be evaluated in any convenient frame [Bodwin et al., 1995].
- The most convenient frame for a production process is usually the CM frame of the colliding particles (CM frame).
- Two possibilities to carry out the matching:
  - ⋆ Evaluate QCD amplitudes in the rest frame. Then boost the short distance coefficients in the NRQCD production amplitude to the CM frame. Use Gremm-Kapustin relations to eliminate heavy quarkonium mass from the matching coefficients.
  - ⋆ Evaluate QCD amplitudes in the CM frame. For the nonrelativistic expansion, express the momenta of the moving constituents in terms of the soft rest frame momenta.
- We applied both approaches and arrived at the same final results.





Matching in the rest frame

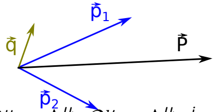
- If we choose to work in the rest frame, our process exhibits strong similarities to the decay $\chi_{cJ} \to 2\gamma$:



- Matching coefficients of the chromoelectric operators in decay are known [Ma & Wang, 2002], [Brambilla et al., 2006], talk of W.L. Sang at QWG 2013.
- We can recover them from our results in the limit $s \to 0$
- After we obtain the NRQCD production amplitudes with matching coefficients from the rest frame, we boost them to the CM frame using heavy quarkonium kinematics.
- The appearance of the physical quarkonium mass M_H is eliminated via following Gremm-Kapustin relations [Ma & Wang, 2002]

$$\langle 0|\mathcal{P}_1(^3P_J)|0\rangle = mE_{\chi_{cJ}} \langle 0|\mathcal{O}_1(^3P_J)|0\rangle + m \langle 0|\mathcal{T}_8(^3P_J)|0\rangle \quad (2)$$

- Of course, we can also work in the CM frame from the very beginning.
- Then the short distance coefficients do not require any further boosts.
- The calculation is slightly more involved as compared to to rest frame:
  - ⋆ In the CM frame, the energies and 3-momenta of Q, $\bar{Q}$ and g are not small.
  - ⋆ Using boost matrix formalism [Braaten & Chen, 1996], we can rewrite all those energies and 3-momenta in terms of soft/ultrasoft rest frame momenta.
  - ⋆ The generalization to the 3-body kinematics is straightforward.
- Braaten-Chen formalism for a 2-body system
  - ⋆ Jacobi momenta: $p_1 = \frac{1}{2}P + \mathcal{Q}, \qquad p_2 = \frac{1}{2}P - \mathcal{Q}$
  - ⋆ CM frame: $P = (\sqrt{P^2 + \mathbf{P}^2}, \mathbf{P}) \qquad \mathcal{Q} = (\sqrt{\mathcal{Q}^2 + \boldsymbol{\mathcal{Q}}^2}, \boldsymbol{\mathcal{Q}})$
  - ⋆ Rest frame ($\mathbf{p}_{1,R} + \mathbf{p}_{2,R} = 0, \quad \mathbf{q} \equiv \mathbf{p}_{1,R} = -\mathbf{p}_{2,R}$):
    $\mathcal{Q}_R \equiv q = (0, \mathbf{q}) \qquad P_R = (2\sqrt{\mathbf{q}^2 + M^2}, 0) \equiv (2E_q, 0)$
  - ⋆ Boost matrix establishes connection between the two frames, e. g. $\mathcal{Q}^\mu = \Lambda^\mu{}_\nu \mathcal{Q}_R^\nu = \Lambda^\mu{}_i \mathbf{q}^i$ with



$$\Lambda^0{}_i = \frac{\mathbf{P}^i}{2E_q}, \quad \Lambda^i{}_j = \delta^{ij} + \left(\frac{P^0}{2E_q} - 1\right)\hat{\mathbf{P}}^i\hat{\mathbf{P}}^j, \quad P^0 = \sqrt{4E_q^2 + \mathbf{P}^2}$$

  - ⋆ So we can expand the CM-frame amplitude in the soft rest frame momentum $|\mathbf{q}|$.

- In a 3-body system the relations become slightly more complicated.
- Jacobi momenta

$$p_1 = \frac{1}{3}P + \mathcal{Q}_1 - \mathcal{Q}_2, \qquad p_2 = \frac{1}{3}P - \mathcal{Q}_1 - \mathcal{Q}_2, \qquad p_g = \frac{1}{3}P + 2\mathcal{Q}_2.$$

- In the rest frame, where $\mathbf{p}_{1,R} + \mathbf{p}_{2,R} + \mathbf{p}_{g,R} = \mathbf{P}_R = 0$, we have

$$p_{1,R} = \frac{1}{3}P_R + q_1 - q_2, \qquad p_{2,R} = \frac{1}{3}P_R - q_1 - q_2, \qquad p_{g,R} = \frac{1}{3}P_R + 2q_2,$$

or

$$q_1 = \frac{1}{2}\left(p_{1,R} - p_{2,R}\right) \equiv (q_1^0, \mathbf{q}_1) \qquad q_2 = \frac{1}{6}\left(2p_{g,R} - p_{1,R} - p_{2,R}\right) \equiv (q_2^0, \mathbf{q}_2)$$

$$q_1^0 = \frac{1}{2}\left(\sqrt{(\mathbf{q}_1 - \mathbf{q}_2)^2 + m^2} - \sqrt{(\mathbf{q}_1 + \mathbf{q}_2)^2 + m^2}\right),$$
$$q_2^0 = \frac{1}{6}\left(4|\mathbf{q}_2| - \sqrt{(\mathbf{q}_1 - \mathbf{q}_2)^2 + m^2} - \sqrt{(\mathbf{q}_1 + \mathbf{q}_2)^2 + m^2}\right)$$

with $|\mathbf{q}_1|$ and $|\mathbf{q}_2|$ being our rest frame soft expansion parameters.

- CM frame vectors $\mathcal{Q}_1$ and $\mathcal{Q}_2$ in terms of $\mathbf{q}_1$ and $\mathbf{q}_2$

$$\mathcal{Q}_1^\mu = \Lambda^\mu{}_\nu q_1^\nu \qquad \mathcal{Q}_2^\mu = \Lambda^\mu{}_\nu q_2^\nu$$

- Boost matrix

$$\Lambda^0{}_0 = \sqrt{1 - \frac{\mathbf{P}^2}{P^2}}, \qquad \Lambda^0{}_i = \Lambda^i{}_0 = \frac{\mathbf{P}^i}{\sqrt{P^2}}, \qquad \Lambda^i_j = \delta^{ij} + \left(\sqrt{1 - \frac{\mathbf{P}^2}{P^2}} - 1 \right) \hat{\mathbf{P}}^i \hat{\mathbf{P}}^j$$

- Boosted Dirac bilinears

$$\bar{u}(p_1)\gamma^\mu v(p_2) = \Lambda^\mu{}_\nu \bar{u}_R(p_{1,R})\gamma^\nu v_R(p_{2,R}),$$
$$\bar{u}(p_1)\gamma^\mu \gamma_5 v(p_2) = \Lambda^\mu{}_\nu \bar{u}_R(p_{1,R})\gamma^\nu \gamma_5 v_R(p_{2,R})$$

- Rest frame bilinears ($N_1 N_2$ is the normalization factor)

$$\bar{u}_R(p_{1,R})\gamma^0 v_R(p_{2,R}) = N_1 N_2 \xi^\dagger \left(\frac{\mathbf{p}_{1,R}\cdot\boldsymbol{\sigma}}{E_{1,R}+m} + \frac{\mathbf{p}_{2,R}\cdot\boldsymbol{\sigma}}{E_{2,R}+m} \right) \eta$$

$$\bar{u}_R(p_{1,R})\gamma^i v_R(p_{2,R}) = N_1 N_2 \xi^\dagger \left(\boldsymbol{\sigma}^i + \frac{1}{E_{1,R}+m}\frac{1}{E_{2,R}+m} \left(\mathbf{p}^i_{1,R}(\mathbf{p}_{2,R}\cdot\boldsymbol{\sigma}) + \mathbf{p}^i_{2,R}(\mathbf{p}_{1,R}\cdot\boldsymbol{\sigma}) + (\mathbf{p}_{1,R}\cdot\mathbf{p}_{2,R})\boldsymbol{\sigma}^i - i(\mathbf{p}_{1,R}\times\mathbf{p}_{2,R})^i \right) \right) \eta$$

$$\bar{u}_R(p_{1,R})\gamma^0 \gamma_5 v_R(p_{2,R}) = N_1 N_2 \xi^\dagger \left(1 + \frac{1}{E_{1,R}+m}\frac{1}{E_{2,R}+m} (\mathbf{p}_{1,R}\cdot\mathbf{p}_{2,R} + i\boldsymbol{\sigma}\cdot(\mathbf{p}_{1,R}\times\mathbf{p}_{2,R}) \right) \eta$$

$$\bar{u}_R(p_{1,R})\gamma^0 \gamma_5 v_R(p_{2,R}) = N_1 N_2 \xi^\dagger \left(\frac{\mathbf{p}^i_{1,R} - i(\mathbf{p}_{1,R}\times\boldsymbol{\sigma})^i}{E_{1,R}+m} + \frac{\mathbf{p}^i_{2,R} + i(\mathbf{p}_{2,R}\times\boldsymbol{\sigma})^i}{E_{2,R}+m} \right) \eta$$

Ablikim, M. et al. (2012).
Two-photon widths of the $\chi_{c0,2}$ states and helicity analysis for $\chi_{c2} \to \gamma\gamma$.
Phys. Rev., D85, 112008.

Aubert, J. J. et al. (1974).
Experimental Observation of a Heavy Particle *J*.
Phys. Rev. Lett., 33, 1404–1406.

Augustin, J. E. et al. (1974).
Discovery of a Narrow Resonance in e^+e^- Annihilation.
Phys. Rev. Lett., 33, 1406–1408.
[Adv. Exp. Phys.5,141(1976)].

Barbieri, R., Caffo, M., Gatto, R., & Remiddi, E. (1980).
Strong QCD Corrections to p Wave Quarkonium Decays.
Phys. Lett., B95, 93–95.

Barbieri, R., Gatto, R., & Kogerler, R. (1976).
Calculation of the Annihilation Rate of P Wave Quark - anti-Quark Bound States.
Phys. Lett., 60B, 183–188.

Bodwin, G. T., Braaten, E., & Lepage, G. P. (1995).
Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium.
Phys. Rev., D51, 1125–1171.
[Erratum: Phys. Rev.D55,5853(1997)].

Bodwin, G. T. & Petrelli, A. (2002).
Order-v^4 corrections to *S*-wave quarkonium decay.
Phys. Rev., D66, 094011.
[Erratum: Phys. Rev.D87,no.3,039902(2013)].

Braaten, E. & Chen, Y.-Q. (1996).
Helicity decomposition for inclusive J / psi production.
Phys. Rev., D54, 3216–3227.

Braaten, E. & Chen, Y.-Q. (1998).
Renormalons in electromagnetic annihilation decays of quarkonium.
Phys. Rev., D57, 4236–4253.
[Erratum: Phys. Rev.D59,079901(1999)].

Braaten, E. & Lee, J. (2003).
Exclusive double charmonium production from e+ e- annihilation into a virtual photon.
Phys. Rev., D67, 054007.
[Erratum: Phys. Rev.D72,099901(2005)].

Braguta, V. V. (2010).
Exclusive C=+ charmonium production in $e^+e^- \to H + \gamma$ at B-factories within light cone formalism.
Phys. Rev., D82, 074009.

Brambilla, N., Mereghetti, E., & Vairo, A. (2006).

Electromagnetic quarkonium decays at order v**7.
JHEP, 08, 039.
[Erratum: JHEP04,058(2011)].

Carlson, C. E. & Suaya, R. (1976).
Hadronic Production of psi/J Mesons.
Phys. Rev., D14, 3115.

Caswell, W. E. & Lepage, G. P. (1986).
Effective Lagrangians for Bound State Problems in QED, QCD, and Other Field Theories.
Phys. Lett., 167B, 437–442.

Chao, K.-T., He, Z.-G., Li, D., & Meng, C. (2013).
Search for $C = +$ charmonium states in $e^+e^- \to \gamma + X$ at BEPCII/BESIII.

Chung, H. S., Lee, J., & Yu, C. (2008).
Exclusive heavy quarkonium + gamma production from e+ e- annihilation into a virtual photon.
Phys. Rev., D78, 074022.

Eichten, E. J. & Quigg, C. (1995).
Quarkonium wave functions at the origin.
Phys. Rev., D52, 1726–1728.

Einhorn, M. B. & Ellis, S. D. (1975).
Hadronic Production of the New Resonances: Probing Gluon Distributions.

Phys. Rev., D12, 2007.

Ellis, S. D., Einhorn, M. B., & Quigg, C. (1976).
Comment on Hadronic Production of Psions.
Phys. Rev. Lett., 36, 1263.

Fritzsch, H. (1977).
Producing Heavy Quark Flavors in Hadronic Collisions: A Test of Quantum Chromodynamics.
Phys. Lett., 67B, 217–221.

Gluck, M., Owens, J. F., & Reya, E. (1978).
Gluon Contribution to Hadronic J/psi Production.
Phys. Rev., D17, 2324.

Gremm, M. & Kapustin, A. (1997).
Annihilation of S wave quarkonia and the measurement of alpha-s.
Phys. Lett., B407, 323–330.

Halzen, F. (1977).
Cvc for Gluons and Hadroproduction of Quark Flavors.
Phys. Lett., B69, 105–108.

Halzen, F. & Matsuda, S. (1978).
Hadroproduction of Quark Flavors.
Phys. Rev., D17, 1344.

Li, B.-Q. & Chao, K.-T. (2009).
Higher Charmonia and X,Y,Z states with Screened Potential.
Phys. Rev., D79, 094004.

Li, D., He, Z.-G., & Chao, K.-T. (2009).
Search for C= charmonium and bottomonium states in e+ e- —> gamma + X at B factories.
Phys. Rev., D80, 114014.

Li, Y.-J., Xu, G.-Z., Liu, K.-Y., & Zhang, Y.-J. (2014).
Search for $C = +$ charmonium and XYZ states in $e^+e^- \to \gamma + H$ at BESIII.
JHEP, 01, 022.

Ma, J. P. & Wang, Q. (2002).
Corrections for two photon decays of chi(c0) and chi(c2) and color octet contributions.
Phys. Lett., B537, 233–240.

Sang, W.-L. & Chen, Y.-Q. (2010).
Higher Order Corrections to the Cross Section of e+e- —> Quarkonium + gamma.
Phys. Rev., D81, 034028.

Wang, X.-P. & Yang, D. (2014).
The leading twist light-cone distribution amplitudes for the S-wave and P-wave quarkonia and their applications in single quarkonium exclusive productions.
JHEP, 06, 121.

Weinberg, S. (1979).
Phenomenological Lagrangians.
Physica, A96, 327–340.

Wilson, K. G. (1974).
Confinement of Quarks.
Phys. Rev., D10, 2445–2459.
[,45(1974)].

Xu, G.-Z., Li, Y.-J., Liu, K.-Y., & Zhang, Y.-J. (2014).
$\alpha_s v^2$ corrections to η_c and χ_{cJ} production recoiled with a photon at e^+e^- colliders.
JHEP, 10, 71.