Exclusive Event Generation for the $t\bar{t}$ Threshold and the Transition to the Continuum

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F. Bach, B. Chokoufé Nejad, A.H. Hoang, WK, J. Reuter, M. Stahlhofen, T. Teubner, C. Weiss: *in preparation*

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Therefore:

- its life is short: $\Gamma_t > \Lambda_{\rm QCD}$
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The top quark is a potential messenger of new (EW/Higgs) physics

$\Rightarrow$ therefore wants to be studied with utmost precision

To produce the top quark:

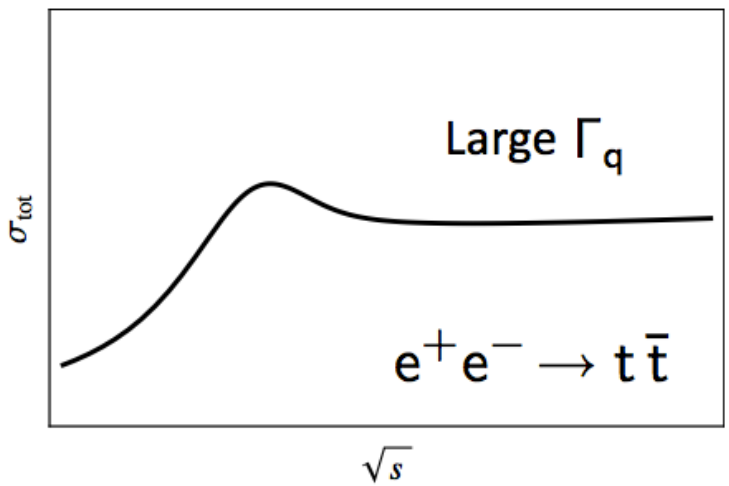
1. at the LHC: $t\bar{t}$ (strong+em/weak), $t\bar{b}$ (strong), ...
   $\Rightarrow$ precision studies vs QCD background

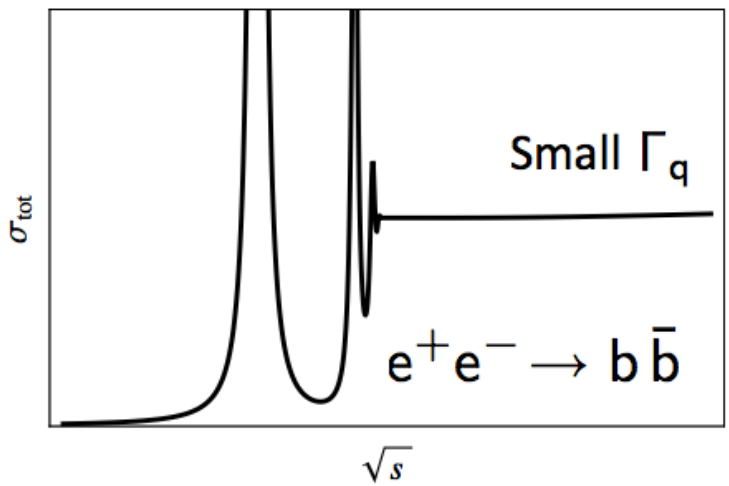
2. in e^+e^- collisions: $t\bar{t}$ (em/weak)
   $\Rightarrow$ need $\sqrt{s} > 350$ GeV.
   $\Rightarrow$ Future lepton colliders: ILC, CLIC (and FCC-ee, CEPC)

$t\bar{t}$ Threshold at a Lepton Collider

Main observation:

- At first glance, like any $q\bar{q}$ production threshold
- Coulombic interactions = Toponium slightly below threshold
- α_s and α_{EW} are reasonably small: observables calculable in perturbation theory.





$t\bar{t}$ Threshold at a Lepton Collider

Details:

- Accurate description of the interplay between threshold and toponium is difficult (but feasible!): non-relativistic approximation
- Non-relativistic approximation fails further beyond threshold
- Top quarks decay immediately

$$e^+e^- \to t\bar{t} \to b\bar{b}W^+W^- \to b\bar{b} + 4f$$

- Non-$t\bar{t}$ Feynman graphs with same final state: relativistic QCD calculation (NLO) for the continuum should include all
- Match both descriptions w/o double counting
- Actual LC analysis: fully exclusive event samples required for detailed handling of detector geometry and response

Our Strategy

1. Make use of a (reasonably) accurate $t\bar{t}$/toponium calculation, for off-shell top momenta: TOPPIK
2. Interpret this as an effective form factor that modifies the $t\bar{t}$ production current but not the irreducible background
3. Add in the irreducible background, computed at NLO QCD: WHIZARD/OpenLoops
4. Also compute the pole part of the NLO QCD in the continuum
5. Combine both for off-shell kinematics. Watch for double counting. Interpolate between threshold and continuum.
6. Embed in the complete e^+e^- event generation framework of WHIZARD.

Top-Pair Interaction Close to Threshold

Perturbative series in two parameters

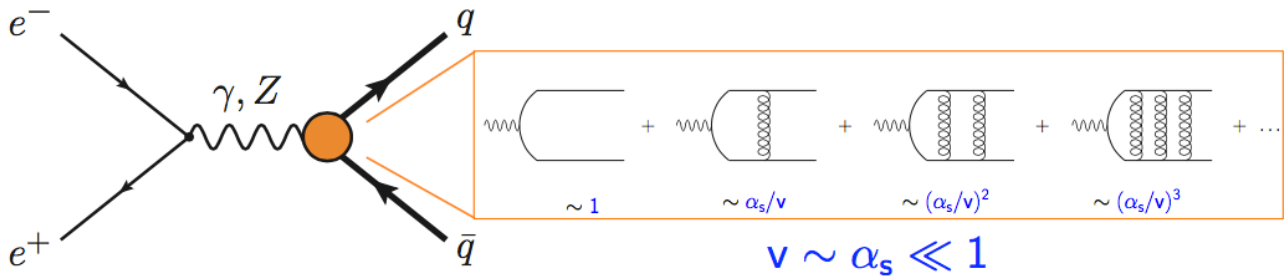
$$v \lesssim 0.1 \qquad \alpha_s \sim 0.1$$

with resummation required:

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times \left\{1\,(\mathrm{LL});\ \alpha_s, v\,(\mathrm{NLL});\ \alpha_s^2, \alpha_s v, v^2\,(\mathrm{NNLL});\ \ldots\right\}.$$

Top-Pair Potential

Resummation: Potential ladder



$t\bar{t}$ potential: pure Coulomb with perturbative corrections

$$\tilde{V}_c(\mathbf{p},\mathbf{p}') = \frac{\mathcal{V}_c^{(s)}}{(\mathbf{p}-\mathbf{p}')^2}$$

and [Fischler 1977]

$$\mathcal{V}_c^{(s)}(\mu_S) = -4\pi C_F \alpha_s(\mu_S) - C_F\left[\alpha_s(\mu_S)\right]^2\left(-\beta_0 \ln\left[\frac{(\mathbf{p}-\mathbf{p}')^2}{\mu_S^2}\right] + \frac{31}{9}C_A - \frac{20}{9}n_f T_F\right)$$

Top-Pair Currents

S-wave and P-wave currents for production:

$$\mathbf{O}_{\mathbf{p},1} = \psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}(\mathrm{i}\sigma_2)\, \chi^*_{-\mathbf{p}}$$

$$\mathbf{O}_{\mathbf{p},3} = \frac{-\mathrm{i}}{2m}\, \psi^\dagger_{\mathbf{p}}\, [\boldsymbol{\sigma}, \boldsymbol{\sigma}\cdot\mathbf{p}]\, (\mathrm{i}\sigma_2)\, \chi^*_{-\mathbf{p}}$$

Solve non-relativistic Lippman-Schwinger equations for current correlators with Coulomb interaction and Breit-Wigner top-quark propagator

Renormalization group for $v \to 0$ resums $\log v$ terms. (NLO $\to$ NLL)
[Hoang, Teubner et al.; Beneke et al.]

$\Rightarrow$ at NLL: numerical code TOPPIK [higher orders known but not used here]

WHIZARD: Overview

Scope

WHIZARD is a stand-alone program for processes at high-energy colliders: scattering and particle decays

- integrate cross sections and decay widths (perturbative, partonic)
- account for LC beam structure
- amplitude code is generated and executed on the fly
- shower/hadrons via internal or external code
- calculate observables, generate event samples

Universal Monte Carlo for elementary processes at (future lepton) colliders

NLO QCD in WHIZARD

WHIZARD is a program for combining matrix elements (as numerical code) with multi-channel twofold-adaptive phase-space integration, for hadron and lepton collider environments.

NLO QCD: Virtual matrix elements

- GoSam [Greiner, Heinrich, Soden-Fraunhofen et al.]
- OpenLoops [Cascioli, Lindert, Maierhfer, Pozzorini]
- Recola [Denner, Hofer, Lang, Uccirati]

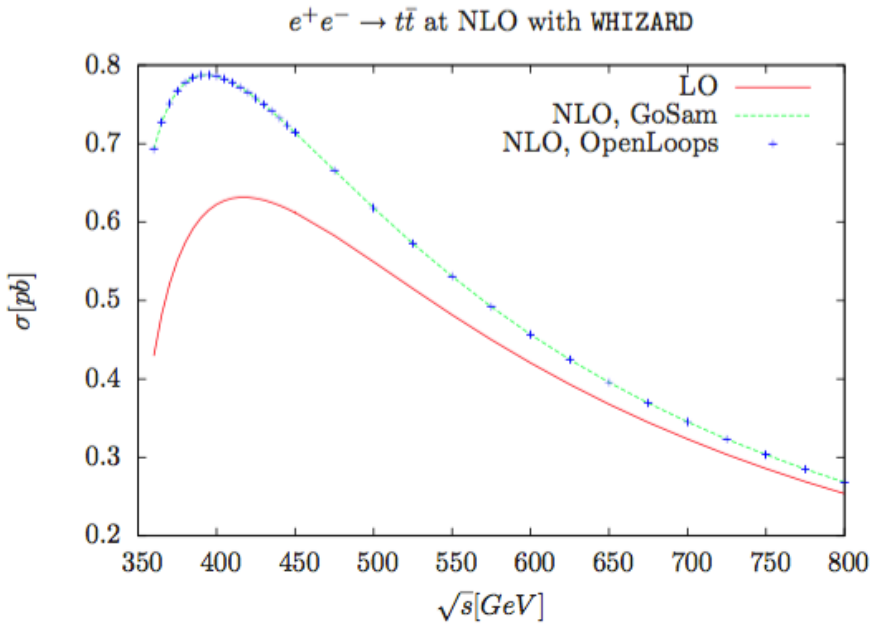
NLO-QCD: WHIZARD Calculation

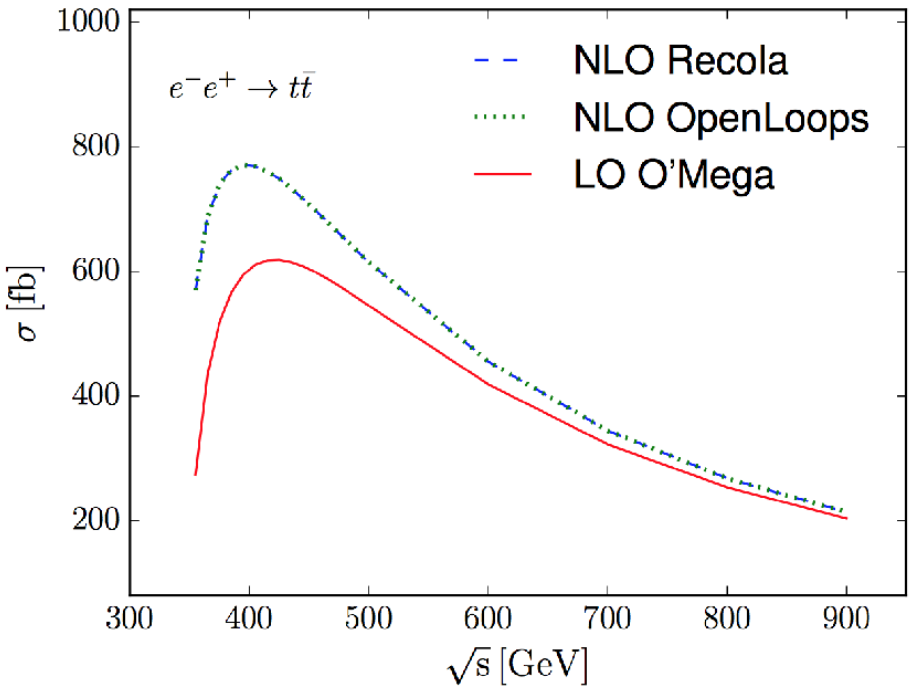
- FKS subtraction [Frixione, Kunszt, Signer]
- Modification for intermediate resonances [Jezo, Nason]
- Matrix elements from external provider
- Real corrections, subtraction terms internal
- NLO decays (Γ_t)
- Fixed-order events
- POWHEG matching to parton shower (PYTHIA 8)

Chokoufé, WK, Lindert, Pozzorini, Reuter, Weiss, JHEP 12 (2016) 075

(Lepton Collider: also account for ISR, beamstrahlung, polarization)

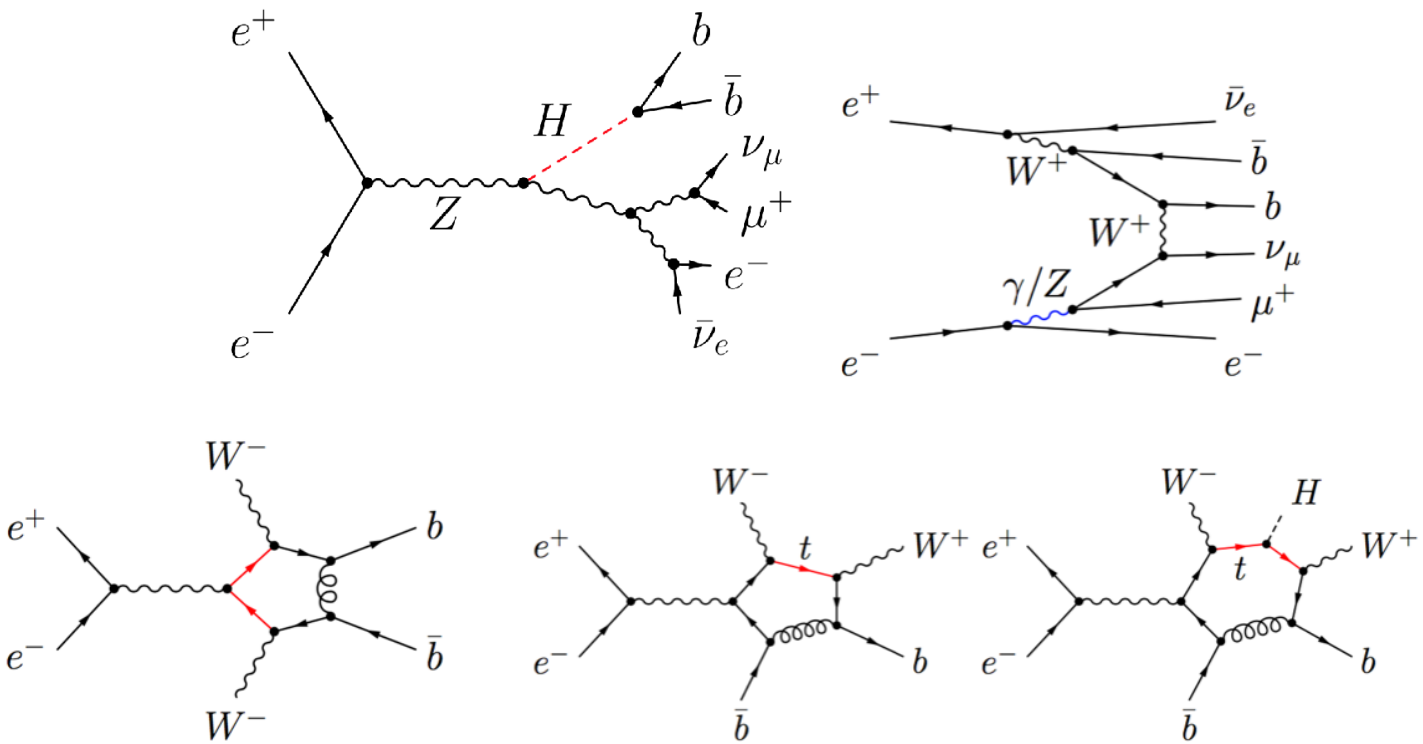
NLO-QCD Continuum (on shell)



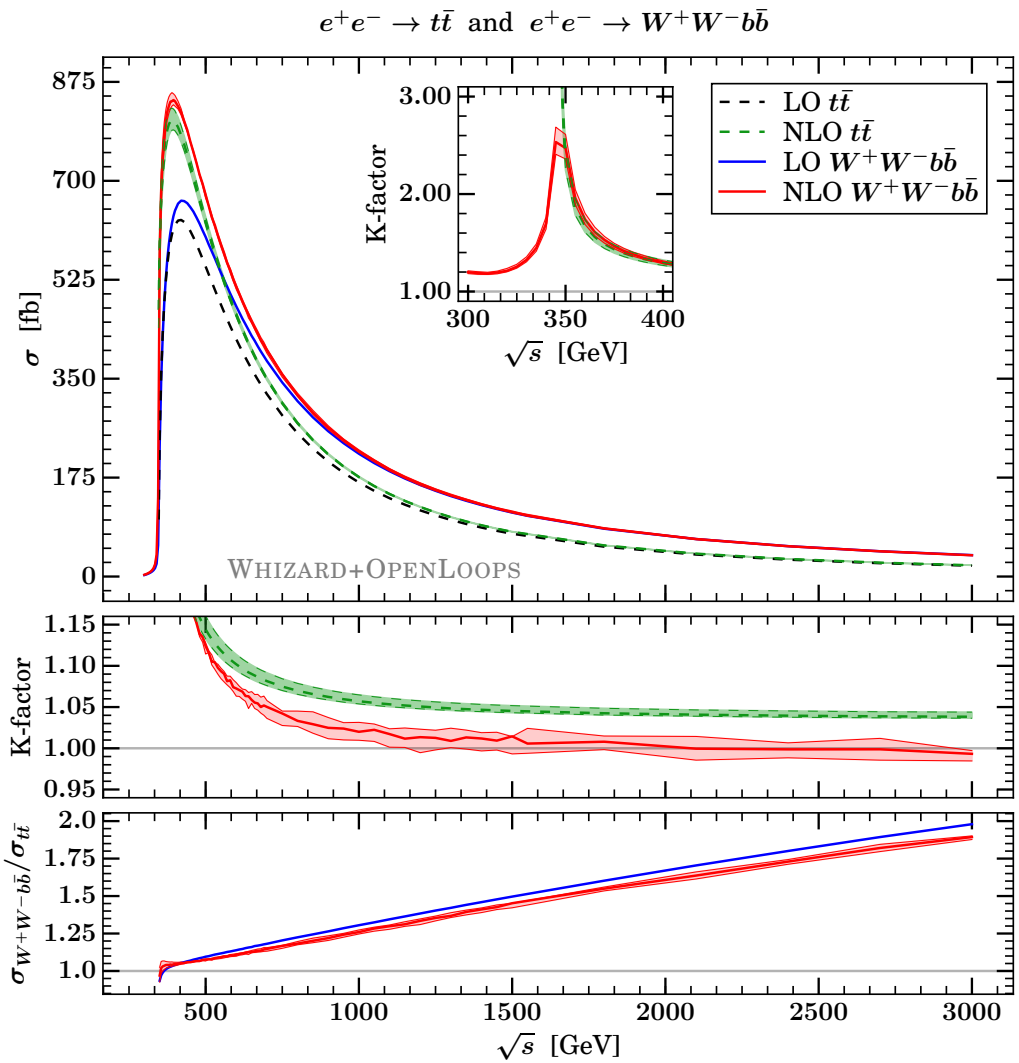


NLO-QCD Continuum (off shell)

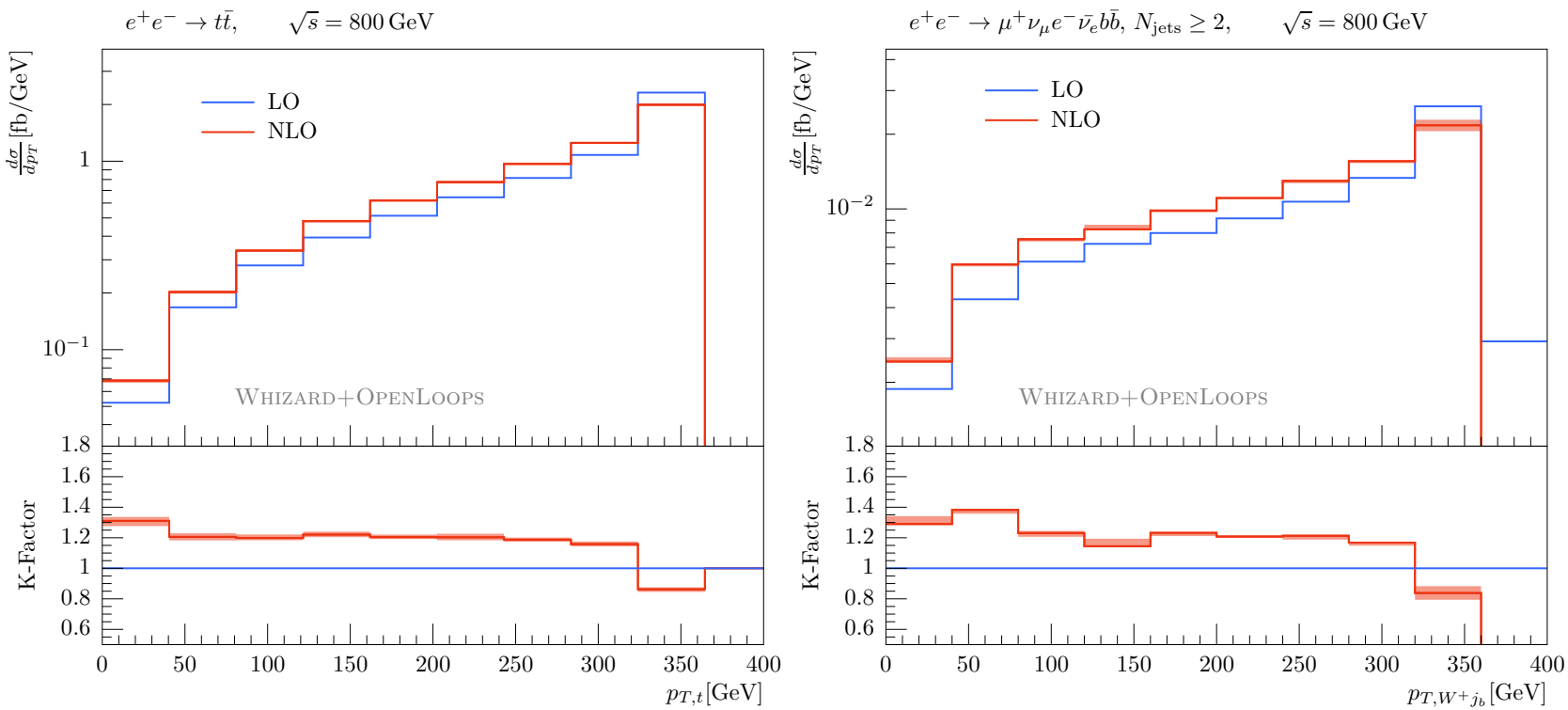
Resonances and irreducible background components



NLO-QCD Continuum: Cross Section (off shell)



NLO-QCD Continuum: Distributions (off shell)



Matching Threshold and continuum

- NRQCD fails for $v \gtrsim 1$

  . . . and is meaningless below threshold

- Relativistic QCD does not account for potential and resonance

$\Rightarrow$ match both calculations (smooth transition!)

Conditions for smooth matching

No double counting

- NRQCD must be expanded in α_s
- Isolate double-pole terms from continuum calculation

Consistent scheme

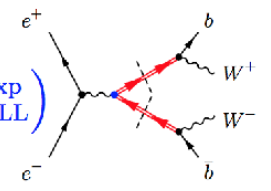
- Relate $\alpha_s(\mu)$ for different μ values in the calculation
- Use calculated NLO width
- Choice for m_t definition

Smooth transition

- Choice for switchoff function

Matching contributions

$$\sigma_{\rm NLO+NLL} = \sigma_{\rm NLO} + \left(\left(\tilde{F}_{\rm NLL} - \tilde{F}^{\rm exp}_{\rm NLL} \right) \right.$$



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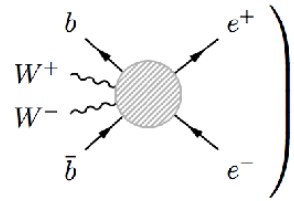
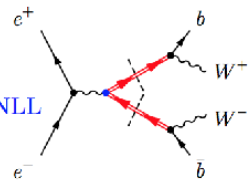


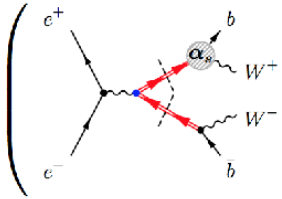
$$\left. \right)$$

$$+ \left| \tilde{F}_{\rm NLL} \right.$$



$$\left. \right|^2$$

$$+ \left\{ \tilde{F}_{\rm NLL} \left(\right. \right.$$



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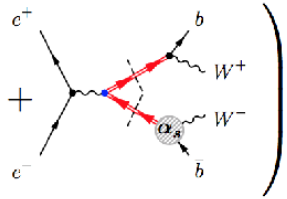

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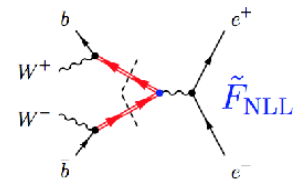

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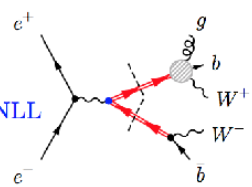

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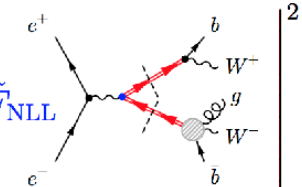
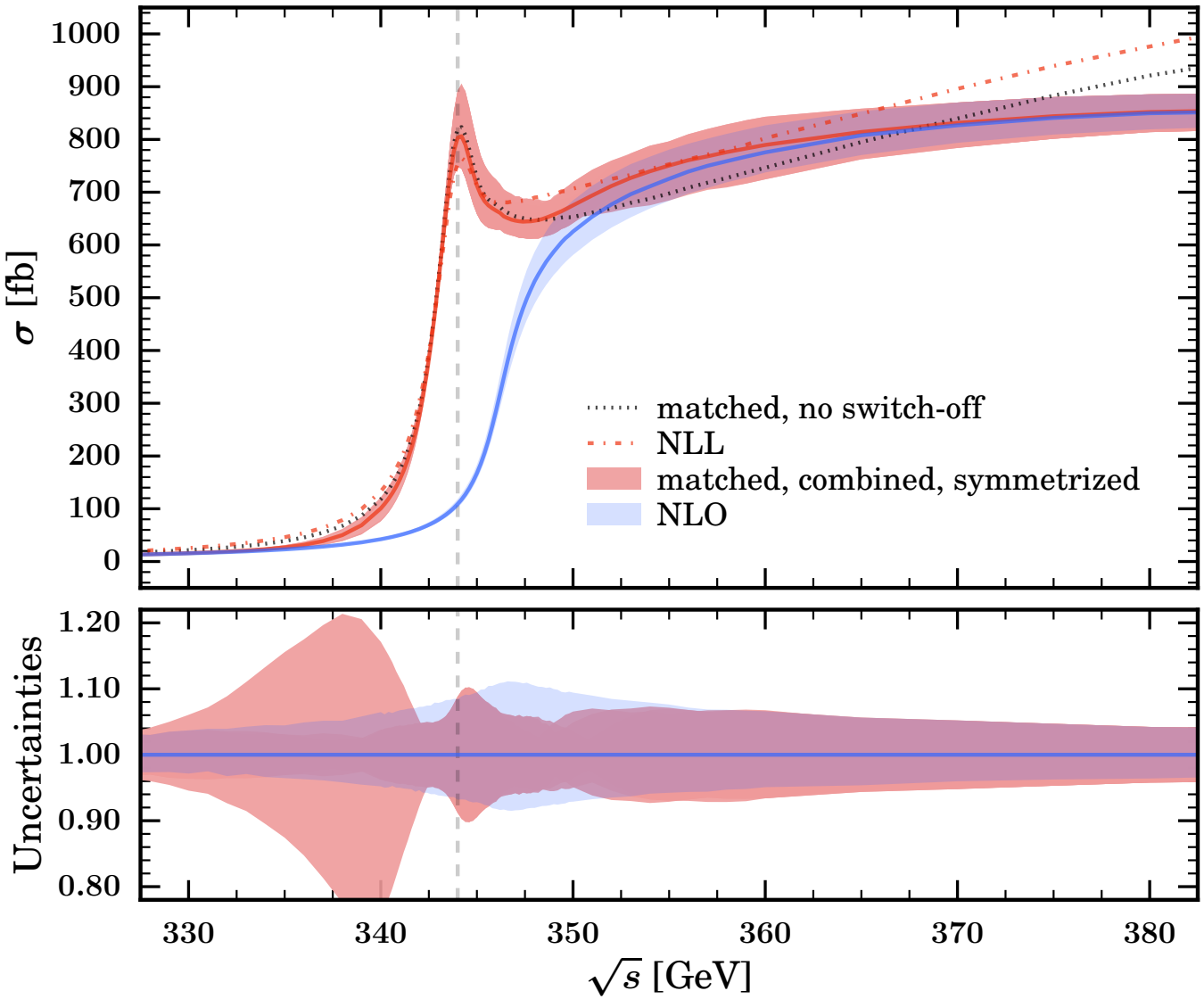


$$\left. \right|^2 ,$$

Result: total cross section



Summary and Outlook

The current setup combines $t\bar{t}$ threshold and continuum calculations with the best precision that can be consistently matched, fully exclusive

- NLO NRQCD in the threshold region, displaying toponium resonance
- NLL resummed very close to threshold
- NLO QCD everywhere

Not shown in this talk:

- Differential distributions
- NLO matched exclusive events (POWHEG scheme)

Future developments

- Higher orders from the threshold calculation, helicity mixing
- $t\bar{t}H$ matched threshold (QCD NLO ok)
- Exclusive photon radiation and electroweak corrections

The Future of the R Ratio