Exotic Quarkonium with Non-Relativistic Effective Field Theories

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Effective theories for heavy particles

EFTs applied to standard quarkonium

- motivating idea: description similar to positronium bound states
- utilizing the existence of a large energy scale: the heavy quark mass
- systematic expansion with effective theory NRQCD
- exploiting further scale hierarchies between relative momentum/distance and energy: $p \sim 1/r \gg E$
- multipole expansion in EFT: pNRQCD
- provides accurate description of quarkonium in perturbative and non-perturbative regime

Adaptation to exotics

- investigate excited spectrum of static Hamiltonian (LO NRQCD)
- study multipole expansion with pNRQCD
- inclusion of largest term beyond static limit leads to Schrödinger description

Static states and $1/M$ expansion

$1/M$ expanded Hamiltonian through EFT (NRQCD)

Caswell and Lepage 1986

Bodwin, Braaten and Lepage 1995

$$H_0 = \int d^3x \left(\mathrm{Tr}\left[\boldsymbol{E}^2 + \boldsymbol{B}^2\right] + \sum_l \bar{q}_l \boldsymbol{\gamma} \cdot (-i\boldsymbol{D}) q_l \right)$$

$$H_1 = \int d^3x\, \psi^\dagger \left(-\frac{\boldsymbol{D}^2}{2M_Q} - c_F \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2M_Q} \right) \psi + \int d^3x\, \chi^\dagger \left(\frac{\boldsymbol{D}^2}{2M_{\bar{Q}}} + c_F \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2M_{\bar{Q}}} \right) \chi$$

ψ: heavy quark, χ: heavy antiquark, q_l: massless quark
H_0: static Hamiltonian, H_1 first $1/M$ correction

Static states $|\underline{n};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle_0$:

- construct with static fields ψ, χ and some light d.o.f. operator $\Phi_n^{(0)}$:

$$|\underline{n};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle_0 = \psi^\dagger(\boldsymbol{x}_1)\Phi_n^{(0)}(\boldsymbol{x}_1,\boldsymbol{x}_2)\chi(\boldsymbol{x}_2)\,|0\rangle$$

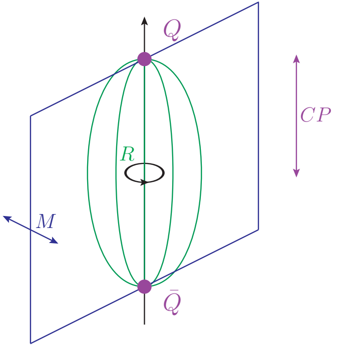
- quantum numbers $\underline{n}$ defined by symmetries of static system
- static energies $\quad H_0\,|\underline{n};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle_0 = E_n^{(0)}(r)\,|\underline{n};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle_0$
  depend only on relative distance r and internal quantum numbers $\underline{n}$

Symmetries of the static system

Static system has cylindric symmetry: $D_{\infty h}$
Elementary group transformations:

- Rotations R around Q-$\bar{Q}$ axis
- CP: Space inversion across center of Q-$\bar{Q}$ combined with charge conjugation
- Reflection M across plane with Q-$\bar{Q}$ axis

All other elements are combinations of these



Static States labeled with associated quantum numbers Λ_η^σ:

- Λ: rotational quantum number; labels Σ, Π, Δ correspond to $\Lambda = 0, 1, 2$
- η: eigenvalue of CP: $g \,\widehat{=}\, +1$ (gerade), $u \,\widehat{=}\, -1$ (ungerade)
- σ: sign of reflections M; σ only relevant for Σ representations

For $r \to 0$ symmetry extends to $O(3) \times C$: Λ_η^σ states form degenerate multiplets

Small distance expansion with pNRQCD

For $r \ll \Lambda_{\rm QCD}^{-1}$ multipole expanded EFT: pNRQCD

Bali and Pineda 1997

Brambilla, Pineda, Soto, Vairo 1999

$$H^{(0,0)} = H_0^{(NRQCD)} + \int d^3r d^3R \, \mathrm{Tr}\left[S^\dagger V_S S + O^\dagger V_O O\right]$$

$$H^{(0,1)} = \int d^3r d^3R \, \mathrm{Tr}\left[V_A\left(S^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}O + O^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}S\right) + \frac{1}{2}V_B O^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, O\}\right]$$

$$H^{(1,-2)} = \int d^3r d^3R \, \mathrm{Tr}\left[-S^\dagger \frac{\boldsymbol{\nabla}_r^2}{M} S - O^\dagger \frac{\boldsymbol{\nabla}_r^2}{M} O\right]$$

with $\quad V_S(r) = -\frac{4}{3}\frac{\alpha_{VS}(r)}{r} \qquad V_O(r) = \frac{1}{6}\frac{\alpha_{VO}(r)}{r} \qquad V_{A/B}(r) = 1 + \mathcal{O}(\alpha_s)$

Multipole expand static states:

$$|\underline{n};\boldsymbol{r},\boldsymbol{R}\rangle_0 = \mathrm{Tr}\left[\left(\Phi_n^{(0,0)}(\hat{\boldsymbol{r}},\boldsymbol{R}) + r\Phi_n^{(0,1)}(\hat{\boldsymbol{r}},\boldsymbol{R}) + \mathcal{O}(r^2)\right)(S/O)^\dagger(\boldsymbol{r},\boldsymbol{R})\right]|0\rangle$$

- $\Phi_n^{(\dots)}$ composed only of gluons or light quarks and have Λ_η^σ quantum numbers
- dependence on $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$ comes from polarization along $Q\bar{Q}$ axis
- static energies $E_n^{(0)}(r) = V_{S/O}(r) + \Lambda_n + \mathcal{O}(r^2)$

Beyond the static limit

We want to diagonalize $H_0 + H^{(1,-2)}$ at leading order in r between the states

$$|\underline{n}; l, m; \boldsymbol{R}\rangle \equiv \int d^3r \; |\underline{n}; \boldsymbol{r}, \boldsymbol{R}\rangle_0 \, \Psi_{nlm}(\boldsymbol{r})$$

with suitable wave functions $\Psi_{nlm}(\boldsymbol{r}) = R_n(r)\,\omega_{nlm}(\theta, \varphi)$

- $|\underline{n}; l, m; \boldsymbol{R}\rangle$ is an (l, m)-eigenstate of angular momentum $\boldsymbol{L} = \boldsymbol{L}_{\text{light}} + \boldsymbol{L}_{\text{heavy}}$
- kinetic term in $H^{(1,-2)}$: $-\frac{\boldsymbol{\nabla}_r^2}{M} = -\frac{1}{Mr^2}\partial_r\, r^2 \partial_r + \frac{\boldsymbol{L}_{\text{heavy}}^2}{Mr^2}$
- radial part acts only on $R_n(r)$, but $\boldsymbol{L}_{\text{heavy}}^2$ does not commute with $\Phi_n^{(0,0)}$
- radial wave function satisfies **coupled Schrödinger equation**

$$\sum_{n'} \left[-\frac{1}{Mr^2}\partial_r\, r^2 \partial_r\, \delta_{nn'} + \frac{1}{Mr^2}\mathcal{M}_{nn'} + E_n^{(0)}(r)\delta_{nn'} \right] R_{n'}(r) = \mathcal{E}_n R_n(r)$$

with matrix elements $\quad \mathcal{M}_{nn'} = \int d\Omega\, \omega_{nlm}^*\, \langle 0|\, \Phi_n^{(0,0)}\, \boldsymbol{L}_{\text{heavy}}^2\, \Phi_{n'}^{(0,0)}\, |0\rangle\, \omega_{n'lm}$

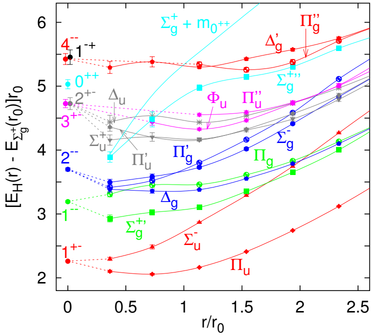
- $\mathcal{M}_{nn'}$ can be expressed exactly through $\underline{n}$, $\underline{n}'$, and l

Numerical determination of static energies

In practice: exact form of light operators Φ_n unknown

- replace Φ_n by some simple light operator X_n with same quantum numbers $\underline{n}$
- calculate the large time correlator of this state:

$$\langle X_n(T)|X_n(0)\rangle = \langle X_n|\, e^{-iH_0T}\sum_{n'}|\underline{n}'\rangle_{0\,0}\langle\underline{n}'|X_n\rangle = \sum_{n'}\underbrace{\left|\langle X_n|\underline{n}'\rangle_0\right|^2}_{\propto\delta_{nn'}} e^{-iE^{(0)}_{n'}T}$$



pic Bali Pineda 2003, data Morningstar et al

For large T smallest static energy dominates

$$E^{(0)}_n(r) = \lim_{T\to\infty}\frac{i}{T}\ln\langle X_n(T)|X_n(0)\rangle$$

- in pNRQCD: short distance expansion in terms of light field correlators
- on the lattice: direct calculation beyond multipole expansion
- short-distance degeneracy reproduced in gluonic excitation spectrum from lattice

Operators for hybrids and $Q\bar{Q}$ tetraquarks

Gluonic excitation spectrum roughly shows scaling of static energies with mass dimension of short distance operators:

Λ_η^σ	pNRQCD operator
Σ_u^-, Π_u	$\hat{\boldsymbol{r}}\cdot\boldsymbol{B}$, $\hat{\boldsymbol{r}}\times\boldsymbol{B}$
Σ_g^+, Π_g	$\hat{\boldsymbol{r}}\cdot\boldsymbol{E}$, $\hat{\boldsymbol{r}}\times\boldsymbol{E}$
Σ_g^-, Π_g, Δ_g	projections of $\left[D_{\{i},B_{j\}}\right]$
Σ_u^+, Π_u, Δ_u	projections of $\left[D_{\{i},E_{j\}}\right]$

By analogy, we expect that lowest tetraquark or pentaquark states are generated by operators with lowest mass dimension: $\bar{q}_i(\boldsymbol{R})q_j(\boldsymbol{R})$

The spin, flavor, and color indices allow for different configurations:

- $q\bar{q}\to\underbrace{(1\oplus 3)}_{\text{spin}}\otimes\underbrace{(1\oplus 3)}_{\text{isospin}}\otimes\underbrace{(1\oplus 8)}_{\text{color}}$
- $Q\bar{Q}\to\underbrace{(1\oplus 3)}_{\text{spin}}\otimes\underbrace{(1\oplus 8)}_{\text{color}}$

light spin	Λ_η^σ	lowest J^{PC}
$s=0$	$\{\Sigma_u^-\}$	$\{0^{--},1^{-+}\}$
$s=1$	$\{\Sigma_g^+,\Pi_g\}$	$\{1^{+-},(0,1,2)^{++}\}$

for both isospin and color configurations

pNRQCD for QQ states and tetraquarks

pNRQCD with 2 heavy quarks: color antitriplet $Q_{\bar{3}}$ or sextet Q_6 diquark fields

$$H^{(0,0)} = H_0^{(HQET)} + \int d^3r d^3R \left[Q_{\bar{3}}^{a\,\dagger} V_{\bar{3}} Q_{\bar{3}}^a + Q_6^{a\,\dagger} V_6 Q_6^a \right]$$

with $$V_{\bar{3}}(r) = -\frac{2}{3}\frac{\alpha_{V\bar{3}}(r)}{r} \qquad V_6(r) = \frac{1}{3}\frac{\alpha_{V6}(r)}{r}$$

Tetraquarks analogous, with light color triplet or antisextet $\bar{q}_i(\boldsymbol{R})\bar{q}_j(\boldsymbol{R})$, but:

- static symmetries have P instead of CP
- $Q(-\boldsymbol{r},\boldsymbol{R}) = (-1)^S Q(\boldsymbol{r},\boldsymbol{R})$, so only (anti)symmetric wave functions allowed
- $(q_i(\boldsymbol{R}))^2 = 0$, so light operator obeys Pauli principle

QQ color state	static energies	light (iso)spin		J^P for heavy spin	
		I	s	$S=0$	$S=1$
antitriplet	Σ_g^+	0	0	1^-	$\mathbf{1^+}$
	$\{\Sigma_g^-, \Pi_g\}$	1	1	0^-	$\mathbf{(0,1,2)^+}$
sextet	$\{\Sigma_g^+\}$	1	0	$\mathbf{0^+}$	$(0,1,2)^-$
	$\{\Sigma_g^-, \Pi_g\}$	0	1	$\mathbf{1^+}$	1^-

QQq baryons and pentaquarks

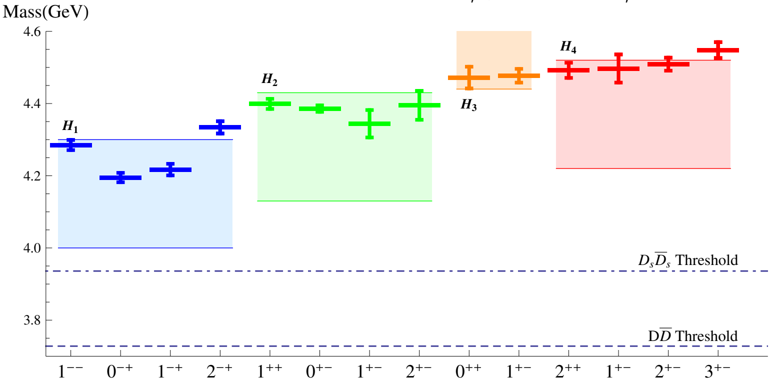
All can be dealt with in the same framework just with different light operators

type	light (iso)spin		J^P for heavy spin	
	I	s	$S=0$	$S=1$
$(QQ)_{\bar{3}}q_3$	$1/2$	$1/2$	$(1/2,3/2)^-$	$\mathbf{(1/2,3/2)^+}$
$(Q\bar{Q})_1(qqq)_1$	$1/2$	$1/2$	$\{(1/2)^-,(1/2,3/2)^-\}$	
	$3/2$	$3/2$	$\{(3/2)^-,(1/2,3/2,5/2)^-\}$	
$(Q\bar{Q})_8(qqq)_8$	any	$1/2$	$\{(1/2)^-,(1/2,3/2)^-\}$	
	$1/2$	$3/2$	$\{(3/2)^-,(1/2,3/2,5/2)^-\}$	
$(QQ)_{\bar{3}}(qq\bar{q})_3$	any	$1/2$	$\mathbf{(1/2)^-}$	$(1/2,3/2,5/2)^+$
	any	$3/2$	$\mathbf{(3/2)^-}$	$(1/2,3/2)^+$
$(QQ)_6(qq\bar{q})_{\bar{6}}$	any	$1/2$	$(1/2,3/2)^+$	$\mathbf{(1/2,3/2)^-}$
	any	$3/2$	$(1/2)^+$	$\mathbf{(1/2,3/2,5/2)^-}$

- all Schrödinger equations for $s=3/2$ are coupled
- shape of orbital term depends on quantum numbers l, P and P_{light}
- Pauli principle for light quarks rules out certain (I,s) combinations
- Pauli principle for QQ rules out some values of l based on P and S

Results for hybrids and comparison to lattice

Results for hybrids published in Phys. Rev. **D92** (2015) 11, 114019 [arXiv:1510.04299]
Nora Brambilla, Jaume Tarrus, Antonio Vairo, M.B.



Hadron Spectrum Collaboration 2012

- in EFT distinction between different spins only at order $1/M^2$
- good agreement for relative distance between spin-averaged multiplets
- some overall shift for absolute values

Conclusions and outlook

Conclusions

- treatment of heavy exotics in EFT framework (NRQCD and pNRQCD)
- study of the spectrum of the static Hamiltonian with non-static corrections
- short-distance limit of static energies: degenerate multiplets
- (un)coupled Schödinger equation (depending on multiplet)
- interplay of other quantum numbers, Pauli principle, etc., determines $J^{P(C)}$

Outlook

- numerical determination of static energies for light quark states
- inclusion of higher order terms (spin dependence, large distance contributions)
- study of other properties like decays

Thank you for your attention!