All-Heavy Tetraquarks and related topics

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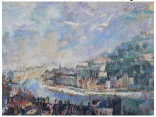

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Based on recent work with J. Vijande, A. Valcarce,
older work with Cafer Ay et Hyam Rubinstein
and ongoing work with Hiyama, Oka & Hosaka.

Introduction

- Clarify some aspects of quark model calculations
- Sorry, a little technical
- But before improving or replacing the quark model, one should know precisely what it predicts
- After a decade of $(Q\bar{Q}\ldots)$
- The flavored $(QQ\ldots)$ states come back
- Unfortunately, with sometimes a poor treatment of the dynamics
- Perhaps news in the pentaquark sector

Doubly-heavy baryons

- Obviously $r(QQ) < r(Qq)$ in (QQq)
- The two heavy quarks are clustered in the ground state
- But the naive diquark model is misleading
- The Hamiltonian

$$H = \frac{\boldsymbol{p}_1^2}{2\,M} + \frac{\boldsymbol{p}_2^2}{2\,M} + \frac{\boldsymbol{p}_3^2}{2\,m} - \text{c.o.m} + v(r_{12}) + [v(r_{13}) + v(r_{23})] \ ,$$

- is not very well approximated by

$$H' = \left[\frac{\boldsymbol{p}_x^2}{M} + v(x)\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + 2\,v(\sqrt{3}\,y/2)\right] \ ,$$

with $\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{y} = (2\,\boldsymbol{r}_3 - \boldsymbol{r}_1 - \boldsymbol{r}_2)/\sqrt{3}$, which factorizes.

- The diquark internal energy is modified by the third quark.

Doubly-heavy baryons

- For instance, in the case of the harmonic oscillator this gives

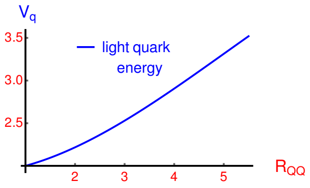
$$H' = \left[\frac{\boldsymbol{p}_x^2}{M} + x^2\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + \frac{3}{2}\,y^2\right] ,$$

instead of the exact

$$H = \left[\frac{\boldsymbol{p}_x^2}{M} + \frac{3}{2}\,x^2\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + \frac{3}{2}\,y^2\right] ,$$

- But the *Born-Oppenheimer* treatment is very good
- Especially if done in $\boldsymbol{y}$ at fixed $\boldsymbol{x}$, instead of $\boldsymbol{r}_3$ at fixed $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$
- For instance, with a linear potential, masses $M/m = 5$,
- $E_{\text{var}} = 4.940$ $E_{\text{BO}} = 4.938$ $E_{Dq} = 4.749$ (arbitrary units)

Doubly-heavy baryons



Born-Oppenheimer potential for (QQq), $M/m = 5$, $V \propto \sum_{i<j} r_{ij}$
Fleck, R., PTP 82 (1989) 760

$(QQ\bar{q}\bar{q})$

- $(QQ\bar{q}\bar{q})$ becomes stable if M/m large
- As shown by Eichten and Quigg in July 2017, Karliner & Rosner, Voloshin et al., . . .

$(QQ\bar{q}\bar{q})$

- $(QQ\bar{q}\bar{q})$ becomes stable if M/m large
- As shown by Eichten and Quigg in July 2017, Karliner & Rosner, Voloshin et al., . . .
- And confirmed 36 years earlier by Ader et al.

Do narrow heavy multiquark states exist?

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(Received 11 August 1981)

- And many others: Heller et al., Rosina et al., Brink et al., Lipkin, Barnea et al., Vijande et al., Oka et al., Bicudo et al., etc.

$(QQ\bar{q}\bar{q})$

- Chromoelectric interaction favors $(QQ\bar{q}\bar{q})$ vs. $(Q\bar{q}) + (Q\bar{q})$
- Chromomagnetic interactions also helps in some cases, e.g., 1^+
- In the chromoelectric limit, analogy with atomic physics

Stable multiquarks: Lessons from atomic physics
J.M. Richard (LPSC, Grenoble). 1992. 8 pp.
Published in **In *Bad Honnef 1992, Quark cluster dynamics* 84-91**
Prepared for Conference: C92-06-29.3, p.84-91 Proceedings

- Delicate 4-body calculation
- Some approximations favor binding artificially, e.g.

$$a\,(r_{12}^2 + r_{34}^2) + b\sum_{\substack{i=1,2\\ j=3,4}} r_{ij}^2 = \begin{cases} (a+b)(x^2+y^2) + 2\,b\,z^2 (\text{exact}) \\ (a \qquad)(x^2+y^2) + 2\,b\,z^2 (\text{diquarks}) \end{cases}$$

if $\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{y} = \boldsymbol{r}_4 - \boldsymbol{r}_3$ and $\boldsymbol{z} = (\boldsymbol{r}_3 + \boldsymbol{r}_4 - \boldsymbol{r}_1 - \boldsymbol{r}_2)/\sqrt{2}$

$(QQ\bar{q}\bar{q})$

- Toy model to probe 4-body techniques

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m_i} - \text{c.o.m.} - \frac{3}{16}\sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij})\ ,$$

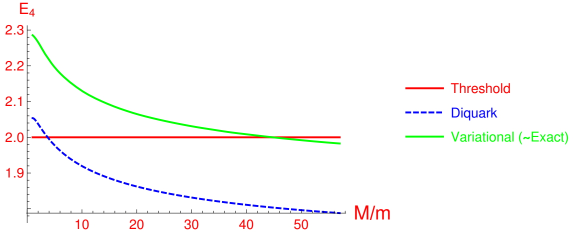
  with masses $\{m_i\} = \{M, M, m, m\}$.
- Two color wave functions (Chan H-M et al, "Color chemistry" in the 70s) $T = \bar{3}3$ and $M = 6\bar{6}$
- Mixing small except if M/m near critical value for binding
- $$H_T = \frac{\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2}{2\,M} + \frac{\boldsymbol{p}_3^2 + \boldsymbol{p}_4^2}{2\,m} - \text{c.o.m.} + \frac{1}{2}(v_{12} + v_{34}) + \frac{1}{4}\sum_{\substack{i=1,2\\ j=3,4}} v_{ij}$$
- BO approximation works very well, even for $M/m \gtrsim 1$
- Diquark-antidiquark approximation fails
- Variational calculation straightforward
- Hyperspherical expansion too (Barnea et al.)

$(QQ\bar{q}\bar{q})$

- For $v(r) = r$, $m = 1$ and $M = 5$, then

$$E_{\text{var}} = 4.331\ , \quad E_{\text{BO}} \sim 4.330\ , E_{D\bar{d}} = 3.899$$

- Scaling such that $1/M + 1/m = 2$, $E_2(M, m) = 1$



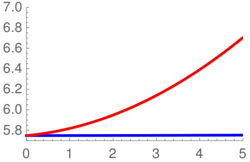
- Require Coulomb part and color mixing to lower the critical M/m

$(QQ\bar{q}\bar{q})$ Eichen-Quigg prescription

- Based on heavy-diquark–heavy-antiquark symmetry
- See Lipkin, Nussinov, . . . : analogies (QQq), (QQq) and $(QQ\bar{q}\bar{q})$
- But more quantitative (spin refinements omitted here)

$$(QQ\bar{q}\bar{q}) \stackrel{?}{=} (QQq) + (Qqq) - (Q\bar{q})$$

- Exact at $M/m \to \infty$ for our toy model H_T
- Overestimates $(QQ\bar{q}\bar{q})$ for finite M/m
- Linear case $m = 1$ and $M = 5$, lhs $= 4.362$ rhs $= 4.335$
- BO approach: exact at $R = 0$, but $V_{\rm BO}$ grows much faster for $(QQ\bar{q}\bar{q})$ than for (QQq) [shifted here by $(Qqq) - (Q\bar{q})$]



- Same in atomic physics for $H_2{}^+$ BO potential vs. H_2

All-heavy tetraquarks

- Many calculations in the past (Heller & Aerts, Vary & Llyod, …and recently (morning session)
- Different dynamics and degrees of rigor for 4-body
- Heller: adiabatic bag model (Kuti-Hasenfratz for $Q\bar{Q}$) extended to multiquark seemingly gives too much attraction
- Consider again

$$H_T = \frac{\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2}{2\,M} + \frac{\boldsymbol{p}_3^2 + \boldsymbol{p}_4^2}{2\,m} - \text{c.o.m.} + \frac{1}{2}\,(v_{12} + v_{34}) + \frac{1}{4}\sum_{\substack{i=1,2\\ j=3,4}} v_{ij}$$

vs. atomic physics

- Improved stability of $M/m \nearrow$
- $M = m$ Ps$_2$ stable vs. $(QQ\bar{Q}\bar{Q})$ unstable in H_T and H_M with $6\bar{6}$

All-heavy tetraquarks: First proof

- Consider the class of Hamiltonians (v is attractive)

$$H = \sum \boldsymbol{p}_i/(2m) + \sum g_{ij}\, v(r_{ij}) \ , \quad \sum g_{ij} = 2 \ .$$

- If g_{ij} are equal: highest energy, and, roughly speaking, the broader the distribution of g_{ij}, the lower the energy.
- Now, if you compare Ps_2 and quark models: Ps_2 favored

$(abcd)$	$v(r)$	g_{ij}	$\bar{g}$	Δg
Thr (1,3)+(2,4)	$-1/r, r$	$\{0,0,1,0,1,0\}$	$1/3$	0.22
Ps_2	$-1/r$	$\{-1,-1,1,1,1,1\}$	$1/3$	0.89
$[(qq)_{\bar{3}}(\bar{q}\bar{q})_3]$	$-1/r, r$	$\{1/2,1/2,1/4,1/4,1/4,1/4\}$	$1/3$	0.01
$[(qq)_6(\bar{q}\bar{q})_{\bar{6}}]$	$-1/r, r$	$\{-1/4,-1/4,5/8,5/8,5/8,5/8\}$	$1/3$	0.17

Mixing effects small

All-heavy tetraquarks: Second proof

- Hall-Post inequalities developed in the 50s in nuclear physics, and discovered in the framework of studies on the stability of matter, or meson/baryon inequalities (Ader et al., Basdevant et al., Nussinov et al., ...). Here (scale set to $M = 1/2$)

$$H_T = \sum \boldsymbol{p}_i^2 + \frac{1}{2}(V_{12} + V_{34}) + \frac{1}{4}(V_{13} + \cdots)\ ,$$

  where $V_{ij} = v(r_{ij})$ is the *quarkonium* potential.

- $$H_T = \frac{1}{2}(h_{12} + h_{34}) + \frac{1}{4}(h_{13} + h_{14} + h_{23} + h_{24})\ ,$$

  where $h_{ij} = \boldsymbol{p}_i^2 + \boldsymbol{p}_j^2 + V_{ij}$ is the quarkonium Hamiltonian.

- $$\min(H_T) \geq 2\min(h_{13}) = 2\,E_{\min}(Q\bar{Q})\ .$$

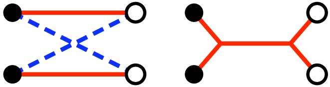
- Improved by removing c.o.m. in H_4 and in each h.

$$\min(H_T) \geq 2.3\,E_{\min}(Q\bar{Q})\ .$$

String potential for $QQ\bar{Q}\bar{Q}$?

- Instead of $\propto \sum \tilde{\lambda}_i.\tilde{\lambda}_j r_{ij}$, use

$$V = \min\left\{ r_{13} + r_{24}, r_{14} + r_{23}, \min_{J,K}(r_{1J} + r_{2J} + r_{JK} + r_{K3} + r_{K4}) \right\} ,$$



- Not so difficult (one does not need to compute the location of the junctions (Ay, R.,Rubinstein (2009), Bicudo et al.)
- gives more attraction (R., Vijande and Valcarce, 2007), and even binding for *equal masses* not submitted to the Pauli principle, say $(QQ'\bar{Q}\bar{Q}')$ with $M(Q) = M(Q')$ but $Q \neq Q'$.
- This restriction was forgotten in some recent papers

Summary for all-heavy

- $(cc\bar{c}\bar{c})$ and $(bb\bar{b}\bar{b})$ not bound in additive model nor in string-inspired variant
- Pity, would be suitable for J/ψ or Υ triggers.
- $(bb\bar{c}\bar{c})$ a little more favorable, mass ratio Q/q perhaps not large enough
- $(bc\bar{b}\bar{c})$ metastable, i.e., below its highest threshold, so a type of $(B_c\text{-}\bar{B}_c)$ molecule that can annihilate or rearrange itself into $(b\bar{b}) + (c\bar{c})$

Hidden-charm pentaquarks

- Two recent contributions:
- Bound states below the threshold
  - Valcarce, Vijande, R., Phys. Lett. B774 (2017) 710-714 [arXiv:1710.08239]
  - $(\bar{c}cqqq)$ with $I = 1/2$ and $J = 5/2$ below the lowest S-wave threshold $\bar{D}^*\Sigma_c^*$ (but above $N\eta_c$ in D-wave)
  - For $I = 3/2$ and $J = 1/2, 3/2$ binding below S- and D-wave thresholds
  - Both chromo-electric and -magnetic parts necessary for binding
- Resonances in the quark model
  - Hiyama et al. (work in progress): real scaling, borrowed from electron-atom and electron-molecule scattering to separate, among the energies above the threshold, actual resonances from fictitious states produced by the variational method. Looks promising.
  - Similar to Luscher criteria for lattice, stability plateau in QCDSR
  - See Hiyama contribution at “Critical Stability”, Dresden, Oct. 2017

Outlook

- The four-body problem is delicate, even for simple models
- Early references are often forgotten, e.g., for $(QQ\bar{q}\bar{q})$
- Stable $(bb\bar{b}\bar{b})$ unlikely
- Perhaps some $(Q\bar{Q}qqq)$ to be discovered
- Quark model can be adapted to resonances, but with care