

Theoretical Issues in Precision Physics at Future e^+e^- Colliders

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High Energy Physics Conference, IAS, Hong Kong, 23/01/2017

N_ν from Z invisible width

$$R_{\text{inv}}^0 = \frac{\Gamma_{\text{inv}}}{\Gamma_{ll}} = \sqrt{\frac{12\pi R_l^0}{\sigma_{\text{had}}^0 m_Z^2}} - R_l^0 - (3 + \delta_\tau)$$

- assuming lepton universality

$$(R_{\text{inv}}^0)_{\text{exp}} = N_\nu \left(\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{ll}} \right)_{\text{SM}}$$

- from LEP Z -peak measurements

$$N_\nu = 2.9840 \pm 0.0082 \quad \text{hint for right handed neutrinos?}$$

$$\delta N_\nu \simeq 10.5 \frac{\delta n_{\text{had}}}{n_{\text{had}}} \oplus 3.0 \frac{\delta n_{\text{lept}}}{n_{\text{lept}}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \implies \delta N_\nu = 0.0046$$

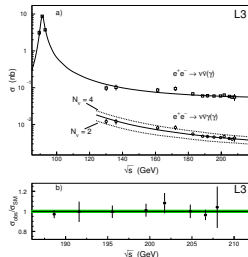
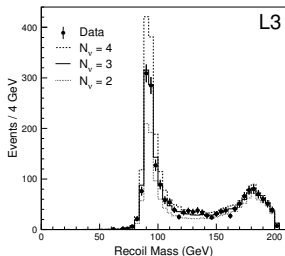
ADLO, SLD and LEPWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

- δN_ν severely affected by luminosity uncertainty through σ_0
(theory dominated at LEP $\implies$ see later)

Independent way for ν count: $\nu\bar{\nu}\gamma$ and LEP2

- no need to tune the collider energy at the Z peak (radiative return)
- provided large enough luminosity is available to be competitive with Γ_{inv} method

$$190 \text{ GeV} \leq \sqrt{s} \leq 208 \text{ GeV}, \mathcal{L} \sim 600 \text{ pb}^{-1}$$



L3 Collab., P. Achard et al., CERN-EP/2003-068 (2003)

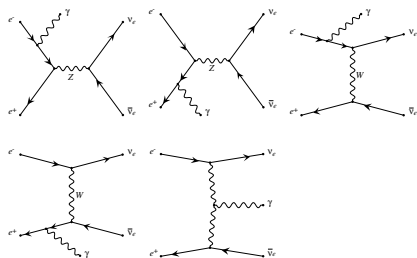
- agreement of data with SM predictions at % level
- $N_\nu = 2.98 \pm 0.05 \pm 0.04$ (L3) (important but not competitive with the Γ_{inv} method)
- similar results for ALEPH, DELPHI and OPAL

$\nu\bar{\nu}\gamma$: ratio measurements

- a factor $10^3/10^4$ of improvement in luminosity @FCC/CEPC w.r.t. LEP allows to exploit the ratios

$$\frac{d\sigma(e^+e^- \rightarrow \nu\bar{\nu}\gamma)}{d\sigma(e^+e^- \rightarrow \mu^+\mu^-\gamma)}$$

in order to cancel common systematics (such as luminosity)



- $\mu^+\mu^-$ only s -channel but ISR and FSR
- ν_μ and ν_τ f.s.: only s -channel ISR
- ν_e f.s.: ISR with t -channel
- ν_e f.s.: also W radiation

- QED and EW corrections do not cancel completely in the ratio**
- but now the technology for full $2 \rightarrow 3$ EW one-loop calculations is available**

Luminosity: theoretical systematics on σ normalization

- theoretical error in small angle Bhabha process at LEP1

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z -exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129

II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245

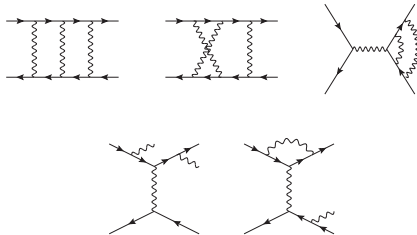
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

- after LEP, progress in complete two-loop pure photonic contributions to QED Bhabha scattering (see following two slides)
 - $\Rightarrow$ building blocks available for MC programs with th. precision below 0.1% on the perturbative side
 - this has already been achieved for large angle Bhabha at flavour factories, with a validation of the BabaYaga event generator below the 0.1% (for the QED higher orders)
- vacuum polarization issue, both for Bhabha as well as for Z observ.

G. Balossini et al., NPB758 (2006) 227; C.M. Carloni Calame et al., JHEP 1107 (2011) 126

NNLO Bhabha calculations (I)

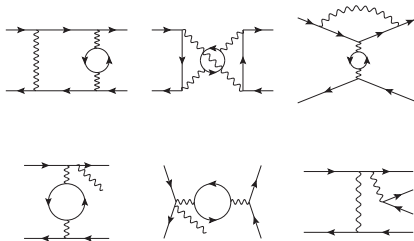
- **Photonic corrections** A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- **Electron loop corrections**

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26

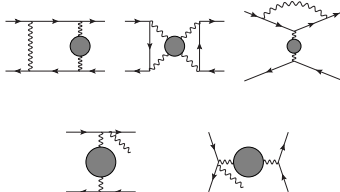


NNLO Bhabha calculations (II)

- Heavy fermion and hadronic loops

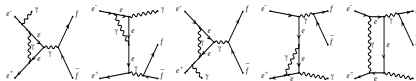
R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601
S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419



Vacuum Polarization: bottleneck for future precision

- $\alpha \rightarrow \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\Delta\alpha(q^2) = \Delta\alpha_{e,\mu,\tau,\text{top}}(q^2) + \Delta\alpha_{\text{had}}^{(5)}(q^2)$
- $\Delta\alpha_{\text{had}}^{(5)}$ is an **intrinsically non-perturbative** contribution. It can be calculated from $e^+e^- \rightarrow \text{hadrons}$ data using dispersion relations

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{q^2\alpha}{3\pi} \left[\oint_{4m_\pi^2}^{E_{\text{cut}}^2} \frac{R_{\text{had}}^{\text{data}}(s)}{s(s-q^2)} ds + \oint_{E_{\text{cut}}^2}^{\infty} \frac{R_{\text{had}}^{\text{pQCD}}(s)}{s(s-q^2)} ds \right]$$

- it is affected by an uncertainty, due to low energy data on $\sigma_{\text{had}}(s)$
 $\implies$ it reflects on Bhabha predictions $\implies Z$ -observables
- an historical perspective on the evolution of the error
 - $\Delta\alpha(M_Z^2) = 0.0280 \pm 0.0007 \implies \alpha^{-1}(M_Z^2) = 128.89 \pm 0.09$
 H. Burkhardt and B. Pietrzyk, Phys. Lett. B356 (1995) 398
 - $\Delta\alpha(M_Z^2) = 0.02750 \pm 0.00033$ H. Burkhardt and B. Pietrzyk, Phys. Rev. D84 (2011) 037502
 - $\Delta\alpha(M_Z^2) = 0.027498 \pm 0.000135 [0.027510 \pm 0.000218]$
 F. Jegerlehner, arXiv:1107.4683
 - $\Delta\alpha(M_Z^2) = 0.02757 \pm 0.0001 \implies \alpha^{-1}(M_Z^2) = 128.952 \pm 0.014$
 Davier, Hoecker, Malaescu, Zhang, arXiv:1010.4180
 - $\Delta\alpha(M_Z^2) = 0.027626 \pm 0.000138$
 T. Teubner et al., Nucl. Phys. Proc. Suppl. 225 (2012) 282

a comprehensive compilation by F. Jegerlehner

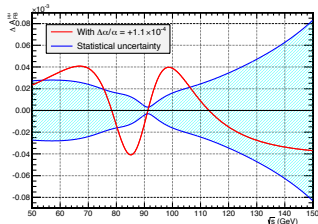
Authors	data	pQCD	sum
Jegerlehner 1985:	247.37 $\pm$ 7.	38.63 $\pm$ 0.37	286. $\pm$ 7.
Lynn et al. 1985:	145. $\pm$ 13.	129. $\pm$ 1.	274. $\pm$ 13.
Burkhardt et al. 1989:	164.32 $\pm$ 8.2	123.68 $\pm$ 3.71	288. $\pm$ 9.
Martin, Zeppenfeld 1994:	51.5 $\pm$ 1.1	221.7 $\pm$ 4.1	273.2 $\pm$ 4.2
Swartz 1995:	232.56 $\pm$ 4.6	42.64 $\pm$ 0.10	275.2 $\pm$ 4.6
Eidelman, Jegerlehner 1995:	237.55 $\pm$ 6.43	42.82 $\pm$ 0.10	280.37 $\pm$ 6.43
Burkhardt, Pietrzyk 1995:	159. $\pm$ 7.	121. $\pm$ 0.2	280. $\pm$ 7.
Adel, Yndurain 1995:	45.99 $\pm$ 0.85	226.6 $\pm$ 4.0	272.59 $\pm$ 4.09
Aleman, Davier, Höcker 1997:	238.01 $\pm$ 6.3	42.82 $\pm$ 0.10	280.9 $\pm$ 6.3
Kühn, Steinhauser 1998:	82.9 $\pm$ 1.40	194.45 $\pm$ 0.96	277.43 $\pm$ 1.70
Davier, Höcker 1998:	56.53 $\pm$ 0.83	219.77 $\pm$ 1.40	276.3 $\pm$ 1.6
Erler 1998 :	56.9 $\pm$ 1.1	220.8 $\pm$ 1.5	277.7 $\pm$ 1.9
Burkhardt, Pietrzyk 2001:	155.8 $\pm$ 3.6	120.3 $\pm$ 0.2	276.1 $\pm$ 3.6
Hagiwara et al 2004:	150.18 $\pm$ 2.3	125.32 $\pm$ 0.15	275.5 $\pm$ 2.3
Jegerlehner 2006 direct:	106.07 $\pm$ 2.24	115.66 $\pm$ 0.11	276.07 $\pm$ 2.25
Jegerlehner 2006 Adler :	73.69 $\pm$ 0.98	201.83 $\pm$ 1.03	275.52 $\pm$ 1.42
Hagiwara et al 2011:	138.70 $\pm$ 1.37	137.56 $\pm$ 0.16	276.26 $\pm$ 1.38
Davier et al 2011:	80.57 $\pm$ 1.00	195.33 $\pm$ 0.09	275.90 $\pm$ 1.00
Jegerlehner 2016 direct:	126.86 $\pm$ 1.78	149.57 $\pm$ 0.05	276.43 $\pm$ 1.78
Jegerlehner 2016 Adler:	60.49 $\pm$ 0.66	214.48 $\pm$ 1.00	275.04 $\pm$ 1.19

possible alternative to Bhabha scattering: $e^+e^- \rightarrow \gamma\gamma$

- $e^+e^- \rightarrow \gamma\gamma$ could be used to cross-check independently $\mathcal{L}$ measurements
 - ★ At present, its theoretical accuracy is similar to Bhabha (NLO + h.o.)
G. Balossini et al., Phys.Lett. B663 (2008) 209
 - ★ Advantages
 - no Z exchange diagrams (at LO)
 - no photon VP corrections (up to NNLO)
 - ★ Disadvantages
 - lower x-section
 - efficiency in detecting $\gamma\gamma$ events

- idea: measuring $\alpha(M_Z^2)$ from a measurement of A_{FB} below and above peak

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta A_{FB}^{\mu\mu}}{A_{FB}^{\mu\mu} - A_{FB,0}^{\mu\mu}} \times \frac{Z + \mathcal{G}}{Z - \mathcal{G}} \simeq \frac{\Delta A_{FB}^{\mu\mu}}{A_{FB}^{\mu\mu}} \times \frac{Z + \mathcal{G}}{Z - \mathcal{G}},$$



- preliminary investigations show that an accuracy of the order of 10^{-5} could be reached
- first studies on the impact of QED corrections, in particular initial-final interference

some theoretical issues for Z peak energy runs at future e^+e^- colliders

- the exceptional recent progress in the calculation of higher order corrections for LHC makes it plausible thinking that future progress in higher order electroweak corrections can meet the projected experimental accuracy
- for a successful physics program, theoretical improvements needed for
 - QED corrections and their unfolding in $e^+e^- \rightarrow f\bar{f}$
 - luminosity determination
- common issue given by the uncertainty in the hadronic contribution to the vacuum polarization
- recent promising proposal to determine $\alpha(M_Z^2)$ from A_{FB} below and above the Z peak