Accessing the gluon Wigner distribution in ultraperipheral pA collisions

arXiv:1706.01765 [hep-ph].

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Nucleon tomography

- Current interests in nucleon tomography
  - multi-dimensional phase space structure of the nucleon
    3-dim : GPD, TMD
    5-dim : GTMD, Wigner

- Can we measure the Wigner distribution?
  - Recently some proposals to probe the small-x gluon Wigner distribution in DIS has appeared.

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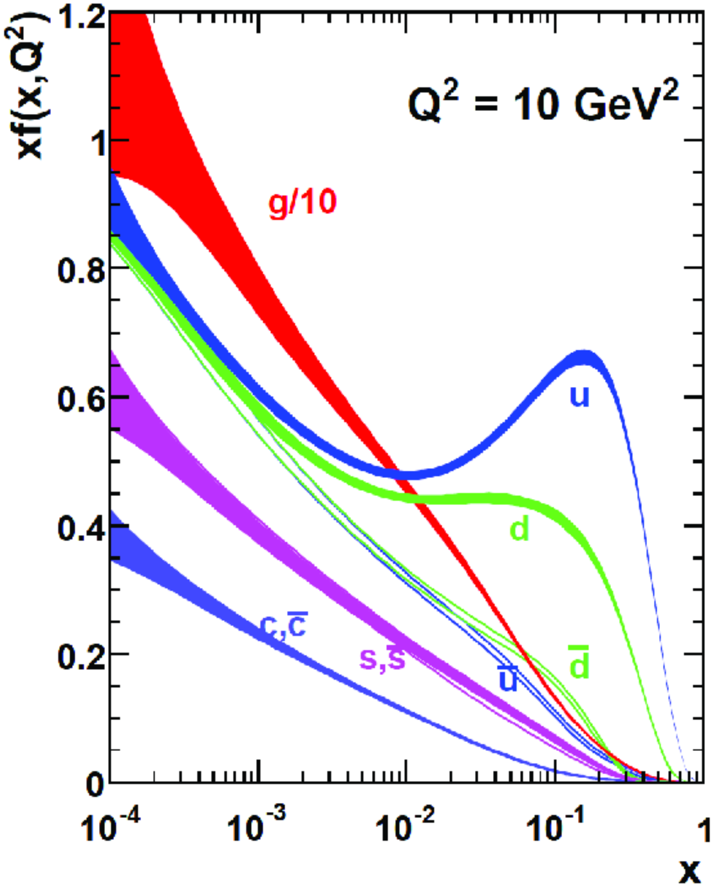
We argue that ultraperipheral pA collision is a promising process to more directly access the gluon Wigner distribution.

Introduction

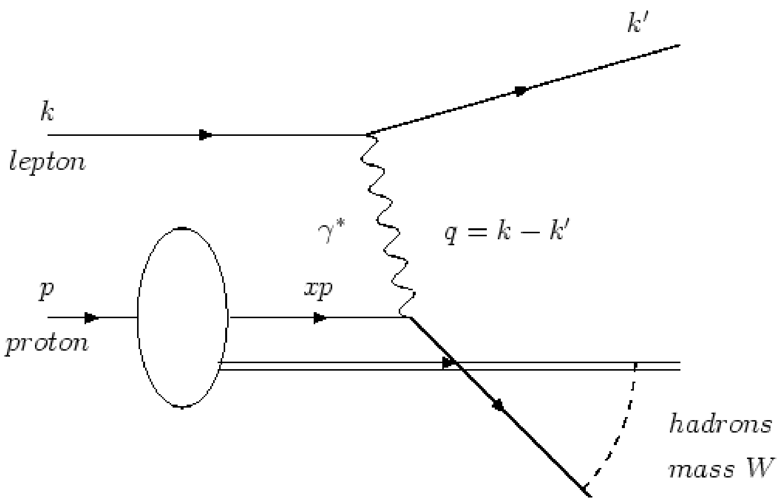
Parton distribution function (PDF)



Martin, A.D. et al. Eur.Phys.J. C63 (2009) 189-285

$f(x, Q^2)$: parton distribution function

x : momentum fraction wrt p
Q : momentum transfer$(-q^2 = Q^2)$



Generalization of the PDF

PDF：$f(x)$

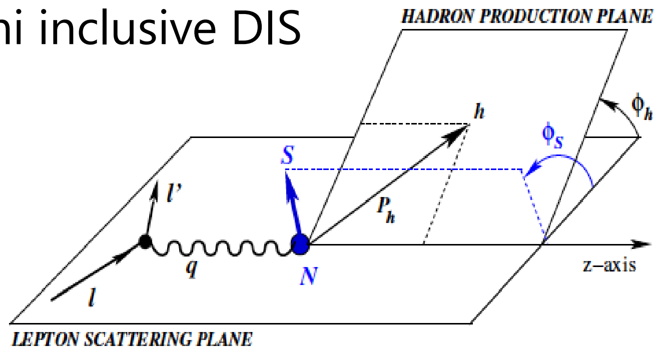
x : momentum fraction wrt P

$\int d^2\mathbf{k}_\perp$

Transverse momentum dependent PDF (TMD)：

$f(x, \mathbf{k}_\perp)$

Semi inclusive DIS

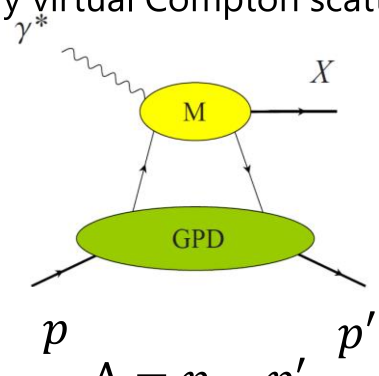


$\Delta_\perp \to 0$

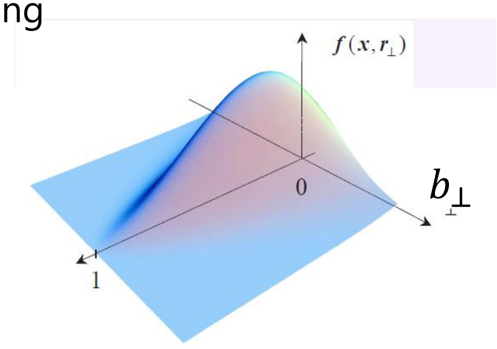
Generalized PDF (GPD)：

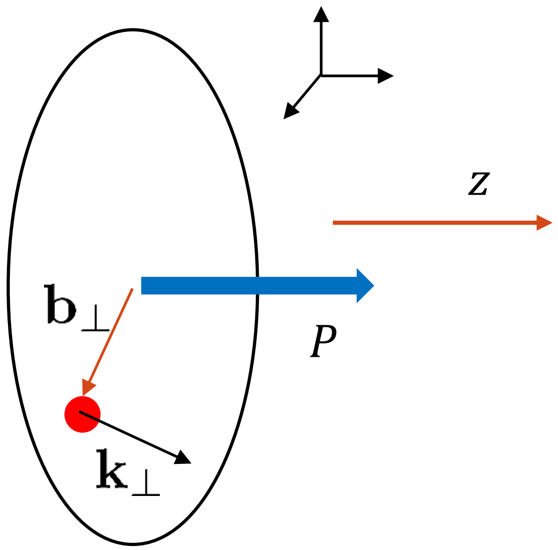
$G(x, \Delta_\perp)$ $\quad \mathbf{b}_\perp \overset{FT}{\leftrightarrow} \Delta_\perp$ $\quad \tilde{G}(x, b_\perp)$

Deeply virtual Compton scattering

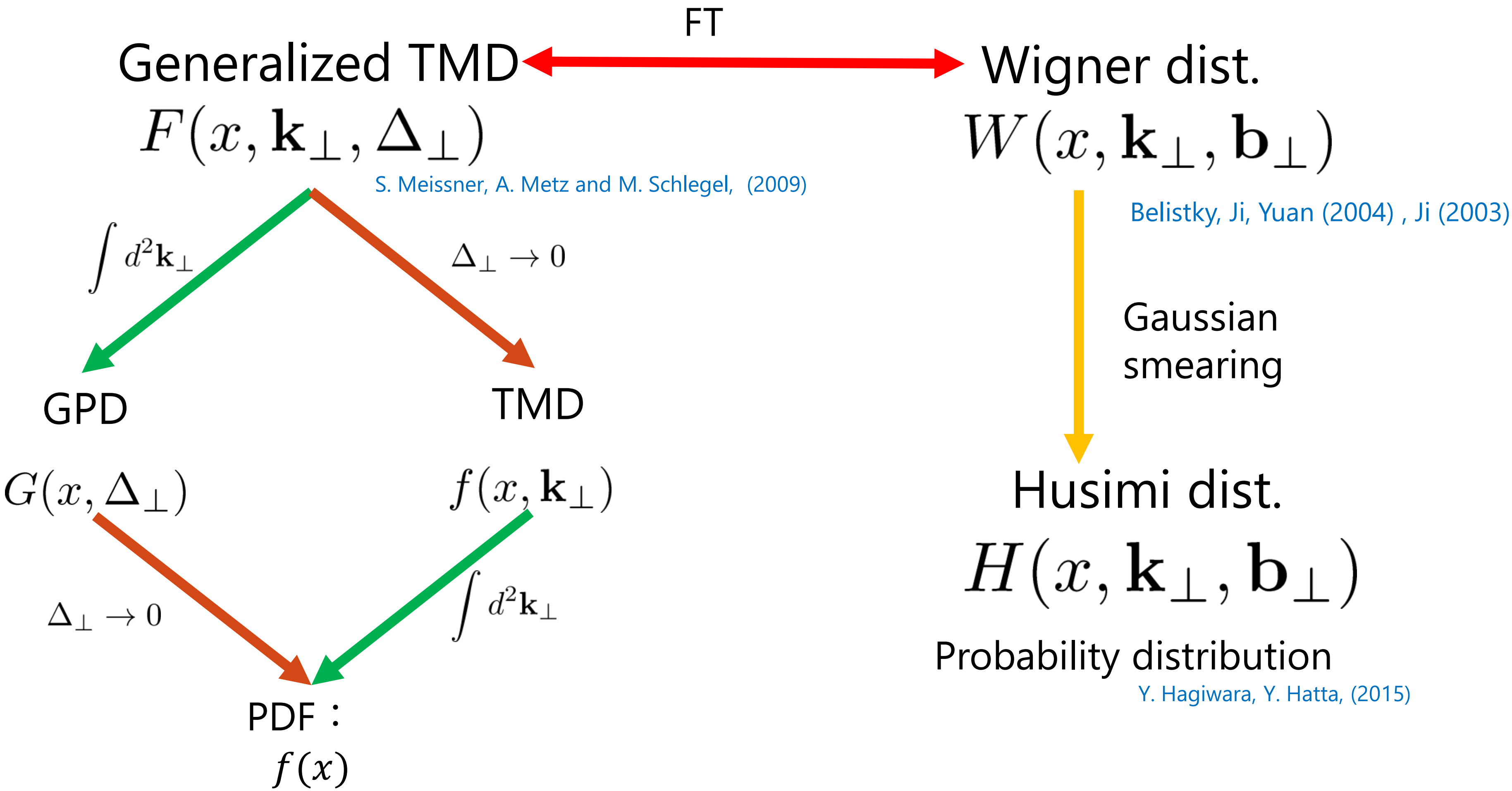


$\Delta = p - p'$





Phase space distributions



Wigner distribution in QM

E. Wigner. *Phys. Rev.* 40:749 (1932)

Wigner distribution

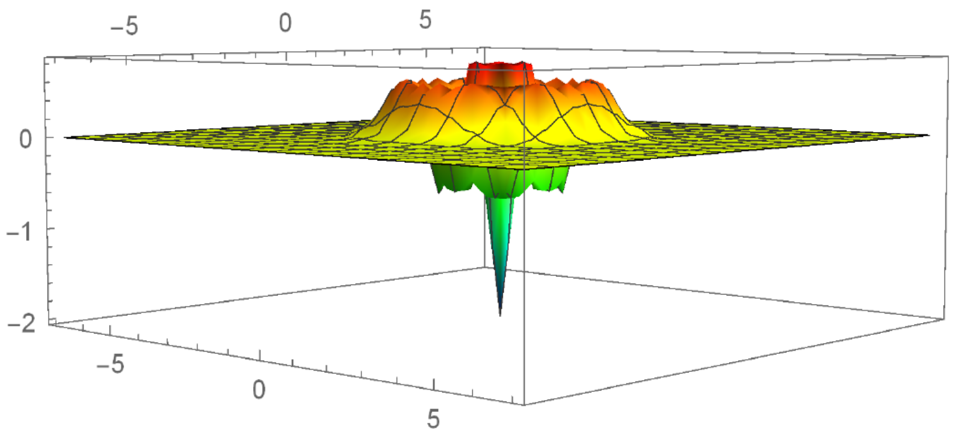
$$W(x,p) = \int d\xi e^{ip\xi}\psi^*(x-\xi/2)\psi(x+\xi/2)$$

$\psi(x)$: wave function

Ex. Harmonic Oscillator in 1D

$$W(q,p)^{(n)} = 2(-1)^n e^{-\frac{2H}{\hbar\omega}} L_n\left(\frac{4H_O}{\hbar\omega}\right)$$

$n = 3$



$$H_O = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

Wigner distribution in QCD

- Quark Wigner distribution

Belistky, Ji, Yuan (2004) , Ji (2003)

$$W_\Gamma(\vec{r}, k) = \frac{1}{2}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \langle \vec{q}/2 | \hat{\mathcal{W}}_\Gamma(0,k) | -\vec{q}/2 \rangle$$

$$\hat{\mathcal{W}}_\Gamma(\vec{r}, k) = \int d^4\xi e^{ik\cdot\xi} \bar{\Psi}(\vec{r} - \xi/2)\Gamma\Psi(\vec{r} + \xi/2)\delta(\xi^+)2\pi$$

Gluon : $\bar{\Psi}(\vec{r} - \xi/2)\Gamma\Psi(\vec{r} + \xi/2) \to F^{+\nu}(\vec{r} - \xi/2)F_\nu^+(\vec{r} + \xi/2)$

- Wigner distribution at high energy

Lorce, Pasquini (2011)

Using infinite Momentum Frame

$$W_\Gamma(\boldsymbol{b}_\perp, k) = \frac{1}{2}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot\boldsymbol{b}_\perp} \langle \Delta_\perp/2 | \hat{\mathcal{W}}_\Gamma(0,k) | -\Delta_\perp/2 \rangle$$

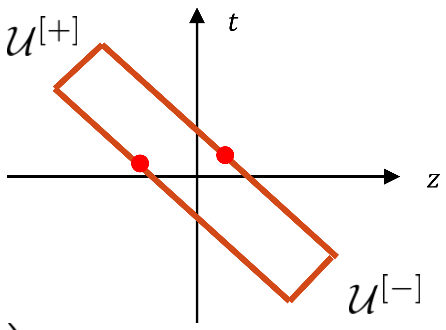
The gluon Wigner distribution

The gluon Wigner distribution

$$xW^g(x,\mathbf{k}_\perp,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot\mathbf{b}_\perp} \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{-ixP^+\xi^- - i\mathbf{k}_\perp\cdot\xi_\perp} \times \left\langle P+\frac{\Delta_\perp}{2}\left|\mathrm{Tr}\left[F^{+j}\left(\mathbf{b}_\perp+\frac{\xi}{2}\right)\mathcal{U}^{[-]\dagger}F^{+j}\left(\mathbf{b}_\perp-\frac{\xi}{2}\right)\mathcal{U}^{[+]}\right]\right|P-\frac{\Delta_\perp}{2}\right\rangle$$

$$\mathcal{U}^{[-]} := U[0,-\infty;0]U[-\infty,\xi^-;\xi_\perp]$$

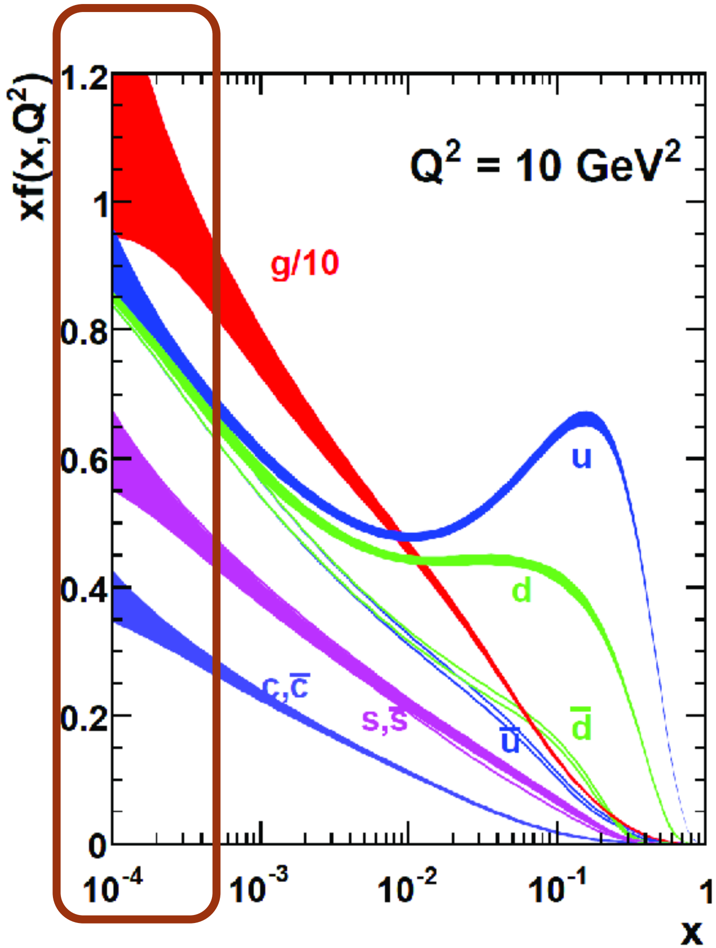
$$\mathcal{U}^{[+]} := U[0,\infty;0]U[\infty,\xi^-;\xi_\perp]$$



$$U[x_1^-,x_2^-;\boldsymbol{x}_\perp] \equiv \mathcal{P}\exp\left(ig\int_{x_1^-}^{x_2^-} dx^- T^c A_c^+(x^-,\boldsymbol{x}_\perp)\right)$$:Wilson line

Small x region

Small x



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Gluon saturation

Increase the CM energy (x becomes small)

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Number of partons increase

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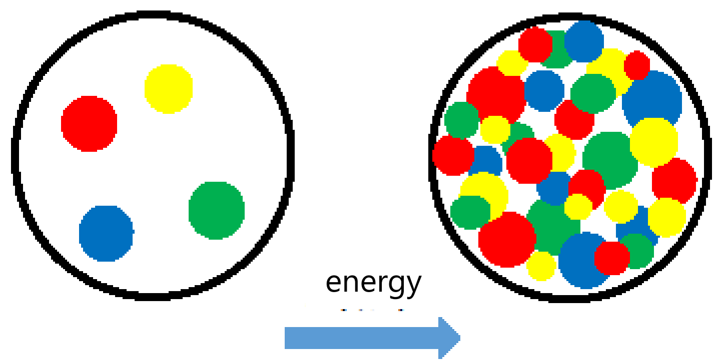
The number of partons become saturate because of the gluon recombination process



The gluon Wigner distribution

- Small x approximation

$$x \ll 1 \rightarrow e^{-ixP^+\xi^-} \approx 1$$

The gluon Wigner distribution at small x

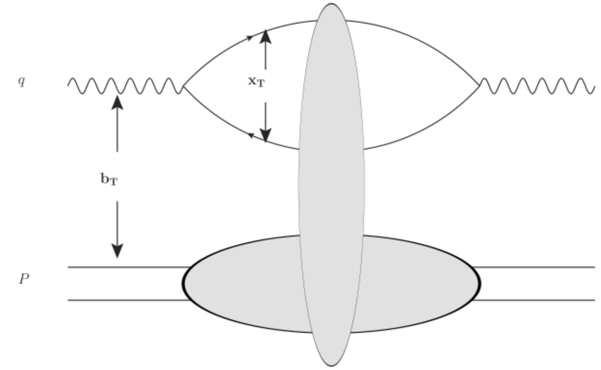
$$xW_g(x,\mathbf{k},\mathbf{b}_\perp) = \frac{2N_c}{\alpha_S}\int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}} \left(\frac{1}{4}\nabla^2_{\mathbf{b}_\perp} - \nabla^2_{\mathbf{r}}\right) S_Y(\mathbf{r},\mathbf{b}_\perp)$$

Y. Hatta, B. W. Xiao, F. Yuan Phys. Rev. Lett. 116, 202301 (2016)

$$S_Y(\mathbf{r},\mathbf{b}_\perp) = \frac{1}{N_c} \mathrm{tr}\left(U(\mathbf{b}_\perp + \mathbf{r}/2)U^\dagger(\mathbf{b}_\perp - \mathbf{r}/2)\right)$$

: S-matrix of the dipole-nucleon scattering

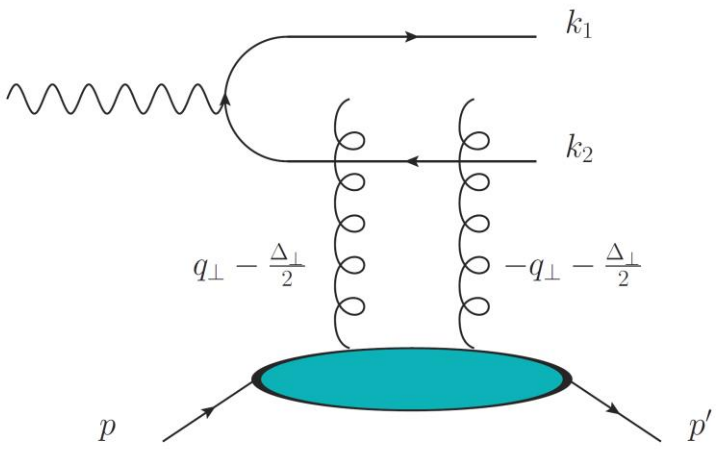
$Y = \ln(1/x)$:rapidity



$S_Y(\mathbf{r},\mathbf{b}_\perp)$ → Wigner distribution!

Accessing
the gluon Wigner distribution

Diffractive di-jet production in DIS



Y. Hatta, B. W. Xiao, F. Yuan Phys. Rev. Lett. 116, 202301 (2016)

$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

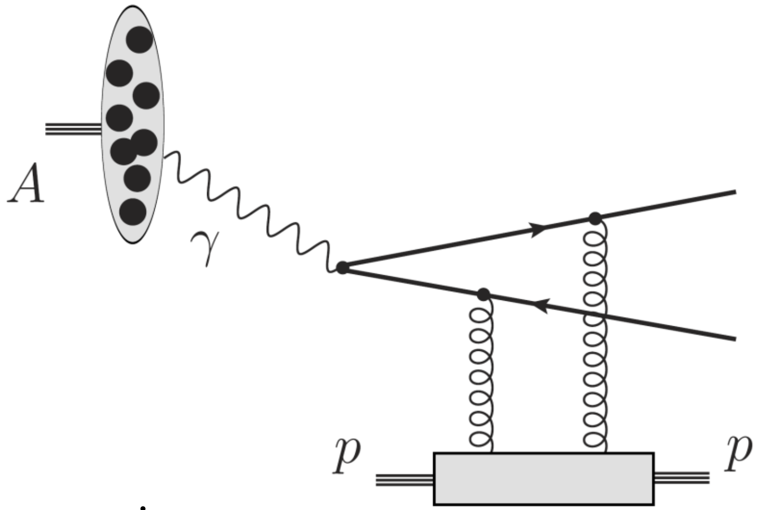
$$\vec{k}_{1\perp} + \vec{k}_{2\perp} = -\vec{\Delta}_\perp$$

$$\frac{d\sigma^{\gamma_T^* A \to q\bar{q}X}}{dy_1 d^2k_{1\perp} dy_2 d^2k_{2\perp}} = 2N_c\alpha_{em}e_q^2\delta(x_{\gamma^*} - 1)z(1-z)[z^2 + (1-z)^2]\int d^2\vec{q}_\perp d^2\vec{q}'_\perp S(\vec{q}_\perp, \vec{\Delta}_\perp)S(\vec{q}'_\perp, \vec{\Delta}_\perp)$$
$$\times \left[\frac{P_\perp}{P_\perp^2 + \epsilon_f^2} - \frac{P_\perp - q_\perp}{(P_\perp - q_\perp)^2 + \epsilon_f^2}\right] \cdot \left[\frac{P_\perp}{P_\perp^2 + \epsilon_f^2} - \frac{P_\perp - q'_\perp}{(P_\perp - q'_\perp)^2 + \epsilon_f^2}\right]$$

$$\epsilon_f^2 \equiv z(1-z)Q^2 \qquad q^+ = \sqrt{2}\omega \qquad z = \frac{k_{1\perp}e^{y_1}}{k_{1\perp}e^{y_1} + k_{2\perp}e^{y_2}}$$

Extraction of S from the data complicated.
Going to small Q^2 helps.

Ultraperipheral pA collisions



Use the Weiszäcker-Williams photons with $Q^2 \sim 0$ in UPC!

- Cross section

$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} = \int d\omega \frac{dN}{d\omega} \frac{d\sigma^{p\gamma}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}}$$

ω : photon energy

$$\frac{dN}{d\omega} = \frac{2 Z^2 \alpha_{em}}{\pi\omega} \left[\xi K_0(\xi) K_1(\xi) - \frac{\xi^2}{2} (K_1^2(\xi) - K_0^2(\xi)) \right]$$

: the photon flux

Z : atomic number

$$\xi = \omega \frac{R_p + R_A}{\gamma} \qquad \gamma = \frac{\sqrt{s_{NN}}}{2m_p}$$

R_p, R_A : radii of the proton and the nucleus

How to access the gluon Wigner Distribution

Amplitude when $Q = 0$

$$\vec{M}(\vec{P}_\perp, \vec{\Delta}_\perp) = \int \frac{d^2\vec{q}_\perp}{2\pi} \frac{\vec{P}_\perp - \vec{q}_\perp}{(\vec{P}_\perp - \vec{q}_\perp)^2} S(\vec{q}_\perp, \vec{\Delta}_\perp)$$

$$S(\vec{q}_\perp, \vec{\Delta}_\perp) = S_0(q_\perp, \Delta_\perp) + 2\cos 2(\phi_q - \phi_\Delta)\tilde{S}(q_\perp, \Delta_\perp)$$

Integrate over the azimuthal angle. Approximate $S_0 \gg \tilde{S}$ (YH, Hatta, Ueda (2016))

$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} \approx \omega \frac{dN}{d\omega} \frac{2(2\pi)^4 N_c \alpha_{em}}{P_\perp^2} \sum_f e_f^2 z(1-z)(z^2 + (1-z)^2)\left(A^2 + 2\cos 2(\phi_P - \phi_\Delta)AB\right)$$

$$\omega = \frac{1}{2}(k_{\perp 1} e^{y_1} + k_{\perp 2} e^{y_2})$$

$$A(P_\perp, \Delta_\perp) \equiv -\int_0^{P_\perp} dq_\perp q_\perp S_0(q_\perp, \Delta_\perp)$$

$$B(P_\perp, \Delta_\perp) \equiv -\int_0^{P_\perp} dq_\perp \frac{q_\perp^3}{P_\perp^2} \tilde{S}(q_\perp, \Delta_\perp) + \int_{P_\perp}^{\infty} dq_\perp \frac{P_\perp^2}{q_\perp} \tilde{S}(q_\perp, \Delta_\perp)$$

How to access the gluon Wigner Distribution

- The angular independent part

$$S_0(P_\perp, \Delta_\perp) = -\frac{1}{P_\perp}\frac{\partial}{\partial P_\perp} A(P_\perp, \Delta_\perp)$$

- The elliptic part

$$B(v) = \int_0^\infty dt\, e^{-t}\left(C(v+t) - C(v-t)\right) \longrightarrow \frac{dC(v)}{dv} = \frac{1}{2}\left(B(v) - \frac{d^2B(v)}{dv^2}\right)$$

$$t = \ln P_\perp^2/q_\perp^2 \quad C(u) = \frac{q_\perp^2}{2}\tilde{S}(q_\perp)$$

$$\tilde{S}(P_\perp, \Delta_\perp) = -\frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} + \frac{2}{P_\perp^2}\int_0^{P_\perp^2} \frac{dP_\perp'^2}{P_\perp'^2} B(P_\perp', \Delta_\perp)$$

→ Wigner distribution!

A model calculation

- Compute S by numerically solving the Balitsky-Kovchegov equation

$$\partial_Y S_Y(\mathbf{x},\mathbf{y}) = \frac{\alpha_S}{2\pi} N_c \int d^2\mathbf{z} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \{S_Y(\mathbf{x},\mathbf{z})S_Y(\mathbf{z},\mathbf{y}) - S_Y(\mathbf{x},\mathbf{y})\}$$

$Y = \ln(\frac{1}{x})$

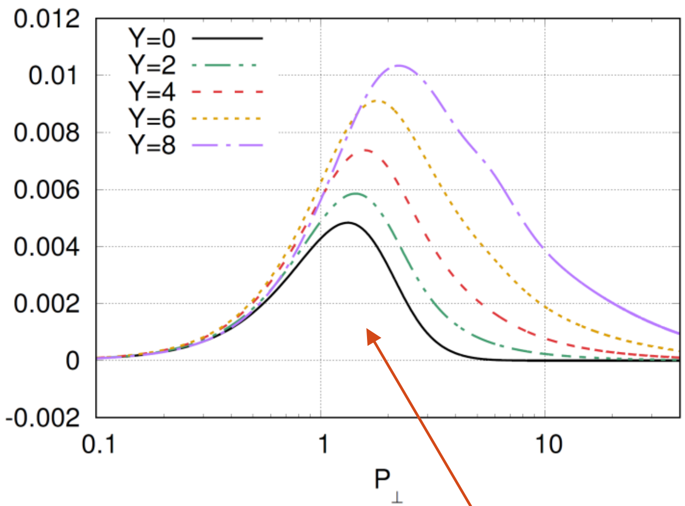
- Keep the dependence on $\mathbf{b}_\perp = (\mathbf{x}+\mathbf{y})/2$
- Assume the SO(3) symmetry (subgroup of conformal group) Gubser (2011)

$$S_Y(\boldsymbol{x},\boldsymbol{y}) = S_Y(d^2(\boldsymbol{x},\boldsymbol{y}))$$

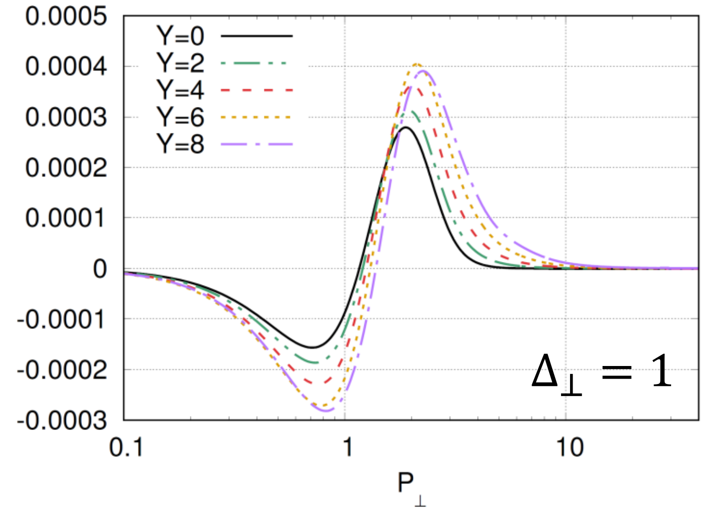
$$d^2(\mathbf{x},\mathbf{y}) = \frac{R^2(\mathbf{x}-\mathbf{y})^2}{(R^2+\mathbf{x}^2)(R^2+\mathbf{y}^2)} = \frac{R^2\mathbf{r}^2}{\left(R^2+\mathbf{b}_\perp^2+\frac{\mathbf{r}^2}{4}\right)^2 - \frac{\mathbf{b}_\perp^2\mathbf{r}^2}{2} - \frac{\mathbf{b}_\perp^2\mathbf{r}^2}{2}\cos 2\phi_{br}}$$

Numerical results

A function

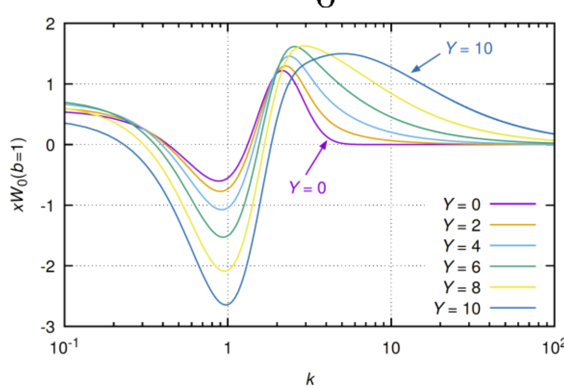


B function

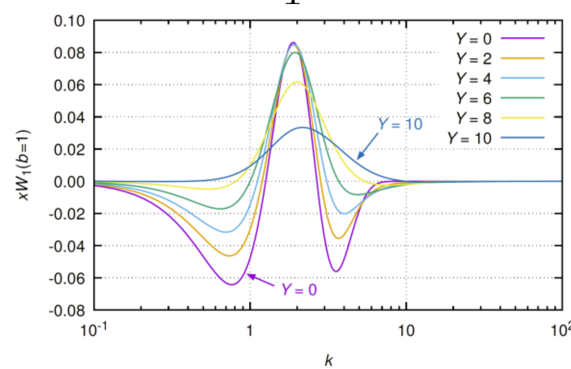


Peak position = Saturation momentum $Q_s(Y)$

W_0



W_1



cf. Wigner distribution

YH, Hatta, Ueda (2016)

Summary

- Measurement of gluon Wigner in UPCs:

  Cross section enhanced by Z^2
  Deconvolution can be done analytically.

- Measurable at RHIC and the LHC.

- Quark Wigner? Bhattacharya, Metz, Zhou (2017)

- Elliptic Wigner affects various observables.

  Elliptic flow in pp, pA, — YH, Hatta, Xiao, Yuan
  $\cos(2\phi)$ in DVCS, — Hatta, Xiao, Yuan, Zhou
  quasielastic scattering