Overview of large momentum effective theory

Jianhui Zhang
University of Regensburg

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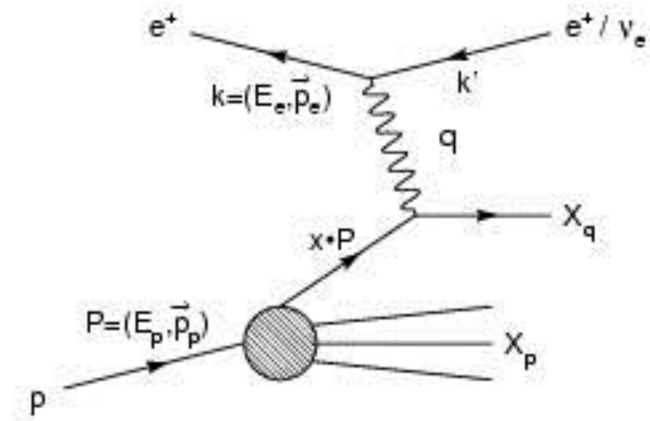
- Large momentum effective theory
- Results on parton distributions
- Summary and outlook

Introduction

- Hadrons are complicated bound systems of quarks and gluons described by the QCD Lagrangian

- A great simplification for the structure of hadron at high energy is provided by parton model
  - Approximation by taking hadron momentum $P_h \to \infty$
  - Hadrons can be viewed as constituted by free, non-interacting partons



  - DIS cross section can be written as an incoherent sum of virtual photon-quark partonic cross sections, weighted by the probability of finding each quark in the proton with a given longitudinal momentum fraction

$$\frac{d^2\sigma}{dx dQ^2} \sim \sum_{q,\bar{q}} \int_x^1 \frac{dz}{z} f_q(z) \hat{\sigma}_{q\gamma^* \to X}(x/z)$$

Introduction

- Physics at infinite momentum limit can be formulated in terms of light-cone quantization
  - Light-cone coordinates $\xi^{\pm} = (t \pm z)/\sqrt{2}$
  - Quantization at equal light-cone time $\xi^{+} = 0$
  - Probability interpretation of parton densities, e.g.

$$\boxed{q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle}$$

  - Non-local correlator, gauge invariance ensured by the insertion of a gauge link
  - Expectation of quark number operator in light-cone gauge $A^+ = 0$
  - Invariant under boost in z direction, valid in rest frame

Introduction

- Precise knowledge of parton observables such as the PDFs are important to
  - Reduce uncertainties in theory prediction of hadronic cross sections
  - Disentangle new physics effects from the standard model

- On the other hand, they are non-perturbative quantities defined in terms of light-cone correlators, and difficult to compute

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  - PDF collaborations (CTEQ, MSTW, NNPDF……)

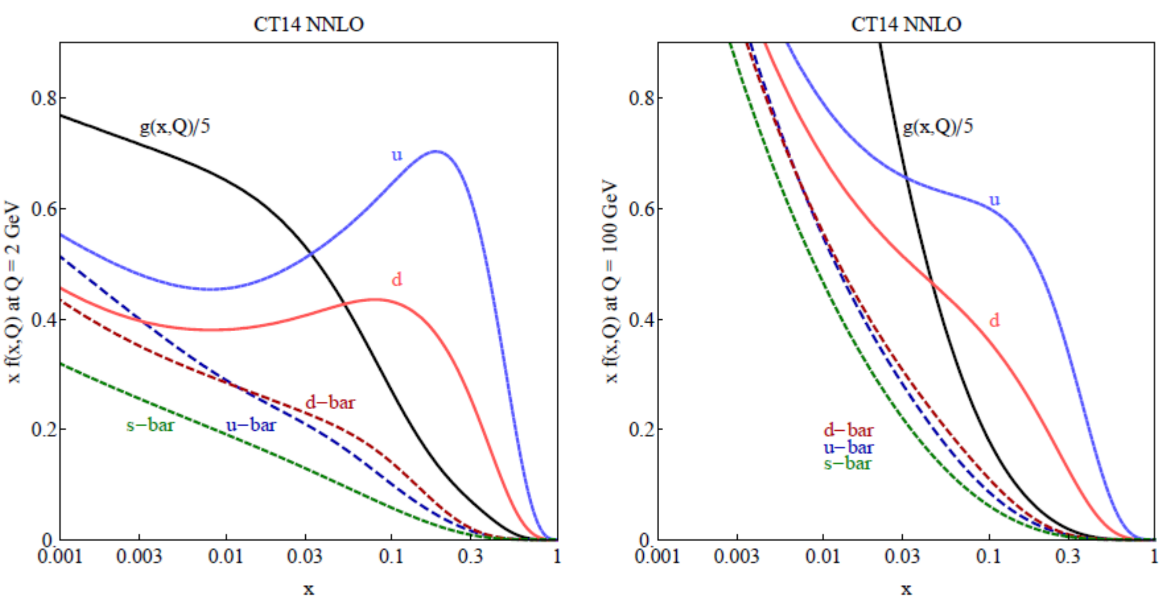

FIG. 4: The CT14 parton distribution functions at $Q = 2$ GeV and $Q = 100$ GeV for $u, \overline{u}, d, \overline{d}, s = \overline{s}$, and g.

Dulat et. al.
arxiv: 1506.07443

$$x\, f_a(x, Q_0) \;=\; x^{a_1}\,(1-x)^{a_2}\, P_a(x)$$

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- On the other hand, they are non-perturbative quantities defined in terms of light-cone correlators, and difficult to compute
  - Lattice QCD (main obstacle: light-cone dependence)
    - Can compute moments which are local operator matrix elements
    - Plagued by operator mixing

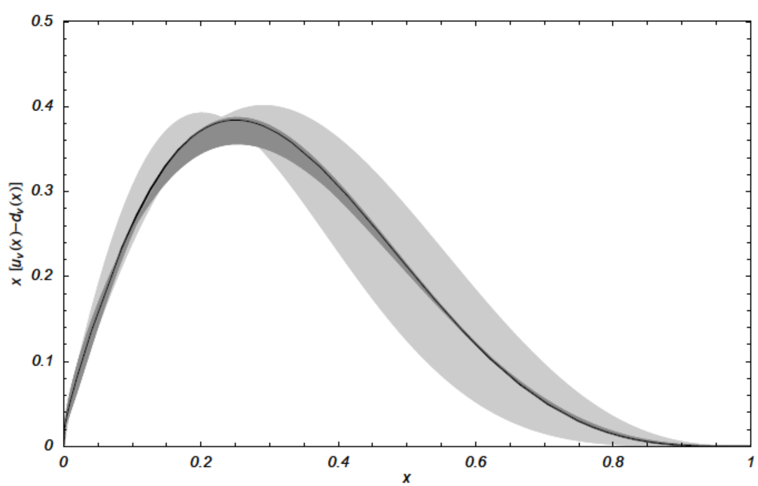

Fig. 5. Reconstructed isovector valence quark distribution $x(u_v - d_v)$ in the proton at $Q^2 = 4\ \mathrm{GeV}^2$. The central fit curve (solid line) and error band (lightly shaded) are compared with the envelope of the phenomenological distributions[20] (darkly shaded).

Detmold et. al.
Mod.Phys.Lett.A, 03'

$$\boxed{xf(x) = Ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)}$$

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    - For pion DA, only the 2^{nd} moment is computed

Introduction

- To study parton physics from lattice QCD, we need to move away from the light-cone
- Observations:
- QFT has UV divergences, which can be characterized by a UV cutoff Λ
  - Parton physics corresponds to taking $P_h \to \infty$ before $\Lambda \to \infty$
    - Leads to light-cone correlations
  - If $\Lambda \to \infty$ is taken prior to $P_h \to \infty$
    - Parton physics will depend on P_h
    - Also parton momentum fraction is not necessarily constrained to 0<x<1
    - Not light-cone correlations, but practically calculable

Introduction

- To study parton physics from lattice QCD, we need to move away from the light-cone

- Observations:
- The two different orders of limit are similar to the difference between an effective field theory/full theory
  - Heavy quark effective theory is obtained by taking $m_q \to \infty$ before $\Lambda \to \infty$
  - A finite m_q is maintained in full theory
  - They can be connected by a perturbative matching

⇒ Large momentum effective theory

Large momentum effective theory

- An effective theory framework that allows to compute light-cone or parton observables from Euclidean quantities [Ji 14′]

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- For a light-cone observable, e.g. the PDF or the pion DA, construct a (frame-dependent) Euclidean quasi observable

$$\boxed{\tilde{O}(P,\mu) \stackrel{P\to\infty}{\longrightarrow} O(\mu)}$$

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$$\boxed{\tilde{O}(P,\mu) \stackrel{P\to\infty}{\longrightarrow} O(\mu)}$$

  - The choice of $\tilde{O}(P,\mu)$ is not necessarily unique

  - Instead of computing the light-cone observable directly, one can compute the quasi observable at a finite hadron momentum P

  - The difference between quasi and light-cone observables is in finite (but large) or infinite momentum, hence they shall have the same IR physics

Large momentum effective theory

- In general, both quasi and light-cone observables suffer from UV divergences.
- They have different UV behavior because of different order of limits (the infinite momentum limit and the UV regularization are not exchangeable)
  - Taking inf. mom. limit first → physical case, light-cone
  - Taking UV regularization first → frame dependent, but practically calculable
- The difference can be perturbatively computed and captured in a matching factor Z

$$\tilde{O}(P,\mu) = Z(\frac{P}{\mu})O(\mu) + \mathcal{O}(\frac{M^2}{P^2})$$

  - M is a typical hadronic mass scale, like hadron mass or Λ_{QCD}

Large momentum effective theory

$$\tilde{O}(P,\mu) = Z(\frac{P}{\mu})O(\mu) + \mathcal{O}(\frac{M^2}{P^2})$$

- Analogous to e.g. the HQET. The role of heavy quark mass is now played by the large hadron momentum

- Parton model is an effective theory for the nucleon moving at large momentum

- The Z factor contains logarithm of $\ln\frac{P}{\mu}$ when P becomes large, which reflects the non-smooth light-cone limit. It can be resummed by RGE

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- Summary:
  - Euclidean quasi observable is practically computable on the lattice
  - Perturbative matching allows to extract light-cone observable from the quasi observable
  - A good approximation of the light-cone observable can be achieved at a moderately large momentum

An example: unpol. quark PDF

- Light-cone PDF

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

- The quasi-PDF can be constructed as

$$\tilde{q}(x,\Lambda,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0,0_\perp,z)\gamma^z \exp\left(-ig\int_0^{z} dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

  - z is a spatial direction
  - It approaches $q(x)$ in the limit $P_z \to \infty$
  - An alternative choice is to replace $\gamma^z \to \gamma^0$

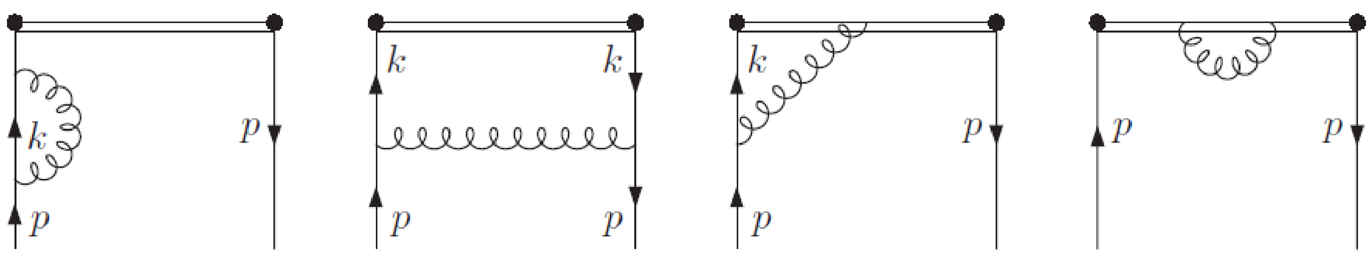
- Matching relation

$$\tilde{q}(x,\Lambda,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) q(y,\mu) + \mathcal{O}\left(\Lambda_{\rm QCD}^2/(P^z)^2, M^2/(P^z)^2\right)$$

- This factorized form holds to all-orders [Ma and Qiu 14′]

From quasi-PDF to PDF

- The matching factor Z can be computed order by order in perturbation theory
  - At one-loop [Xiong et al. 13′]



  - The quasi distribution does not vanish outside [-1,1]
  - Both distributions have soft and coll. div., soft div. cancels in themselves, coll. div. identical

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}\,, \qquad \xi > 1$$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln\left[4\xi(1-\xi)\right] - \frac{2\xi}{1-\xi} + 1 + \frac{\Lambda}{(1-\xi)^2 P^z}\,, \qquad 0 < \xi < 1$$

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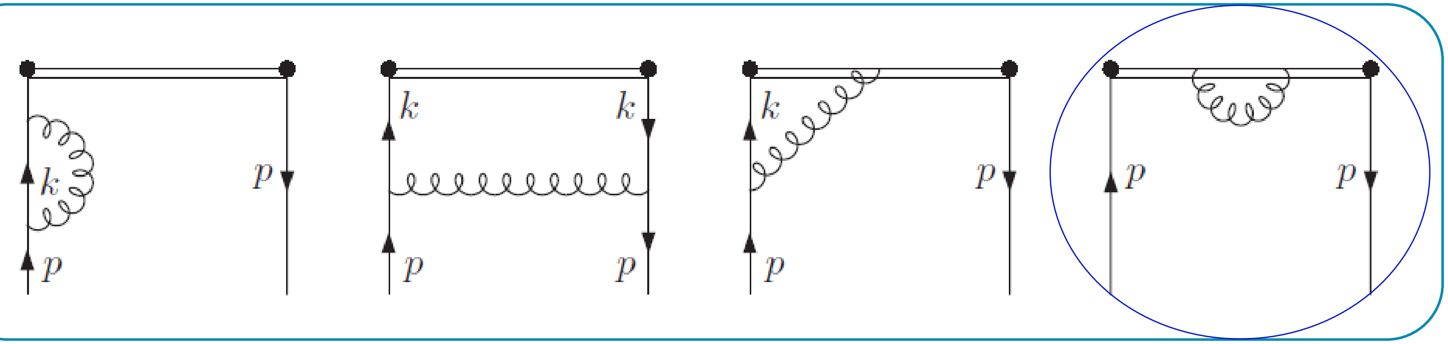

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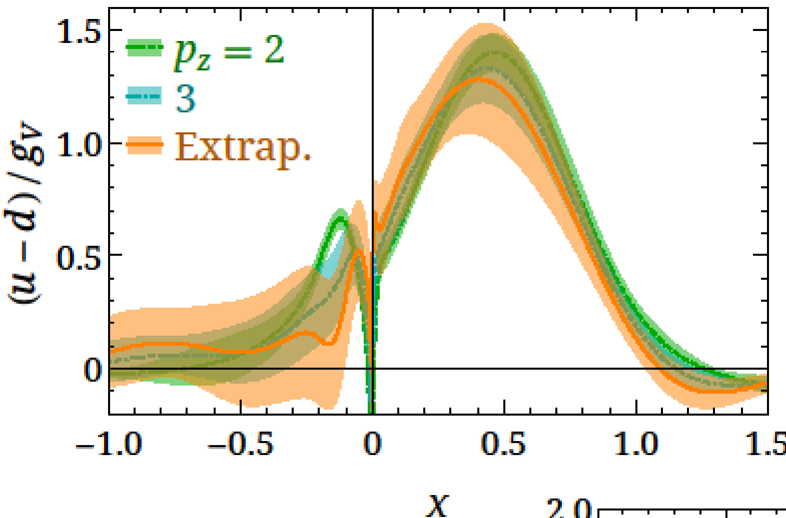
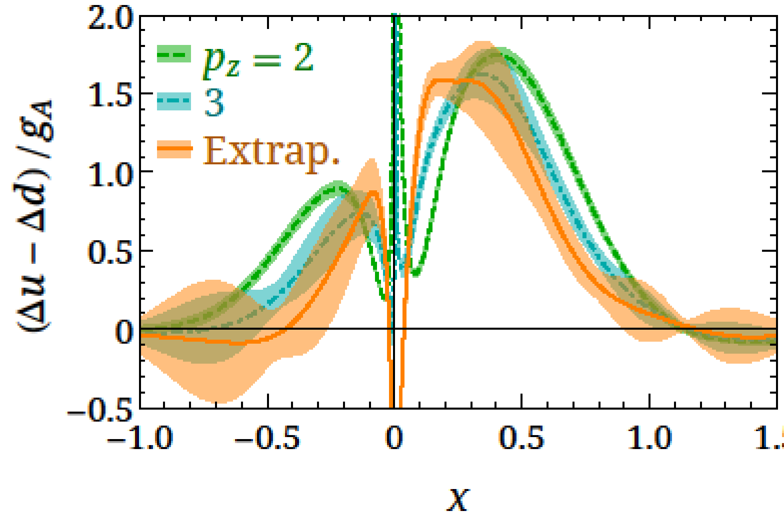
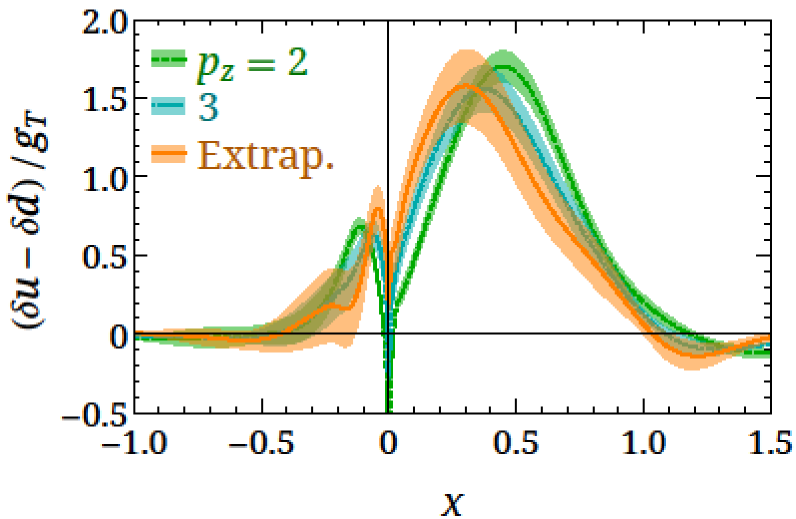
Results from lattice simulations

- Unpolarized, helicity and transversity distributions
  - HYP smearing to smoothen Wilson line gauge links
  - One-loop matching + mass corrections + higher-twist corrections







Chen et. al. 16'
$m_\pi = 310\ MeV, L = 3fm, a = 0.12fm$

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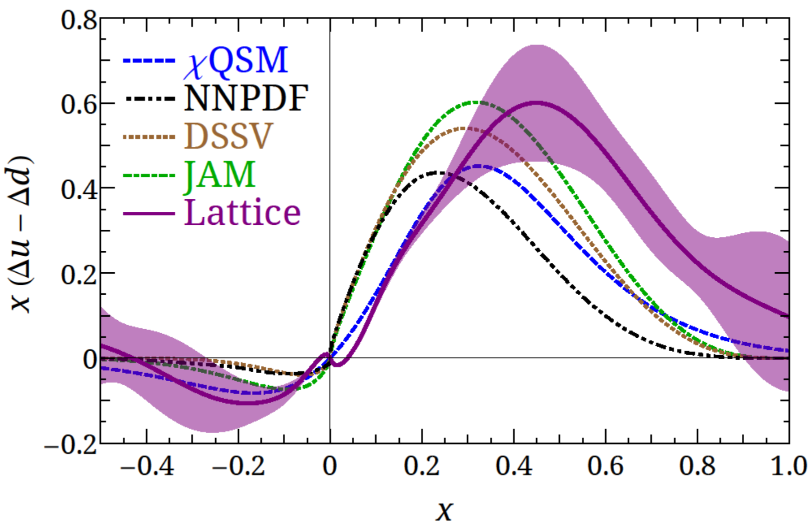
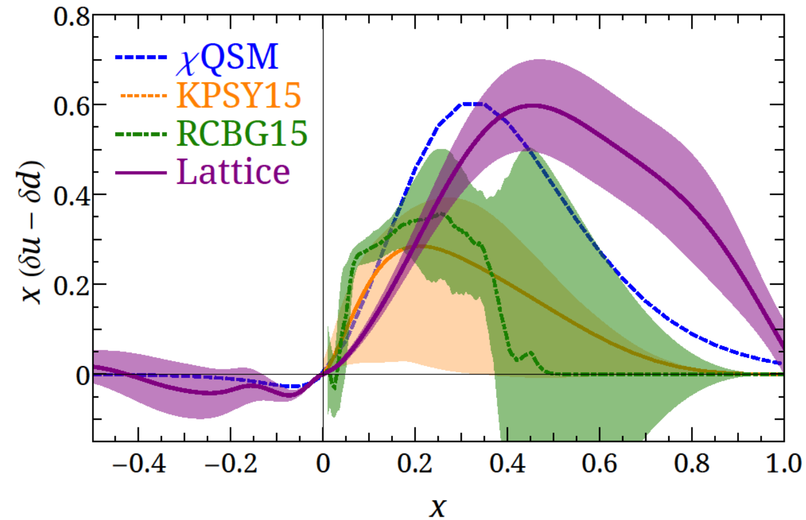




Chen et. al. 16′
$m_\pi = 310\ MeV, L = 3fm, a = 0.12fm$

  - Sea flavor asymmetry

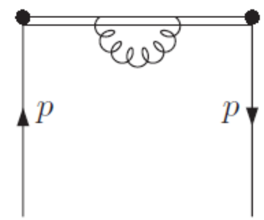
- Renormalization effects not yet taken into account

Renormalization of quasi-PDF

- Studies on its renormalization property:
  - Ji and Zhang, 1505.07699,
    - Multiplicative renormalization up to two-loop in DR
  - Ishikawa, Ma, Qiu, Yoshida, 1609.02018,
  - Chen, Ji and Zhang, 1608.08102,
  - Constantinou and Panagopoulos, 1705.11193,
    - Renormalization of power divergence and factorization of renormalization factors
  - Alexandrou, Cichy, Constantinou, Hadjiyiannakou, Jansen, Panagopoulos and Steffens, 1706.00265,
  - Chen, Ishikawa, Jin, Lin, Yang, Zhang and Zhao, 1706.01295,
    - Regularization-independent Momentum subtraction
  - Ji, Zhang and Zhao, 1706.08962,
  - Ishikawa, Ma, Qiu and Yoshida, 1707.03107,
    - Multiplicative renormalization proof to all-orders

Renormalization of power divergence

- Power divergence comes from Wilson line self energy [Ishikawa et al. 16′, Chen et al. 16′]
  - At one-loop, a linear div. is associated with



  - It is well-known that linear divergence associated with Wilson line can be removed by a mass renormalization (e.g. in auxiliary z-field formalism)
  - In a sense, the auxiliary field can be understood as a Wilson line extending between $[z, \infty]$

$$Z(z) = L(z, \infty), \qquad [\partial_z - igA_z(z)]\, Z(z) = 0$$

  - Analogous to a heavy quark field
  - Non-local Wilson line can be interpreted as a two-point function of z-field

$$L(z,0) = Z(z)Z^{\dagger}(0)$$

  - Renormalizes analogously to a heavy quark two-point function [Dotsenko and Vergeles 80′, Dorn 86′]

$$L^{\mathrm{ren}}(z,0) = \mathcal{Z}_Z^{-1} e^{-\delta m|z|} L(z,0)$$

Renormalization of power divergence

- One-loop illustration
  - The Wilson line self energy diagram gives ($\bar{x}$=1-x)

$$\lim_{\epsilon\to 0}\int dk_z \frac{\alpha_s C_F \Lambda}{2\pi}\frac{[\delta(k_z-\bar{x}p_z)-\delta(\bar{x}p_z)]p_z}{k_z^2+\epsilon^2}$$

  - Mass counterterm contributes

$$-\int \frac{dz}{2\pi}p_z\, e^{i(x-1)p_z z}|z|\,\delta m = -\lim_{\epsilon\to 0}\int \frac{dz}{2\pi}p_z\, e^{-i\bar{x}p_z z}\frac{1-e^{-\epsilon|z|}}{\epsilon}\delta m$$
$$= -\lim_{\epsilon\to 0}\int \frac{dk_z}{\pi}p_z \frac{\delta(\bar{x}p_z)-\delta(k_z-\bar{x}p_z)}{k_z^2+\epsilon^2}\delta m.$$

  - Therefore

$$\delta m = -\frac{\alpha_s C_F}{2\pi}(\pi\Lambda)$$

  - It is gauge-independent

Improved quasi-PDF

- We can define an improved quasi-PDF without power divergence [Chen et al. 16′]

$$\tilde{q}_{\text{imp}}(x, \Lambda, p^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z - \delta m|z|} \langle p|\overline{\psi}(0, 0_\perp, z)\gamma^z L(z, 0)\psi(0)|p\rangle$$

- Similarly for the pion quasi-DA [Zhang et al. 17′]

$$\tilde{\phi}_{\text{imp}}(x, P_z) = \frac{i}{f_\pi} \int \frac{dz}{2\pi} e^{-i(x-1)P_z z - \delta m|z|} \langle \pi(P)|\bar{\psi}(0)\gamma^z \gamma_5 \Gamma(0, z)\psi(z)|0\rangle$$

- Apart from the exponential mass renormalization to remove power divergence, there supposed to be other renormalization factors at the endpoint, which are local

- Normalization condition roughly means an implementation of renormalization

Improved quasi-PDF

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- The matching factor for the improved quasi-PDF is obtained from the previous result by removing linear divergence (similarly for pion DA)

- Intuitively, the exponential renormalization factor $e^{-\delta m|z|}$ will increase the weight of matrix elements with relatively large z, and therefore will increase the contribution at relatively small momentum when Fourier transformed to momentum space

Determination of δm

- Can be done following the method in [Musch et al. 11′]
  - Static heavy quark-antiquark potential can be obtained from asymptotic behavior of a rectangular Wilson loop

$$W(R,T) = c(R)e^{-V(R)T} + \text{higher excitations},$$

  - Choose a Wilson loop long in t-direction such that higher excitations are sufficiently suppressed

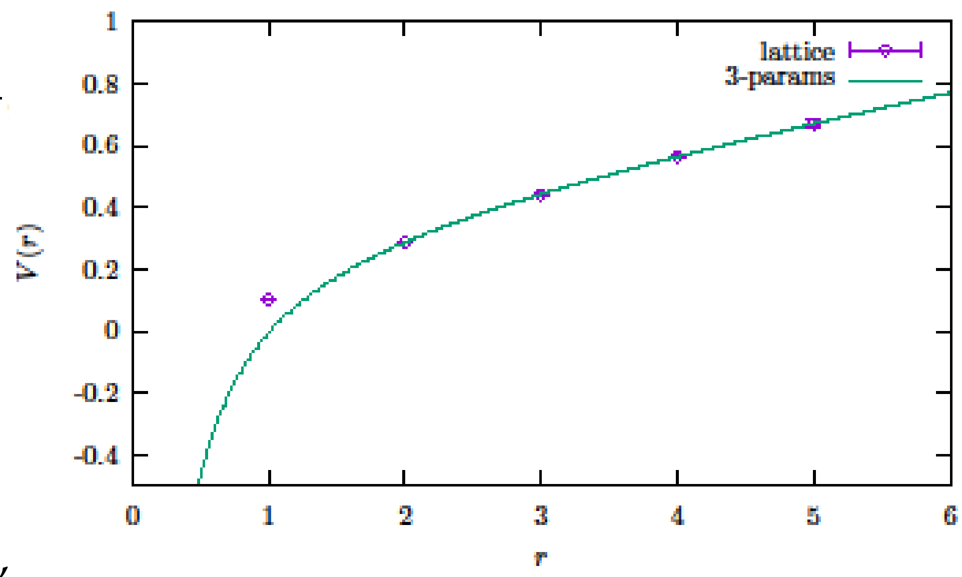
  - Fit the quark potential [Zhang et al. 17′]

$$V(r) = -\frac{1}{a}\lim_{t\to\infty}\ln\frac{\langle \mathrm{Tr}[W(t,r)]\rangle}{\langle \mathrm{Tr}[W(t-a,r)]\rangle}$$

  - to

$$V(r) = \frac{c_1}{r} + c_2 + c_3 r$$

  - c_1 is the Coulomb potential
  - c_3 is the confinement linear potential
  - c_2 twice the rest mass of heavy quark, expected to be $c_2 = \tilde{c}/a + O(\Lambda_{QCD})$,

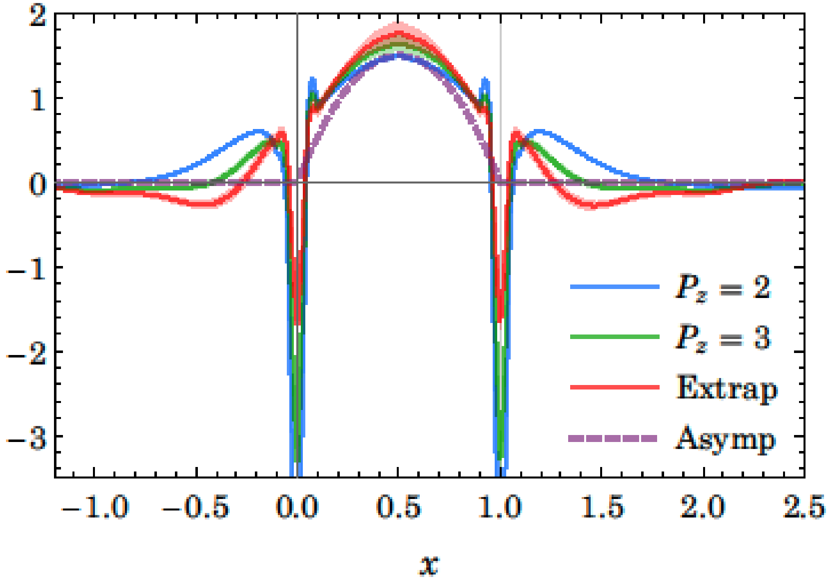


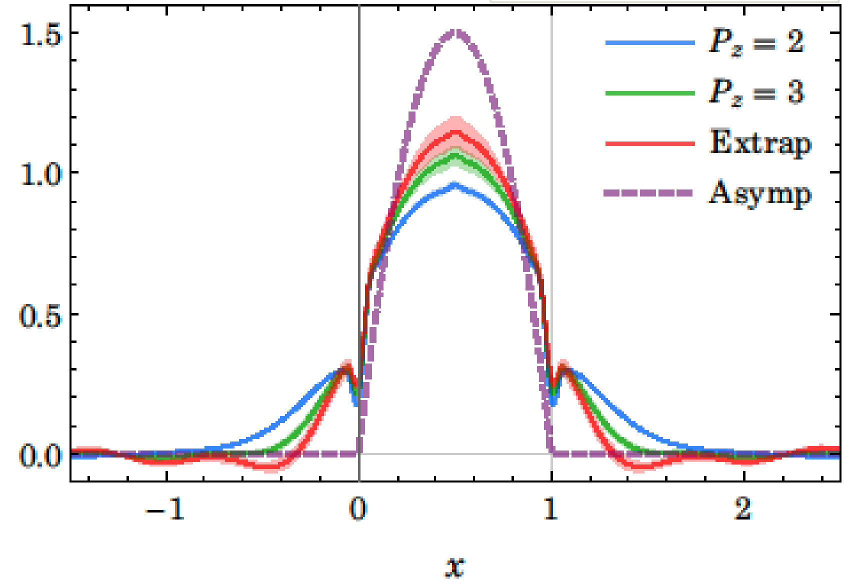
$\delta m = -c_2/2 \approx -260 \pm 200$ MeV

Results on pion DA

- Unimproved vs. improved

Zhang et. al. 17'
$m_\pi = 310\ MeV, L = 3fm, a = 0.12fm$

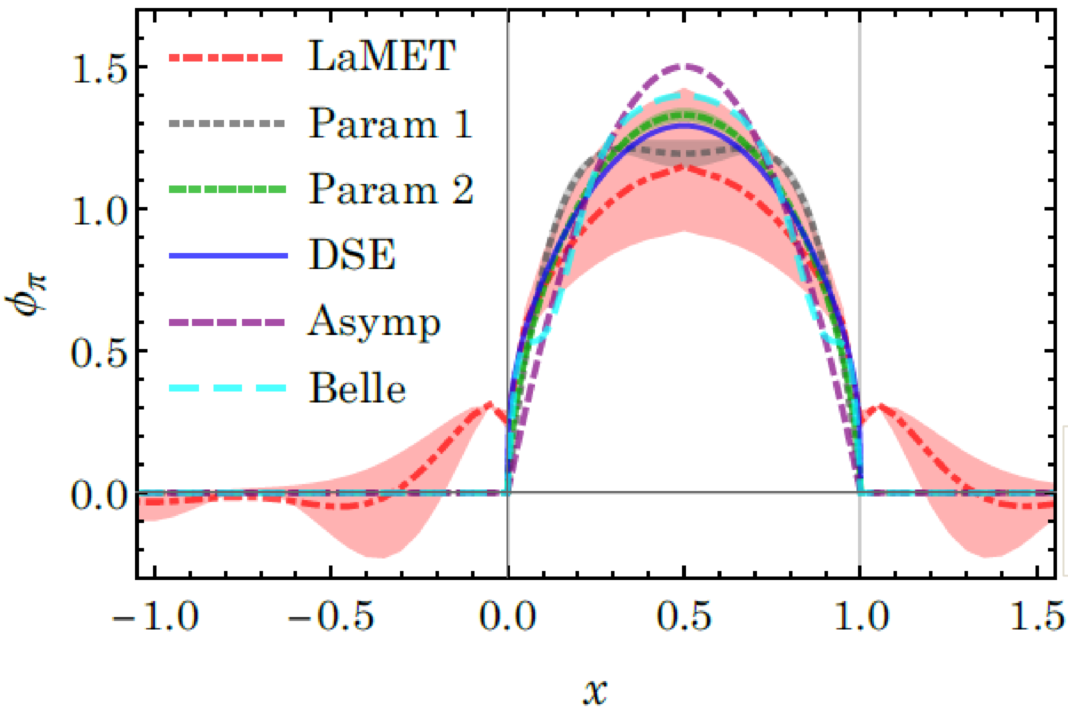




- One-loop correction, and also mass correction included, $P_z = \{2, 3\} \times 2\pi/L$
- Extrapolation to infinite momentum using a simple form $\alpha(x) + \beta(x)/P_z^2$, but has residual contribution outside [0,1]
- Left: using the unrenormalized quasi correlation, linear divergence contained in the matching factor, unphysical oscillatory behavior near $x = 0, 1$
- Right: improved, there are still small kinks in the unphysical region, expected to vanish when higher-order matching is included/higher momentum is reached

Results on pion DA

- Improved pion DA result



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- Param 1: $\phi_\pi(x) = 6x(1-x)[1 + a_2 C_2^{3/2}(2x-1)]$
- Param 2: $\phi_\pi(x) = A[x(1-x)]^B$
- Consistent with previous studies, favors a single-hump shape
- Error dominated by uncertainty in δm, can be improved by computing at different lattice spacing

Results on PDF

- Non-perturbative renormalization in RI/MOM scheme
  - For a local operator

$$O_\Gamma = \bar{\psi}\Gamma\psi$$

  - The renormalization factor is defined as

$$Z_O \langle p|O_\Gamma|p\rangle_{p^2=\mu^2} = \langle p|O_\Gamma|p\rangle_{\text{free}}$$

  - Generalization to non-local operator [Zhao and Stewart]

$$\tilde{Z}^{\rm OM}(z, p^z, \Lambda, \mu_R)^{-1} \sum_s \langle ps|\bar{\psi}(z)\gamma^z W(z,0)\psi(0)|ps\rangle\Big|_{p^2=-\mu_R^2}$$
$$= \sum_s \langle ps|\bar{\psi}(z)\gamma^z W(z,0)\psi(0)|ps\rangle\Big|_{\text{tree}}$$

  - The renormalization factor and bare matrix element depend on the choice of regulator, the renormalized matrix element does not

Results on PDF

- Non-perturbative renormalization in RI/MOM scheme
  - Vector quasi-PDF can mix with scalar one due to breaking of chiral symmetry

$$\begin{pmatrix} O_{\gamma_z}(z) \\ O_{\mathcal{I}}(z) \end{pmatrix} = \tilde{Z} \times \begin{pmatrix} O^R_{\gamma_z}(z) \\ O^R_{\mathcal{I}}(z) \end{pmatrix}, \\ = \begin{pmatrix} Z_{11}(z) & Z_{12}(z) \\ Z_{21}(z) & Z_{22}(z) \end{pmatrix} \begin{pmatrix} O^R_{\gamma_z}(z) \\ O^R_{\mathcal{I}}(z) \end{pmatrix}$$

  - Renormalization conditions

$$\frac{\mathrm{Tr}[\not{p}\Lambda(p,z,\gamma_z)]^R}{\mathrm{Tr}[\not{p}\Lambda(p,z,\gamma_z)_{\mathrm{tree}}]}\Big|_{p^2=\mu_R^2,\ p_z=P_z} = 1,$$

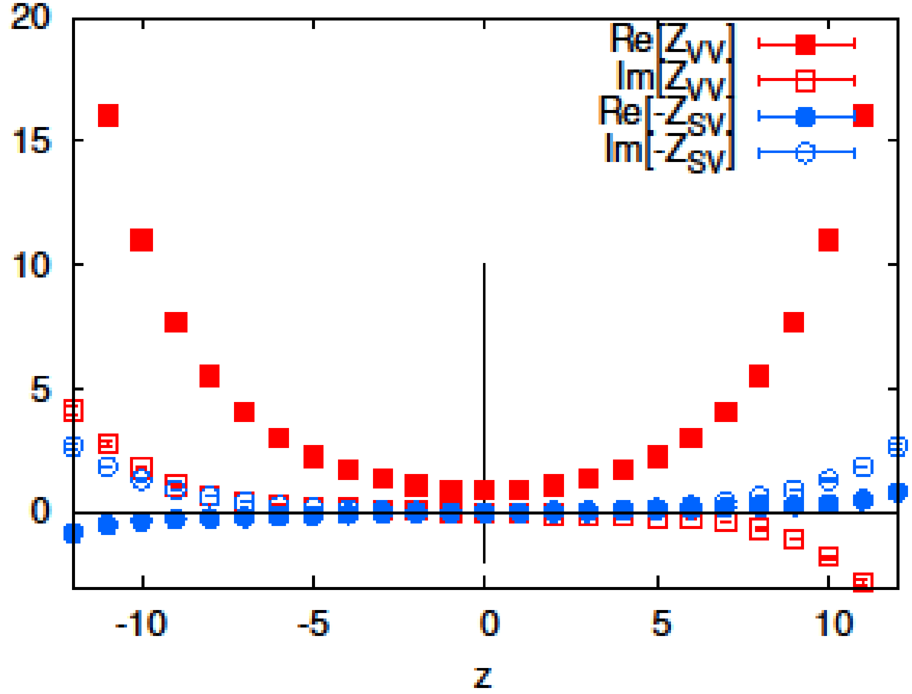
$$\frac{\mathrm{Tr}[\Lambda(p,z,\mathcal{I})]^R}{\mathrm{Tr}[\Lambda(p,z,\mathcal{I})_{\mathrm{tree}}]}\Big|_{p^2=\mu_R^2,\ p_z=P_z} = 1,$$

$$\mathrm{Tr}[[\not{p}\Lambda(p,z,\mathcal{I})]^R_{p^2=\mu_R^2,\ p_z=P_z} = 0,$$

$$\mathrm{Tr}[\Lambda(p,z,\gamma_z)]^R_{p^2=\mu_R^2,\ p_z=P_z} = 0,$$

Results on PDF

- Non-perturbative renormalization in RI/MOM scheme
  - Renormalization factors



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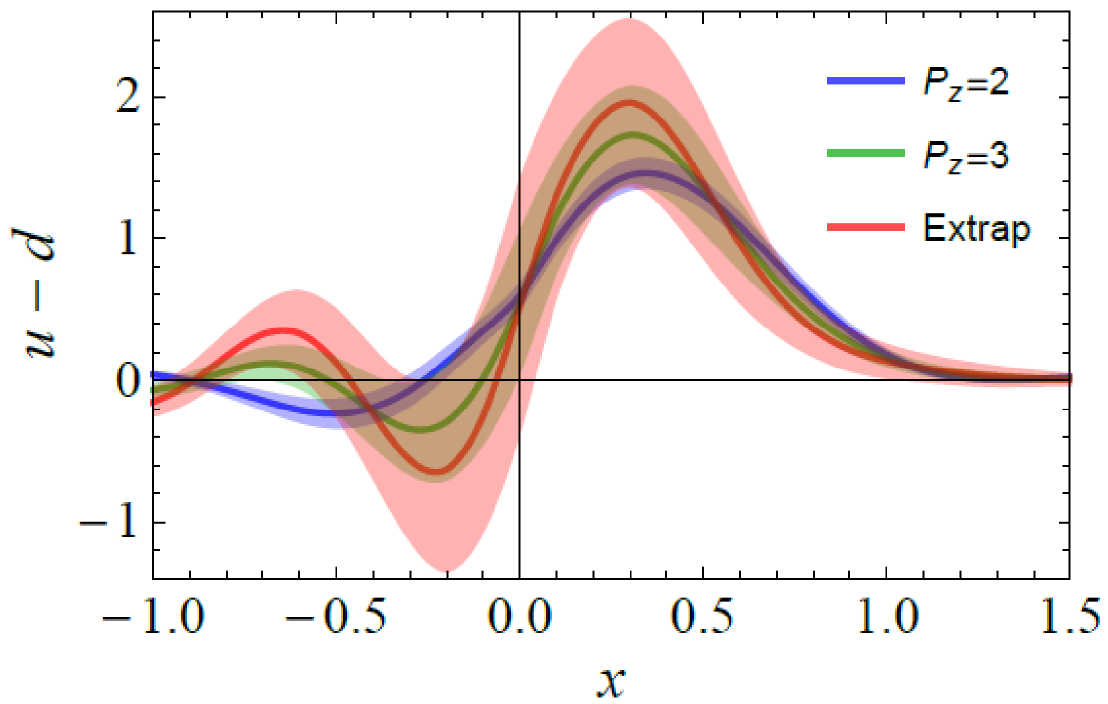

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- Renormalization effects induce large uncertainties

- See Huey-Wen's talk

All-order renormalization of quasi-PDF

- To study the renormalization property of the non-local operator

$$O(x,y) = \overline{\psi}(x)\Gamma L(x,y)\psi(y)$$

  - We are motivated to introduce the following auxiliary heavy quark Lagrangian [Ji et al. 17']

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \overline{Q}(x) i n \cdot D Q(x)$$

  - For a real heavy quark, n is timelike, and Q is a dynamical field
  - For an auxiliary heavy quark, n is spacelike, no dynamical evolution

  - After integrating out the heavy quark field, we have

$$\int \mathcal{D}\overline{Q}\mathcal{D}Q\, Q(x)\overline{Q}(y) e^{i\int d^4x \mathcal{L}} = S_Q(x,y) e^{i\int d^4x \mathcal{L}_{\mathrm{QCD}}}$$

up to a constant that can be absorbed into the overall normalization

All-order renormalization of quasi-PDF

- $S_Q(x, y)$ satisfies

$$n \cdot D\, S_Q(x, y) = \delta^{(4)}(x - y),$$

  with the solution

$$\begin{aligned} S_Q(x, y) &= \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x, y) \\ &= \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x^z, y^z) \end{aligned}$$

  - δ-function ensures that the time and transverse components are equal, and thereby generates a spacelike Wilson line

- We can do the replacement (restrict to $x^z > y^z$ for the moment)

$$O(x, y) = \overline{\psi}(x)\Gamma Q(x)\overline{Q}(y)\psi(y)$$

  - The non-local operator can be replaced by a product of two heavy-light currents

All-order renormalization of quasi-PDF

- Renormalization in HQET [Bagan et al. 94′]
  - Green's functions can be renormalized to all-orders in perturbation theory
  - So is the local composite operator
  - The arguments are valid independent of whether the heavy quark field in the Lagrangian is defined by a timelike or spacelike vector

- In DR, our non-local operator renormalization can be done as (for a spacelike separation)

$$O(z_2, z_1) = Z_{\bar{j}} Z_j O_R(z_2, z_1)$$

  - After integrating over the heavy quark field, we have

$$O_R = Z_{\bar{j}}^{-1} Z_j^{-1} \overline{\psi}(z_2) \Gamma L(z_2, z_1) \psi(z_1)$$

  - The above renormalization is consistent with explicit computations [Ji and Zhang, 15′]
    - For example

$$Z_O = 1 + \frac{\alpha_s}{\pi \epsilon} \qquad\qquad Z_j = 1 + \frac{\alpha_s}{2\pi \epsilon}$$

All-order renormalization of quasi-PDF

- Renormalization in HQET [Maiani et al. 92′]
  - The HQET Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{Q}(x) i n \cdot D Q(x)$$

  - Takes infinite heavy quark mass limit, and does not contain any mass term

  - Mass correction can receive power divergent contributions, which are absent in DR

  - In cutoff regularization like lattice regularization, heavy quark self-energy does introduce a linear divergence, which has to be absorbed into an effective mass counterterm

$$\delta\mathcal{L}_m = -\delta m \overline{Q} Q$$

with $\delta m \sim 1/a$

All-order renormalization of quasi-PDF

- Renormalization in HQET [Maiani et al. 92′]
  - The appearance of linear divergence can be understood as following:
    - Infinitely heavy quark behaves like a static color source, its energy will have a Coulomb-like form 1/r, and will diverge linearly if the source is a pointlike particle
  - This physical picture is lost for a auxiliary heavy quark defined by a spacelike vector, but the linear divergence can be removed in the same way
  - The mass counterterm shall not be understood as a physical mass, but as a parameter with mass dimension
  - The total heavy quark Lagrangian now becomes

$$\mathcal{L}_Q = \bar{Q}(in \cdot D - \delta m)Q$$

  - Integrating over the auxiliary heavy quark field, our non-local operator renormalization becomes

$$O_R = Z_{\bar{j}}^{-1} Z_{j}^{-1} e^{\delta \bar{m}|z_2 - z_1|} \overline{\psi}(z_2)\Gamma L(z_2, z_1)\psi(z_1)$$

  - See Xiangdong's and Yanqing's talk

Summary and outlook

- Parton observables like PDFs or hadron DAs are difficult to compute due to their non-perturbative and intrinsically Minkowskian nature

- Large momentum effective theory offers a practical possibility to directly compute them from Euclidean lattice
  - Perturbative matching
  - Non-perturbative renormalization

- A lot more effort needed for lattice results to reach an accuracy comparable with phenomenological studies
  - Finer lattice spacing
  - Larger hadron momentum
  - ……