Color entanglement effect in collinear twist-3 factorization

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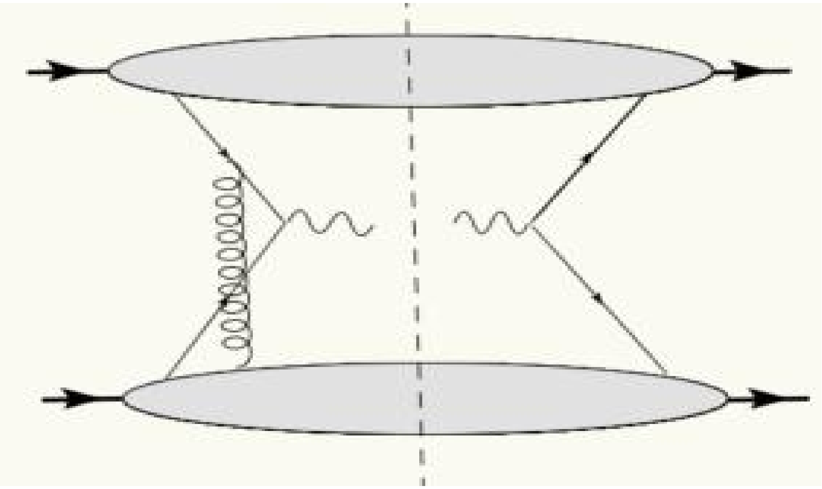
Based on: arXiv:1706.02842

Outline:

- Background and motivations
- Color entanglement in collinear twist-3 factorization
- Summary

What is color entanglement?

Gauge link

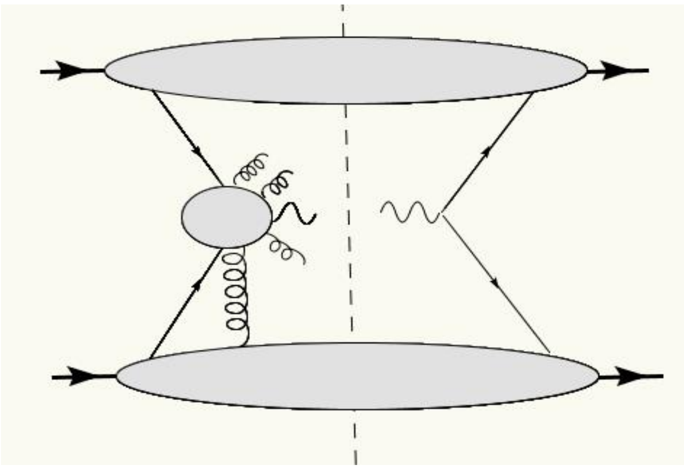


$$f_q^{unsub.}(x,k_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ix\xi^- P^+ + i\vec{\xi}_\perp\cdot\vec{k}_\perp} \langle PS \left| \overline{\psi}(\xi)\mathcal{L}_n^\dagger(\xi)\gamma^+\mathcal{L}_n(0)\psi(0) \right| PS\rangle$$

$$\mathcal{L}_n(\xi) \equiv \exp\left(-ig\int_0^{-\infty} d\lambda\, v\cdot A(\lambda n+\xi)\right)$$

J. Collins, D. Soper 1982

All order analysis: the Ward identity argument



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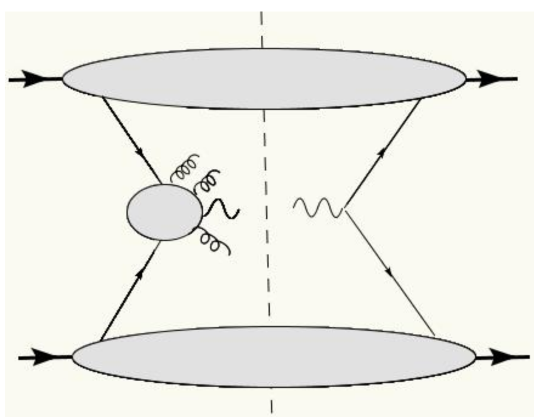

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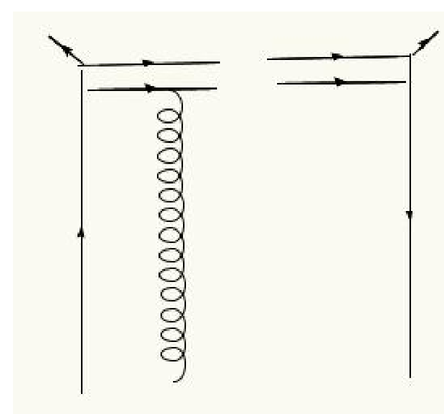
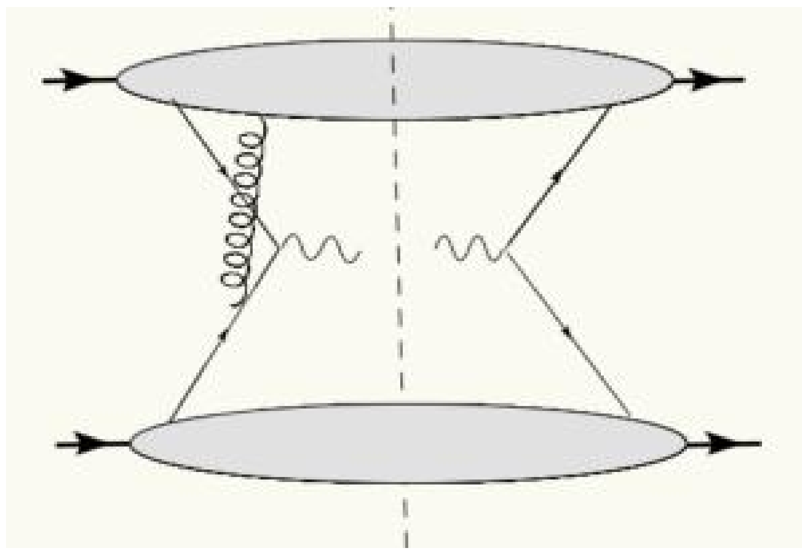
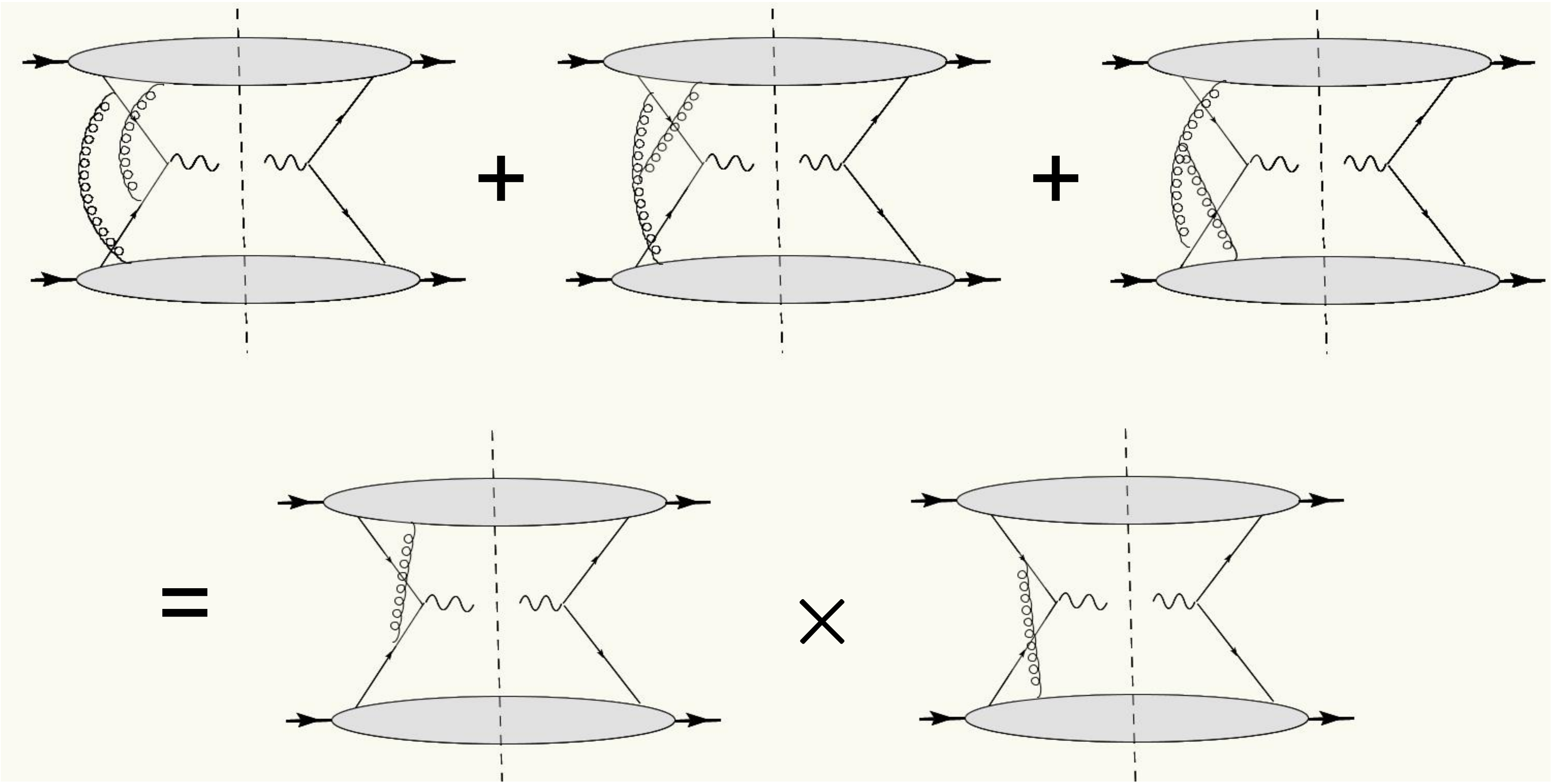


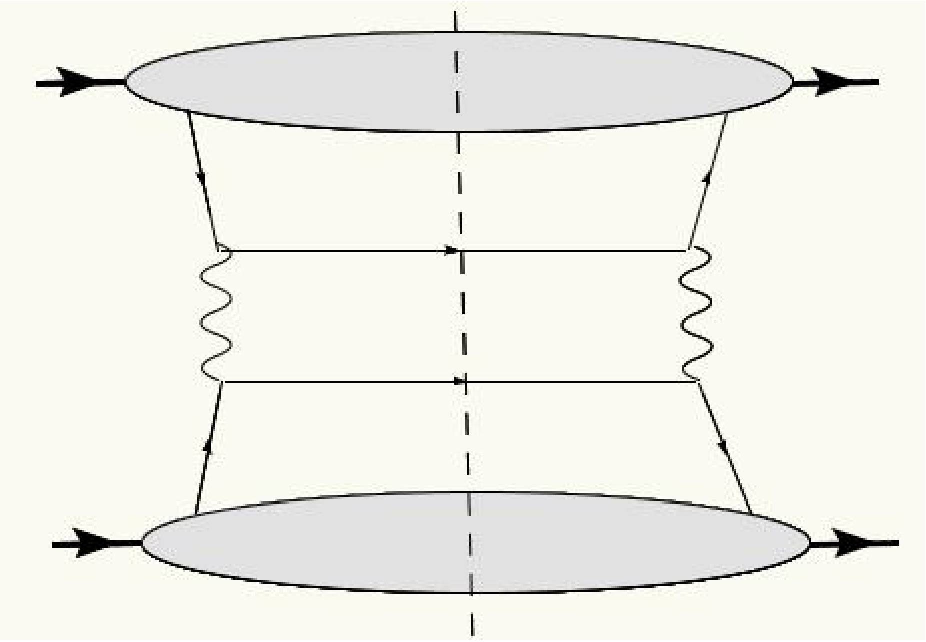
Gauge link in anti-quark distribution



Longitudinal gluon attachment from each nucleon side

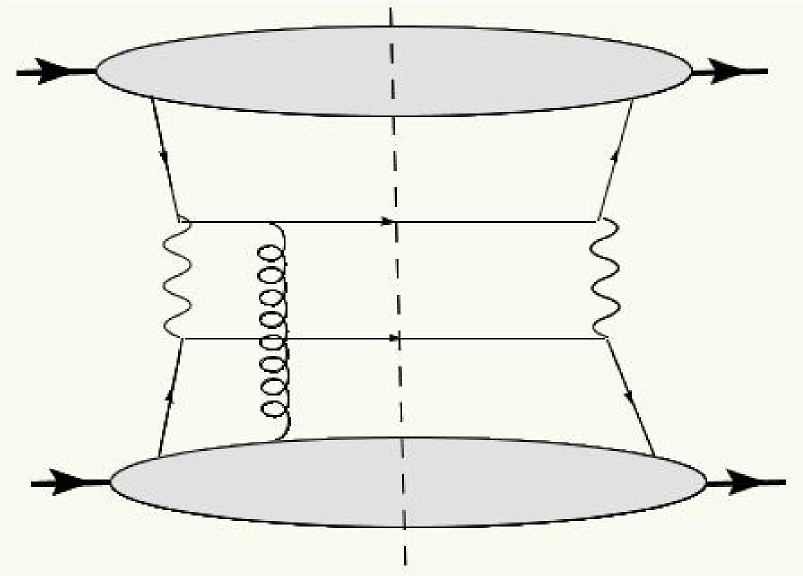


Di-jet production in pp collisions



photon exchange

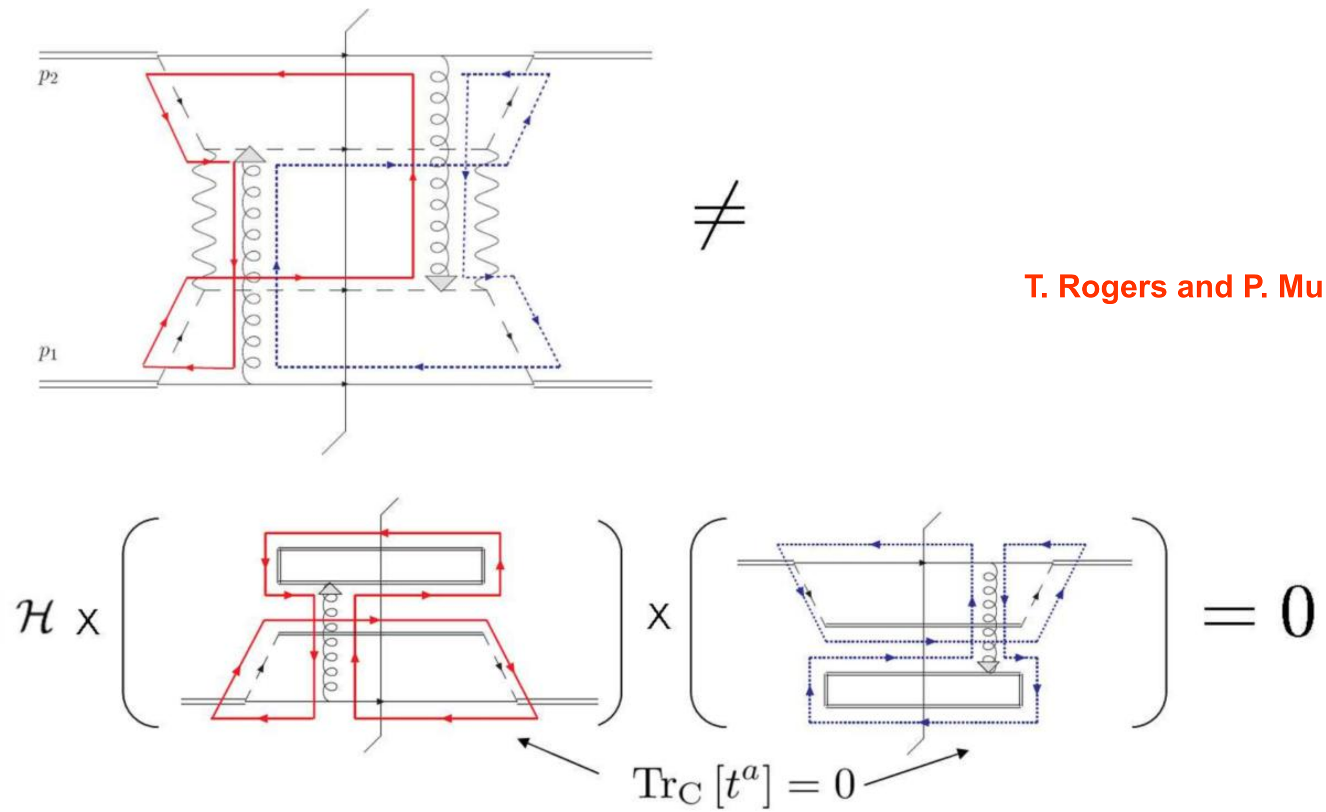
Di-jet imbalance kt can be described in the context of TMD factorization



$= 0$

due to $\mathrm{Tr}[T^a]=0$

Color Entanglement(CE) Effect



T. Rogers and P. Mulders, 2010

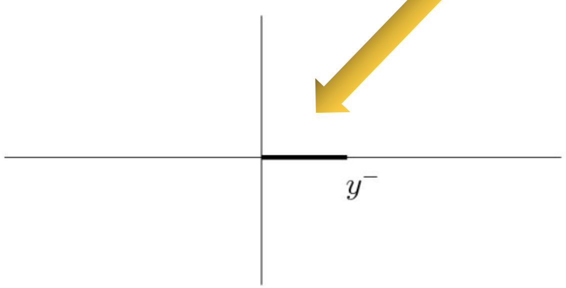
The Ward Identity argument fails.

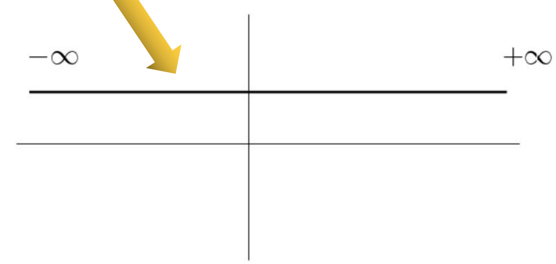
Why the WI argument fails

To decouple the longitudinal gluon, rely on $P^\mu \mathcal{M}_\mu = 0$

But the WI is: $x_g P^\mu \mathcal{M}_\mu = 0$

Gluon pole $$\frac{1}{x_g + i\epsilon} = P\frac{1}{x_g} - i\pi\delta(x_g)$$





$x_g P^\mu \mathcal{M}_\mu = 0$ When

$x_g \neq 0$ $\quad P^\mu \mathcal{M}_\mu = 0$

$x_g = 0$ $\quad P^\mu \mathcal{M}_\mu \stackrel{?}{=} 0$

No color entanglement in collinear factorization at leading power

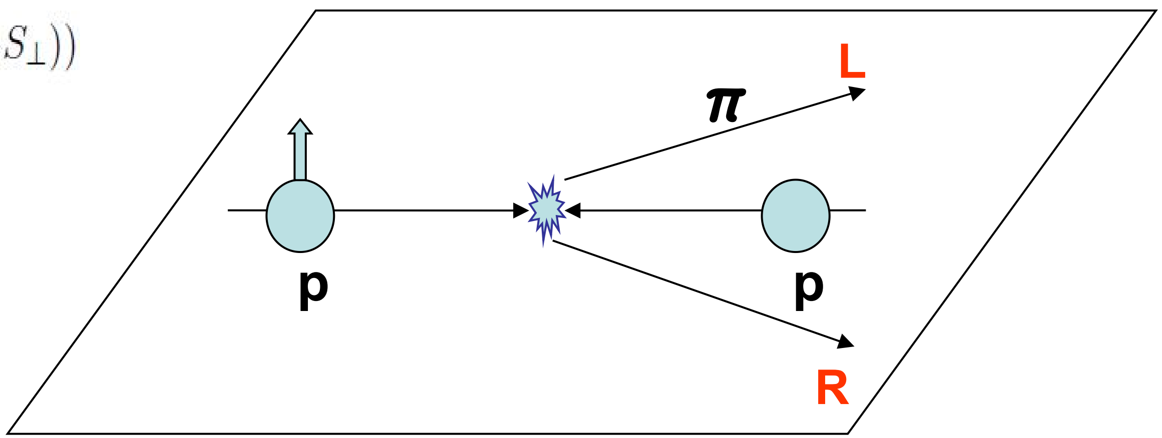
How about twist-3 collinear factorization?

Single spin asymmetry in collinear twist-3 approach

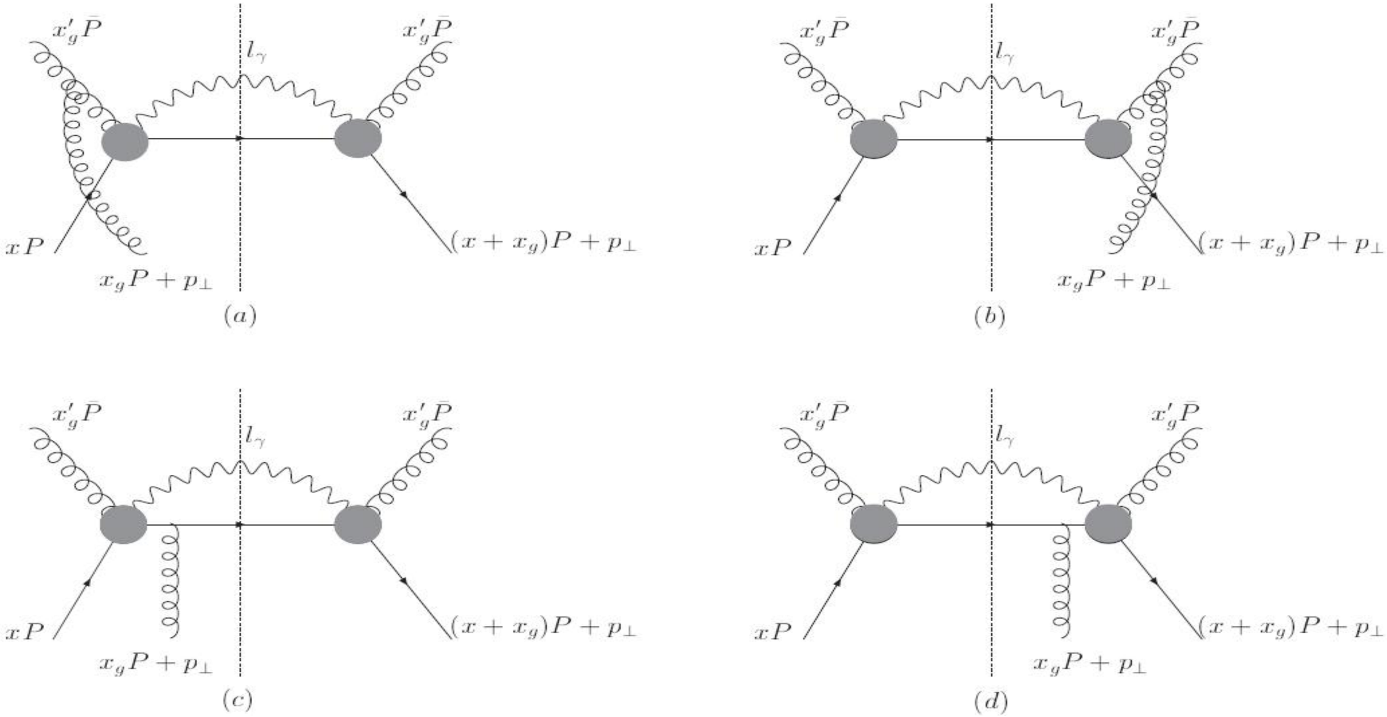
Transverse single spin asymmetries

$$p(\uparrow) + p \rightarrow \pi + X$$

$$A_N \equiv (\sigma(S_\perp) - \sigma(-S_\perp))/(\sigma(S_\perp) + \sigma(-S_\perp))$$



SSA for photon production in polarized pp collisions



Based on the collinear twist-3 approach

Qiu, Sterman 1992
Kouvaris, Qiu, Vogelsang, Yuan 2006

Hybrid approach: Collinear twist-3 & CGC

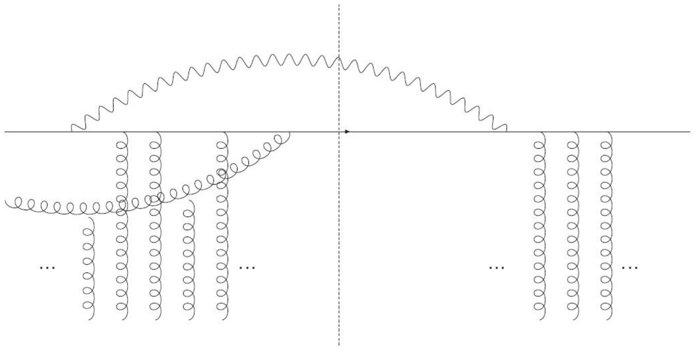
Formulated in a hybrid approach:

one extra gluon exchange with proton remnants;

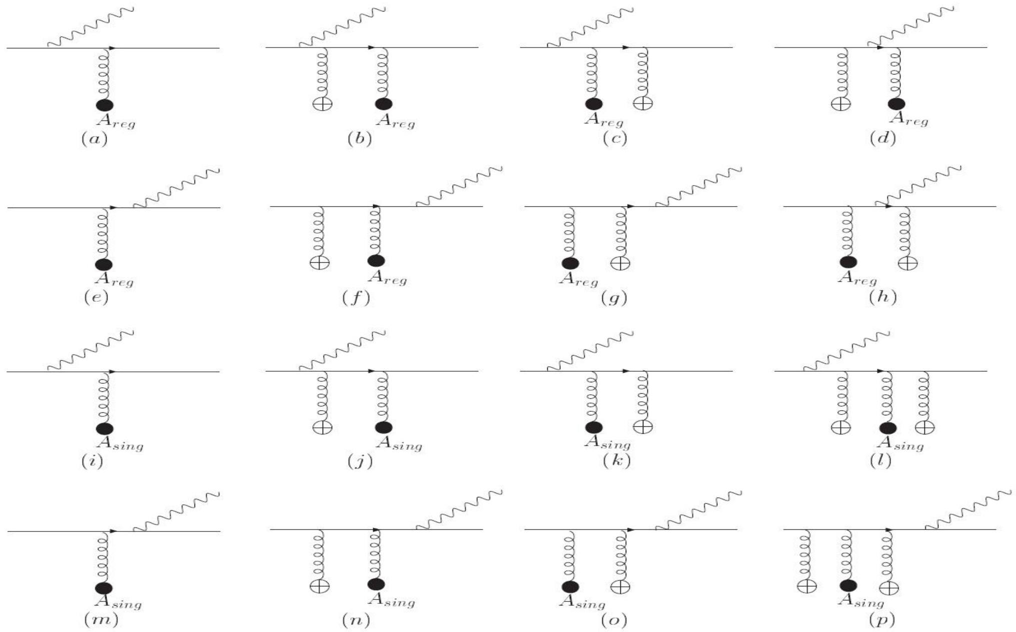
resuming multiple gluon scattering inside nucleus to all orders.

→ **Collinear twist-3 & CGC**

A typical diagram



In the hybrid approach:



Color entanglement effect for SSA

The derivative term contribution:

$$\frac{d^3\Delta\sigma}{d^2 l_{\gamma\perp} dz} = \frac{\alpha_s \alpha_{em} N_c}{N_c^2 - 1} \epsilon^{l_\gamma S_\perp n p} \frac{z[1+(1-z)^2]}{l_{\gamma\perp}^6}(z-1)$$
$$\times \sum_q e_q^2 \int_{x_{min}}^1 \frac{dx}{x}\left[-x\frac{d}{dx}T_{F,q}(x,x)\right]\left[x'_g G_{DP}(x'_g,k_\perp) - x'_g G_4(x'_g,k_\perp)\right]$$

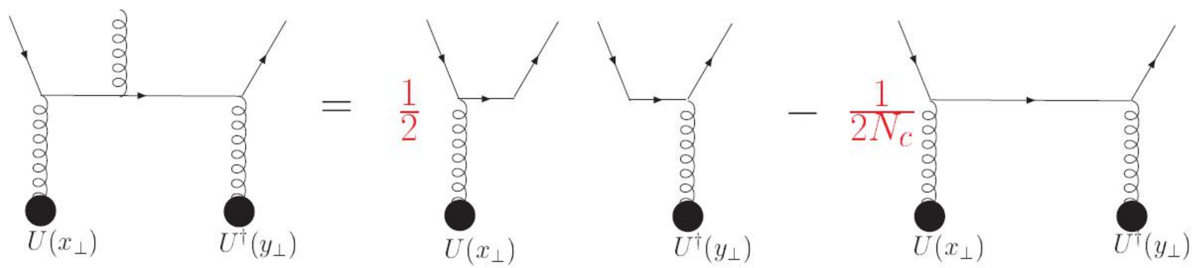
Color entanglement effect

Collinear twist-3

Schafer, ZJ 2014

Dipole gluon TMD: $G_{DP} \propto \langle \mathrm{Tr}_c[U(x_\perp)U^\dagger(y_\perp)]\rangle$

New gluon TMD: $G_4 \propto \langle \mathrm{Tr}_c[U(x_\perp)]\mathrm{Tr}_c[U^\dagger(y_\perp)]\rangle$

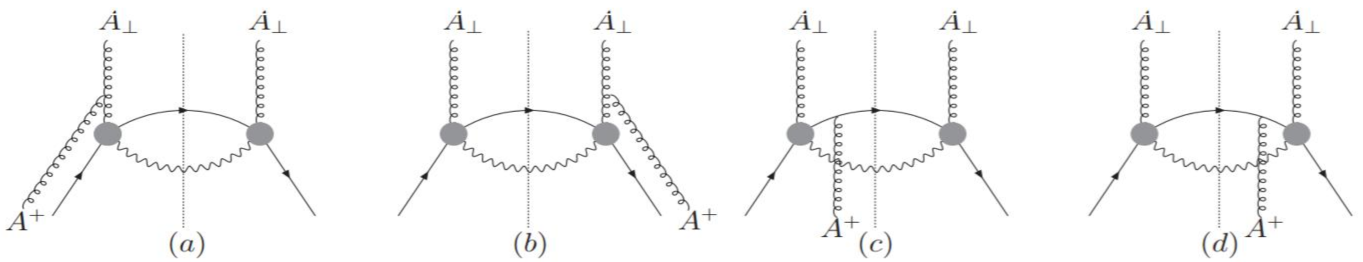


$\mathrm{Tr}_c[U(x_\perp)]\mathrm{Tr}_c[U^\dagger(y_\perp)]$ $\quad$ $\mathrm{Tr}\left[U(x_\perp)U^\dagger(y_\perp)\right]$

Other applications:

Schafer, ZJ 2015, ZJ 2015; Hatta, Xiao, Yuan 2016, Hatta, Xiao, Yuan 2017, ZJ 2017

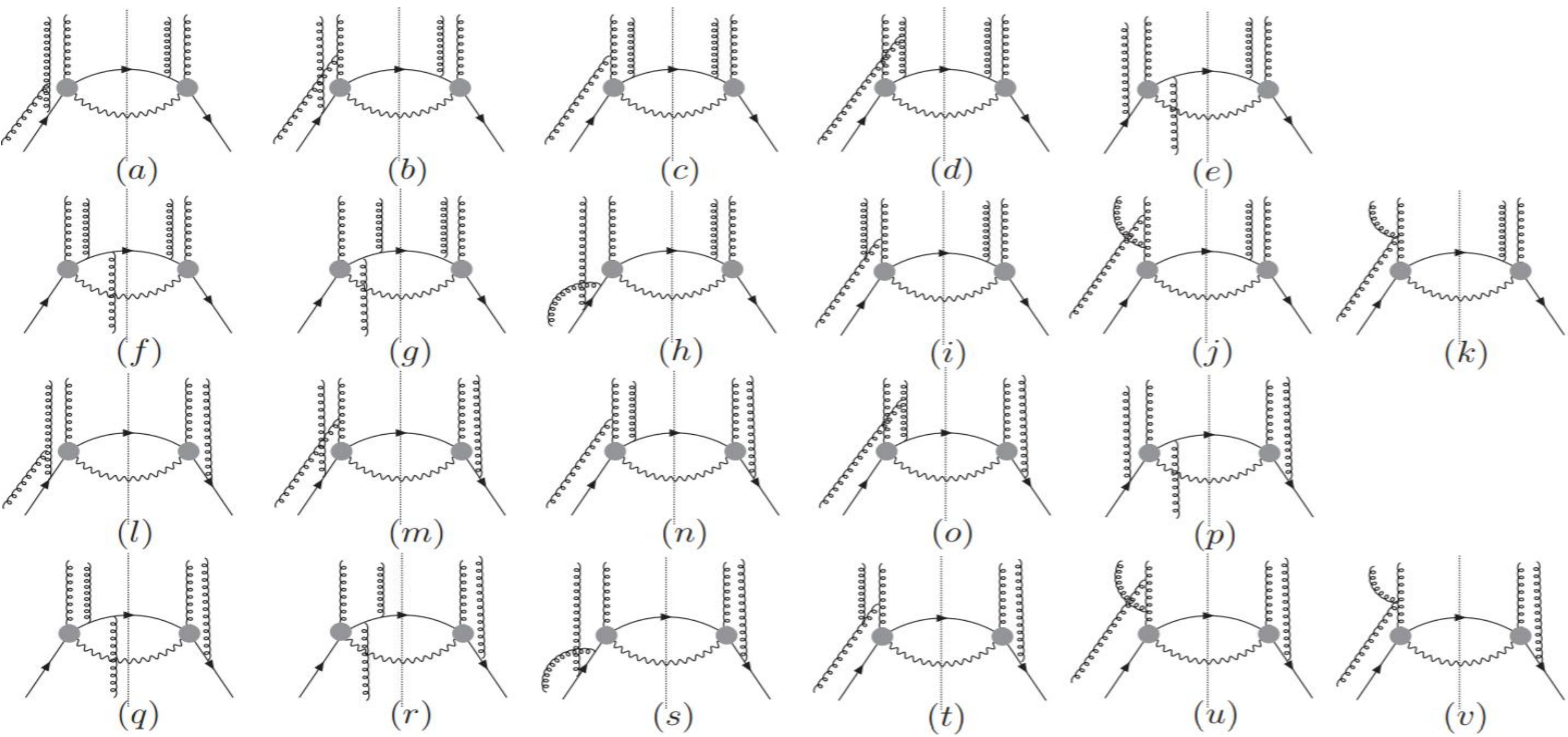
How can one recover this result in pure collinear twist-3 approach?



Color entanglement in collinear twist-3?

Lowest nontrival order diagrams

$$G_4 \propto \langle \mathrm{Tr_c}[U(x_\perp)]\mathrm{Tr_c}[U^\dagger(y_\perp)]\rangle \qquad \mathrm{Tr_C}\,[t^a] = 0$$



Mirror diagrams are not shown.

We focus on the derivative term $\frac{d}{dx}T_{F,q}(x,x)$

$$\mathcal{H}(p_\perp)\delta(l_q^2) = \mathcal{H}(p_\perp)\delta(l_q^2)|_{p_\perp=0} + \frac{\partial\mathcal{H}(p_\perp)\delta(l_q^2)}{\partial p_\perp^\rho}|_{p_\perp=0}\, p_\perp^\rho + \ldots$$

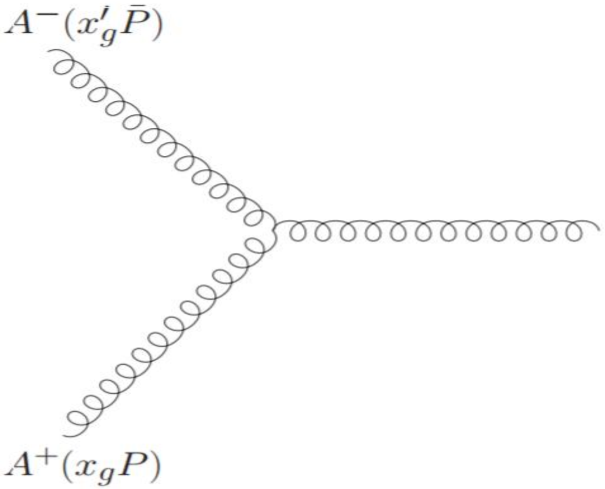
$$\frac{-l_{\gamma\perp}^\rho}{l_q\cdot P}\mathcal{H}(p_\perp=0)\left[\frac{\partial\delta(l_q^2)}{\partial x}\right]_{p_\perp=0} p_{\perp,\rho}$$

It is sufficient to only analyze the color structure and gluon pole structure of hard parts

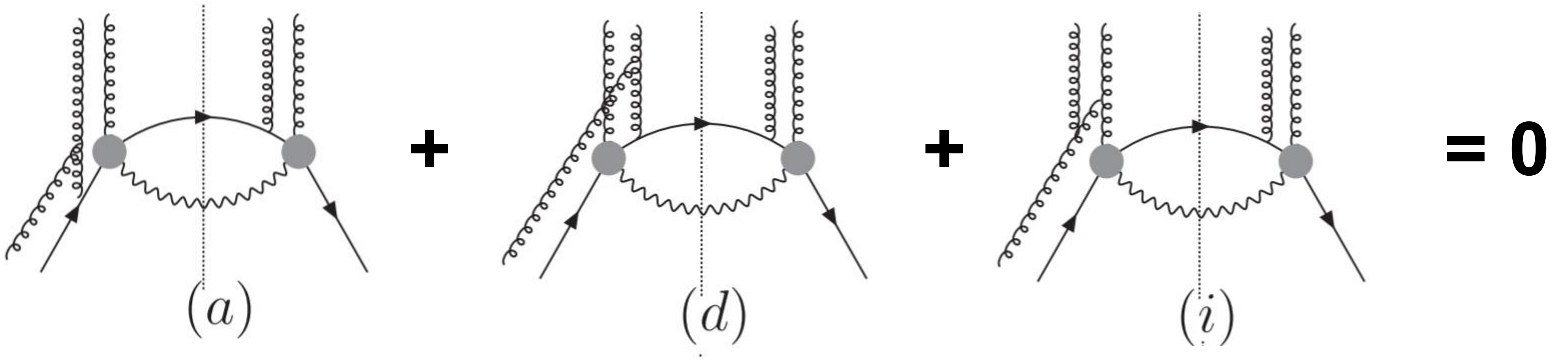
A trick

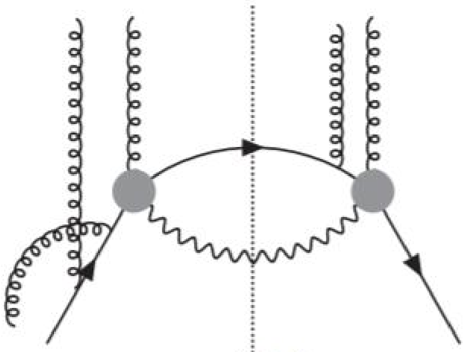
$$\frac{-igf^{abc}}{(x_gP+x'_g\bar{P})^2+i\epsilon}\left[g^{\mu\nu}(x'_g\bar{P}-x_gP)^\rho+g^{\nu\rho}(2x_gP+x'_g\bar{P})^\mu+g^{\rho\mu}(-2x'_g\bar{P}-x_gP)^\nu\right]n^\mu p^\nu$$

$$=\frac{-igf^{abc}\left(x_gP+x'_g\bar{P}\right)^\rho}{(x_gP+x'_g\bar{P})^2+i\epsilon}-\frac{-igf^{abc}\bar{P}^\rho}{x_gP\cdot\bar{P}+i\epsilon}$$



For instance:

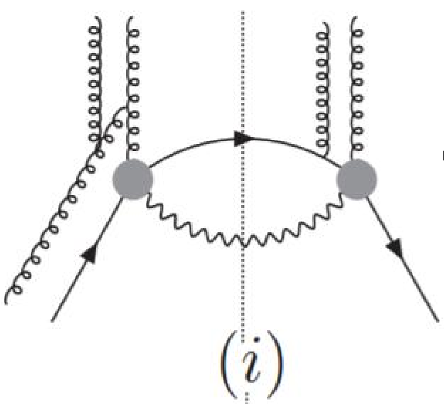




(h)

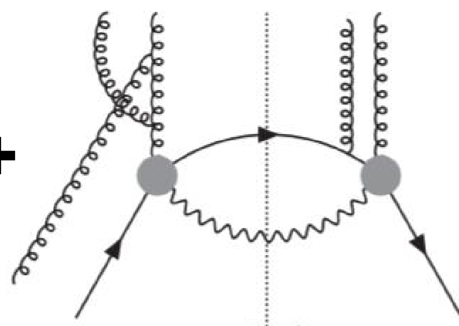
= 0

$$\not{p}(a\not{p} + b\not{n})\not{n}$$



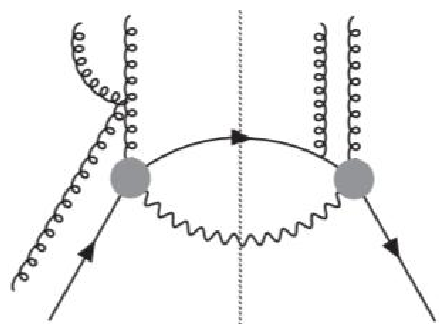
(i)

+



(j)

+



(k)

= 0

The color strucutre and the pole structure

$$\mathcal{H}_a \propto \frac{1}{x_g+i\epsilon}\frac{1}{-x'_{g1}-i\epsilon}\frac{1}{x'_g+i\epsilon}\mathrm{Tr}\left[T^aT^bT^cT^fT^e\right]if^{def}$$

$$\mathcal{H}_b \propto \frac{1}{x_g+i\epsilon}\frac{1}{-x'_{g1}-i\epsilon}\frac{1}{x'_g+i\epsilon}\mathrm{Tr}\left[T^aT^bT^fT^dT^e\right]if^{cef}$$

$$\mathcal{H}_c \propto \frac{1}{x_g+i\epsilon}\frac{1}{-x'_{g1}-i\epsilon}\frac{1}{-x'_g+i\epsilon}\mathrm{Tr}\left[T^aT^bT^dT^fT^e\right]if^{cef}$$

$$\mathcal{H}_d \propto \frac{1}{x_g+i\epsilon}\frac{1}{-x'_{g1}-i\epsilon}\frac{2l_q\cdot\bar{P}}{(l_q-x'_g\bar{P}-x_gP)^2+i\epsilon}\mathrm{Tr}\left[T^aT^bT^fT^cT^e\right]if^{def}$$

$$\mathcal{H}_e \propto \frac{-1}{-x_g+i\epsilon}\frac{1}{-x'_{g1}-i\epsilon}\frac{1}{x'_g+i\epsilon}\mathrm{Tr}\left[T^aT^bT^eT^cT^dT^e\right]$$

$$\mathcal{H}_f \propto \frac{-1}{-x_g+i\epsilon}\frac{1}{-x'_{g1}-i\epsilon}\frac{2l_q\cdot\bar{P}}{(l_q-x'_g\bar{P}-x_gP)^2+i\epsilon}\mathrm{Tr}\left[T^aT^bT^eT^dT^cT^e\right]$$

$$\mathcal{H}_g \propto \frac{-1}{-x'_g+i\epsilon}\frac{1}{-x'_{g1}-i\epsilon}\frac{2l_q\cdot P}{(l_q-x'_g\bar{P}-x_gP)^2+i\epsilon}\mathrm{Tr}\left[T^aT^bT^dT^eT^cT^e\right]$$

The resulting gauge link structure

$$\mathcal{H}_{a+b} \Rightarrow C_F \langle P|\mathrm{Tr}\left[F_a^{-\mu}(\xi^+)T^a\left(ig\int_{\xi^+}^{\infty}dz^+T^bA_b^-(z)\right)F_c^{-\mu}(0)T^c\left(-ig\int_{-\infty}^{0}dz^+T^dA_d^-(z)\right)\right]|P\rangle$$
$$-\langle P|\mathrm{Tr}\left[F_a^{-\mu}(\xi^+)T^a\left(ig\int_{\xi^+}^{\infty}dz^+T^bA_b^-(z)\right)T^eF_c^{-\mu}(0)T^c\left(-ig\int_{-\infty}^{0}dz^+T^dA_d^-(z)\right)T^e\right]|P\rangle$$

$$\mathcal{H}_{c+d1} \Rightarrow C_F \langle P|\mathrm{Tr}\left[F_a^{-\mu}(\xi^+)T^a\left(ig\int_{\xi^+}^{\infty}dz^+T^bA_b^-(z)\right)\left(-ig\int_{0}^{\infty}dz^+T^dA_d^-(z)\right)F_c^{-\mu}(0)T^c\right]|P\rangle$$
$$-\langle P|\mathrm{Tr}\left[F_a^{-\mu}(\xi^+)T^a\left(ig\int_{\xi^+}^{\infty}dz^+T^bA_b^-(z)\right)T^e\left(-ig\int_{0}^{\infty}dz^+T^dA_d^-(z)\right)F_c^{-\mu}(0)T^cT^e\right]|P\rangle$$

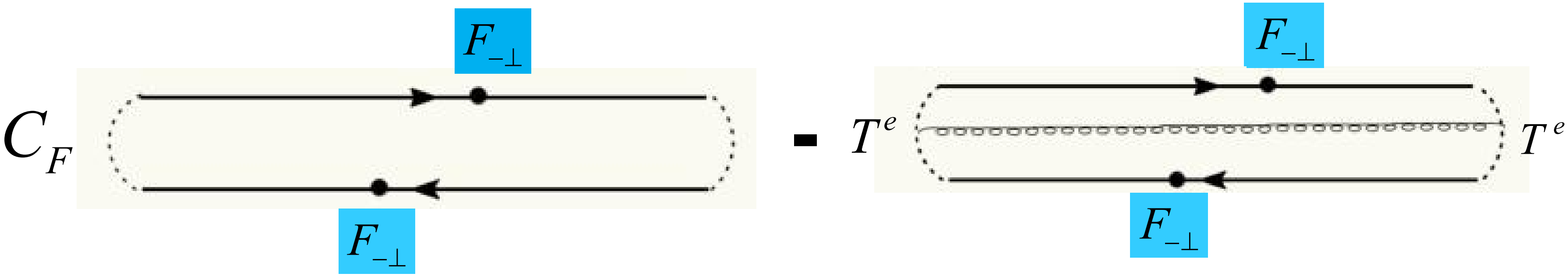
$$\mathcal{H}_{l+m} \Rightarrow C_F \langle P|\mathrm{Tr}\left[\left(ig\int_{-\infty}^{\xi^+}dz^+T^bA_b^-(z)\right)F_a^{-\mu}(\xi^+)T^aF_c^{-\mu}(0)T^c\left(-ig\int_{-\infty}^{0}dz^+T^dA_d^-(z)\right)\right]|P\rangle$$
$$-\langle P|\mathrm{Tr}\left[\left(ig\int_{-\infty}^{\xi^+}dz^+T^bA_b^-(z)\right)F_a^{-\mu}(\xi^+)T^aT^eF_c^{-\mu}(0)T^c\left(-ig\int_{-\infty}^{0}dz^+T^dA_d^-(z)\right)T^e\right]|P\rangle$$

$$\mathcal{H}_{n+o1} \Rightarrow C_F \langle P|\mathrm{Tr}\left[\left(ig\int_{-\infty}^{\xi^+}dz^+T^bA_b^-(z)\right)F_a^{-\mu}(\xi^+)T^a\left(-ig\int_{0}^{\infty}dz^+T^dA_d^-(z)\right)F_c^{-\mu}(0)T^c\right]|P\rangle$$
$$-\langle P|\mathrm{Tr}\left[\left(ig\int_{-\infty}^{\xi^+}dz^+T^bA_b^-(z)\right)F_a^{-\mu}(\xi^+)T^aT^e\left(-ig\int_{0}^{\infty}dz^+T^dA_d^-(z)\right)F_c^{-\mu}(0)T^cT^e\right]|P\rangle$$

Puting them all together:

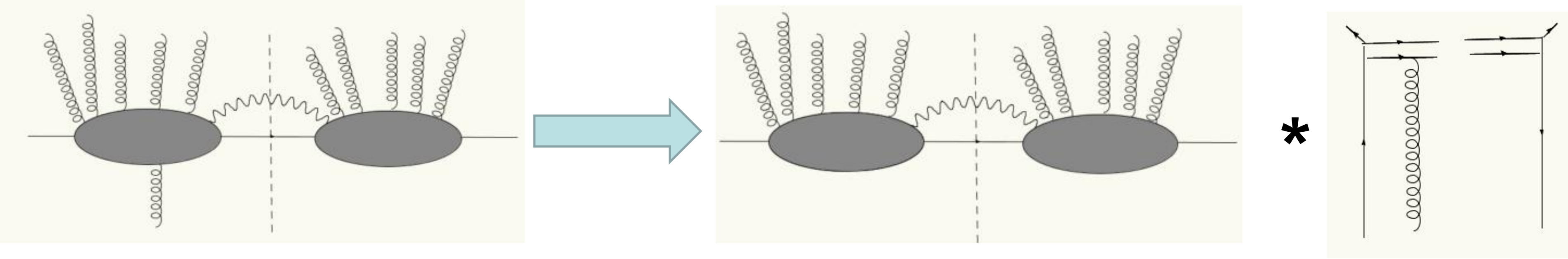
$$C_F \langle P | \mathrm{Tr}\left[F_a^{-\mu}(\xi^+) T^a \mathcal{L}(\xi^+, 0) F_c^{-\mu}(0) T^c \mathcal{L}^\dagger(\xi^+, 0)\right] |P\rangle$$
$$-\langle P | \mathrm{Tr}\left[\mathcal{L}^\dagger(-\infty, \xi^+) F_a^{-\mu}(\xi^+) T^a \mathcal{L}^\dagger(\xi^+, \infty) T^e \mathcal{L}(\infty, 0) F_c^{-\mu}(0) T^c \mathcal{L}(0, -\infty) T^e\right] |P\rangle$$

Schematically:



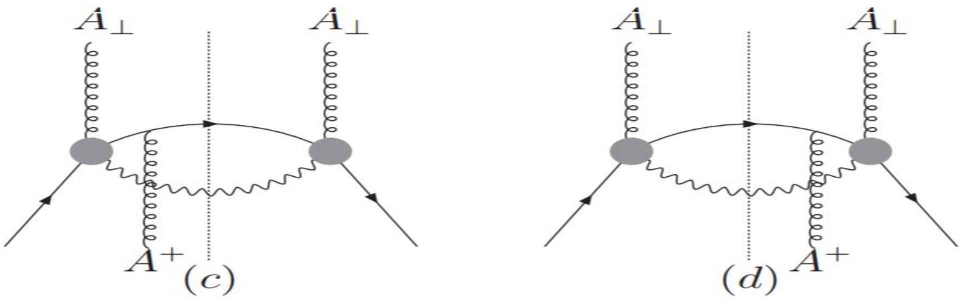
All order analysis I

The Ward identity:



All possible insertions

Final state interaction doesn't contribute

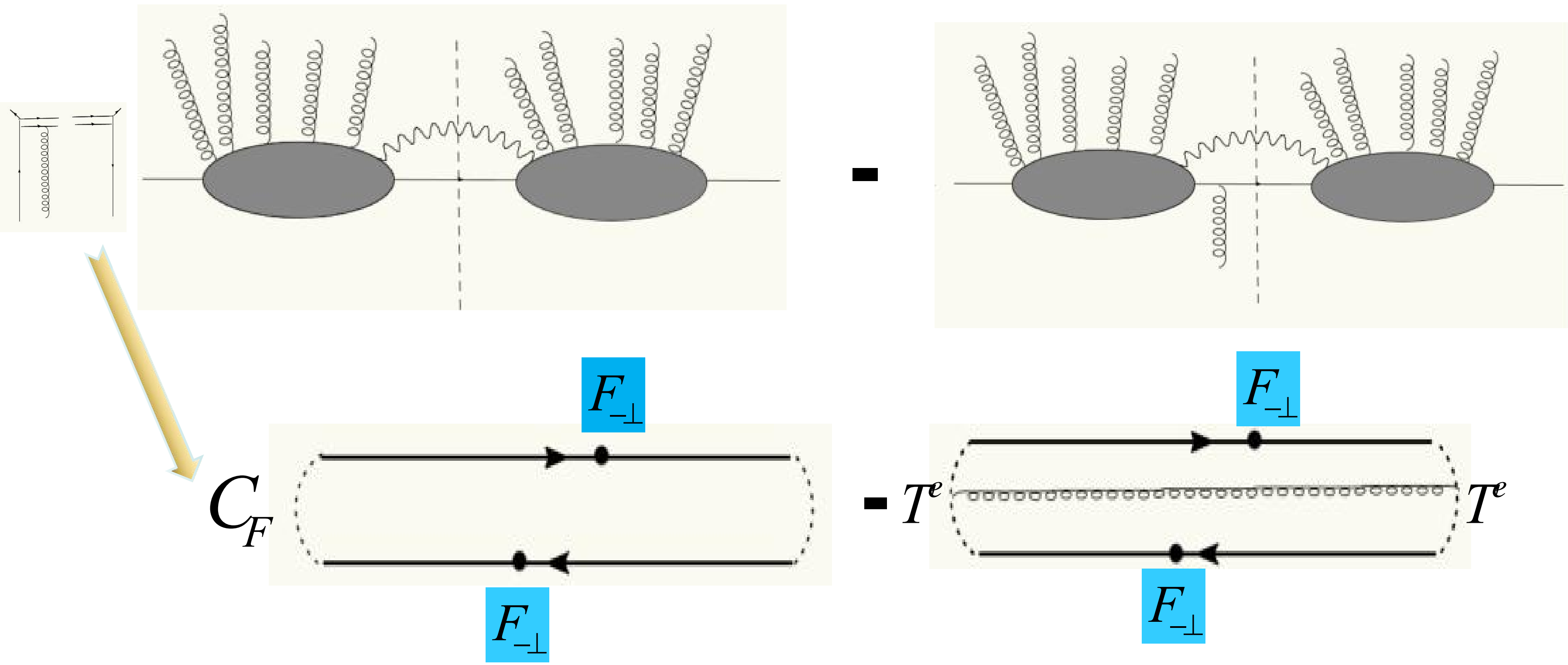


$$\frac{1}{(l_q - x_g P - k_\perp)^2 + i\epsilon}|_{\rm pole}\delta(l_q^2) = -i\pi\delta\left((l_q - x_g P - k_\perp)^2\right)\delta(l_q^2)$$

$$\frac{1}{l_q^2 - i\epsilon}|_{\rm pole}\delta\left((l_q - x_g P - k_\perp)^2\right) = +i\pi\delta(l_q^2)\delta\left((l_q - x_g P - k_\perp)^2\right)$$

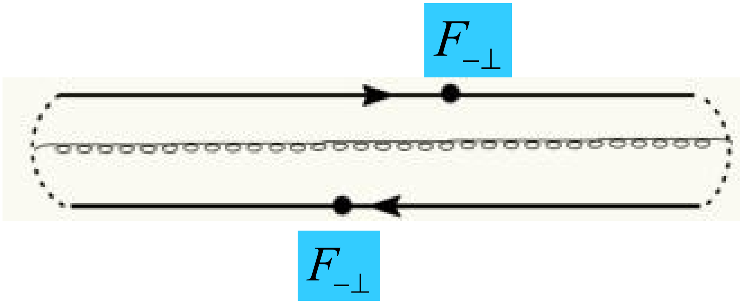
All order analysis II

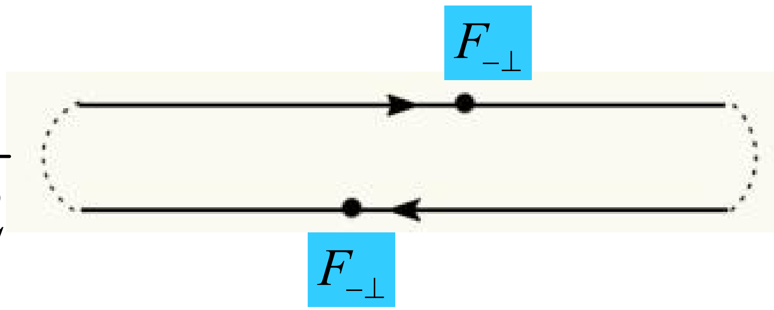
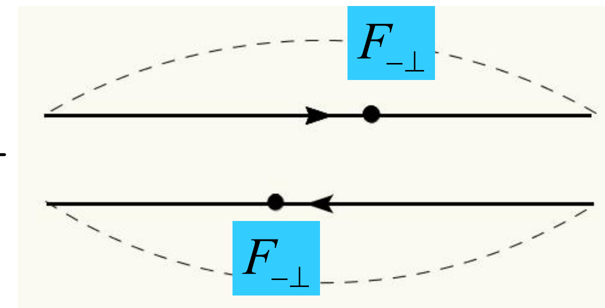
Subtract final state interaction contribution:



Color entanglement effect is absent in collinear twist-2 factorization!

Using the Fierz identity:

T^e  T^e $=$

$\frac{1}{2}$  $-\,\frac{1}{2N_C}$ 

One arrives at,

$$\frac{d^3\Delta\sigma}{d^2l_{\gamma\perp}dz}=\frac{\alpha_s\alpha_{em}N_c}{N_c^2-1}\frac{(z^2-z)[1+(1-z)^2]}{l_{\gamma\perp}^4}\frac{\epsilon^{l_\gamma S_\perp np}}{l_{\gamma\perp}^2}\sum_q e_q^2\int_{x_{\min}}^1 dx\,[x'G(x')-x'G_4(x')]\left[-x\frac{d}{dx}T_{F,q}(x,x)\right]$$

ZJ, 2017

in full agreement with the hybrid approach

Summary

- ➢ Color entanglement effect arises in collinear twist-3 factorization for T-odd case.

Outlook

- ➢ Color entanglement; T-even case at twist-3 level, Hard gluon pole contribution, Soft fermion pole contribution.
- ➢ Color entanglement in inclusive hardron production in forward pp/pA collisions. Welcom to my talk in OCPA meeting.

Thank you !