PDF from Hadronic Tensor on the Lattice and Connected Sea Evolution

- Path-integral Formulation of Hadronic Tensor in DIS
- Parton Degrees of Freedom
- Evolution of Connected Sea Partons
- Numerical Challenges

χ QCD Collaboration

Workshop on PDF at Beida, July 14-16, 2017

Experimental Data

- New Muon Collaboration (NMC – PRL 66, 2712 (1991)) μ+ p(n) → μX

$$S_G(x_0,x_1;Q^2)=\int_{x_0}^{x_1} dx\ \frac{F_2^{\mu P}(x,Q^2)-F_2^{\mu n}(x,Q^2)}{x}$$

Quark parton model + Isospin symmetry

$$S_G(0,1;Q^2)=\frac{1}{3}+\frac{2}{3}\int_0^1 dx\ (\bar{u}_P(x)-\bar{d}_P(x));\quad S_G(0,1;Q^2)=\frac{1}{3}\text{(Gottfried Sum Rule)}$$

NMC : $S_G(0,1;4\ \text{GeV}^2)=0.240\pm 0.016\ (5\sigma\ \text{from GSR})$

- $\bar{d}\,/\,\bar{u}$ asymmetry from Drell-Yan Production (PRL 69, 1726 (1992))

- NuTeV experiment (PRL 88, 091802 (2002))

$$\sin^2\theta_w (3\sigma\ \text{from Standard Model}) \xrightarrow{?} \text{s(x)} \neq \bar{\text{s}}\text{(x)}$$

Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering
  In Minkowski space

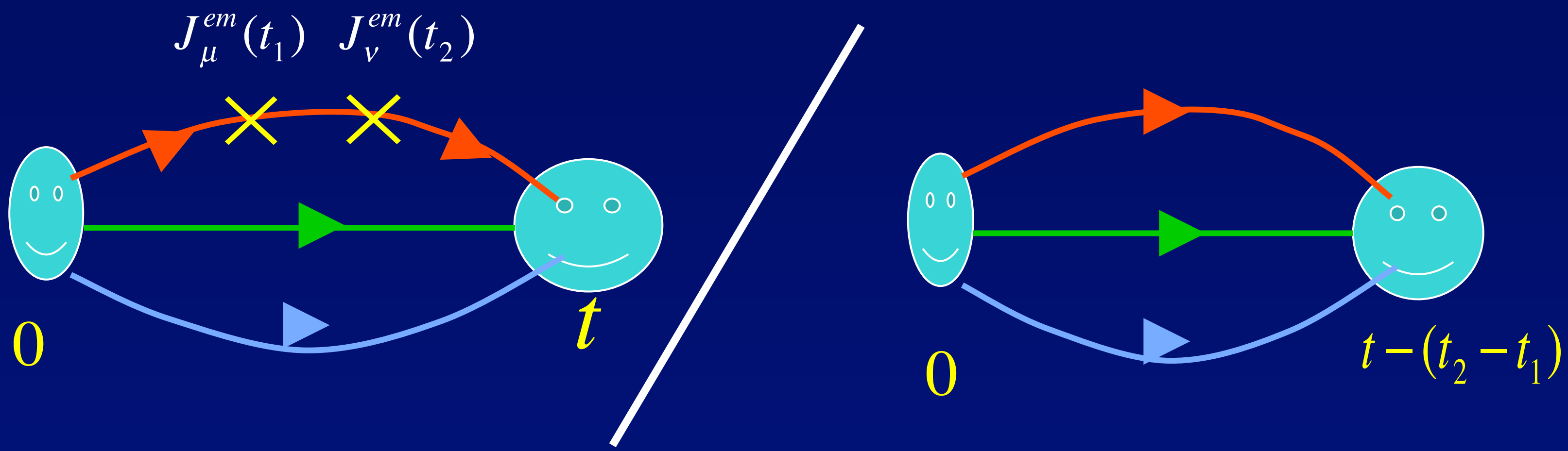
$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{\pi}\mathrm{Im}\,T_{\mu\nu} = \left\langle N(\vec{p})\right| \int \frac{d^4x}{4\pi} e^{iq\cdot x} J_\mu(x) J_\nu(0) \left| N(\vec{p})\right\rangle_{\text{spin avg}}$$

$$= \frac{1}{2}\sum_n \int \prod_{i=1}^{n}\left[\frac{d^3 p_i}{(2\pi)^3 2E_{pi}}\right](2\pi)^3 \delta^4(p_n - p - q) < N(\vec{p}) | J_\mu | n >< n | J_\nu | N(\vec{p}) >_{\text{spin avg}}$$

- Euclidean path-integral

KFL and S.J. Dong, PRL 72, 1790 (1994)
KFL, PRD 62, 074501 (2000)



$W_{\mu\nu}$ in Euclidean Space

$$\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau = t_2 - t_1) = \frac{\frac{E_P}{M_N} \mathrm{Tr} < \Gamma_e \chi_N(\vec{p},t) \sum_{\vec{x}} \frac{1}{4\pi} e^{-i\vec{q}\cdot\vec{x}} J_\mu(\vec{x},t_2) J_\nu(0,t_1) \chi_N^\dagger(\vec{p},0) >}{\mathrm{Tr} < \Gamma_e \chi_N(\vec{p},t) \chi_N^\dagger(\vec{p},0) >}$$

$$\xrightarrow[t-t_2 >> 1/\Delta E_P,\ t_1 >> 1/\Delta E_P]{}$$

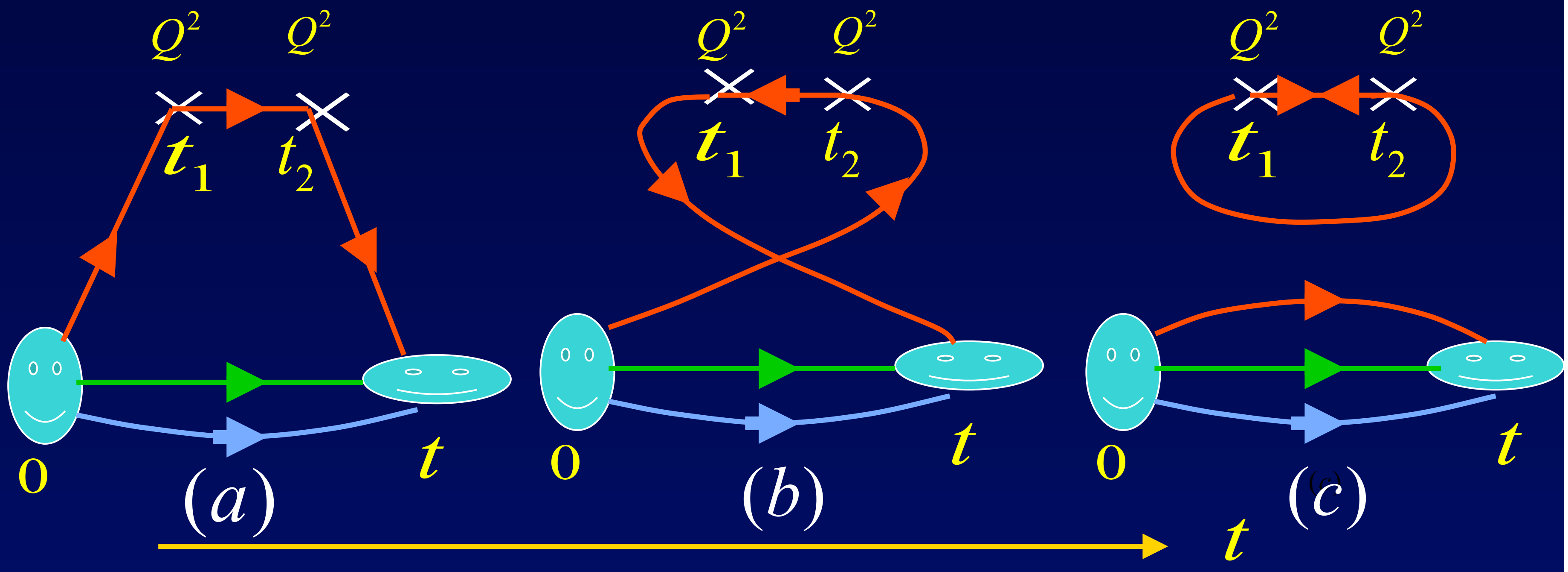
$$= \frac{1}{4\pi} \sum_n \left(\frac{2m_N}{2E_n}\right) \delta_{\vec{p}_n - \vec{p} - \vec{q}} \quad < N(\vec{p}) | J_\mu | n >< n | J_\nu | N(\vec{p}) >_{\text{spin avg}} \quad e^{-(E_n - E_P)\tau}$$

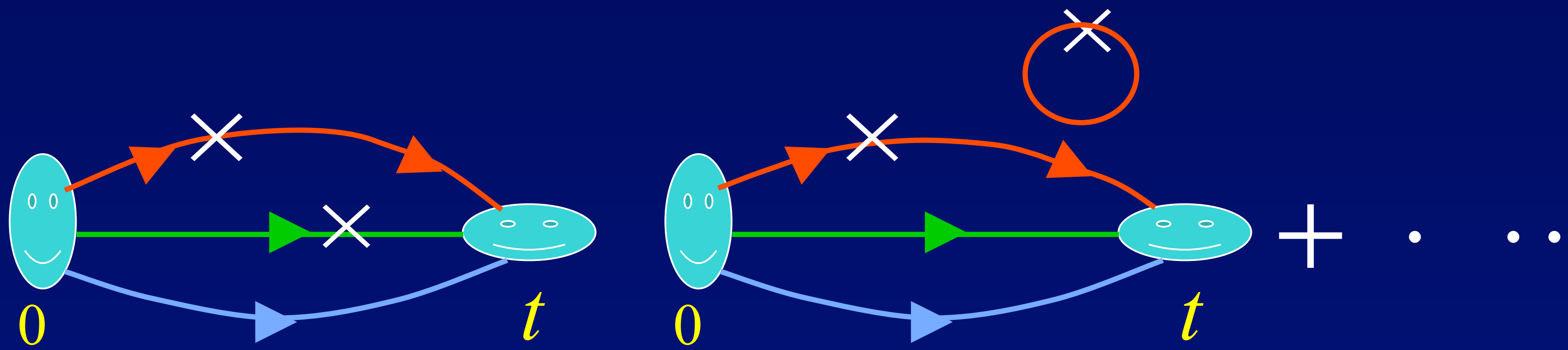
$$= \quad < N(\vec{p}) | \sum_{\vec{x}} \frac{e^{-i\vec{q}\cdot\vec{x}}}{4\pi} J_\mu(\vec{x},\tau) J_\nu(0,0) | N(\vec{p}) >_{\text{spin avg}}$$

Laplace transform

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{i} \int_{c-i\infty}^{c+i\infty} d\tau \; e^{\nu\tau} \; \tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$$

$q = q_V + q_{CS}$ $\qquad \bar{q}_{CS}$ $\qquad q_{DS} = (\neq ?)\ \bar{q}_{DS}$





Cat’s ears diagrams are suppressed by O(1/Q²).

- $$W_{\mu\nu}(p,q) = -W_1(q^2,\nu)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + W_2(q^2,\nu)(p_\mu - \frac{p\cdot q}{q^2}q_\mu)(p_\nu - \frac{p\cdot q}{q^2}q_\nu)$$

- Large momentum frame

$$\nu W_2(q^2,\nu) \longrightarrow F_2(x,Q^2) = x\sum_i e_i^2(q_i(x,Q^2) + \bar{q}_i\left(x,Q^2\right)); \;\; x = \frac{Q^2}{2p\cdot q}$$

- Parton degrees of freedom: valence, connected sea and disconnected sea

u	d	s
$u_V(x) + u_{CS}(x)$	$d_V(x) + d_{CS}(x)$	
$\bar{u}_{CS}(x)$	$\bar{d}_{CS}(x)$	
$u_{DS}(x) + \bar{u}_{DS}(x)$	$d_{DS}(x) + \bar{d}_{DS}(x)$	$s_{DS}(x) + \bar{s}_{DS}(x)$

Properties of this separation

- No renormalization
- Gauge invariant
- Topologically distinct as far as the quark lines are concerned
- $W_1(x, Q^{2})$ and $W_2(x, Q^2)$ are frame independent.
- Small x behavior of CS and DS are different.

$$q_V\,,\ q_{CS}\ ,\ \bar{q}_{CS} \sim_{x\to 0}\ x^{-\alpha_R}\,(x^{-1/2})$$

$$q_{DS}\ ,\ \bar{q}_{DS} \sim_{x\to 0}\ x^{-1}$$

- Short distance expansion (Taylor expansion) ⟹ OPE
-

Operator Product Expansion -> Taylor Expansion

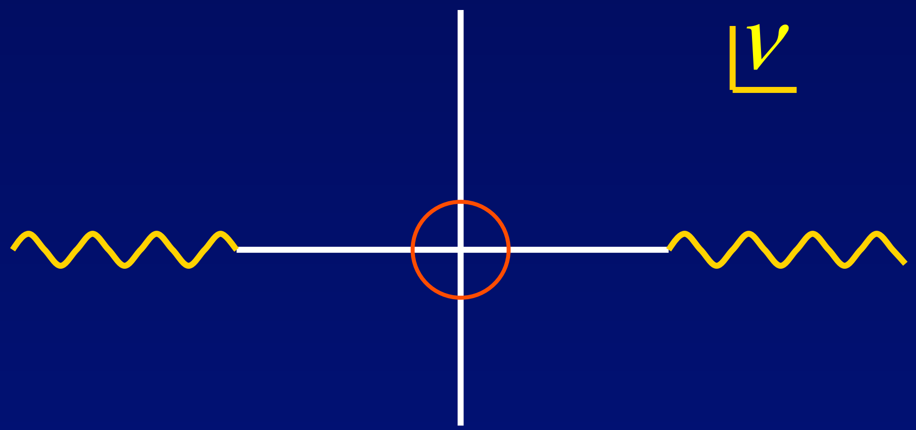
- Operator product expansion

$$W_{\mu\nu} = \frac{1}{\pi} \operatorname{Im} T_{\mu\nu}$$

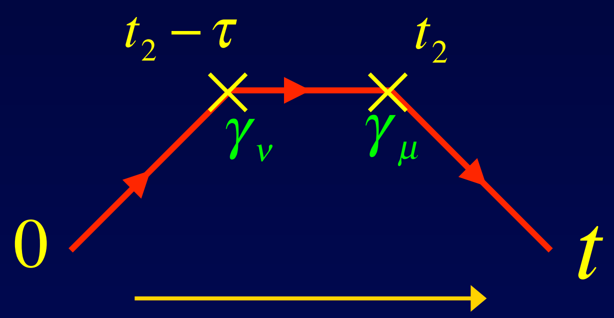
  - Dispersion relation

$$T_{\mu\nu} = \frac{1}{\pi} \int_{Q^2/2M_N}^{\infty} d\nu' \frac{\nu' W_{\mu\nu}(q^2, \nu')}{\nu'^2 - \nu^2}$$

  - Expand in the unphysical region $\frac{2M_N \nu}{Q^2} = \frac{2p \cdot q}{Q^2} < 1 \quad (\mathrm{x} > 1)$



- Euclidean path-integral
  - Consider



$$\tilde{W}_{\mu\nu}(q^2,\tau)|_{(a)} \propto \int D[A]\det M(A)\, e^{-S_g}$$

$$\times Tr\left[...M^{-1}(t,t_2)\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\, i\gamma_\mu M^{-1}(t_2,t_2-\tau)\, i\gamma_\nu M^{-1}(t_2-\tau,0)...\right]$$

  - Short-distance expansion ($|\vec{x}|,\tau \to 0$ from $|\vec{q}|,\nu \to \infty$)

$$M^{-1}(t_2,t_2-\tau) \xrightarrow[\text{free quark}]{} \frac{1}{4\pi^2}\frac{\partial\!\!\!/}{\vec{x}^2+\tau^2};$$

$$M^{-1}(t_2-\tau,0) \xrightarrow[|\vec{x}|,\tau\to 0]{} e^{\vec{x}\cdot\vec{D}+\tau D_\tau} M^{-1}(t_2,0)$$

  - Laplace transform

$$W_{\mu\nu}(q^2,\nu) \propto Tr\left[...M^{-1}(t,t_2)\, i\gamma_\mu \frac{-i\pi(q\!\!\!/+iD\!\!\!\!/)}{|\vec{q}+i\vec{D}|}\delta(\nu+D_\tau-|\vec{q}+i\vec{D}|)\, i\gamma_\nu M^{-1}(t_2,0)...\right]$$

- Dispersion relation

$$T_{\mu\nu}(q^2,\nu)=\frac{1}{\pi}\int_{Q^2/2M_N+D_\tau}^{\infty}d\nu'\frac{\nu' W_{\mu\nu}(q^2,\nu'-D_\tau)}{\nu'^2-(\nu+D_\tau)^2},$$

$$\propto\ Tr\left[\ldots M^{-1}(t,t_2)i\gamma_\mu\frac{-i(\not q+i\not D)}{(Q^2+2iq\cdot D-D^2)}i\gamma_\nu M^{-1}(t_2,0)\ldots\right],$$

where $\tau=it$ and $D_t=iD_\tau$

so that $D=(\vec{D},-iD_t)$ is covariant derivative in Minkowski space.

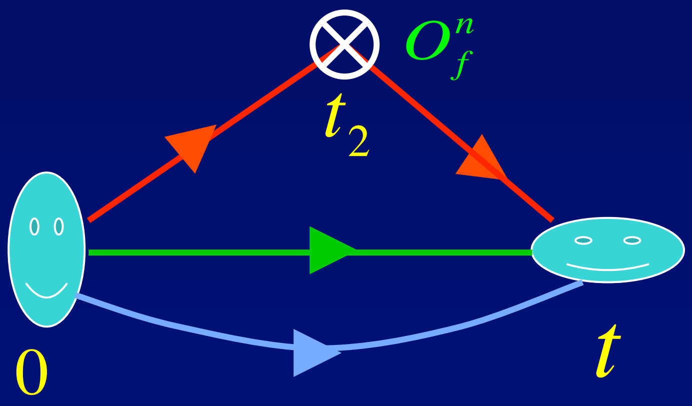
- Expansion about the unphysical region ($2q\cdot p/Q^2<1$)

$$T_{\mu\nu}(q_V+q_{CS})=\sum_f e_f^2\left[8p_\mu p_\nu\sum_{n=2}\frac{(-2q\cdot p)^{n-2}}{(Q^2)^{n-1}}A_f^n(CI)-2g_{\mu\nu}\sum_{n=2}\frac{(-2q\cdot p)^n}{(Q^2)^n}A_f^n(CI)\right]$$

even + odd n terms

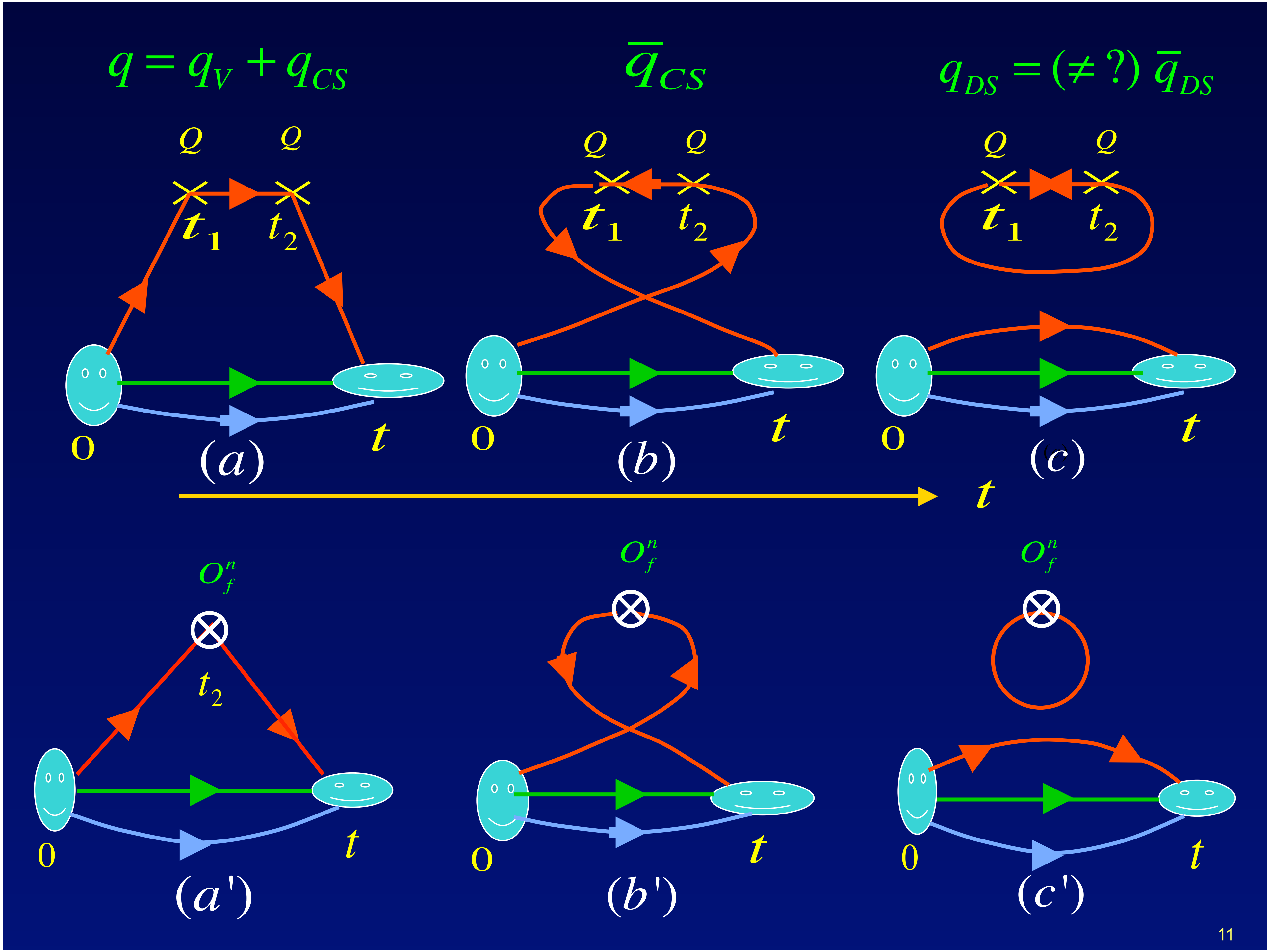
- $A_f^n=?$

$$A_f^n(CI)\propto\int D[A]\det M(A)\,e^{-S_g}Tr\left[\ldots M^{-1}(t,t_2)O_f^nM^{-1}(t_2,0)\ldots\right]$$



$$O_f^n=i\gamma_{\mu_1}\left(\frac{-i}{2}\right)^{n-1}\overleftrightarrow{D}_{\mu_2}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n},$$

$$<p\,|\,\bar\psi\ O_f^n\psi\,|\,p>=A_f^n(CI)\ 2p_{\mu_1}p_{\mu_2}\ldots p_{\mu_n}$$

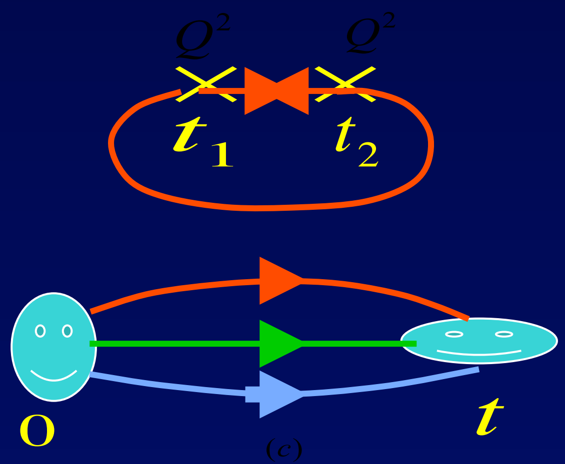


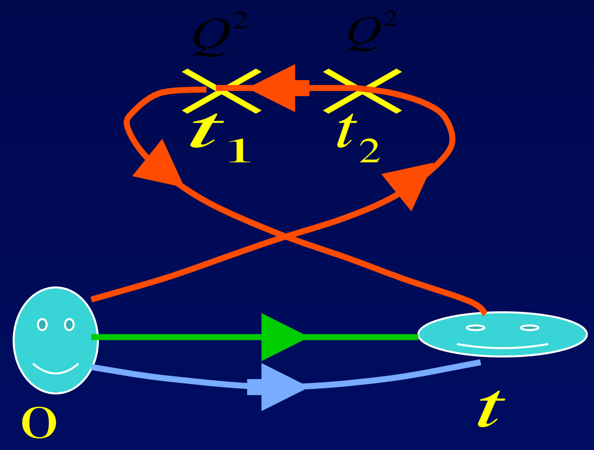
$q = q_V + q_{CS}$
$\bar{q}_{CS}$
$q_{DS} = (\neq ?)\ \bar{q}_{DS}$
Q Q
t_1 t_2
0
t
(a)
(b)
(c)
O_f^n
t_2
(a')
(b')
(c')

Gottfried Sum Rule Violation

$$S_G(0,1;Q^2) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_p(x) - \bar{d}_p(x)); \quad S_G(0,1;Q^2) = \frac{1}{3} \text{(Gottfried Sum Rule)}$$

NMC: $S_G(0,1;4\ \text{GeV}^2) = 0.240 \pm 0.016$ (5σ from GSR)





two flavor traces ($\bar{u}_{DS} = \bar{d}_{DS}$) one flavor trace ($\bar{u}_{CS} \neq \bar{d}_{CS}$)

K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$Sum = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_{CS}(x) - \bar{d}_{CS}(x)),$$

$$= \frac{1}{3} + \frac{2}{3}\left[n_{\bar{u}_{CS}} - n_{\bar{d}_{CS}}\right] (1 + O(\alpha_s^2))$$

Comments

- The results are the same as derived from the conventional operator product expansion.
- Contrary to conventional OPE, the path-integral formalism admits separation of CI and DI.
- The OPE turns out to be Taylor expansion of functions in the path-integral formalism.
- For O_f^n with definite n, there is only one CI and one DI in the three-point function, i.e. (a') is the same as (b'). Thus, one cannot separate quark contribution from that of antiquark in matrix elements.

K.F. Liu, PRD 62, 074501 (2000)

3) Fitting of experimental data

$$\bar{u} - \bar{d} \xrightarrow[x \to 0]{} x^{-1/2} \qquad \text{O.K.}$$

But $\bar{u} + \bar{d} \propto \bar{s}$ is not adequate.

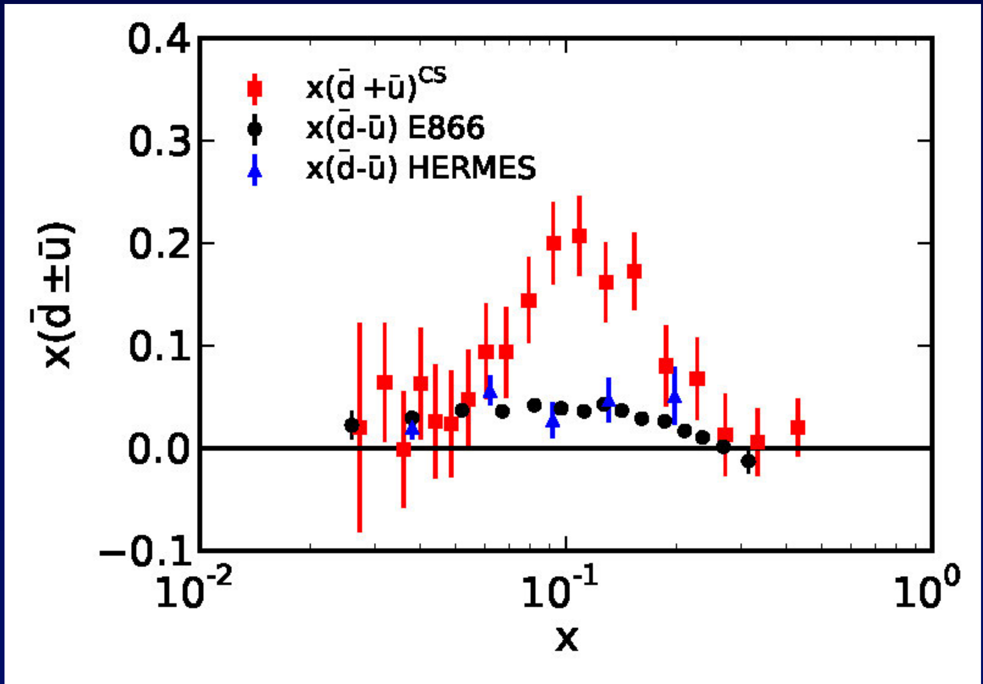
A better fit $$\frac{\bar{u}(x) + \bar{d}(x)}{2} = f\,\bar{s}(x) + c(x), \quad f \approx 1$$

where $c(x) \xrightarrow[x \to 0]{} x^{-1/2}$ like in $\bar{u}(x) - \bar{d}(x)$

4) Unlike DS, CS evolves the same way as the valence.

Connected Sea Partons

K.F. Liu, W.C. Chang, H.Y. Cheng, J.C. Peng, PRL 109, 252002 (2012)



Assume $(u+\bar{u})_{DS} = (d+\bar{d})_{DS} = \frac{1}{R}(s+\bar{s})$

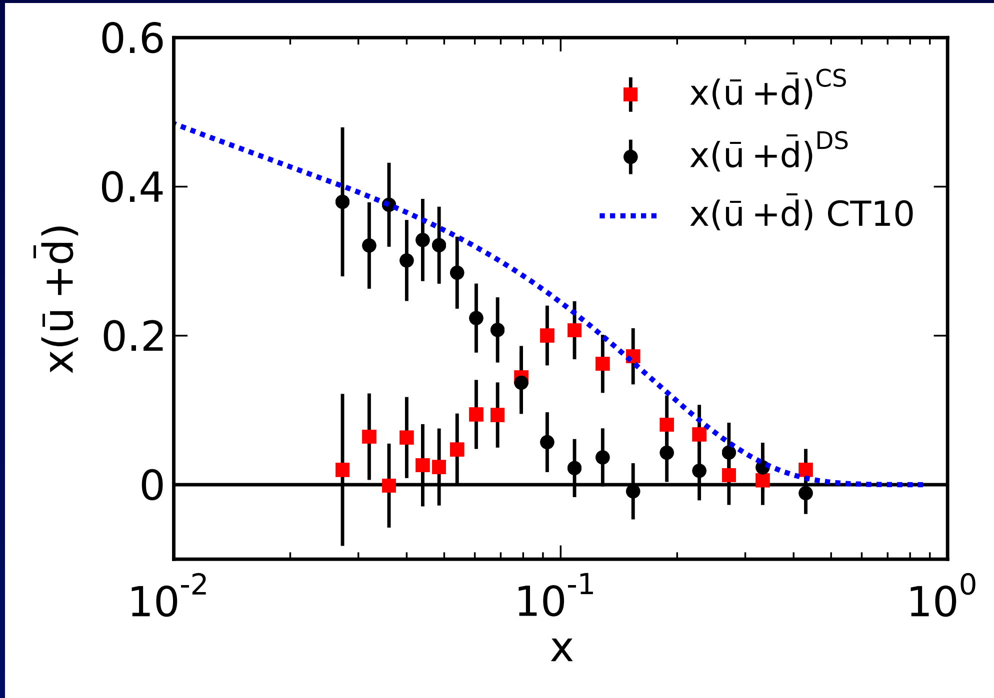
$$R = \frac{\langle x \rangle_s}{\langle x \rangle_u (DI)} \text{(lattice)} \sim 0.857$$

$$x(\bar{d}+\bar{u})_{CS}(x) = x(\bar{d}+\bar{u})(x) - \frac{1}{R} x(s+\bar{s})(x)$$

CT10

lattice

expt

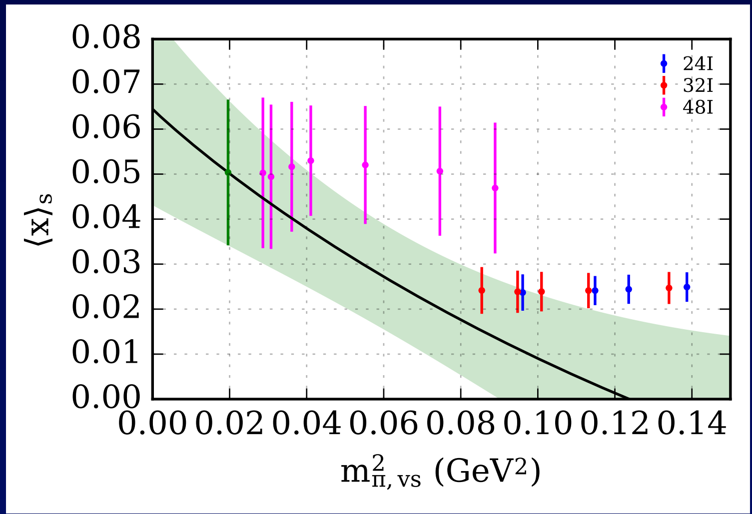


$$q_V\,,\ q_{CS}\ ,\ \bar{q}_{CS} \sim_{x\to 0}\ x^{-\alpha_R}(x^{-1/2})$$

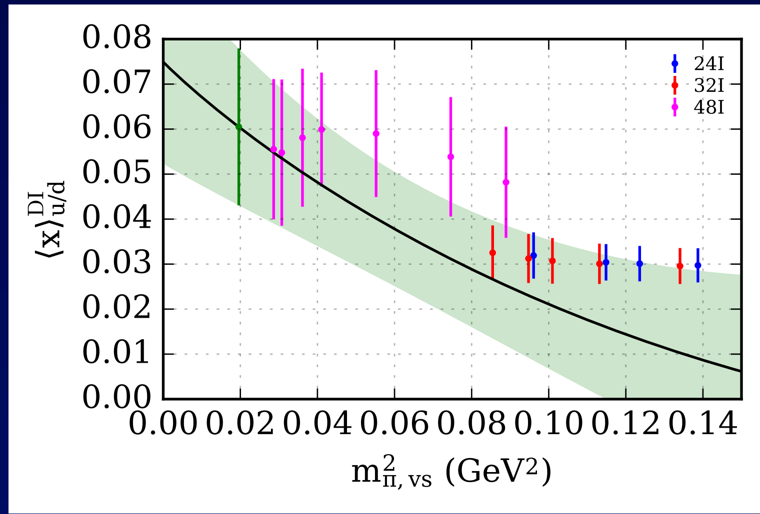
$$q_{DS}\ ,\ \bar{q}_{DS} \sim_{x\to 0}\ x^{-1}$$

Lattice input to global fitting of PDF

$<x>_s$



$<x>_{u/d}$ (DI)

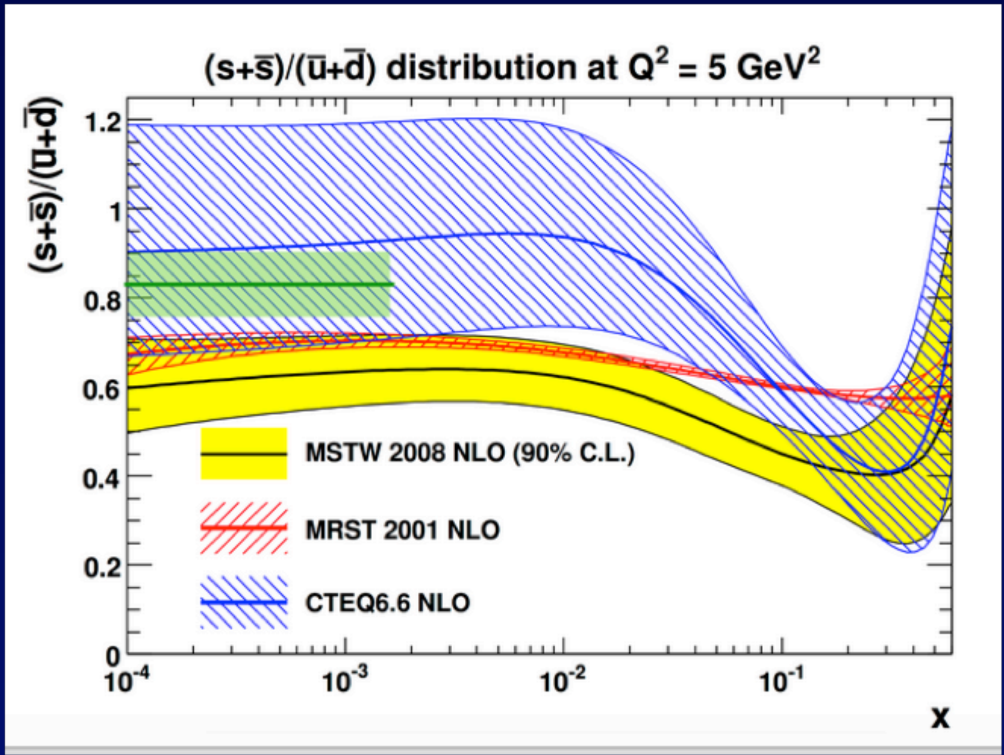


Lattice calculation with Overlap fermion on 3 lattices including on at m_π ~ 140 MeV (Mingyang Sun, χQCD Collaboration)

$<x>_{s=}$ 0.050(16), $<x>_{u/d}$ (DI=0.060(17)

$<x>_s$ /$<x>_{u/d}$(DI) = 0.83(7)

Lattice input to global fitting of PDF



$\langle x \rangle_s / \langle x \rangle_{u/d}$(DI) 0.83(7)

Operator Mixing

- Connected insertion

$$\frac{d\ M_f^n(CI)}{d\log Q^2} = \frac{a_f^n}{2b_0}\frac{1}{\log(Q^2/\Lambda^2)}M_f^n(CI)$$

- Disconnected insertion

$$\frac{d\ M_f^n(DI)}{d\log Q^2} = \frac{1}{2b_0}\frac{1}{\log(Q^2/\Lambda^2)}\left[a_{qq}^n M_f^n(CI) + \frac{1+(-)^n}{2}a_{qG}^n M_G^n\right]$$

Evolution Equations

S. Moch et al., hep/0403192,0404111
A. Cafarella et al., 0803.0462

NNLO

$$dq_i / dt = \sum_k (P_{ik} \otimes q_k + P_{i\bar{k}} \otimes q_{\bar{k}}) + P_{ig} \otimes g;$$

$$d\bar{q}_i / dt = \sum_k (P_{\bar{i}k} \otimes q_k + P_{\bar{i}\bar{k}} \otimes q_{\bar{k}}) + P_{\bar{i}g} \otimes g;$$

$$dg / dt = \sum_k (P_{gk} \otimes q_k + P_{g\bar{k}} \otimes q_{\bar{k}}) + P_{gg} \otimes g.$$

$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + \frac{P_{ns}^s}{N_f} \otimes \Sigma_v;$$

where $\mathrm{q}_i^- \equiv q_i - \bar{q}_i, \quad \Sigma_v \equiv \sum_k (q_k - \bar{q}_k),$

and $P_{ns}^s \sim O(\alpha_s^3)$

Valence u can evolve into valence d ?

Note: $q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$

Evolution equations separating CS from the DS partons:
11 equations for the general case

$$u^{val},\ d^{val},\ u^{cs} \equiv \bar{u}^{cs},\ d^{cs} \equiv \bar{d}^{cs},\ u^{ds} \neq \bar{u}^{ds},\ d^{ds} \neq \bar{d}^{ds},\ s^{ds} \neq \bar{s}^{ds},\ g$$

K.F. Liu 1703.046690

$$dq_i^{v+cs} / dt = P_{ii}^{c} \otimes q_i^{v+cs} + P_{i\bar{i}}^{c} \otimes \bar{q}_i^{cs};$$

$$d\bar{q}_i^{cs} / dt = P_{\bar{i}\bar{i}}^{c} \otimes \bar{q}_i^{cs} + P_{\bar{i}i}^{c} \otimes q_i^{v+cs};$$

$$dq_i^{ds} / dt = \sum_k (P_{ik}^{cd} \otimes q_k^{ds} + P_{i\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{ik}^{d} \otimes q_k^{v+cs} + P_{i\bar{k}}^{d} \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$d\bar{q}_i^{ds} / dt = \sum_k (P_{\bar{i}\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{\bar{i}k}^{cd} \otimes q_k^{ds} + P_{\bar{i}k}^{d} \otimes q_k^{v+cs} + P_{\bar{i}\bar{k}}^{d} \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$dg / dt = \sum_k [P_{gk} \otimes (q_k^{v+cs} + q_k^{ds}) + P_{g\bar{k}} \otimes (\bar{q}_k^{cs} + \bar{q}_k^{ds}) + P_{gg} \otimes g.$$

Hadronic Tensor on the Lattice (Inverse Laplace Transform)

- Improved Maximum Entropy method
- Backus-Gilbert method
- Fitting with a prescribed functional form for the spectral distribution.
- OPE w/o OPE

Improved Maximum Entropy Method

- Inverse problem

$$D(\tau) = \int K(\tau,\nu)\rho(\nu)d\nu,$$

$$D(\tau) = \tilde{W}_{\mu\nu}(\tau),\ \ K(\tau,\nu) = e^{-\nu\tau},\ \ \rho(\nu) = \mathrm{W}_{\mu\nu}(q^2,\nu)$$

- Bayes' theorem

$$P[\rho \mid D] = \frac{P[D \mid \rho]P[\rho]}{P[D]}$$

- Maximum entropy method: find $\rho(\nu)$ from

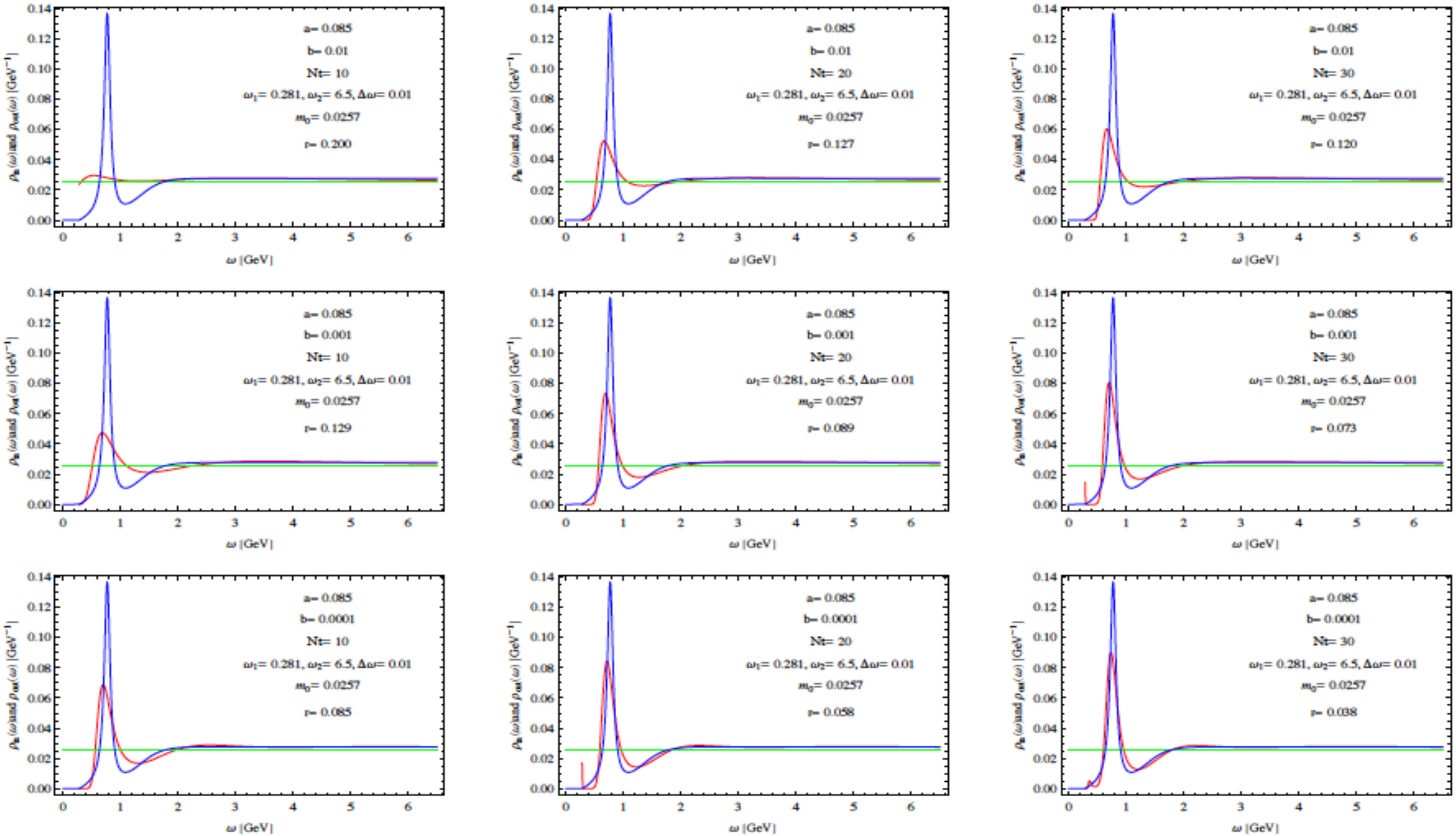
$$\frac{\partial P[\rho \mid D]}{\partial \rho} = 0$$

- Improved MEM (Burnier and Rothkpf, PRL 111, 182003 (2013))

$$P[\rho \mid D] \propto e^{\alpha S - L - \gamma(L - N_\tau)^2},\ \ L = \frac{\chi^2}{2}$$

$$S = \int d\nu \left[1 - \frac{\rho(\nu)}{m(\nu)} - \ln\left(\frac{\rho(\nu)}{m(\nu)}\right)\right]$$

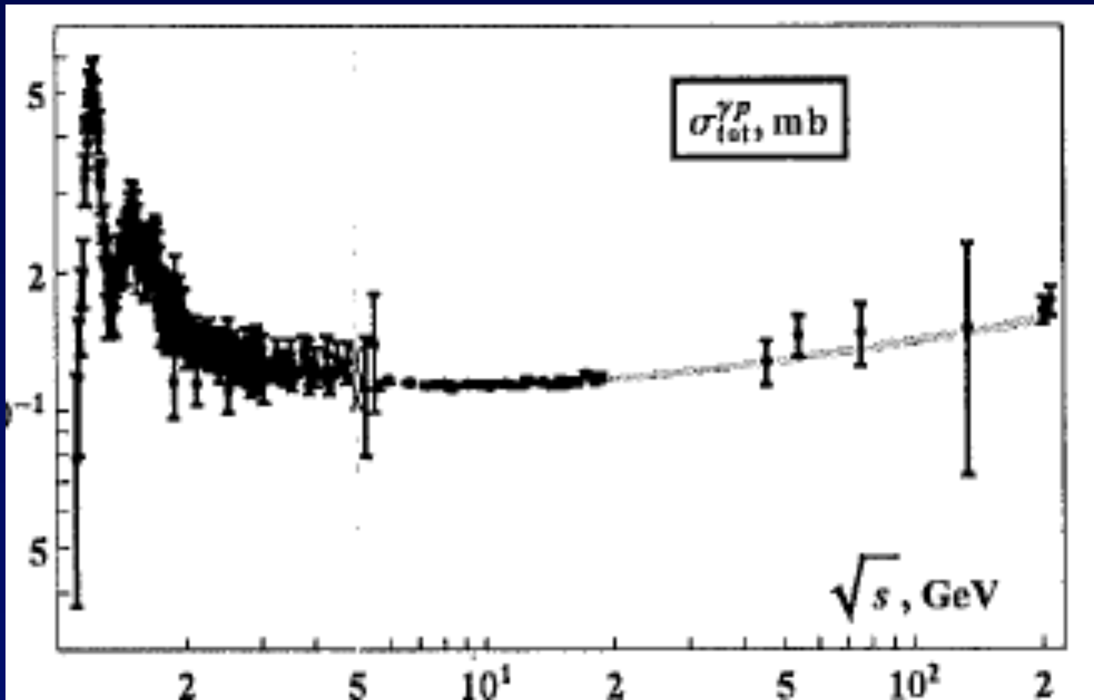
Reconstruction of realistic data



$e^+e^- \to \rho$

Frank X. Lee

Photo-proton Inclusive X-section



Numerical Challenges

- Lattice calculation of the hadronic tensor – no renormalization, continuum and chiral limits, direct comparison with expts ⟹ PDF.

- Bjorken x

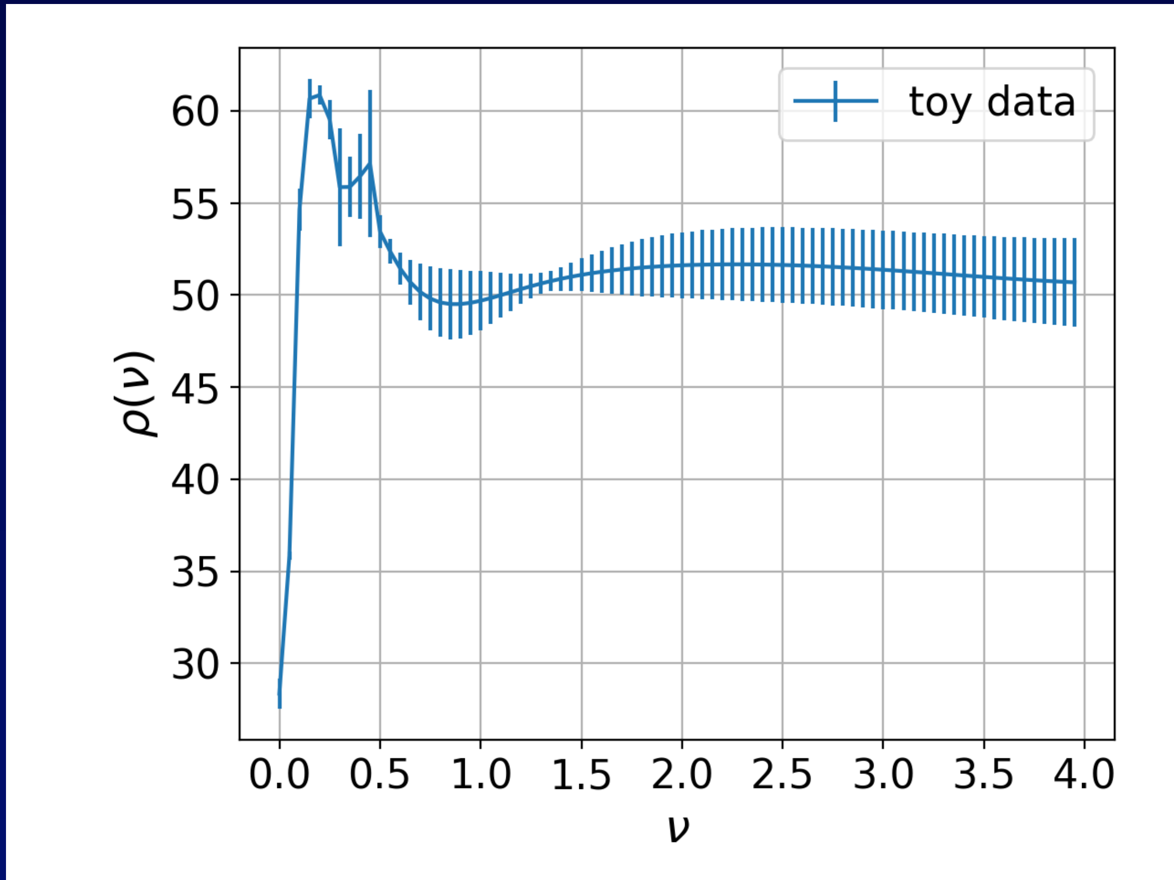
$$x = \frac{Q^2}{2p\cdot q} = \frac{\vec{q}^{\,2} - \nu^2}{2(\nu E_p - \vec{p}\cdot\vec{q})}$$

- Range of x: $Q^2 = 2\ \mathrm{GeV}^2$

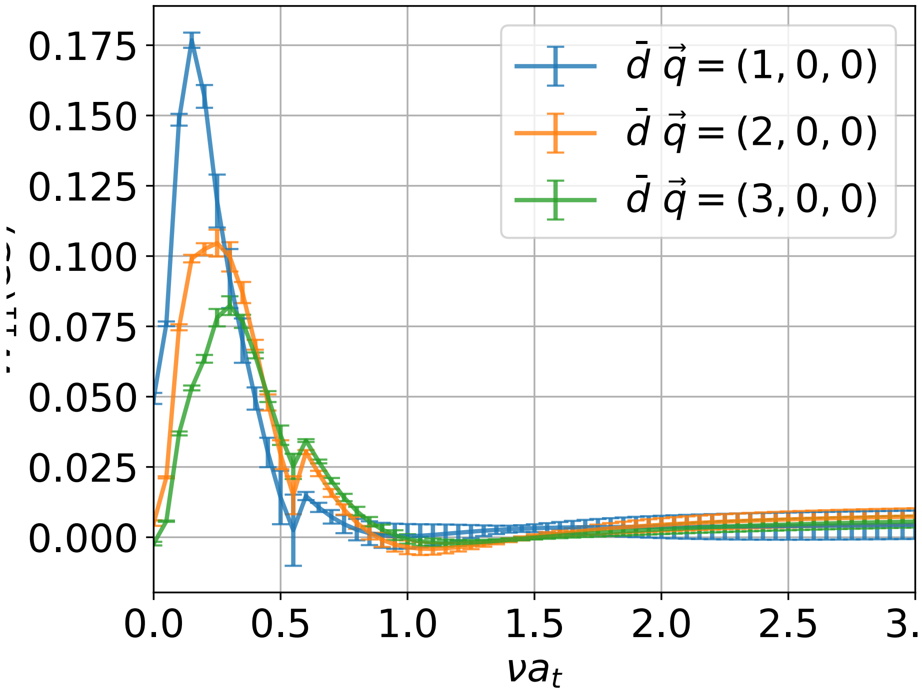
$-\vec{q} \parallel \vec{p}$ $\quad |\vec{p}| = 3\ \mathrm{GeV}, |\vec{q}| = 3\ \mathrm{GeV}, \Rightarrow\ x = 0.058$

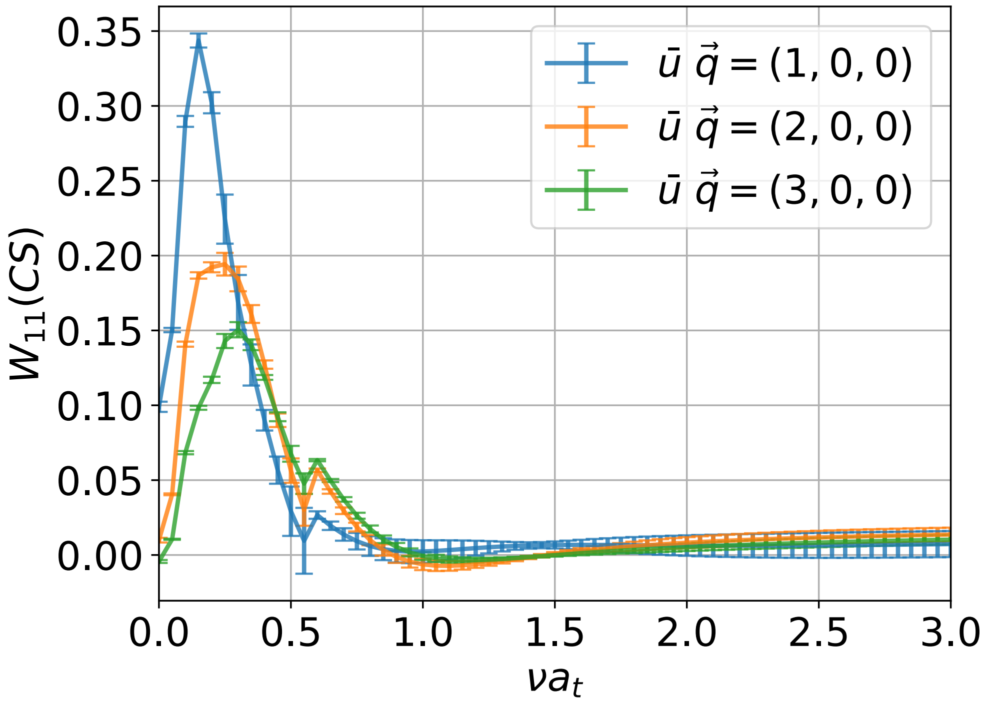
$$\vec{p} = 0,\ |\vec{q}| = 2\ \mathrm{GeV}\ \Rightarrow\ x = 0.75$$

Backus-Gilbert reconstruction test

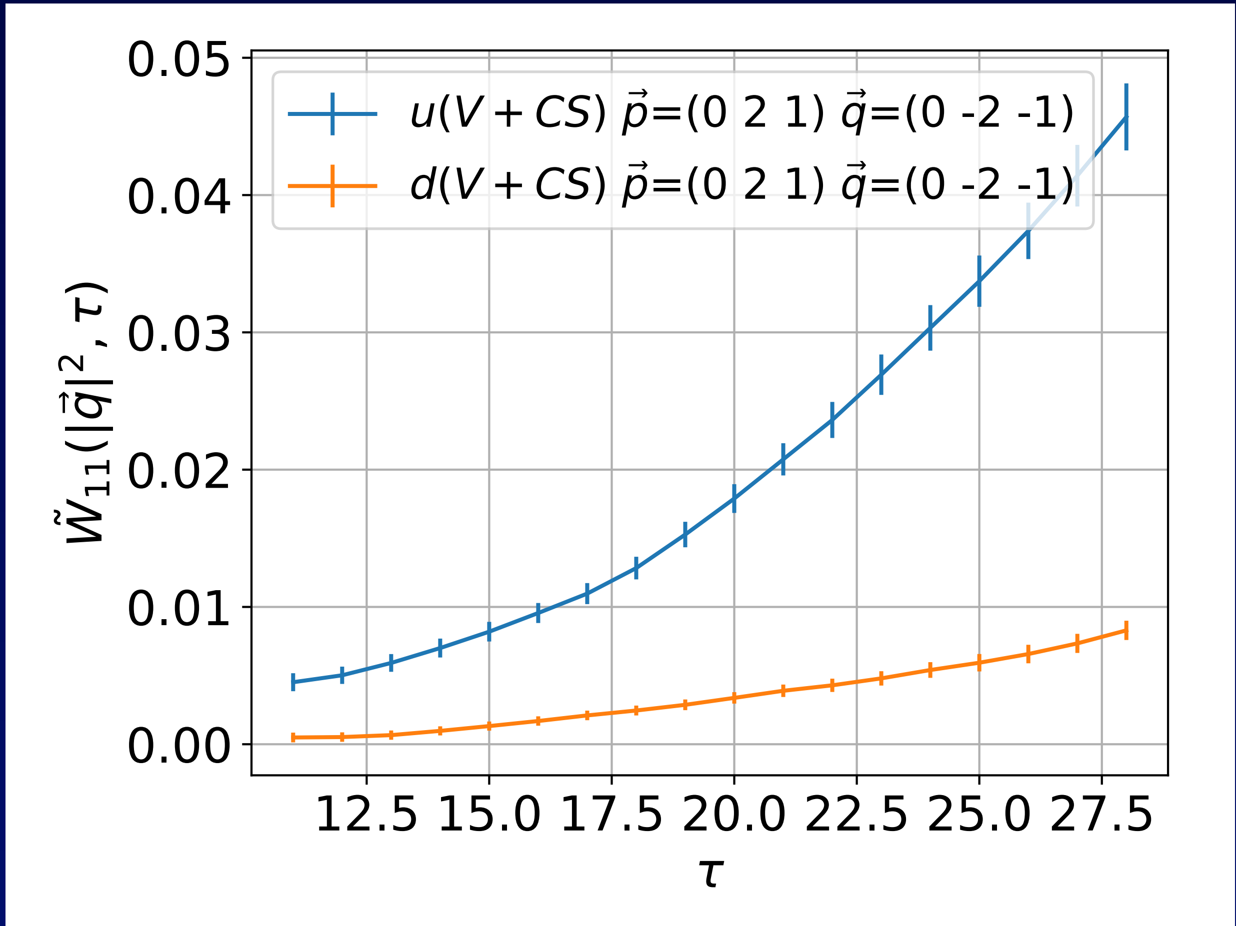


Model data with two peaks at 0.1 and 0.3,
and a continuum with height = 50

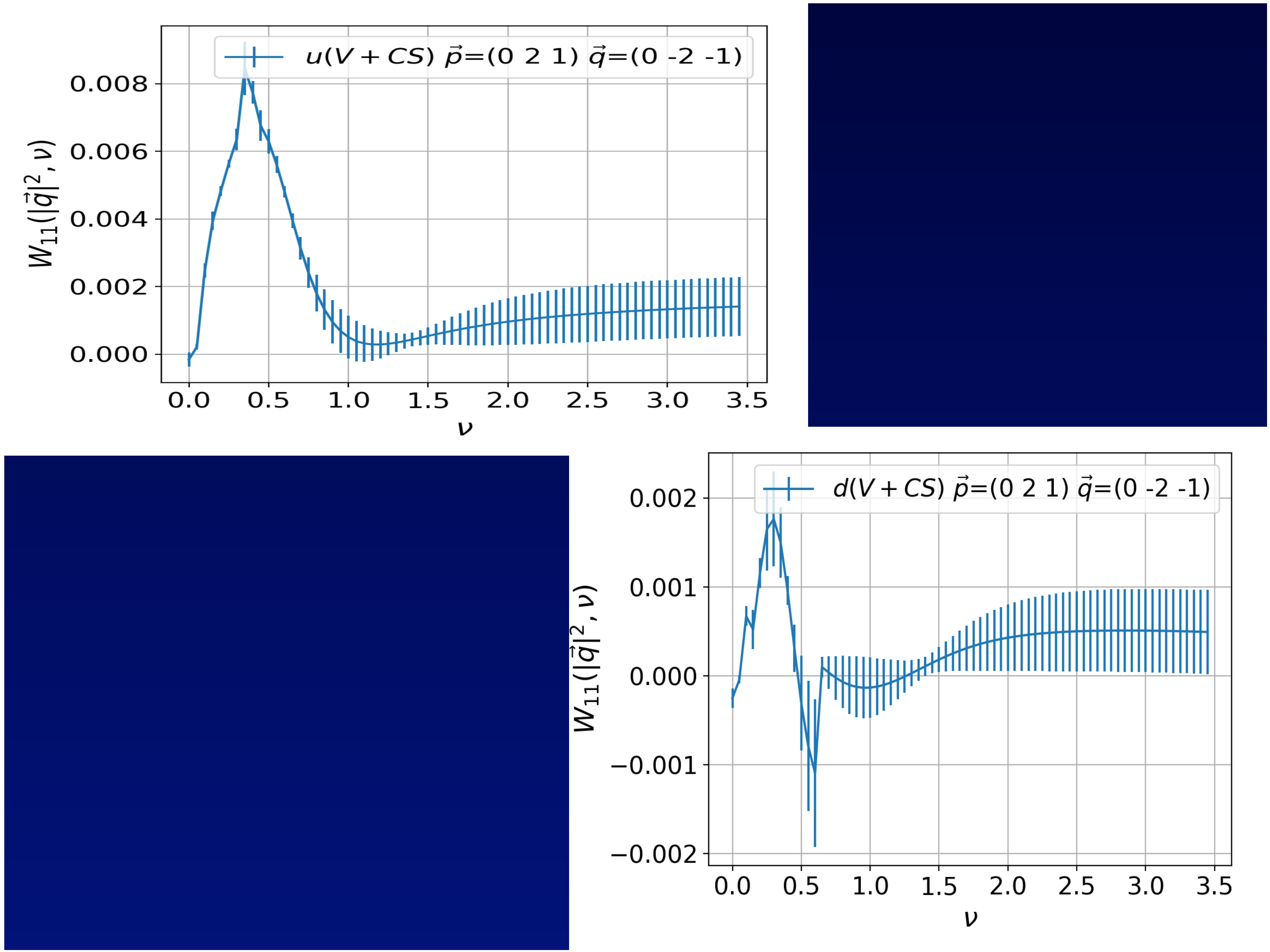




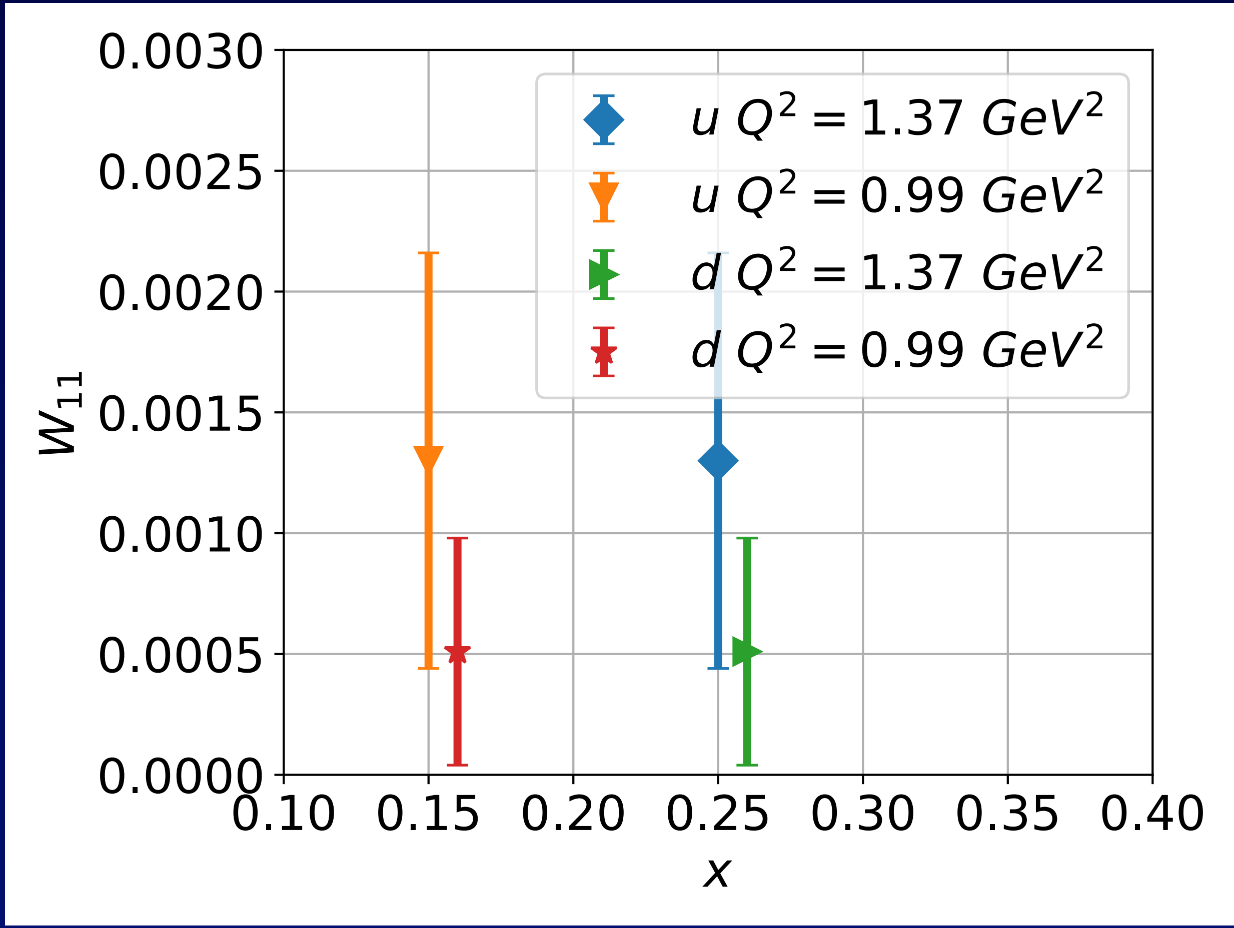
12^3 x 128 lattice (CLQCD), a ~ 0.15 fm, m_π ~ 600 MeV



$u(V+CS)$ $\vec{p}$=(0 2 1) $\vec{q}$=(0 -2 -1)
$d(V+CS)$ $\vec{p}$=(0 2 1) $\vec{q}$=(0 -2 -1)
$\tilde{W}_{11}(|\vec{q}|^2, \tau)$
τ
0.00
0.01
0.02
0.03
0.04
0.05
12.5
15.0
17.5
20.0
22.5
25.0
27.5

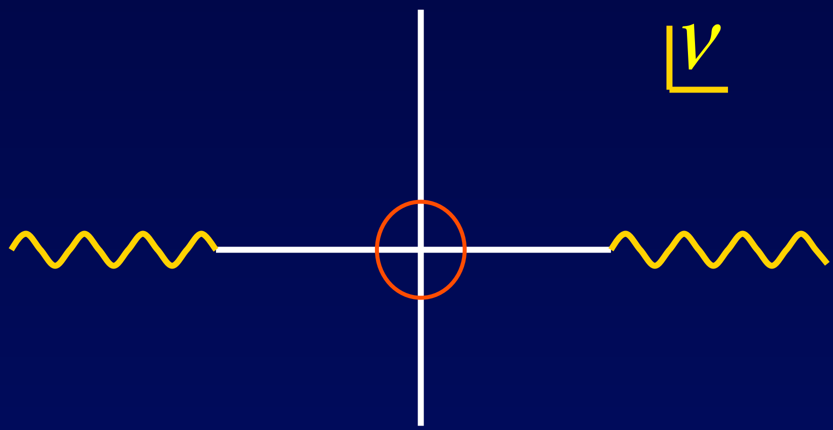


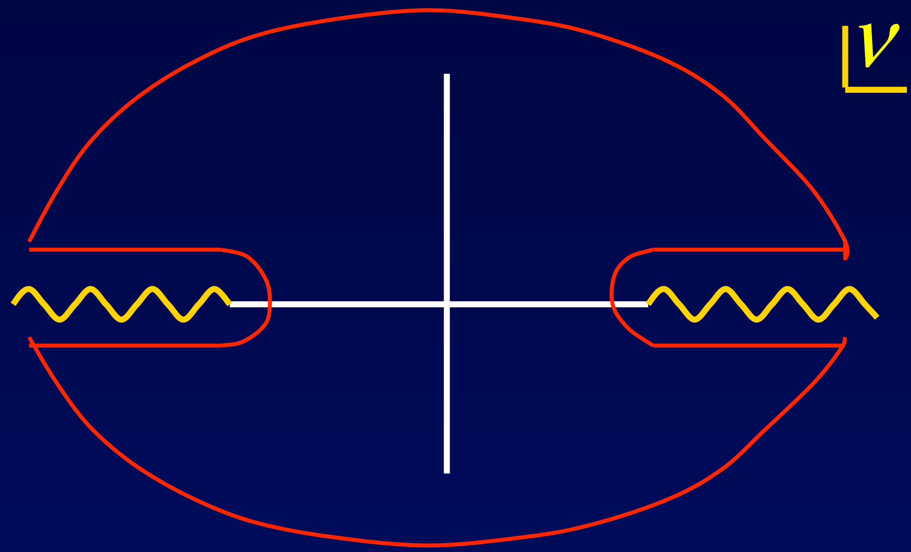
u(V + CS) p⃗=(0 2 1) q⃗=(0 -2 -1)
W11(|q⃗|2, ν)
ν
d(V + CS) p⃗=(0 2 1) q⃗=(0 -2 -1)
W11(|q⃗|2, ν)
ν



0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
W_{11}
0.10
0.15
0.20
0.25
0.30
0.35
0.40
x
$u\ Q^2 = 1.37\ GeV^2$
$u\ Q^2 = 0.99\ GeV^2$
$d\ Q^2 = 1.37\ GeV^2$
$d\ Q^2 = 0.99\ GeV^2$

Quark Parton Model





$$I_n = \oint \frac{d\nu}{2\pi i}\frac{1}{\nu^{n-1}}T_2(Q^2,\nu), \qquad I_n = 2\int_{Q^2}^{\infty}\frac{d\nu 2M_N}{2\pi i}\frac{2i}{\nu^{n-1}}W_2(Q^2,\nu),$$

$$= \sum_f 8\, e_f^2 \left(\frac{2M_N}{Q^2}\right)^{n-1} A_f^n \qquad = 8\left(\frac{2M_N}{Q^2}\right)^{n-1}\int_0^1 dx\; x^{n-2}\frac{2M_N \nu W_2(Q^2,\nu)}{4}$$

- $A_f^{n=even}(CI) \equiv M_f^n(CI) = \int_0^1 dx\; x^{n-1}(q_V(x) + q_{CS}(x) + \bar{q}_{CS}(x))_f$
- $A_f^{n=odd}(CI) \equiv M_f^n(CI) = \int_0^1 dx\; x^{n-1} q_V(x)_f$
- $A_f^{n=even}(DI) \equiv M_f^n(DI) = \int_0^1 dx\; x^{n-1}(q_{DS}(x) + \bar{q}_{DS}(x))_f$

OPE w/o OPE

- U. Aglietti, et al., PLB 432, 411 (1998)
- A. J. Chambers et al. (QCDSF), PRL 118, 24001 (2007)

$$T_{\mu\nu}(p,q) = \rho_{\lambda\lambda'} \int d^4x \; e^{iq\cdot x} \left\langle p,\lambda' \,|\, T(J_\mu(x) J_\nu(0)) \,|\, p,\lambda \right\rangle$$

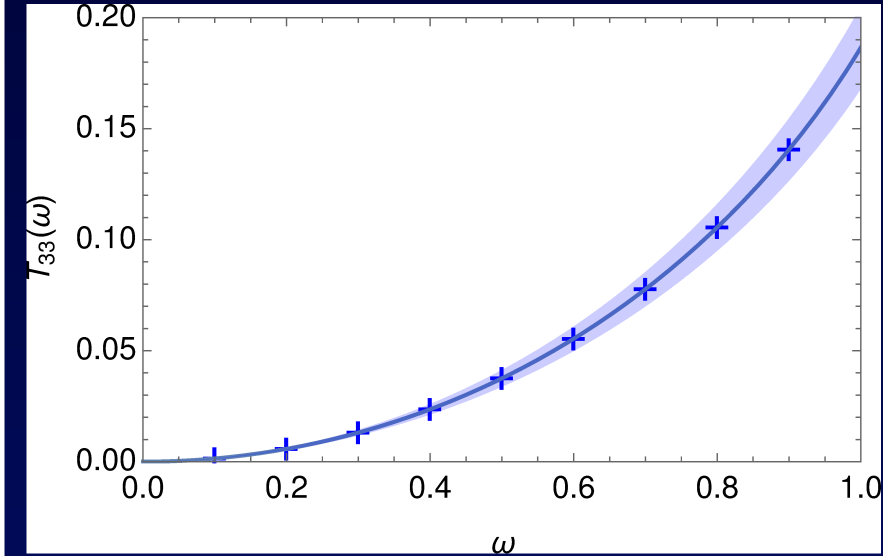
$$q_4 = \nu = 0 \rightarrow \text{ no } \tau \text{ dependence}$$

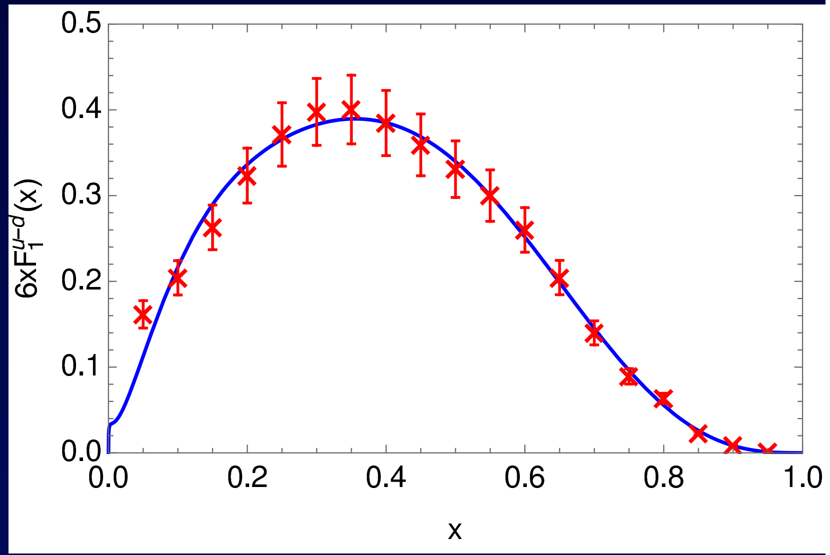
Can show that

$$T_{33}(p,q) = \sum_{n=2,4,\cdots} 4\omega^n \int_0^1 dx x^{n-1} F_1(x,q^2),$$

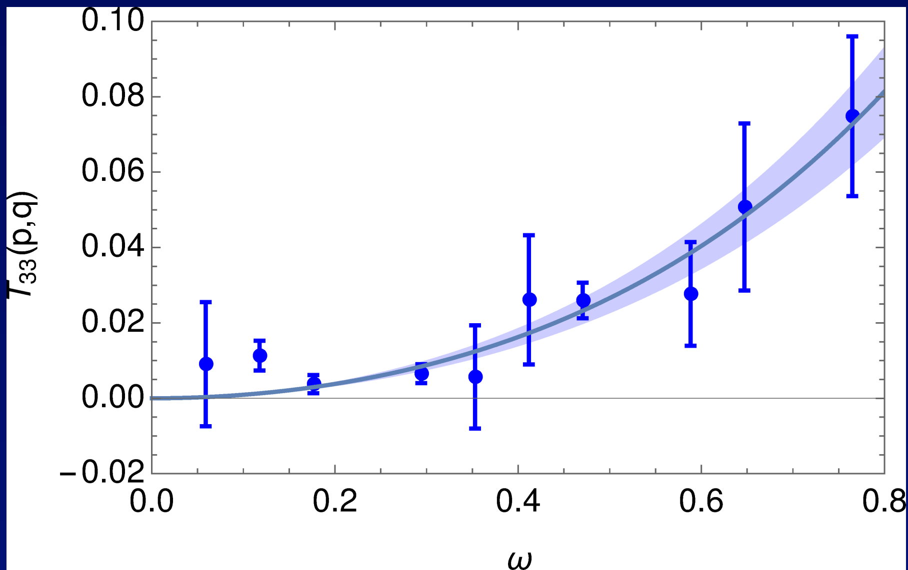
$$= 4\omega \int_0^1 dx \frac{\omega x}{1-(\omega x)^2} F_1(x,q^2),$$

$$\omega = 2p\cdot q / q^2$$





$6xF_1^{u-d}(x)$ from LO MSTW at $Q^2 = 1$ GeV^2



?

Limited to LO

Conclusion

- The connected sea partons (CSP) found in path-integral formulation are extracted by combining PDF, experimental data and ratio of lattice matrix elements.
- It would be better to have separate evolution equations for the CSP and DSP. The separation will remain at different Q^2.
- This way one can facilitate the comparison with lattice calculation of moments in the CI and DI to the corresponding moments from PDF.
- Lattice calculation of hadronic tensor is numerically tougher, but theoretical interpretation is relatively easy. No renormalization is needed and the structure function is frame independent.
- Hadronic tensor on the lattice is being tackled.

Comments

- CS and DS are explicitly separated, leading to more equations (11 vs 7) which can accommodate $s \neq \bar{s},\ \mathrm{u}^{ds} \neq \bar{\mathrm{u}}^{ds}$
- There is no flavor-changing evolution of the valence partons.

$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + P_{ds}^- \otimes \sum_k (q_k - \bar{q}_k);$$

is the sum of two equations

$$dq_i^v / dt = P_{qq}^- \otimes q_i^v, \quad q^v \equiv q^{v+cs} - \bar{q}^{cs}$$

$$d(q_i^{ds} - \bar{q}_i^{ds}) / dt = \sum_k P_{ik}^{cd-} \otimes (q_k^{ds} - \bar{q}_k^{ds}) + \sum_k P_{ds}^{d-} \otimes q_k^v$$

- Once the CS is separated at one Q^2 , it will remain separated at other Q^2.
- Gluons can split into DS, but not to valence and CS.
- It is necessary to separate out CS from DS when quark and antiquark annihilation (higher twist) is included in the evolution eqs. (Annihilation involves only DS.)

Improved Maximum Entropy Method

- Inverse problem

$$D(\tau) = \int K(\tau, \nu)\rho(\nu)d\nu,$$

$$D(\tau) = \tilde{W}_{\mu\nu}(\tau),\ \ K(\tau,\nu) = e^{-\nu\tau},\ \ \rho(\nu) = \mathrm{W}_{\mu\nu}(q^2, \nu)$$

- Bayes' theorem $P[\rho \mid D] = \frac{P[D \mid \rho] P[\rho]}{P[D]}$

- Maximum entropy method: find $\rho(\nu)$ from

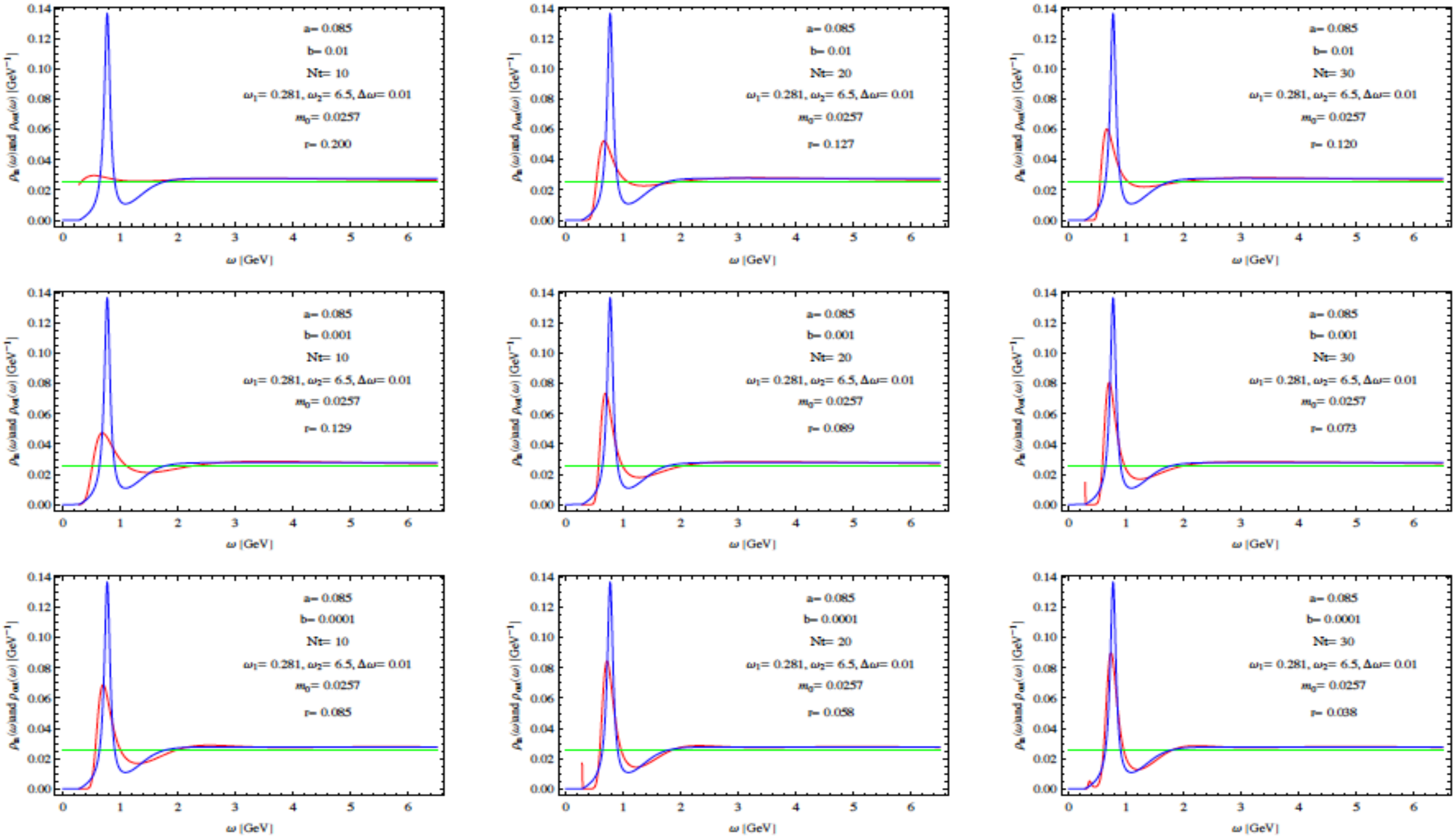
$$\frac{\partial P[\rho \mid D]}{\partial \rho} = 0$$

- Improved MEM (Burnier and Rothkpf, PRL 111, 182003 (2013))

$$P[\rho \mid D] \propto e^{\alpha S - L - \gamma (L - N_\tau)^2},\ \ L = \frac{\chi^2}{2}$$

$$S = \int d\nu \left[1 - \frac{\rho(\nu)}{m(\nu)} - \ln\left(\frac{\rho(\nu)}{m(\nu)} \right) \right]$$

Reconstruction of realistic data



$e^+e^- \to \rho$

Numerical Challenges

- Lattice calculation of the hadronic tensor – no renormalization, continuum and chiral limits, direct comparison with expts ⟹ PDF.

- Bjorken x

$$x = \frac{Q^2}{2p\cdot q} = \frac{\vec{q}^{\,2} - \nu^2}{2(\mathrm{vE}_p - \vec{p}\cdot\vec{q})}$$

- Range of x: $Q^2 = 2\ \mathrm{GeV}^2$

$-\vec{q}\ \|\ \vec{p}$ $\quad |\vec{p}\,| = 3\ \mathrm{GeV}, |\vec{q}\,| = 3\ \mathrm{GeV}, \Rightarrow\ x = 0.058$

$$\vec{p} = 0,\ \ |\vec{q}\,| = 2\ \mathrm{GeV}\ \Rightarrow\ \mathrm{x} = 0.75$$

Large Momentum Approach

X. Ji, PRL, 110, 262002 (2013)

§ Take the large-P_z limit:

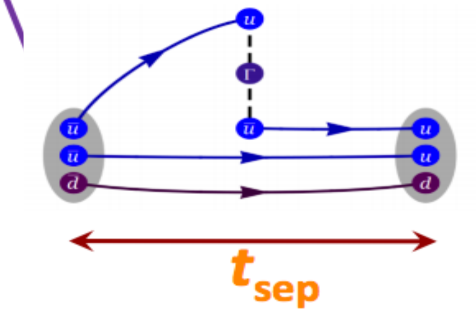
$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

$$+\mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

$x = k^z/P^z$

Lattice z coordinate

Nucleon momentum $P^\mu = \{P^0, 0, 0, P^z\}$



Product of lattice gauge links

- At $P^z \longrightarrow \infty$ limit, twist-2 parton distribution is recovered
- For finite P^z, corrections are needed

Xiangdong Ji, this Thursday; HWL et al in progress

Theoretical Issues

- Relatively simple numerically
- Renormalization of quasi-distribution

$$\tilde{q}(x,\mu^2,P_z) = \int_0^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + O\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

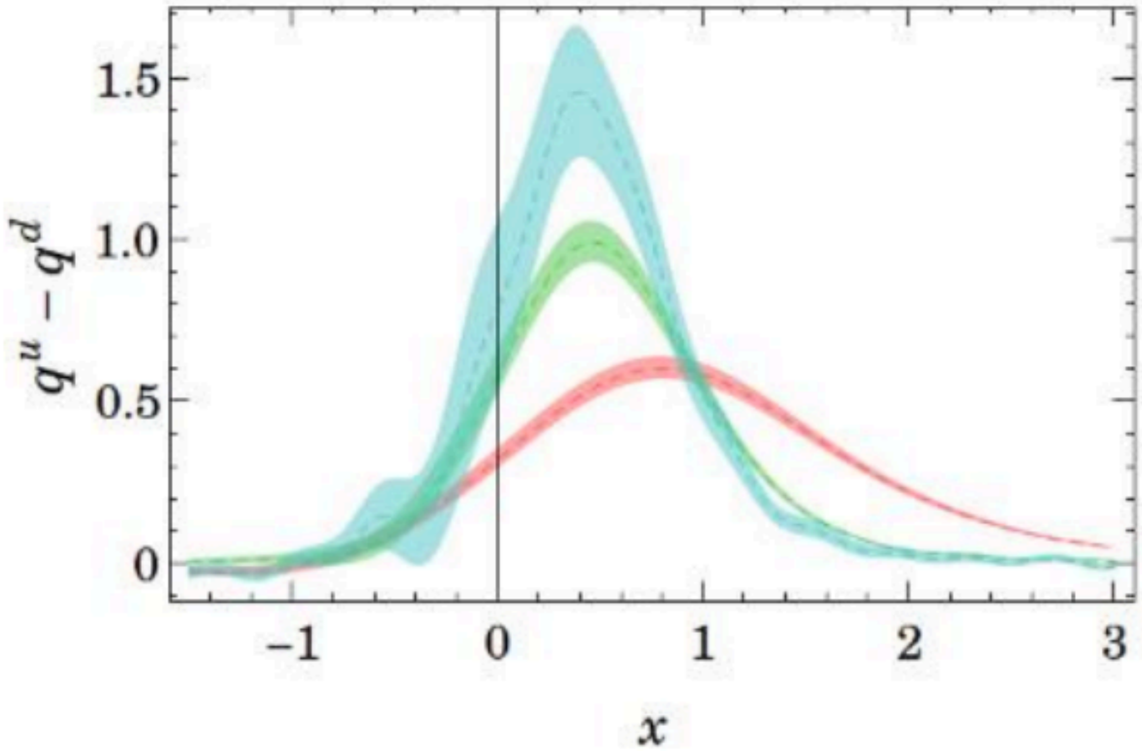
  - Perturbative and non-perturbatice lattice renormalization
  - Linear divergence of the Wilson line
- How large P_Z needs to be?

Quasi-PDF u(x) – d(x)

§ Do the integral

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\,\gamma_z \exp\left(-ig\int_0^z dz'\, A_z(z')\right)\psi(0) \middle| P \right\rangle$$

$P_z \in \{0.43, 0.86, 1.29\}$ GeV

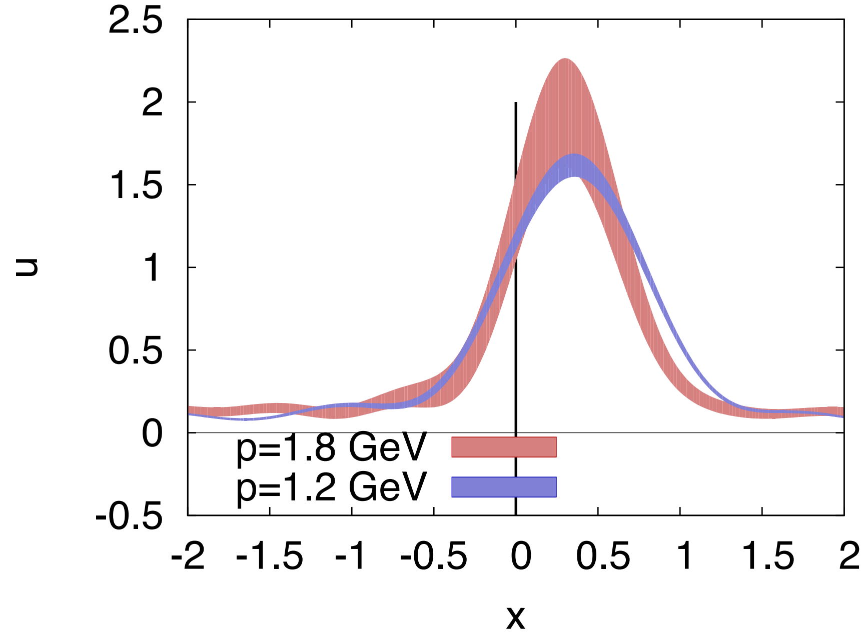


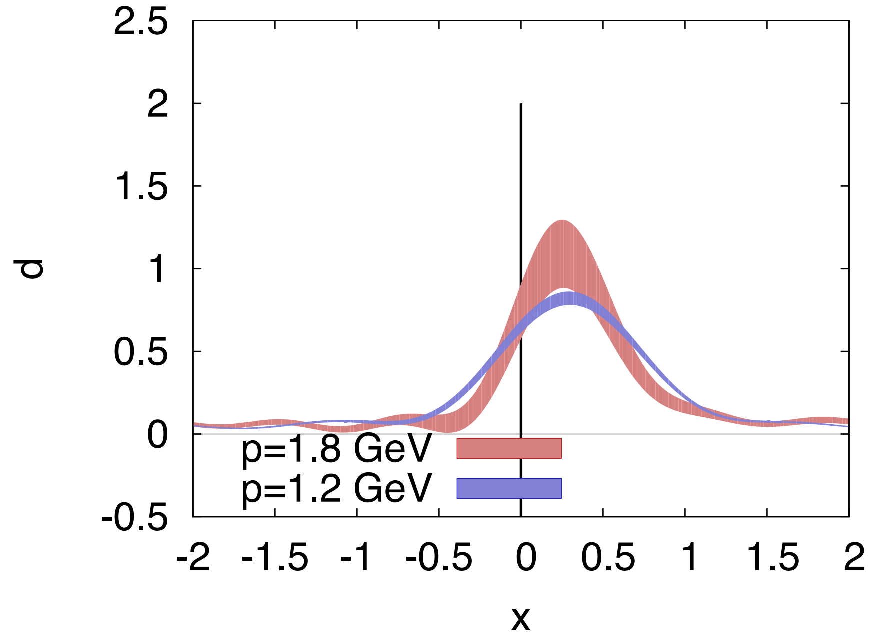
Uncorrected bare lattice results

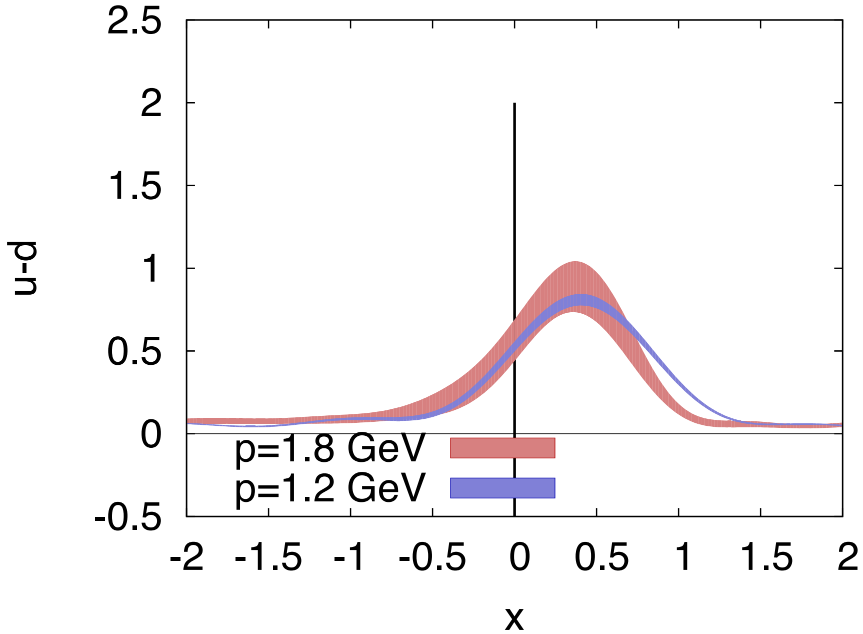
$x = k_z/P_z$

Distribution should sharper as P_z increases

Artifacts due to finite P_z on the lattice



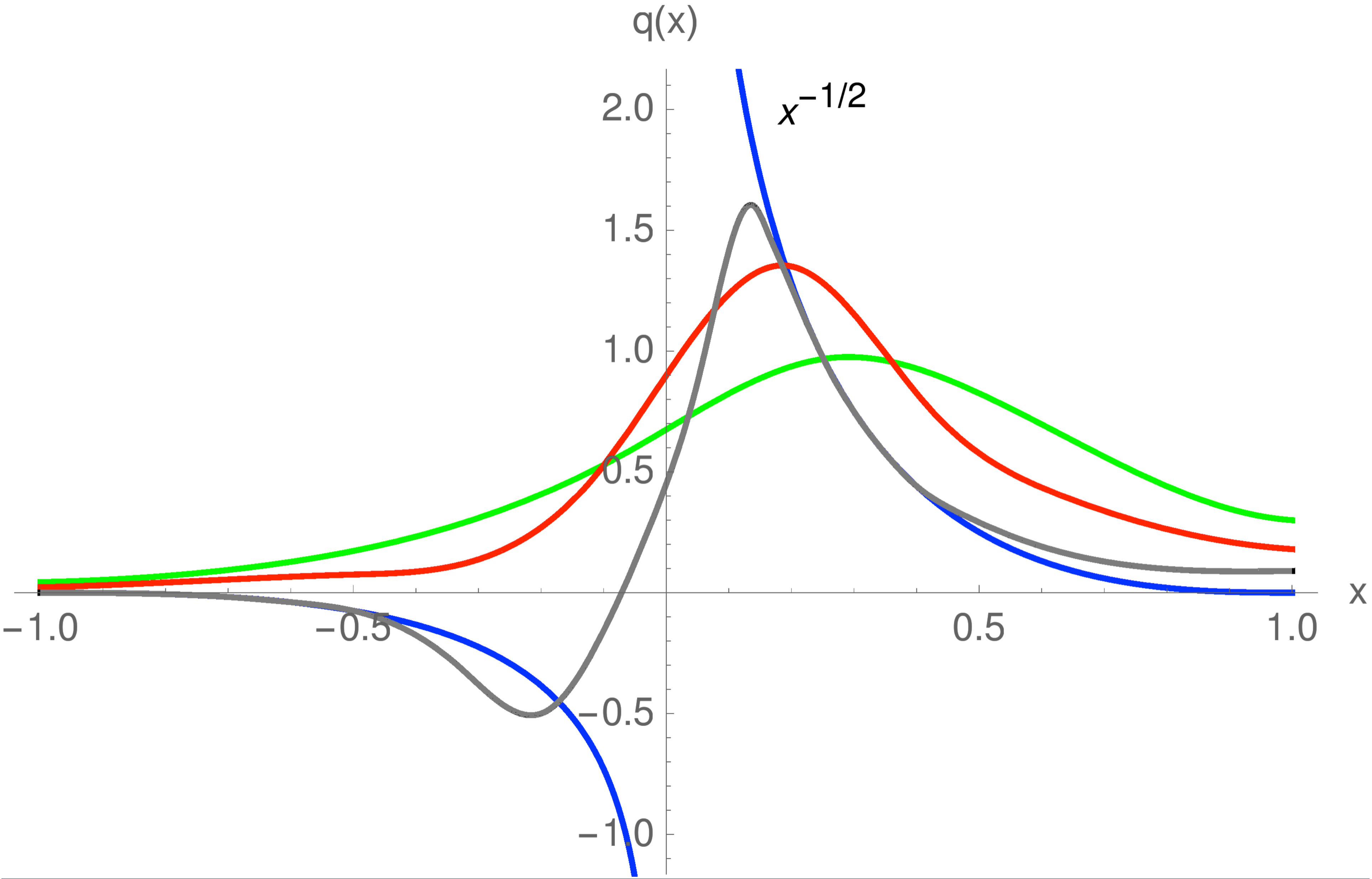


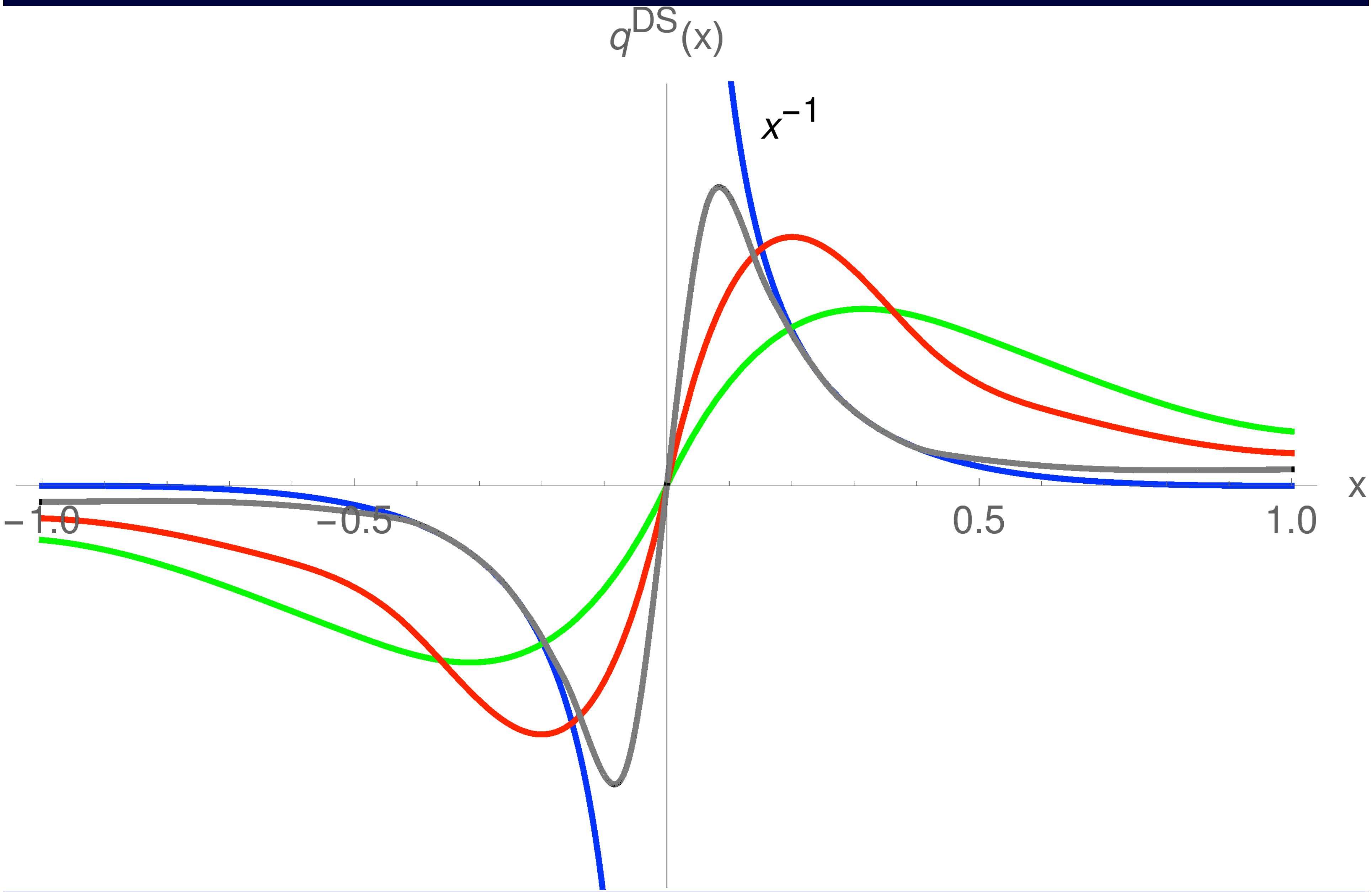


$$\bar{q}(x) = -f(-x)$$

32^3 x 64 lattice at a = 0.06 fm
Clover on DWF configurations

$m_\pi(val) = 500 \text{ MeV}, \ m_\pi(sea) = 400 \text{ MeV}$


q(x)
$x^{-1/2}$
2.0
1.5
1.0
0.5
−0.5
−1.0
x
−1.0
−0.5
0.5
1.0

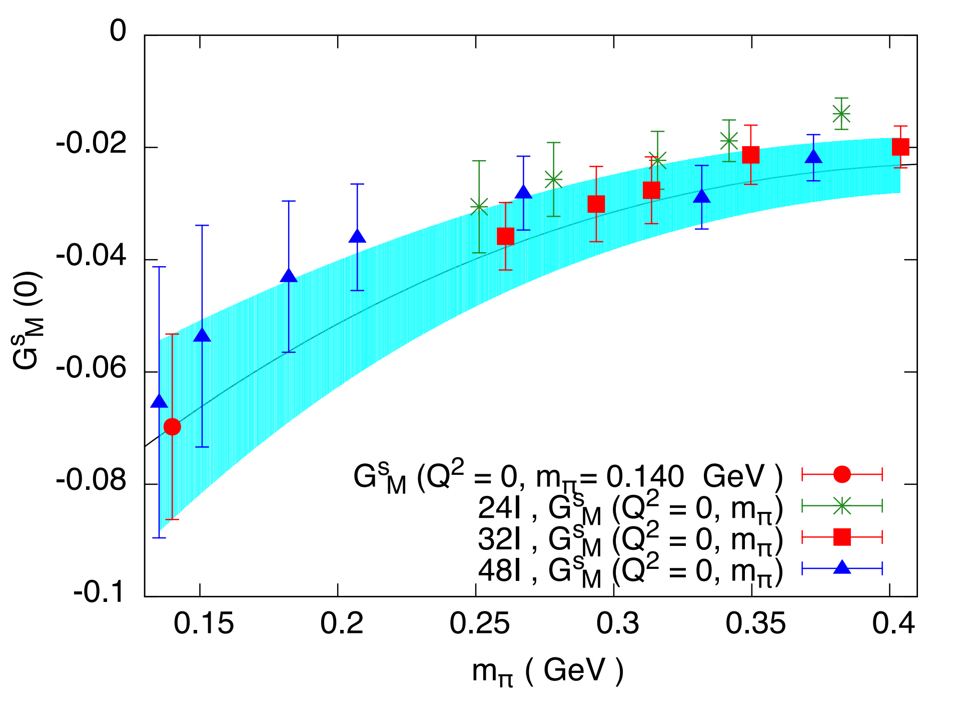


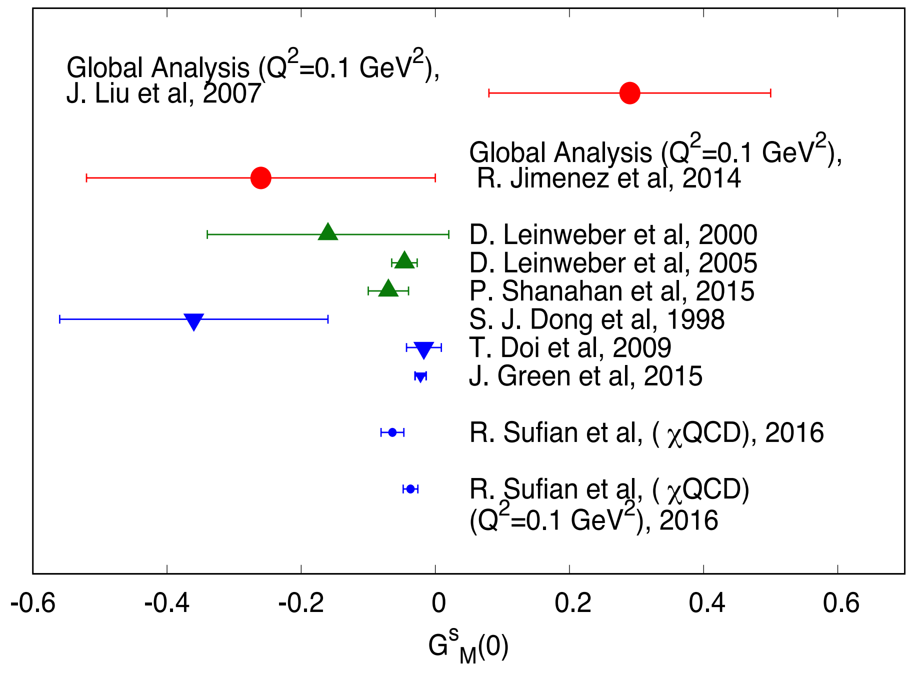
$q^{DS}(x)$
x^{-1}
x
-1.0
-0.5
0.5
1.0

Strange quark magnetic moment

Parity-violating ep scattering
with radiative correction

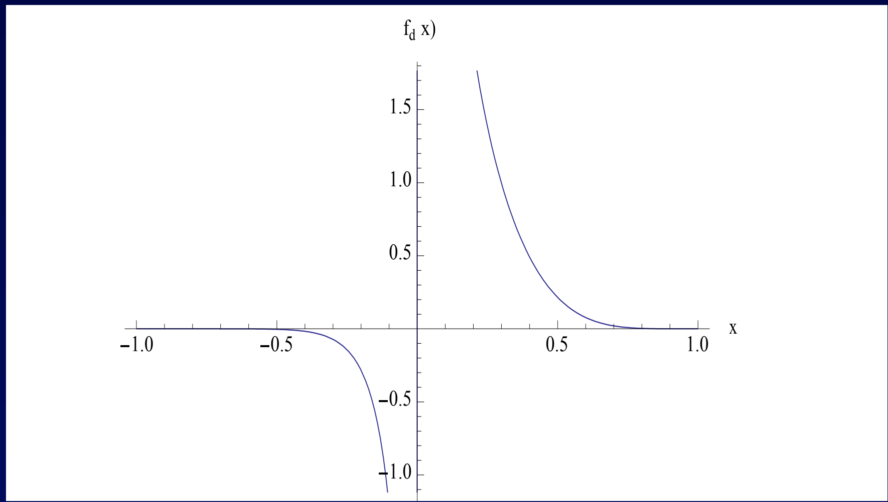
R. Sufian et al, 1606.07075
PRL – editor's choice
Nature – Ross Young





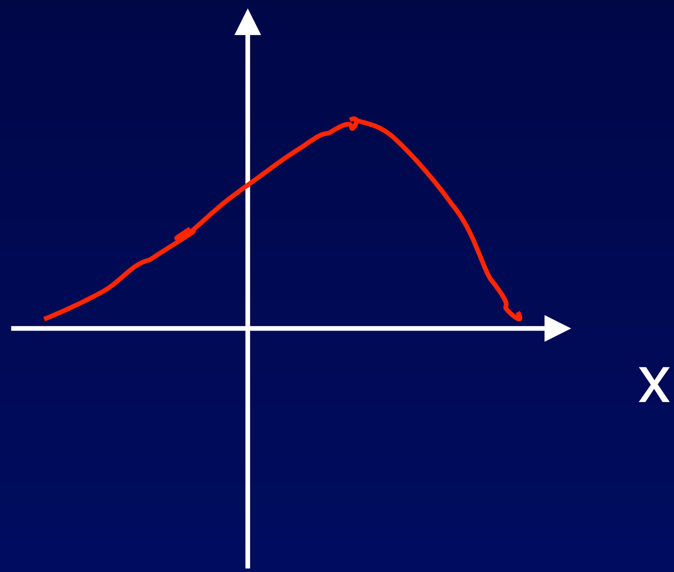
$G_M^S(0) = -\,0.064(14)(9)\ \mu_N$

- Negative $\bar{q}(x)$ puzzle



d and $\bar{d}$ from CTEQ6 (JW Chen)

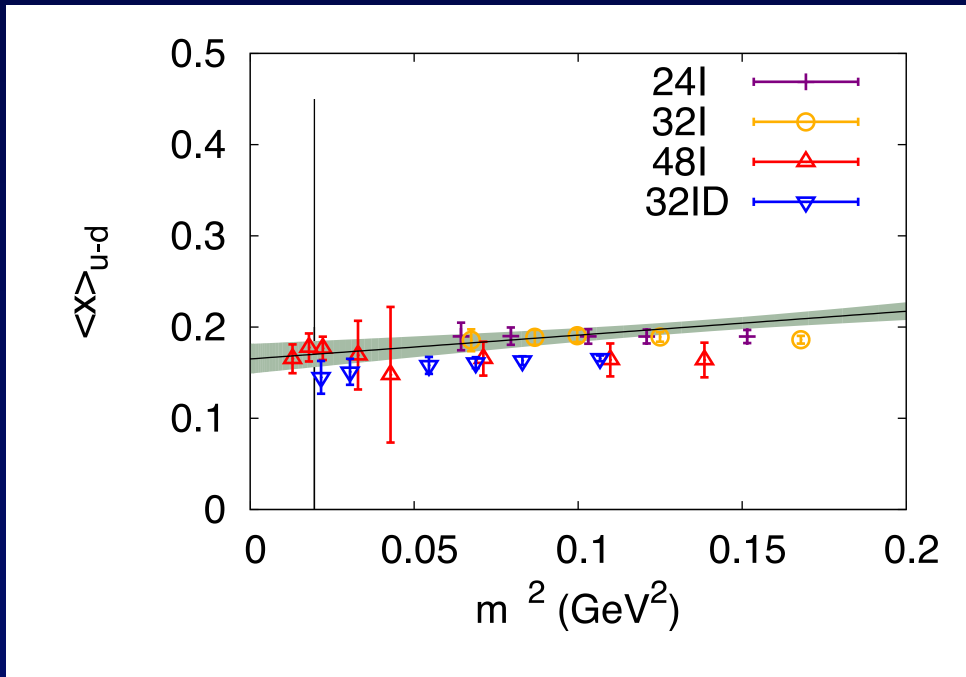
$\bar{d}(|x|) = -d(-|x|)$



present P_z ~ 1 GeV

H.W. Lin, 1402.1462

- Larger P_z ? (How large)
- Lattice scale (a^{-1} ~ 2 GeV) too small?
- Range of x limited?



0.5
0.4
0.3
0.2
0.1
0
0
0.05
0.1
0.15
0.2
<X>u-d
m 2 (GeV2)
24I
32I
48I
32ID

Comments

- *The results are the same as derived from the conventional operator product expansion.*
- *The OPE turns out to be Taylor expansion of functions in the path-integral formalism.*
- *Contrary to conventional OPE, the path-integral formalism admits separation of CI and DI.*
- *For* O_f^n *with definite n, there is only one CI and one DI in the three-point function, i.e. (a') is the same as (b'). Thus, one cannot separate quark contribution from that of antiquark in matrix elements.*

Kinematics

- Bjorken x

$$x = \frac{Q^2}{2p \cdot q} = \frac{\vec{q}^{\,2} - \nu^2}{2(\mathrm{vE}_p - \vec{p} \cdot \vec{q})}$$

- Range of x:

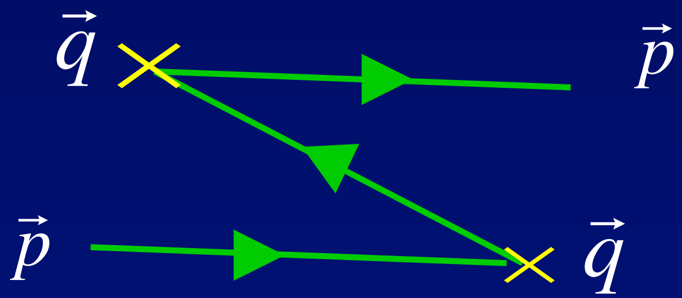
$$Q^2 = 2\ \mathrm{GeV}^2$$

$$-\vec{q} \parallel \vec{p} \qquad |\vec{p}| = 3\ \mathrm{GeV}, |\vec{q}| = 3\ \mathrm{GeV}, \Rightarrow x = 0.058$$

$$\vec{p} = 0,\ |\vec{q}| = 2\ \mathrm{GeV} \Rightarrow \mathrm{x} = 0.75$$

- Note that diagram (b) are from pre-existing connected sea antipartons the same way as in (c) which involves pre-existing disconnected sea partons and antipartons.

- Whereas, current induced pair productions are suppressed as O($\vec{q}^2 / \vec{p}^2$).



$$\delta(p \cdot q + 2p_o^2)$$

Operator Product Expansion -> Taylor Expansion

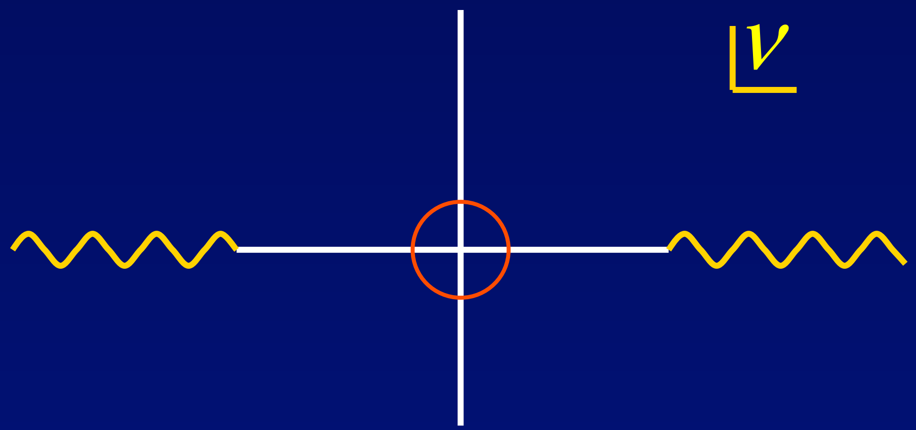
- Operator product expansion

$$W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$$

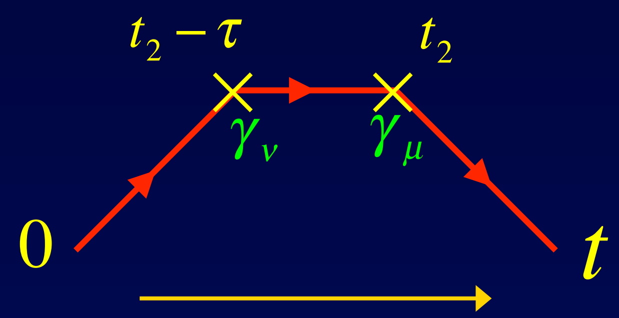
  - Dispersion relation

$$T_{\mu\nu} = \frac{1}{\pi}\int_{Q^2/2M_N}^{\infty} d\nu' \frac{\nu' W_{\mu\nu}(q^2,\nu')}{\nu'^2-\nu^2}$$

  - Expand in the unphysical region $\frac{2M_N\nu}{Q^2} = \frac{2p\cdot q}{Q^2} < 1 \quad (\mathrm{x} > 1)$



- Euclidean path-integral
  - Consider



$$\tilde{W}_{\mu\nu}(q^2,\tau)|_{(a)} \propto \int D[A]\det M(A)\, e^{-S_g}$$

$$\times Tr\left[\ldots M^{-1}(t,t_2)\int d^3x\, e^{-i\vec{q}\cdot\vec{x}} i\gamma_\mu M^{-1}(t_2,t_2-\tau) i\gamma_\nu M^{-1}(t_2-\tau,0)\ldots\right]$$

  - Short-distance expansion ($|\vec{x}|,\tau \to 0$ from $|\vec{q}|,\nu \to \infty$)

$$M^{-1}(t_2,t_2-\tau)\xrightarrow[\text{free quark}]{}\frac{1}{4\pi^2}\frac{\not{\partial}}{\vec{x}^2+\tau^2};$$

$$M^{-1}(t_2-\tau,0)\xrightarrow[|\vec{x}|,\tau\to 0]{} e^{\vec{x}\cdot\vec{D}+\tau D_\tau} M^{-1}(t_2,0)$$

  - Laplace transform

$$W_{\mu\nu}(q^2,\nu) \propto Tr\left[\ldots M^{-1}(t,t_2) i\gamma_\mu \frac{-i\pi(\not{q}+i\not{D})}{|\vec{q}+i\vec{D}|}\delta(\nu+D_\tau-|\vec{q}+i\vec{D}|) i\gamma_\nu M^{-1}(t_2,0)\ldots\right]$$

- Dispersion relation

$$T_{\mu\nu}(q^2,\nu) = \frac{1}{\pi}\int_{Q^2/2M_N + D_\tau}^{\infty} d\nu' \frac{\nu' W_{\mu\nu}(q^2,\nu' - D_\tau)}{\nu'^2 - (\nu + D_\tau)^2},$$

$$\propto \; Tr\left[...M^{-1}(t,t_2) i\gamma_\mu \frac{-i(\not{q} + i\not{D})}{(Q^2 + 2iq\cdot D - D^2)} i\gamma_\nu M^{-1}(t_2,0)... \right],$$

where $\tau = it$ and $D_t = iD_\tau$

so that $D = (\vec{D}, -iD_t)$ is covariant derivative in Minkowski space.

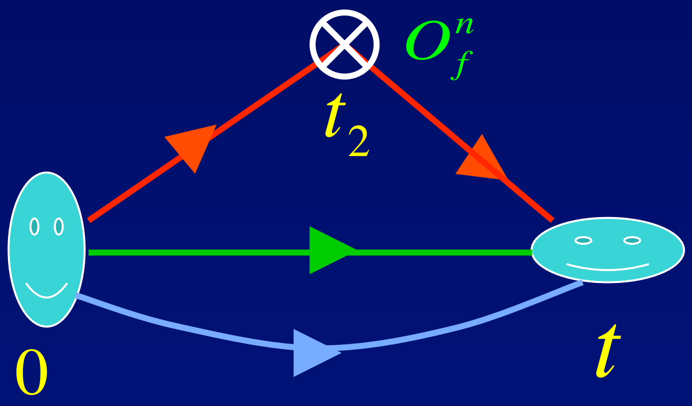
- Expansion about the unphysical region ($2q\cdot p/Q^2 < 1$)

$$T_{\mu\nu}(q_V + q_{CS}) = \sum_f e_f^2 \left[8 p_\mu p_\nu \sum_{n=2} \frac{(-2q\cdot p)^{n-2}}{(Q^2)^{n-1}} A_f^n(CI) - 2 g_{\mu\nu} \sum_{n=2} \frac{(-2q\cdot p)^n}{(Q^2)^n} A_f^n(CI) \right]$$

even + odd n terms

- $A_f^n = ?$

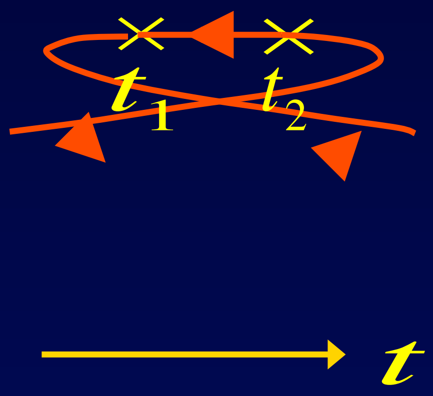
$$A_f^n(CI) \propto \int D[A] \det M(A)\, e^{-S_g} Tr\left[...M^{-1}(t,t_2) O_f^n M^{-1}(t_2,0)... \right]$$



$$O_f^n = i\gamma_{\mu_1} \left(\frac{-i}{2}\right)^{n-1} \overleftrightarrow{D}_{\mu_2} \overleftrightarrow{D}_{\mu_3} ... \overleftrightarrow{D}_{\mu_n},$$

$$< p | \bar{\psi} \; O_f^n \psi | p >= A_f^n(CI) \; 2 p_{\mu_1} p_{\mu_2} ... p_{\mu_n}$$

- Similarly for $\bar{q}_{CS}$
  except with $q \to -q$



$$T_{\mu\nu}(\bar{q}_{CS}) = \sum_{even,\, n=2} \ldots A_f^n(CI) - \sum_{odd,\, n=3} \ldots A_f^n(CI)$$

even – odd

- For $q_{DS} / \bar{q}_{DS}$

$$T_{\mu\nu}(q_{DS} / \bar{q}_{DS}) = \sum_{even,\, n=2} \ldots A_f^n(DI) \pm \sum_{odd,\, n=3} \ldots A_f^n(DI)$$

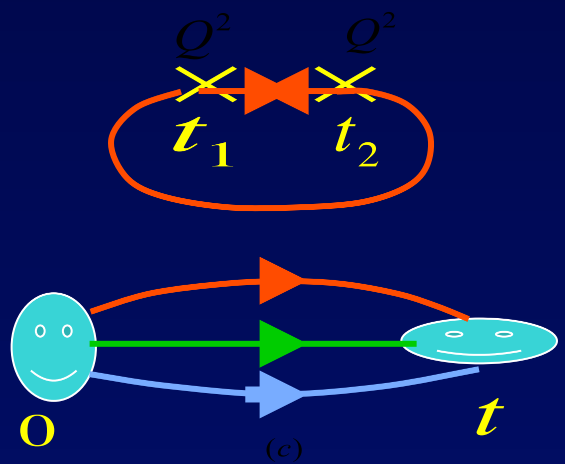
- DIS with electromagnetic currents J_μ^{em}

$$T_{\mu\nu} = T_{\mu\nu}(q_V + q_{CS}) + T_{\mu\nu}(\bar{q}_{CS}) + T_{\mu\nu}(q_{DS}) + T_{\mu\nu}(\bar{q}_{DS}),$$
$$= 2 \sum_{even,\, n=2} \ldots [A_f^n(CI) + A_f^n(DI)]$$

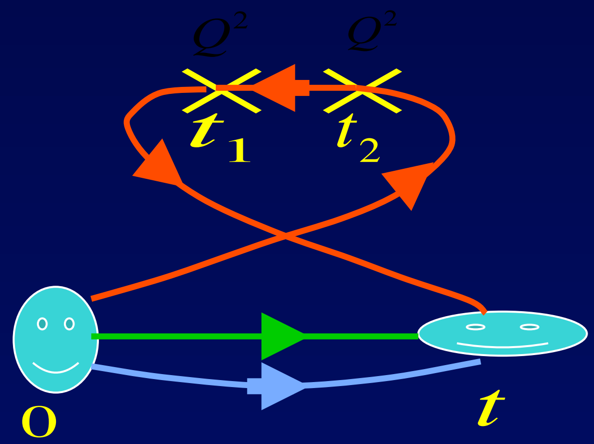
Gottfried Sum Rule Violation

$$S_G(0,1;Q^2)=\frac{1}{3}+\frac{2}{3}\int_0^1 dx\ (\bar{u}_P(x)-\bar{d}_P(x));\quad S_G(0,1;Q^2)=\frac{1}{3}\text{(Gottfried Sum Rule)}$$

NMC: $S_G(0,1;4\ \mathrm{GeV}^2)=0.240\pm0.016\ (5\sigma\ \text{from GSR})$



two flavor traces ($\bar{u}_{DS}=\bar{d}_{DS}$)



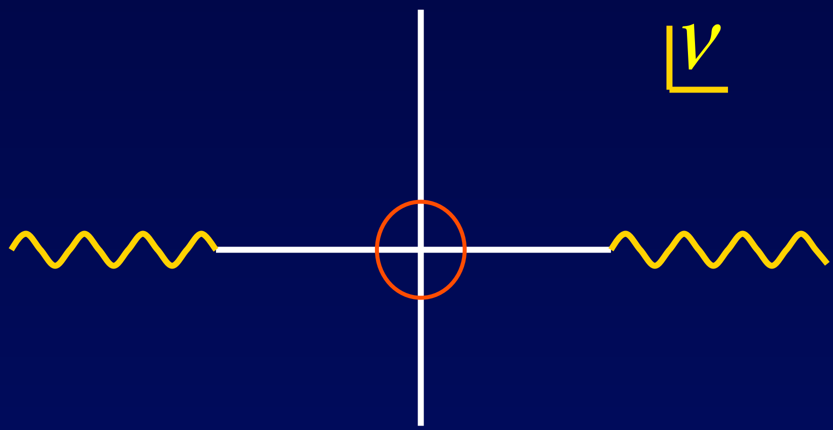
one flavor trace ($\bar{u}_{CS}\neq\bar{d}_{CS}$)

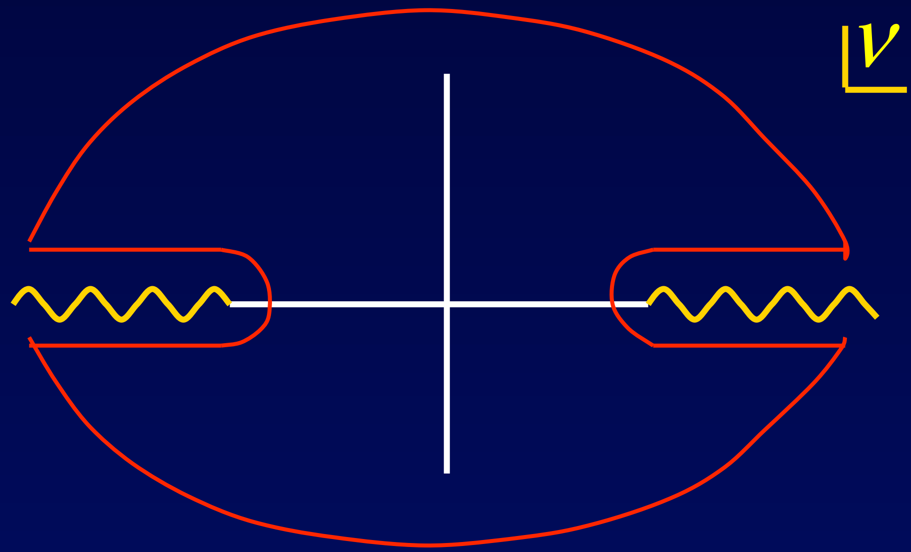
K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$Sum\ =\frac{1}{3}+\frac{2}{3}\int_0^1 dx\ (\bar{u}_{CS}(x)-\bar{d}_{CS}(x)),$$

$$=\frac{1}{3}+\frac{2}{3}\left[n_{\bar{u}_{CS}}-n_{\bar{d}_{CS}}\right]\ (1+O(\alpha_s^2))$$

Quark Parton Model





$$I_n = \oint \frac{d\nu}{2\pi i}\frac{1}{\nu^{n-1}} T_2(Q^2,\nu),$$

$$= \sum_f 8\, e_f^2 \left(\frac{2M_N}{Q^2}\right)^{n-1} A_f^n$$

$$I_n = 2\int_{Q^2}^{\infty} \frac{d\nu 2M_N}{2\pi i}\frac{2i}{\nu^{n-1}} W_2(Q^2,\nu),$$

$$= 8\left(\frac{2M_N}{Q^2}\right)^{n-1}\int_0^1 dx\, x^{n-2}\frac{2M_N\nu W_2(Q^2,\nu)}{4}$$

- $A_f^{n=even}(CI) \equiv M_f^n(CI) = \int_0^1 dx\, x^{n-1}(q_V(x) + q_{CS}(x) + \bar{q}_{CS}(x))_f$
- $A_f^{n=odd}(CI) \equiv M_f^n(CI) = \int_0^1 dx\, x^{n-1} q_V(x)_f$
- $A_f^{n=even}(DI) \equiv M_f^n(DI) = \int_0^1 dx\, x^{n-1}(q_{DS}(x) + \bar{q}_{DS}(x))_f$